T - Run

[5, 5, 1, 5, 2], [3, 4, 5, 2, 1]

$\tilde{\pi}$ = [2, 2, 1, 1, 3]
$\boldsymbol{\delta}$ = [2, 2, 1, 1, 4]

POSSIBLE RANKS

1 x 9
3 x 3

BASE DETERMINANT 351/4096, .8569335938e-1

NullSpace of Δ

{1, 2, 3, 4, 5}

Nullspace of A

` det(A) = ` -1/16

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[2] v[4] + v[2] v[5] + v[1] v[3] + v[1] v[5]

Degree 3
v[2] v[4] v[5] + v[1] v[3] v[5]

Degree 4
v[1] v[2] v[4] v[5] + v[1] v[2] v[3] v[5] + v[1] v[2] v[3] v[4]

Degree 5
2 v[1] v[2] v[3] v[4] v[5]

==

{2}

R: [5, 4, 1, 5, 2]
B: [3, 5, 5, 2, 1]

TRACE TWO = 1

det AT = $\frac{3}{16}(-1+t)(1+3t^2)(1+t)$

$$AT = \begin{pmatrix} 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{-3}{65536}\,(3+s)\,(-1+s)\,(-52-55s-4s^2+3s^3)\,(-36-13s+12s^2+s^3)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 1, "vs", 3

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 3

"R CYCLES", 1 + v[2] v[4] v[5]
"B CYCLES", 1 + v[1] v[3] v[5]

Eigenvalues

R: [0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

B: [0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

NullSpace of R

{[0, 0, 1, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0]}

NullSpace of R^*

{[-1, 0, 0, 1, 0]}

NullSpace of B^*

{[0, -1, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 8 & 0 & 16 \\ 8 & 0 & 0 & 8 & 16 \\ 8 & 0 & 0 & 0 & 8 \\ 0 & 8 & 0 & 0 & 8 \\ 16 & 16 & 8 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 5

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{9}$ ($2v[1] + 2v[2] + v[3] + v[4] + 3v[5]$)
degree 2: $\frac{1}{9}$ ($v[1]v[2] + v[1]v[3] + 2v[1]v[5] + v[2]v[4] + 2v[2]v[5] + v[3]v[5] + v[4]v[5]$)
degree 3 : $\frac{1}{3}$ ($v[5]$) ($v[1]v[2] + v[1]v[3] + v[2]v[4]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 4}, {5}, {2, 3}}

"RG1" = {2, 4, 5}

"RG2" = {1, 3, 5}

"RG3" = {1, 2, 5}

$$\pi 3 = [0, 0, 1, 0, 1, 0, 0, 0, 1, 0]$$

supp $\pi 3$ = {3, 5, 9}

$$u3 = [0, 0, 1, 0, 1, 0, 0, 0, 1, 1]$$

supp u3 = {3, 5, 9, 10}

Action of R on ranges, [[1], [3], [1]]
Action of B on ranges, [[3], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 1, 2]

B-BLOCKS,

[2, 3, 1]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1 = \{1, 4\}$
$b_2 = \{5\}$
$b_3 = \{2, 3\}$

dim(span of partition vectors), rank(N_0), rank(N):   3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 11, Shape: 3 ⊕ 8/6

$$\mathrm{CLB} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4, 5}}, true

Ω_B in Vec(K)? , {{1, 3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{9} & \frac{2}{9} & \frac{-5}{9} & \frac{1}{9} & \frac{1}{3} \\ \frac{-2}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{5}{9} & \frac{-1}{3} \\ \frac{4}{9} & \frac{-2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{3} \\ \frac{2}{9} & \frac{-4}{9} & \frac{1}{9} & \frac{-2}{9} & \frac{1}{3} \\ \frac{-2}{3} & \frac{2}{3} & \frac{-1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

Omega–checks true, true

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 4}, {5}, {2, 3}}

1, "range", [2, 4, 5], [[5, 4, 4, 5, 2], [4, 2, 2, 4, 5], [2, 5, 5, 2, 4]]

2, "range", [1, 3, 5], [[5, 1, 1, 5, 3], [3, 5, 5, 3, 1], [1, 3, 3, 1, 5]]

3, "range", [1, 2, 5], [[5, 1, 1, 5, 2], [2, 5, 5, 2, 1], [1, 2, 2, 1, 5]]

"group has", 3, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 3]]

$g_2 =$ []

$g_3 =$ [[1, 3, 2]]

linear dimension, 3

"Symmetric?", false

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] & h[3] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

3, "coeff", 1

Z[3] = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 1. & -0.5000000000 + 0.8660254040I & -0.5000000000 - 0.8660254040I \\ 1. & -0.5000000000 + 0.8660254040I & -0.5000000000 - 0.8660254040I \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 4t^3 + 5t^4 + 7t^5 + 10t^6 + 12t^7 + 15t^8 + 19t^9 + 22t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 3, 4]}, {5, [1, 3, 5]}, {6, [1, 4, 5]}, {7, [2, 3, 4]}, {8, [2, 3, 5]}, {9, [2, 4, 5]}, {10, [3, 4, 5]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

{3, 5, 9}

$u3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

{3, 5, 9, 10}

picheck $\begin{pmatrix} 2 & 2 & 1 & 1 & 3 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{3} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 1 & 1 & 0 & 2 & 0 & 1 & 2 & 0 & 1 & 1 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 4 & 2 & 2 & 6 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 4 & 4 & 2 & 2 & 6 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 4 & 2 & 2 & 6 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

Row Projections

$PP_1 = \begin{pmatrix} \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks

$T = \begin{pmatrix} \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ $NM = \begin{pmatrix} \frac{32}{9} & \frac{16}{9} & \frac{8}{9} & \frac{16}{9} & \frac{8}{3} \\ \frac{16}{9} & \frac{32}{9} & \frac{16}{9} & \frac{8}{9} & \frac{8}{3} \\ \frac{16}{9} & \frac{32}{9} & \frac{16}{9} & \frac{8}{9} & \frac{8}{3} \\ \frac{32}{9} & \frac{16}{9} & \frac{8}{9} & \frac{16}{9} & \frac{8}{3} \\ \frac{16}{9} & \frac{16}{9} & \frac{8}{9} & \frac{8}{9} & \frac{16}{3} \end{pmatrix}$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, -1, 1, 0]$

ker $N_C = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & -t & t & 0 & 0 \\ -s & 0 & 0 & s & 0 \end{pmatrix}$ RB checks

$\pi\Delta$ via ker NC $\begin{pmatrix} -1 & 1 \end{pmatrix}$

M0 is invertible. det= 6500/19683

ker $M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} t+s \\ t+s \\ t+s \\ t+s \\ t+s \end{pmatrix}$ RB checks

$n\pi x^{\dagger} = (5)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 3

$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & \frac{-8}{9} & -1 & \frac{8}{9} \\ 0 & 0 & -1 & \frac{-8}{9} & \frac{8}{9} \\ \frac{8}{9} & 1 & 0 & 0 & \frac{16}{9} \\ 1 & \frac{8}{9} & 0 & 0 & \frac{16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & 0 \end{pmatrix}$ Skew T = $\begin{pmatrix} 0 & 0 & 0 & \frac{-1}{3} & 0 \\ 0 & 0 & \frac{-1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ Skew Omega =

$$\begin{pmatrix} 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} \\ 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{2}{9} \\ \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{-2}{9} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & \frac{8}{9} & \frac{8}{9} & 0 & \frac{16}{9} \\ \frac{8}{9} & 2 & 0 & \frac{8}{9} & \frac{16}{9} \\ \frac{8}{9} & 0 & 1 & 0 & \frac{8}{9} \\ 0 & \frac{8}{9} & 0 & 1 & \frac{8}{9} \\ \frac{16}{9} & \frac{16}{9} & \frac{8}{9} & \frac{8}{9} & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = \frac{8}{3} T + 8\Omega$$

$\Omega \left(\frac{1}{9} \;\; \frac{1}{9} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{3} \;\; \frac{1}{9} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{2}{9}\right)$

$T \left(0 \;\; 0 \;\; \frac{1}{3} \;\; 0 \;\; 0 \;\; \frac{1}{3} \;\; 0 \;\; 0 \;\; \frac{2}{3}\right)$

"IS NM in Vec(K)?", true

$\text{NM} \left(\frac{8}{9} \;\; \frac{8}{9} \;\; \frac{16}{9} \;\; \frac{16}{9} \;\; \frac{8}{3} \;\; \frac{16}{9} \;\; \frac{8}{9} \;\; \frac{16}{9} \;\; \frac{32}{9}\right)$

"IS MN in Vec(K)?", false

$\text{MN} \left(0 \;\; 0 \;\; 0 \;\; \frac{8}{7} \;\; \frac{16}{7} \;\; \frac{8}{3} \;\; \frac{8}{7} \;\; \frac{8}{7} \;\; \frac{8}{3}\right)$

$$\boldsymbol{\tau} = 9/1 \text{ , rank} = 3 \text{ , ratio} = 3/1 \text{ , } n^2 / r = 25/3$$

$$\boldsymbol{\tau}' = 16/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 9/25$$

$$p^2 = 19/81 \text{ , min } \boldsymbol{\tau} = 475/81 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 254/81$$

$$\text{max r} = 81/19 \text{ , } r\text{–check is positive? } 8/27$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 8\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 3

KERNEL HAS LINEAR DIMENSION 9
out of total no. of elements equal to 9

dim span idems 3 vs no. of idems 3

"PT1" = {{1, 4}, {5}, {2, 3}}

"RG1" = {2, 4, 5}

"RG2" = {1, 3, 5}

"RG3" = {1, 2, 5}

$$M_C = \begin{pmatrix} \frac{62}{81} & \frac{-28}{81} & \frac{22}{81} & \frac{-50}{81} & \frac{-2}{27} \\ \frac{-28}{81} & \frac{62}{81} & \frac{-50}{81} & \frac{22}{81} & \frac{-2}{27} \\ \frac{22}{81} & \frac{-50}{81} & \frac{56}{81} & \frac{-25}{81} & \frac{-1}{27} \\ \frac{-50}{81} & \frac{22}{81} & \frac{-25}{81} & \frac{56}{81} & \frac{-1}{27} \\ \frac{-2}{27} & \frac{-2}{27} & \frac{-1}{27} & \frac{-1}{27} & \frac{2}{9} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{62}{81} & \frac{-19}{81} & \frac{-19}{81} & \frac{62}{81} & \frac{-19}{81} \\ \frac{-19}{81} & \frac{62}{81} & \frac{62}{81} & \frac{-19}{81} & \frac{-19}{81} \\ \frac{-19}{81} & \frac{62}{81} & \frac{62}{81} & \frac{-19}{81} & \frac{-19}{81} \\ \frac{62}{81} & \frac{-19}{81} & \frac{-19}{81} & \frac{62}{81} & \frac{-19}{81} \\ \frac{-19}{81} & \frac{-19}{81} & \frac{-19}{81} & \frac{-19}{81} & \frac{62}{81} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-14}{31} & \frac{11}{31} & \frac{-25}{31} & \frac{-3}{31} \\ \frac{-14}{31} & 1 & \frac{-25}{31} & \frac{11}{31} & \frac{-3}{31} \\ \frac{11}{28} & \frac{-25}{28} & 1 & \frac{-25}{56} & \frac{-3}{56} \\ \frac{-25}{28} & \frac{11}{28} & \frac{-25}{56} & 1 & \frac{-3}{56} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{6} & \frac{-1}{6} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-19}{62} & \frac{-19}{62} & 1 & \frac{-19}{62} \\ \frac{-19}{62} & 1 & 1 & \frac{-19}{62} & \frac{-19}{62} \\ \frac{-19}{62} & 1 & 1 & \frac{-19}{62} & \frac{-19}{62} \\ 1 & \frac{-19}{62} & \frac{-19}{62} & 1 & \frac{-19}{62} \\ \frac{-19}{62} & \frac{-19}{62} & \frac{-19}{62} & \frac{-19}{62} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{4}{27} & \frac{-2}{27} & \frac{-1}{27} & \frac{2}{27} & \frac{-1}{9} \\ \frac{-2}{27} & \frac{4}{27} & \frac{2}{27} & \frac{-1}{27} & \frac{-1}{9} \\ \frac{-2}{27} & \frac{4}{27} & \frac{2}{27} & \frac{-1}{27} & \frac{-1}{9} \\ \frac{4}{27} & \frac{-2}{27} & \frac{-1}{27} & \frac{2}{27} & \frac{-1}{9} \\ \frac{-2}{27} & \frac{-2}{27} & \frac{-1}{27} & \frac{-1}{27} & \frac{2}{9} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{4}{27} & \frac{-2}{27} & \frac{-2}{27} & \frac{4}{27} & \frac{-2}{27} \\ \frac{-2}{27} & \frac{4}{27} & \frac{4}{27} & \frac{-2}{27} & \frac{-2}{27} \\ \frac{-1}{27} & \frac{2}{27} & \frac{2}{27} & \frac{-1}{27} & \frac{-1}{27} \\ \frac{2}{27} & \frac{-1}{27} & \frac{-1}{27} & \frac{2}{27} & \frac{-1}{27} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{-1}{27} & \frac{2}{27} & \frac{1}{27} \\ 0 & 0 & \frac{2}{27} & \frac{-1}{27} & \frac{1}{27} \\ \frac{1}{27} & \frac{-2}{27} & 0 & 0 & \frac{2}{27} \\ \frac{-2}{27} & \frac{1}{27} & 0 & 0 & \frac{2}{27} \\ \frac{-1}{27} & \frac{-1}{27} & \frac{-2}{27} & \frac{-2}{27} & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 1.946178864, 0.1649322482, 0.7505405108, 0.2741508472]

Eigenvalues N_C

[0., 0., 2., 1.405552070, 0.4216084235]

Eigenvalues M_C–scaled

[0., 1.135874376, 0.9660841503, 2.670943107, 0.227098367]

Eigenvalues N_C–scaled

[0., 0., 2.612903226, 1.836285769, 0.5508110047]

NullSpace M_C

{[1, 1, 1, 1, 1]}

NullSpace N_C

{[0, −1, 1, 0, 0], [−1, 0, 0, 1, 0]}

Eigenvalues M_0

[1.946178864, 0.1649322482, 5.929100987, 0.2416082767, 0.7181796227]

Eigenvalues N_0

[1., 0., 0., 2., 2.]

NullSpace M_0

{}

NullSpace N_0

{[−1, 0, 0, 1, 0], [0, 1, −1, 0, 0]}

Eigenvalues M

[0.5493635453, −1.438252434, 3.615324520, −2.195546624, −0.5308890059]

Eigenvalues N

[0., 0., −2., 3.236067977, −1.236067977]

NullSpace M

{}

NullSpace N

{[1, 0, 0, −1, 0], [0, 1, −1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

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{2, 3, 4, 5}

R: [5, 4, 5, 2, 1]
B: [3, 5, 1, 5, 2]

TRACE TWO = 1

det AT = $\frac{3}{16}(-1+t)(1+3t^2)(1+t)$

$$AT = \begin{pmatrix} 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{-3}{65536}(-36-13s+12s^2+s^3)(-52+57s-20s^2+3s^3)(3+s)(-1+s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", (1 + v[1] v[5]) (1 + v[2] v[4])
"B CYCLES", (1 + v[2] v[5]) (1 + v[1] v[3])

Eigenvalues

R: [0., 1., −1., 1., −1.]

B: [0., 1., −1., 1., −1.]

NullSpace of R

{[0, 0, 1, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0]}

NullSpace of R^*

{[−1, 0, 1, 0, 0]}

NullSpace of B^*

{[0, 1, 0, −1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO–M} = \begin{pmatrix} 0 & 8 & 0 & 8 & 16 \\ 8 & 0 & 8 & 0 & 16 \\ 0 & 8 & 0 & 0 & 8 \\ 8 & 0 & 0 & 0 & 8 \\ 16 & 16 & 8 & 8 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 5

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{9}\left(2v[1] + 2v[2] + v[3] + v[4] + 3v[5] \right)$
degree 2: $\frac{1}{9}\left(v[1]v[2] + v[1]v[4] + 2v[1]v[5] + v[2]v[3] + 2v[2]v[5] + v[3]v[5] + v[4]v[5] \right)$
degree 3 : $\frac{1}{3}\left(v[5] \right)\left(v[1]v[2] + v[1]v[4] + v[2]v[3] \right)$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 3}, {5}, {2, 4}}

"RG1" = {2, 3, 5}

"RG2" = {1, 4, 5}

"RG3" = {1, 2, 5}

$$\pi 3 = [0, 0, 1, 0, 0, 1, 0, 1, 0, 0]$$

supp $\pi 3$ = {3, 6, 8}

$$u3 = [0, 0, 1, 0, 0, 1, 0, 1, 0, 1]$$

supp u3 = {3, 6, 8, 10}

Action of R on ranges, [[2], [3], [2]]
Action of B on ranges, [[3], [1], [1]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 1, 3]

B–BLOCKS,

[1, 3, 2]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1 = \{1, 3\}$
$b_2 = \{5\}$
$b_3 = \{2, 4\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 11, Shape: 3 ⊕ 8/6

$$\mathrm{CLB} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 5}, {2, 4}}, true

Ω_B in Vec(K)? , {{1, 3}, {2, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{9} & \frac{2}{9} & \frac{-5}{9} & \frac{1}{9} & \frac{1}{3} \\ \frac{-2}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{5}{9} & \frac{-1}{3} \\ \frac{-4}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{1}{9} & \frac{1}{3} \\ \frac{-2}{9} & \frac{4}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{3} \\ \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} & \frac{-1}{3} & 0 \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{3}{10} & \frac{1}{5} & 0 & \frac{1}{5} & \frac{3}{10} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{5} & \frac{3}{10} & \frac{1}{5} & 0 & \frac{3}{10} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3}, {5}, {2, 4}}

1, "range", [2, 3, 5], [[5, 3, 5, 3, 2], [5, 2, 5, 2, 3], [3, 5, 3, 5, 2], [3, 2, 3, 2, 5], [2, 5, 2, 5, 3], [2, 3, 2, 3, 5]]

2, "range", [1, 4, 5], [[5, 4, 5, 4, 1], [5, 1, 5, 1, 4], [4, 5, 4, 5, 1], [4, 1, 4, 1, 5], [1, 5, 1, 5, 4], [1, 4, 1, 4, 5]]

3, "range", [1, 2, 5], [[5, 2, 5, 2, 1], [5, 1, 5, 1, 2], [2, 5, 2, 5, 1], [2, 1, 2, 1, 5], [1, 5, 1, 5, 2], [1, 2, 1, 2, 5]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 3]]

$g_2 =$ [[2, 3]]

$g_3 =$ [[1, 3]]

$g_4 =$ []

$g_5 =$ [[1, 3, 2]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & 2h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 3, 4]}, {5, [1, 3, 5]}, {6, [1, 4, 5]}, {7, [2, 3, 4]}, {8, [2, 3, 5]}, {9, [2, 4, 5]}, {10, [3, 4, 5]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$

{3, 6, 8}

$u3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$

{3, 6, 8, 10}

picheck $\begin{pmatrix} 2 & 2 & 1 & 1 & 3 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{3} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 1 & 0 & 1 & 2 & 1 & 0 & 2 & 0 & 1 & 1 \end{pmatrix}$

$u2 = \left(\frac{1}{3} \ \ 0 \ \ \frac{1}{3} \ \ \frac{1}{3} \ \ \frac{1}{3} \ \ 0 \ \ \frac{1}{3} \ \ \frac{1}{3} \ \ \frac{1}{3} \ \ \frac{1}{3}\right)$

picheck $(4 \ \ 4 \ \ 2 \ \ 2 \ \ 6)$

$\pi 1 = (4 \ \ 4 \ \ 2 \ \ 2 \ \ 6)$

$u1 = \left(\frac{2}{9} \ \ \frac{2}{9} \ \ \frac{2}{9} \ \ \frac{2}{9} \ \ \frac{2}{9}\right)$

picheck $(4 \ \ 4 \ \ 2 \ \ 2 \ \ 6)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks

$$T = \begin{pmatrix} \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} \frac{32}{9} & \frac{16}{9} & \frac{16}{9} & \frac{8}{9} & \frac{8}{3} \\ \frac{16}{9} & \frac{32}{9} & \frac{8}{9} & \frac{16}{9} & \frac{8}{3} \\ \frac{32}{9} & \frac{16}{9} & \frac{16}{9} & \frac{8}{9} & \frac{8}{3} \\ \frac{16}{9} & \frac{32}{9} & \frac{8}{9} & \frac{16}{9} & \frac{8}{3} \\ \frac{16}{9} & \frac{16}{9} & \frac{8}{9} & \frac{8}{9} & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, -1, 1, 0]$

$\ker N_C = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} -t & 0 & t & 0 & 0 \\ 0 & -s & 0 & s & 0 \end{pmatrix}$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 \end{pmatrix}$

M0 is invertible. det= 6500/19683

$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} t+s \\ t+s \\ t+s \\ t+s \\ t+s \end{pmatrix}$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 5 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 3

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & -1 & \frac{-8}{9} & \frac{8}{9} \\ 0 & 0 & \frac{-8}{9} & -1 & \frac{8}{9} \\ 1 & \frac{8}{9} & 0 & 0 & \frac{16}{9} \\ \frac{8}{9} & 1 & 0 & 0 & \frac{16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{-1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} \\ 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{2}{9} \\ \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{-2}{9} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & \frac{8}{9} & 0 & \frac{8}{9} & \frac{16}{9} \\ \frac{8}{9} & 2 & \frac{8}{9} & 0 & \frac{16}{9} \\ 0 & \frac{8}{9} & 1 & 0 & \frac{8}{9} \\ \frac{8}{9} & 0 & 0 & 1 & \frac{8}{9} \\ \frac{16}{9} & \frac{16}{9} & \frac{8}{9} & \frac{8}{9} & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = \frac{8}{3} T + 8 \Omega$$

$$\Omega \left(\frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{3} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; \frac{1}{3} \;\; \frac{1}{9} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{2}{9} \right)$$

$$T \left(0 \;\; \frac{2}{3} \;\; 1 \;\; 0 \;\; \frac{2}{3} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{3} \;\; 0 \;\; \frac{2}{3} \right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \left(\frac{8}{9} \;\; \frac{32}{9} \;\; \frac{16}{3} \;\; \frac{8}{9} \;\; \frac{32}{9} \;\; \frac{16}{9} \;\; \frac{8}{3} \;\; \frac{8}{9} \;\; \frac{16}{9} \;\; \frac{16}{9} \;\; \frac{32}{9} \right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN} \left(\frac{-2}{11} \;\; \frac{92}{33} \;\; \frac{184}{33} \;\; \frac{14}{11} \;\; \frac{86}{33} \;\; \frac{14}{11} \;\; \frac{24}{11} \;\; \frac{14}{11} \;\; \frac{86}{33} \;\; \frac{14}{11} \;\; \frac{86}{33} \right)$$

$\tau = 9/1$, rank = 3 , ratio = 3/1 , $n^2 / r = 25/3$

$\tau' = 16/1$, $r' = 2/3$, $\tau / n^2 = 9/25$

$p^2 = 19/81$, min $\tau = 475/81$, τ–check is positive? 254/81

max r = 81/19 , r–check is positive? 8/27

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 8\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 11
out of total no. of elements equal to 18

dim span idems 3 vs no. of idems 3

"PT1" = {{1, 3}, {5}, {2, 4}}

"RG1" = {2, 3, 5}

"RG2" = {1, 4, 5}

"RG3" = {1, 2, 5}

$$M_C = \begin{pmatrix} \frac{62}{81} & \frac{-28}{81} & \frac{-50}{81} & \frac{22}{81} & \frac{-2}{27} \\ \frac{-28}{81} & \frac{62}{81} & \frac{22}{81} & \frac{-50}{81} & \frac{-2}{27} \\ \frac{-50}{81} & \frac{22}{81} & \frac{56}{81} & \frac{-25}{81} & \frac{-1}{27} \\ \frac{22}{81} & \frac{-50}{81} & \frac{-25}{81} & \frac{56}{81} & \frac{-1}{27} \\ \frac{-2}{27} & \frac{-2}{27} & \frac{-1}{27} & \frac{-1}{27} & \frac{2}{9} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{62}{81} & \frac{-19}{81} & \frac{62}{81} & \frac{-19}{81} & \frac{-19}{81} \\ \frac{-19}{81} & \frac{62}{81} & \frac{-19}{81} & \frac{62}{81} & \frac{-19}{81} \\ \frac{62}{81} & \frac{-19}{81} & \frac{62}{81} & \frac{-19}{81} & \frac{-19}{81} \\ \frac{-19}{81} & \frac{62}{81} & \frac{-19}{81} & \frac{62}{81} & \frac{-19}{81} \\ \frac{-19}{81} & \frac{-19}{81} & \frac{-19}{81} & \frac{-19}{81} & \frac{62}{81} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-14}{31} & \frac{-25}{31} & \frac{11}{31} & \frac{-3}{31} \\ \frac{-14}{31} & 1 & \frac{11}{31} & \frac{-25}{31} & \frac{-3}{31} \\ \frac{-25}{28} & \frac{11}{28} & 1 & \frac{-25}{56} & \frac{-3}{56} \\ \frac{11}{28} & \frac{-25}{28} & \frac{-25}{56} & 1 & \frac{-3}{56} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{6} & \frac{-1}{6} & 1 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-19}{62} & 1 & \frac{-19}{62} & \frac{-19}{62} \\ \frac{-19}{62} & 1 & \frac{-19}{62} & 1 & \frac{-19}{62} \\ 1 & \frac{-19}{62} & 1 & \frac{-19}{62} & \frac{-19}{62} \\ \frac{-19}{62} & 1 & \frac{-19}{62} & 1 & \frac{-19}{62} \\ \frac{-19}{62} & \frac{-19}{62} & \frac{-19}{62} & \frac{-19}{62} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{4}{27} & \frac{-2}{27} & \frac{2}{27} & \frac{-1}{27} & \frac{-1}{9} \\ \frac{-2}{27} & \frac{4}{27} & \frac{-1}{27} & \frac{2}{27} & \frac{-1}{9} \\ \frac{4}{27} & \frac{-2}{27} & \frac{2}{27} & \frac{-1}{27} & \frac{-1}{9} \\ \frac{-2}{27} & \frac{4}{27} & \frac{-1}{27} & \frac{2}{27} & \frac{-1}{9} \\ \frac{-2}{27} & \frac{-2}{27} & \frac{-1}{27} & \frac{-1}{27} & \frac{2}{9} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{4}{27} & \frac{-2}{27} & \frac{4}{27} & \frac{-2}{27} & \frac{-2}{27} \\ \frac{-2}{27} & \frac{4}{27} & \frac{-2}{27} & \frac{4}{27} & \frac{-2}{27} \\ \frac{2}{27} & \frac{-1}{27} & \frac{2}{27} & \frac{-1}{27} & \frac{-1}{27} \\ \frac{-1}{27} & \frac{2}{27} & \frac{-1}{27} & \frac{2}{27} & \frac{-1}{27} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{2}{27} & \frac{-1}{27} & \frac{1}{27} \\ 0 & 0 & \frac{-1}{27} & \frac{2}{27} & \frac{1}{27} \\ \frac{-2}{27} & \frac{1}{27} & 0 & 0 & \frac{2}{27} \\ \frac{1}{27} & \frac{-2}{27} & 0 & 0 & \frac{2}{27} \\ \frac{-1}{27} & \frac{-1}{27} & \frac{-2}{27} & \frac{-2}{27} & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 1.946178864, 0.1649322482, 0.7505405108, 0.2741508472]

Eigenvalues N_C

[0., 0., 2., 1.405552070, 0.4216084235]

Eigenvalues M_C–scaled

[0., 1.135874376, 0.9660841503, 2.670943107, 0.227098367]

Eigenvalues N_C–scaled

[0., 0., 2.612903226, 1.836285769, 0.5508110047]

NullSpace M_C

{[1, 1, 1, 1, 1]}

NullSpace N_C

{[0, −1, 0, 1, 0], [−1, 0, 1, 0, 0]}

Eigenvalues M_0

[1.946178864, 0.1649322482, 5.929100987, 0.2416082767, 0.7181796227]

Eigenvalues N_0

[1., 0., 0., 2., 2.]

NullSpace M_0

{}

NullSpace N_0

{[0, 1, 0, −1, 0], [−1, 0, 1, 0, 0]}

Eigenvalues M

[0.5493635453, −1.438252434, 3.615324520, −2.195546624, −0.5308890059]

Eigenvalues N

[0., 0., −2., 3.236067977, −1.236067977]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 1, 0], [−1, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$