T - Run

[4, 5, 6, 6, 3, 3], [5, 4, 1, 1, 2, 2]

$\tilde{\pi}$ = [1, 1, 1, 1, 1, 1]
$\boldsymbol{\delta}$ = [2, 2, 2, 2, 2, 2]

POSSIBLE RANKS

1 x 6
2 x 3

BASE DETERMINANT 49/512, .9570312500e-1

NullSpace of Δ

{1, 6}, {2, 3}, {4, 5}

Nullspace of A

[{3},{2}] \`,\` [{5},{4}] \`,\` [{6},{1}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[3] v[6] + v[2] v[5] + v[1] v[4]

Degree 3
v[1] v[3] v[5] + v[2] v[4] v[6]

Degree 4
2 v[2] v[3] v[5] v[6] + 2 v[1] v[3] v[4] v[6] + 2 v[1] v[2] v[4] v[5]

Degree 5
0

Degree 6
8 v[1] v[2] v[3] v[4] v[5] v[6]

==

{}

R: [4, 5, 6, 6, 3, 3]
B: [5, 4, 1, 1, 2, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{512}(7 + 2s + s^2)^2 (1 + s)(-1 + s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[3] v[6]
"B CYCLES", 1 + v[1] v[2] v[4] v[5]

Eigenvalues

R: [0., 0., 0., 0., 1., −1.]

B: [0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 1, −1]}

NullSpace of B^*

{[0, 0, −1, 1, 0, 0], [0, 0, 0, 0, −1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 1 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 1 & 1 \\ \frac{1}{3} & \frac{2}{3} & 1 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{3}$ ($v[1]v[2] + v[3]v[6] + v[4]v[5]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 5, 6}, {2, 3, 4}}

"PT2" = {{1, 3, 4}, {2, 5, 6}}

"RG1" = {3, 6}

"RG2" = {4, 5}

"RG3" = {1, 2}

$$\pi 2 = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0]$$

supp $\pi 2$ = {1, 12, 13}

$$u2 = [3, 2, 2, 1, 1, 1, 1, 2, 2, 0, 3, 3, 3, 3, 0]$$

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14}

Action of R on ranges, [[1], [1], [2]]
Action of B on ranges, [[3], [3], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[3, 1, 1, 3]

B−BLOCKS,

[2, 3, 4, 1]

with invariant measure, [2, 1, 2, 1]

N by blocks, N − check: true

$b_1 = \{1, 5, 6\}$
$b_2 = \{1, 3, 4\}$
$b_3 = \{2, 3, 4\}$
$b_4 = \{2, 5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & h[1] \\ 0 & 0 & 0 & h[2] & h[1] & 0 \\ 0 & 0 & 0 & h[1] & h[2] & 0 \\ 0 & 0 & h[1] & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: 3 ⊕ 7/5

$$\text{CLB} = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 6}}, true

Ω_B in Vec(K)? , {{1, 2, 4, 5}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 5, 6}, {2, 3, 4}}

1, "range", [3, 6], [[6, 3, 3, 3, 6, 6], [3, 6, 6, 6, 3, 3]]

2, "range", [4, 5], [[5, 4, 4, 4, 5, 5], [4, 5, 5, 5, 4, 4]]

3, "range", [1, 2], [[2, 1, 1, 1, 2, 2], [1, 2, 2, 2, 1, 1]]

2, "partition", {{1, 3, 4}, {2, 5, 6}}

1, "range", [3, 6], [[6, 3, 6, 6, 3, 3], [3, 6, 3, 3, 6, 6]]

2, "range", [4, 5], [[5, 4, 5, 5, 4, 4], [4, 5, 4, 4, 5, 5]]

3, "range", [1, 2], [[2, 1, 2, 2, 1, 1], [1, 2, 1, 1, 2, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$

{1, 12, 13}

$u2 = \begin{pmatrix} 3 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 2 & 0 & 3 & 3 & 3 & 3 & 0 \end{pmatrix}$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ idem−checks N0−checks

$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \end{pmatrix}$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{9} & \frac{2}{9} & \frac{2}{9} \\ 0 & \frac{1}{3} & \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{2}{9} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{9} & \frac{2}{9} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{2}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{2}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 3 & 0 & 1 & 1 & 2 & 2 \\ 0 & 3 & 2 & 2 & 1 & 1 \\ 1 & 2 & 3 & 3 & 0 & 0 \\ 1 & 2 & 3 & 3 & 0 & 0 \\ 2 & 1 & 0 & 0 & 3 & 3 \\ 2 & 1 & 0 & 0 & 3 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 1, 0, 0, 1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ t & t & s & -s-t & -s-t & s \\ t & t & s & -s-t & -s-t & s \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & -s+t & 0 \\ 0 & -t+s & 0 \\ t & 0 & s \\ t & 0 & s \\ -t & 0 & -s \\ -t & 0 & -s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} t & s & 0 & 0 \\ s & t & 0 & 0 \\ t & t & -t & s \\ t & t & -t & s \\ s & s & t & -s \\ s & s & t & -s \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 3 & 3 & 0 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 1 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 1 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 3 & 3 & 0 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 3 & 0 & 0 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 1 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 1 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 9T + 0\Omega$$

$\Omega \left(\frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{6}\right)$

$T \left(\frac{1}{3} \ \ \frac{1}{3} \ \ \frac{1}{9} \ \ \frac{2}{9} \ \ \frac{2}{9} \ \ \frac{1}{9} \ \ \frac{1}{9} \ \ 0 \ \ \frac{1}{3}\right)$

"IS NM in Vec(K)?", true

NM $(3 \ \ 3 \ \ 1 \ \ 2 \ \ 2 \ \ 1 \ \ 1 \ \ 0 \ \ 3)$

"IS MN in Vec(K)?", true

MN $(3 \ \ 3 \ \ 1 \ \ 2 \ \ 2 \ \ 1 \ \ 1 \ \ 0 \ \ 3)$

$\boldsymbol{\tau} = 18/1$, rank = 2 , ratio = 9/1 , $n^2 / r = 18/1$

$\boldsymbol{\tau}' = 18/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 1/6$, min $\boldsymbol{\tau} = 6/1$, $\boldsymbol{\tau}$–check is positive? 12/1

max r = 6/1 , r–check is positive? 2/3

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 3, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 9
out of total no. of elements equal to 12

dim span idems 6 vs no. of idems 6

"PT1" = {{1, 5, 6}, {2, 3, 4}}

"PT2" = {{1, 3, 4}, {2, 5, 6}}

"RG1" = {3, 6}

"RG2" = {4, 5}

"RG3" = {1, 2}

$$M_C = \begin{pmatrix} 2 & 2 & -1 & -1 & -1 & -1 \\ 2 & 2 & -1 & -1 & -1 & -1 \\ -1 & -1 & 2 & -1 & -1 & 2 \\ -1 & -1 & -1 & 2 & 2 & -1 \\ -1 & -1 & -1 & 2 & 2 & -1 \\ -1 & -1 & 2 & -1 & -1 & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{2} \\ \frac{-1}{6} & \frac{5}{6} & \frac{1}{2} & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{2} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{1}{6} & \frac{1}{2} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ 1 & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & 1 \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & 1 & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & 1 & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} & \frac{3}{5} \\ \frac{-1}{5} & 1 & \frac{3}{5} & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{3}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{1}{5} & \frac{3}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{3}{5} & \frac{1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \\ \frac{3}{5} & \frac{1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[6., 6., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 2., 2.187184271, 0.8128157289]

Eigenvalues M_C–scaled

[3., 3., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 2.400000000, 2.624621125, 0.9753788748]

NullSpace M_C

{[−1, 1, 0, 0, 0, 0], [1, 0, 1, 0, 1, 0], [1, 0, 1, 1, 0, 0], [0, 0, −1, 0, 0, 1]}

NullSpace N_c

{[0, 0, −1, 1, 0, 0], [−1, −1, 1, 0, 1, 0], [−1, −1, 1, 0, 0, 1]}

Eigenvalues M_0

[6., 6., 6., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[1, −1, 0, 0, 0, 0], [0, 0, 0, 1, −1, 0], [0, 0, 1, 0, 0, −1]}

NullSpace N_0

{[−1, −1, 0, 1, 0, 1], [−1, −1, 0, 1, 1, 0], [0, 0, 1, −1, 0, 0]}

Eigenvalues M

[3., −3., 3., −3., 3., −3.]

Eigenvalues N

[0., 0., 0., 3., −0.8128157289, −2.187184271]

NullSpace M

{}

NullSpace N

{[0, 0, 0, 0, 1, −1], [−1, −1, 0, 1, 0, 1], [−1, −1, 1, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 2 & 1 & 1 \\ 3 & 0 & 1 & 1 & 2 & 2 \\ 2 & 1 & 0 & 0 & 3 & 3 \\ 2 & 1 & 0 & 0 & 3 & 3 \\ 1 & 2 & 3 & 3 & 0 & 0 \\ 1 & 2 & 3 & 3 & 0 & 0 \end{pmatrix}$$

==

{4}

R: [4, 5, 6, 1, 3, 3]
B: [5, 4, 1, 6, 2, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{512} (7 + s^2) (-1 + s) (-7 + s)$

RANK of R is 5

R ranking is 2, "vs", 5

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[3] v[6]) (1 + v[1] v[4])
"B CYCLES", 1 + v[2] v[4] v[6]

Eigenvalues

R: [1., -1., 1., -1., 0., 0.]

B: [0., 0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

NullSpace of R

{[0, 1, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, -1, 1]}

NullSpace of B^*

{[0, 0, 0, 0, -1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 3

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3   "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{12}$ ($v[1]v[2] + v[1]v[3] + v[1]v[5] + v[1]v[6] + v[2]v[4] + v[2]v[5] + v[2]v[6] + v[3]v[4] + v[3]v[5] + v[3]v[6] + v[4]v[5] + v[4]v[6]$)
degree 3 : $\frac{1}{8}$ ($v[1] + v[4]$) ($v[2] + v[3]$) ($v[5] + v[6]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 4}, {2, 3}, {5, 6}}

"RG1" = {3, 4, 6}

"RG2" = {3, 4, 5}

"RG3" = {2, 4, 6}

"RG4" = {2, 4, 5}

"RG5" = {1, 3, 6}

"RG6" = {1, 3, 5}

"RG7" = {1, 2, 6}

"RG8" = {1, 2, 5}

$$\pi 3 = [0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]$$

supp $\pi 3$ = {3, 4, 6, 7, 14, 15, 17, 18}

$$u3 = [0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]$$

supp u3 = {3, 4, 6, 7, 14, 15, 17, 18}

Action of R on ranges, [[5], [5], [6], [6], [1], [1], [2], [2]]
Action of B on ranges, [[7], [7], [3], [3], [8], [8], [4], [4]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\alpha = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[1, 3, 2]

B–BLOCKS,

[2, 3, 1]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1 = \{1, 4\}$
$b_2 = \{2, 3\}$
$b_3 = \{5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N):   3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 17, Shape: 3 ⊕ 14/12

$$\mathrm{CLB} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4}, {3, 6}}, true

Ω_B in Vec(K)? , {{2, 4, 6}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 4}, {2, 3}, {5, 6}}

1, "range", [3, 4, 6], [[6, 4, 4, 6, 3, 3], [6, 3, 3, 6, 4, 4], [4, 6, 6, 4, 3, 3], [4, 3, 3, 4, 6, 6], [3, 6, 6, 3, 4, 4], [3, 4, 4, 3, 6, 6]]

2, "range", [3, 4, 5], [[5, 4, 4, 5, 3, 3], [5, 3, 3, 5, 4, 4], [4, 5, 5, 4, 3, 3], [4, 3, 3, 4, 5, 5], [3, 5, 5, 3, 4, 4], [3, 4, 4, 3, 5, 5]]

3, "range", [2, 4, 6], [[6, 4, 4, 6, 2, 2], [6, 2, 2, 6, 4, 4], [4, 6, 6, 4, 2, 2], [4, 2, 2, 4, 6, 6], [2, 6, 6, 2, 4, 4], [2, 4, 4, 2, 6, 6]]

4, "range", [2, 4, 5], [[5, 4, 4, 5, 2, 2], [5, 2, 2, 5, 4, 4], [4, 5, 5, 4, 2, 2], [4, 2, 2, 4, 5, 5], [2, 5, 5, 2, 4, 4], [2, 4, 4, 2, 5, 5]]

5, "range", [1, 3, 6], [[6, 3, 3, 6, 1, 1], [6, 1, 1, 6, 3, 3], [3, 6, 6, 3, 1, 1], [3, 1, 1, 3, 6, 6], [1, 6, 6, 1, 3, 3], [1, 3, 3, 1, 6, 6]]

6, "range", [1, 3, 5], [[5, 3, 3, 5, 1, 1], [5, 1, 1, 5, 3, 3], [3, 5, 5, 3, 1, 1], [3, 1, 1, 3, 5, 5], [1, 5, 5, 1, 3, 3], [1, 3, 3, 1, 5, 5]]

7, "range", [1, 2, 6], [[6, 2, 2, 6, 1, 1], [6, 1, 1, 6, 2, 2], [2, 6, 6, 2, 1, 1], [2, 1, 1, 2, 6, 6], [1, 6, 6, 1, 2, 2], [1, 2, 2, 1, 6, 6]]

8, "range", [1, 2, 5], [[5, 2, 2, 5, 1, 1], [5, 1, 1, 5, 2, 2], [2, 5, 5, 2, 1, 1], [2, 1, 1, 2, 5, 5], [1, 5, 5, 1, 2, 2], [1, 2, 2, 1, 5, 5]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 3]]

$g_2 =$ [[2, 3]]

$g_3 =$ [[1, 3]]

$g_4 =$ []

$g_5 =$ [[1, 3, 2]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[2] & 0 & 0 & 2h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$

{3, 4, 6, 7, 14, 15, 17, 18}

$u3 = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$

{3, 4, 6, 7, 14, 15, 17, 18}

picheck $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 4 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 2 & 2 & 0 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$

picheck $\begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 2 & 2 & 4 & 2 & 2 \\ 2 & 4 & 4 & 2 & 2 & 2 \\ 2 & 4 & 4 & 2 & 2 & 2 \\ 4 & 2 & 2 & 4 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 4 \\ 2 & 2 & 2 & 2 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, -1, 1, 0, 0, 0]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ t & 0 & 0 & -t & -s & s \\ s & 0 & 0 & -s & -t & t \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \\ -1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} t & s \\ s & t \\ s & t \\ t & s \\ -t-s & -t-s \\ -t-s & -t-s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & t \\ 0 & t & s \\ 0 & t & s \\ 0 & s & t \\ t+s & 0 & 0 \\ t+s & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 3

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathrm{Skew\ Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 1 & 1 & 0 & 1 & 1 \\ 1 & 2 & 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 0 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 12\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$

"IS NM in Vec(K)?", true

$\mathrm{NM} \begin{pmatrix} 4 & 2 & 2 & 2 & 8 & 4 & 4 & 2 & 2 & 2 & 4 & 2 & 2 & 4 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 4 & 2 & 2 & 2 & 8 & 4 & 4 & 2 & 2 & 2 & 4 & 2 & 2 & 4 \end{pmatrix}$

$$\boldsymbol{\tau} = 12/1 \text{ , rank} = 3 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 6/1$$

$$\text{max r} = 6/1 \text{ , } r\text{–check is positive? } 1/2$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 4 vs no. of idems 8

"PT1" = {{1, 4}, {2, 3}, {5, 6}}

"RG1" = {3, 4, 6}

"RG2" = {3, 4, 5}

"RG3" = {2, 4, 6}

"RG4" = {2, 4, 5}

"RG5" = {1, 3, 6}

"RG6" = {1, 3, 5}

"RG7" = {1, 2, 6}

"RG8" = {1, 2, 5}

$$M_c = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C

[1., 2., 2., 0., 0., 0.]

Eigenvalues M_C–scaled

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C–scaled

[1.200000000, 2.400000000, 2.400000000, 0., 0., 0.]

NullSpace M_C

{[0, 1, 1, 0, 0, 0], [1, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 1]}

NullSpace N_C

{[1, 0, 0, −1, 0, 0], [0, 0, 0, 0, 1, −1], [0, 1, −1, 0, 0, 0]}

Eigenvalues M_0

[6., 0., 0., 2., 2., 2.]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[1, −1, −1, 1, 0, 0], [0, −1, −1, 0, 1, 1]}

NullSpace N_0

{[1, 0, 0, −1, 0, 0], [0, 1, −1, 0, 0, 0], [0, 0, 0, 0, −1, 1]}

Eigenvalues M

[4., −2., −2., 0., 0., 0.]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{[0, −1, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0], [0, 0, 0, 0, −1, 1]}

NullSpace N

{[0, −1, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0], [0, 0, 0, 0, −1, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

==

{5}

R: [4, 5, 6, 6, 2, 3]
B: [5, 4, 1, 1, 3, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{512}\left(7+s^2\right)\left(-7+s\right)\left(-1+s\right)$

RANK of R is 5

R ranking is 2, "vs", 5

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[2] v[5]) (1 + v[3] v[6])
"B CYCLES", 1 + v[1] v[3] v[5]

Eigenvalues

R: [1., −1., 1., −1., 0., 0.]

B: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[1, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1]}

NullSpace of R^*

{[0, 0, −1, 1, 0, 0]}

NullSpace of B^*

{[0, 0, −1, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 3

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{12}$ ($v[1]v[2] + v[1]v[3] + v[1]v[4] + v[1]v[5] + v[2]v[3] + v[2]v[4] + v[2]v[6] + v[3]v[5] + v[3]v[6] + v[4]v[5] + v[4]v[6] + v[5]v[6]$)
degree 3 : $\frac{1}{8}$ ($v[3] + v[4]$) ($v[1] + v[6]$) ($v[2] + v[5]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {3, 4}}

"RG1" = {4, 5, 6}

"RG2" = {3, 5, 6}

"RG3" = {2, 4, 6}

"RG4" = {2, 3, 6}

"RG5" = {1, 4, 5}

"RG6" = {1, 3, 5}

"RG7" = {1, 2, 4}

"RG8" = {1, 2, 3}

$$\pi 3 = [1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1]$$

supp $\pi 3$ = {1, 2, 6, 8, 13, 15, 19, 20}

$$u3 = [1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1]$$

supp u3 = {1, 2, 6, 8, 13, 15, 19, 20}

Action of R on ranges, [[4], [4], [2], [2], [3], [3], [1], [1]]
Action of B on ranges, [[8], [8], [7], [7], [6], [6], [5], [5]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 2, 1]

B-BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

b_1 = {1, 6}
b_2 = {2, 5}
b_3 = {3, 4}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 17, Shape: 3 ⊕ 14/12

$$CLB = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 6}, {2, 5}}, true

Ω_B in Vec(K)? , {{1, 3, 5}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {3, 4}}

1, "range", [4, 5, 6], [[6, 5, 4, 4, 5, 6], [6, 4, 5, 5, 4, 6], [5, 6, 4, 4, 6, 5], [5, 4, 6, 6, 4, 5], [4, 6, 5, 5, 6, 4], [4, 5, 6, 6, 5, 4]]

2, "range", [3, 5, 6], [[6, 5, 3, 3, 5, 6], [6, 3, 5, 5, 3, 6], [5, 6, 3, 3, 6, 5], [5, 3, 6, 6, 3, 5], [3, 6, 5, 5, 6, 3], [3, 5, 6, 6, 5, 3]]

3, "range", [2, 4, 6], [[6, 4, 2, 2, 4, 6], [6, 2, 4, 4, 2, 6], [4, 6, 2, 2, 6, 4], [4, 2, 6, 6, 2, 4], [2, 6, 4, 4, 6, 2], [2, 4, 6, 6, 4, 2]]

4, "range", [2, 3, 6], [[6, 3, 2, 2, 3, 6], [6, 2, 3, 3, 2, 6], [3, 6, 2, 2, 6, 3], [3, 2, 6, 6, 2, 3], [2, 6, 3, 3, 6, 2], [2, 3, 6, 6, 3, 2]]

5, "range", [1, 4, 5], [[5, 4, 1, 1, 4, 5], [5, 1, 4, 4, 1, 5], [4, 5, 1, 1, 5, 4], [4, 1, 5, 5, 1, 4], [1, 5, 4, 4, 5, 1], [1, 4, 5, 5, 4, 1]]

6, "range", [1, 3, 5], [[5, 3, 1, 1, 3, 5], [5, 1, 3, 3, 1, 5], [3, 5, 1, 1, 5, 3], [3, 1, 5, 5, 1, 3], [1, 5, 3, 3, 5, 1], [1, 3, 5, 5, 3, 1]]

7, "range", [1, 2, 4], [[4, 2, 1, 1, 2, 4], [4, 1, 2, 2, 1, 4], [2, 4, 1, 1, 4, 2], [2, 1, 4, 4, 1, 2], [1, 4, 2, 2, 4, 1], [1, 2, 4, 4, 2, 1]]

8, "range", [1, 2, 3], [[3, 2, 1, 1, 2, 3], [3, 1, 2, 2, 1, 3], [2, 3, 1, 1, 3, 2], [2, 1, 3, 3, 1, 2], [1, 3, 2, 2, 3, 1], [1, 2, 3, 3, 2, 1]]

"group has", 6, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

$g_3 =$ [[2, 3]]

$g_4 =$ [[1, 3, 2]]

$g_5 =$ [[1, 2, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} 2h[1] & 0 & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[1] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$$\pi 3 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

{1, 2, 6, 8, 13, 15, 19, 20}

$$u3 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

{1, 2, 6, 8, 13, 15, 19, 20}

picheck $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 4 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 2 = \begin{pmatrix} 2 & 2 & 2 & 2 & 0 & 2 & 2 & 0 & 2 & 0 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem−checks

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 2 & 2 & 2 & 2 & 4 \\ 2 & 4 & 2 & 2 & 4 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 4 & 2 & 2 & 4 & 2 \\ 4 & 2 & 2 & 2 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 0, 0, 0, 0, 1]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & t & -t & -s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & t & s & -s & -t & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ -1 & -1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -s+t & -s \\ -t+s & -t \\ 0 & t+s \\ 0 & t+s \\ -t+s & -t \\ -s+t & -s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} t & s & 0 \\ s & t & 0 \\ 0 & 0 & t+s \\ 0 & 0 & t+s \\ s & t & 0 \\ t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \ RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 1 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 4T + 12\Omega$$

$$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$T \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

"IS NM in Vec(K)?", true

$$NM \begin{pmatrix} 4 & 2 & 2 & 8 & 2 & 2 & 4 & 2 & 4 & 2 & 2 & 2 & 2 & 4 \end{pmatrix}$$

"IS MN in Vec(K)?", true

$$MN \begin{pmatrix} 4 & 2 & 2 & 8 & 2 & 2 & 4 & 2 & 4 & 2 & 2 & 2 & 2 & 4 \end{pmatrix}$$

$$\boldsymbol{\tau} = 12/1 \text{ , rank} = 3 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 6/1$$

$$\text{max r} = 6/1 \text{ , } r\text{–check is positive? } 1/2$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 4 vs no. of idems 8

"PT1" = {{1, 6}, {2, 5}, {3, 4}}

"RG1" = {4, 5, 6}

"RG2" = {3, 5, 6}

"RG3" = {2, 4, 6}

"RG4" = {2, 3, 6}

"RG5" = {1, 4, 5}

"RG6" = {1, 3, 5}

"RG7" = {1, 2, 4}

"RG8" = {1, 2, 3}

$$M_C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 \\ \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} \\ 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C

[1., 2., 2., 0., 0., 0.]

Eigenvalues M_C–scaled

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C–scaled

[1.200000000, 2.400000000, 2.400000000, 0., 0., 0.]

NullSpace M_C

{[0, 1, 0, 0, 1, 0], [0, 0, 1, 1, 0, 0], [1, 0, 0, 0, 0, 1]}

NullSpace N_C

{[−1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0], [0, 0, −1, 1, 0, 0]}

Eigenvalues M_0

[6., 0., 0., 2., 2., 2.]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[0, 1, −1, −1, 1, 0], [1, 0, −1, −1, 0, 1]}

NullSpace N_0

{[0, 0, −1, 1, 0, 0], [0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M

[4., −2., −2., 0., 0., 0.]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{[0, 0, −1, 1, 0, 0], [0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

NullSpace N

{[0, 0, −1, 1, 0, 0], [0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{2, 3}

R: [4, 4, 1, 6, 3, 3]
B: [5, 5, 6, 1, 2, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{512} (1 + s) (-1 + s) (7 + 2s + s^2)^2$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[1] v[3] v[4] v[6]

"B CYCLES", 1 + v[2] v[5]

Eigenvalues

R: [0., 0., −1., 1., 1. I, −1. I]

B: [0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[−1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1]}

NullSpace of B^*

{[−1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO–M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 1 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 1 \\ \frac{1}{3} & \frac{1}{3} & 0 & 1 & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & 1 & 0 & \frac{1}{3} & \frac{1}{3} \\ 1 & 1 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 1 & 1 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{3}$ ($v[1]v[6] + v[2]v[5] + v[3]v[4]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 4}, {3, 5, 6}}

"PT2" = {{4, 5, 6}, {1, 2, 3}}

"RG1" = {3, 4}

"RG2" = {2, 5}

"RG3" = {1, 6}

$$\pi 2 = [0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0]$$

supp $\pi 2$ = {5, 8, 10}

$$u2 = [0, 1, 2, 3, 3, 1, 2, 3, 3, 3, 2, 2, 1, 1, 0]$$

supp u2 = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}

Action of R on ranges, [[3], [1], [1]]
Action of B on ranges, [[3], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[3, 1, 4, 2]

B–BLOCKS,

[2, 3, 2, 3]

with invariant measure, [1, 2, 2, 1]

N by blocks, N – check: true

b_1 = {1, 2, 4}
b_2 = {4, 5, 6}
b_3 = {1, 2, 3}
b_4 = {3, 5, 6}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & 0 & h[1] \\ 0 & h[2] & 0 & 0 & h[1] & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & h[1] & 0 & 0 & h[2] & 0 \\ h[1] & 0 & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: 3 ⊕ 7/5

$$CLB = \begin{pmatrix} 1 & 1 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3, 4, 6}}, true

Ω_B in Vec(K)? , {{2, 5}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega–checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 4}, {3, 5, 6}}

1, "range", [3, 4], [[4, 4, 3, 4, 3, 3], [3, 3, 4, 3, 4, 4]]

2, "range", [2, 5], [[5, 5, 2, 5, 2, 2], [2, 2, 5, 2, 5, 5]]

3, "range", [1, 6], [[6, 6, 1, 6, 1, 1], [1, 1, 6, 1, 6, 6]]

2, "partition", {{4, 5, 6}, {1, 2, 3}}

1, "range", [3, 4], [[4, 4, 4, 3, 3, 3], [3, 3, 3, 4, 4, 4]]

2, "range", [2, 5], [[5, 5, 5, 2, 2, 2], [2, 2, 2, 5, 5, 5]]

3, "range", [1, 6], [[6, 6, 6, 1, 1, 1], [1, 1, 1, 6, 6, 6]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{5, 8, 10}

$u2 = \begin{pmatrix} 0 & 1 & 2 & 3 & 3 & 1 & 2 & 3 & 3 & 3 & 2 & 2 & 1 & 1 & 0 \end{pmatrix}$

{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem−checks} \quad \text{N0−checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem−checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem−checks}$$

$$T = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{2}{9} & \frac{1}{9} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{9} & \frac{1}{9} & 0 & 0 \\ \frac{2}{9} & \frac{2}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & 0 & \frac{1}{3} & \frac{2}{9} & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} & \frac{2}{9} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{9} & \frac{2}{9} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 3 & 3 & 2 & 1 & 0 & 0 \\ 3 & 3 & 2 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 1 \\ 1 & 1 & 0 & 3 & 2 & 2 \\ 0 & 0 & 1 & 2 & 3 & 3 \\ 0 & 0 & 1 & 2 & 3 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, -1, 1, 1, -1, 0]$

$\ker N_C = \begin{pmatrix} 0 & 1 & -1 & -1 & 0 & 1 \\ 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$ $\begin{pmatrix} -s-t & t & s & s & t & -s-t \\ -s-t & t & s & s & t & -s-t \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 0 & -1 \end{pmatrix}$

$\ker M_0 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} s & t & 0 \\ s & t & 0 \\ 0 & 0 & -s+t \\ 0 & 0 & -t+s \\ -s & -t & 0 \\ -s & -t & 0 \end{pmatrix}$ RB checks

$\ker M_C = \begin{pmatrix} 0 & 1 & 1 & -1 \\ -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} t & 0 & s & 0 \\ t & 0 & s & 0 \\ 0 & s & s & -s+t \\ 0 & t & t & -t+s \\ -t & s+t & t & 0 \\ -t & s+t & t & 0 \end{pmatrix}$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 3 & 3 & 0 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 1 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & 0 & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 1 & 1 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 3&0&0&0&0&3\\0&3&0&0&3&0\\0&0&3&3&0&0\\0&0&3&3&0&0\\0&3&0&0&3&0\\3&0&0&0&0&3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&1&\frac{2}{3}&\frac{1}{3}&0&0\\1&1&\frac{2}{3}&\frac{1}{3}&0&0\\\frac{2}{3}&\frac{2}{3}&1&0&\frac{1}{3}&\frac{1}{3}\\\frac{1}{3}&\frac{1}{3}&0&1&\frac{2}{3}&\frac{2}{3}\\0&0&\frac{1}{3}&\frac{2}{3}&1&1\\0&0&\frac{1}{3}&\frac{2}{3}&1&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 9T + 0\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} \frac{1}{3} & \frac{2}{9} & \frac{2}{9} & 0 & 0 & \frac{1}{9} & \frac{2}{9} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

"IS NM in Vec(K)?", true

NM $\begin{pmatrix} 3 & 2 & 2 & 0 & 0 & 1 & 2 & 3 & 3 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 3 & 2 & 2 & 0 & 0 & 1 & 2 & 3 & 3 \end{pmatrix}$

$$\boldsymbol{\tau} = 18/1 \text{ , rank} = 2 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 12/1$$

$$\text{max r} = 6/1 \text{ , } r\text{–check is positive? } 2/3$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 3, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 9
out of total no. of elements equal to 12

dim span idems 6 vs no. of idems 6

"PT1" = {{1, 2, 4}, {3, 5, 6}}

"PT2" = {{4, 5, 6}, {1, 2, 3}}

"RG1" = {3, 4}

"RG2" = {2, 5}

"RG3" = {1, 6}

$$M_C = \begin{pmatrix} 2 & -1 & -1 & -1 & -1 & 2 \\ -1 & 2 & -1 & -1 & 2 & -1 \\ -1 & -1 & 2 & 2 & -1 & -1 \\ -1 & -1 & 2 & 2 & -1 & -1 \\ -1 & 2 & -1 & -1 & 2 & -1 \\ 2 & -1 & -1 & -1 & -1 & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{5}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{1}{2} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{1}{2} & \frac{1}{2} & \frac{5}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{1}{2} & \frac{1}{2} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{5}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{5}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 \\ \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & 1 & 1 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & 1 & 1 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} \\ 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & 1 & \frac{3}{5} & \frac{1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ 1 & 1 & \frac{3}{5} & \frac{1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{3}{5} & \frac{3}{5} & 1 & \frac{-1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{-1}{5} & 1 & \frac{3}{5} & \frac{3}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{1}{5} & \frac{3}{5} & 1 & 1 \\ \frac{-1}{5} & \frac{-1}{5} & \frac{1}{5} & \frac{3}{5} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[6., 6., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 2., 2.187184271, 0.8128157289]

Eigenvalues M_C–scaled

[3., 3., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 2.400000000, 2.624621125, 0.9753788748]

NullSpace M_C

{[0, 1, 1, 0, 0, 1], [0, 0, −1, 1, 0, 0], [0, −1, 0, 0, 1, 0], [1, 1, 1, 0, 0, 0]}

NullSpace N_C

{[1, 0, −1, −1, 0, 1], [0, 0, 0, 0, 1, −1], [0, 1, −1, −1, 0, 1]}

Eigenvalues M_0

[6., 6., 6., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1], [0, 0, −1, 1, 0, 0]}

NullSpace N_0

{[0, 0, 0, 0, −1, 1], [−1, 0, 1, 1, −1, 0], [−1, 1, 0, 0, 0, 0]}

Eigenvalues M

[3., −3., 3., −3., 3., −3.]

Eigenvalues N

[0., 0., 0., 3., −0.8128157289, −2.187184271]

NullSpace M

{}

NullSpace N

{[1, 0, −1, −1, 0, 1], [−1, 1, 0, 0, 0, 0], [1, 0, −1, −1, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 3 & 3 \\ 0 & 0 & 1 & 2 & 3 & 3 \\ 1 & 1 & 0 & 3 & 2 & 2 \\ 2 & 2 & 3 & 0 & 1 & 1 \\ 3 & 3 & 2 & 1 & 0 & 0 \\ 3 & 3 & 2 & 1 & 0 & 0 \end{pmatrix}$$

==

{2, 4}

R: [4, 4, 6, 1, 3, 3]
B: [5, 5, 1, 6, 2, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{512}(-7+s^2)^2(-1+s)(1+s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 2

"R CYCLES", (1 + v[3] v[6]) (1 + v[1] v[4])
"B CYCLES", 1 + v[2] v[5]

Eigenvalues

R: [1., −1., 1., −1., 0., 0.]

B: [0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[−1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1]}

NullSpace of B^*

{[0, 0, 0, 0, 1, −1], [1, −1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{3}$ ($v[1]v[6] + v[2]v[5] + v[3]v[4]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 4}, {3, 5, 6}}

"RG1" = {3, 4}

"RG2" = {2, 5}

"RG3" = {1, 6}

$$\pi 2 = [0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0]$$

supp $\pi 2$ = {5, 8, 10}

$$u2 = [0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0]$$

supp u2 = {2, 4, 5, 6, 8, 9, 10, 13, 14}

Action of R on ranges, [[3], [1], [1]]
Action of B on ranges, [[3], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[1, 2]

B–BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N – check: true

$b_1 = \{1, 2, 4\}$
$b_2 = \{3, 5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & 0 & 0 & h[2] \\ 0 & h[1] & 0 & 0 & h[2] & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & h[2] & 0 & 0 & h[1] & 0 \\ h[2] & 0 & 0 & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 7, Shape: 0 ⊕ 7/5

$$\text{CLB} = \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4}, {3, 6}}, true

Ω_B in Vec(K)? , {{2, 5}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega–checks true, true

"I–V is singular"

$\pi_R = \left(\frac{1}{4} \;\; 0 \;\; \frac{1}{4} \;\; \frac{1}{4} \;\; 0 \;\; \frac{1}{4}\right)$ vs $\left(0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right)$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \left(0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; \frac{1}{2} \;\; 0\right)$ vs $\left(0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; \frac{1}{2} \;\; 0\right)$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 4}, {3, 5, 6}}

1, "range", [3, 4], [[4, 4, 3, 4, 3, 3], [3, 3, 4, 3, 4, 4]]

2, "range", [2, 5], [[5, 5, 2, 5, 2, 2], [2, 2, 5, 2, 5, 5]]

3, "range", [1, 6], [[6, 6, 1, 6, 1, 1], [1, 1, 6, 1, 6, 6]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\left(h[1] \;\; h[2]\right)$

"Basis for Z(G)"

1, "coeff", 1

$Z[\,1\,] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{5, 8, 10}

$u2 = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$

{2, 4, 5, 6, 8, 9, 10, 13, 14}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$ idem−checks

$$\mathrm{T} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \mathrm{NM} = \begin{pmatrix} 3 & 3 & 0 & 3 & 0 & 0 \\ 3 & 3 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 3 & 3 \\ 3 & 3 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 3 & 3 \\ 0 & 0 & 3 & 0 & 3 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [0, −1, 1, 1, −1, 0]

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ t & -t & -s & 0 & 0 & s \\ -s & 0 & 0 & s & t & -t \\ -s & 0 & 0 & s & t & -t \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & -1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & t & 0 \\ s & t & 0 \\ 0 & 0 & -t+s \\ 0 & 0 & -s+t \\ -s & -t & 0 \\ -s & -t & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ -1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & -t & s+t & t \\ 0 & -t & s+t & t \\ -s+t & 0 & s & s \\ -t+s & 0 & t & t \\ 0 & t & 0 & s \\ 0 & t & 0 & s \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 3 & 3 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 \\ 3 & 0 & 0 & 0 & 0 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 9T + 0\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

"IS NM in Vec(K)?", true

NM $\begin{pmatrix} 0 & 0 & 3 & 0 & 3 & 3 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 0 & 0 & 3 & 0 & 3 & 3 \end{pmatrix}$

$$\boldsymbol{\tau} = 18/1 \text{ , rank} = 2 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 12/1$$

$$\text{max r} = 6/1 \text{ , } r\text{–check is positive? } 2/3$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 6
out of total no. of elements equal to 6

dim span idems 3 vs no. of idems 3

"PT1" = {{1, 2, 4}, {3, 5, 6}}

"RG1" = {3, 4}

"RG2" = {2, 5}

"RG3" = {1, 6}

$$M_C = \begin{pmatrix} 2 & -1 & -1 & -1 & -1 & 2 \\ -1 & 2 & -1 & -1 & 2 & -1 \\ -1 & -1 & 2 & 2 & -1 & -1 \\ -1 & -1 & 2 & 2 & -1 & -1 \\ -1 & 2 & -1 & -1 & 2 & -1 \\ 2 & -1 & -1 & -1 & -1 & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 \\ \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & 1 & 1 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & 1 & 1 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} \\ 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} \\ 1 & 1 & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & 1 & 1 \\ 1 & 1 & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & 1 & 1 \\ \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[6., 6., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 3., 2.]

Eigenvalues M_C–scaled

[3., 3., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 3.600000000, 2.400000000]

NullSpace M_C

{[1, 1, 1, 0, 0, 0], [1, 1, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

NullSpace N_C

{[−1, 1, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0], [0, 0, −1, 0, 1, 0], [0, 0, −1, 0, 0, 1]}

Eigenvalues M_0

[6., 6., 6., 0., 0., 0.]

Eigenvalues N_0

[3., 3., 0., 0., 0., 0.]

NullSpace M_0

{[0, 1, 0, 0, −1, 0], [0, 0, 1, −1, 0, 0], [1, 0, 0, 0, 0, −1]}

NullSpace N_0

{[0, 0, 1, 0, 0, −1], [0, 0, 0, 0, 1, −1], [−1, 0, 0, 1, 0, 0], [−1, 1, 0, 0, 0, 0]}

Eigenvalues M

[3., −3., 3., −3., 3., −3.]

Eigenvalues N

[0., 0., 0., 0., 3., −3.]

NullSpace M

{}

NullSpace N

{[1, 0, 0, −1, 0, 0], [0, 0, 1, 0, 0, −1], [0, 1, 0, −1, 0, 0], [0, 0, 0, 0, 1, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

==

{3, 5}

R: [4, 5, 1, 6, 2, 3]
B: [5, 4, 6, 1, 3, 2]

TRACE TWO = 2

det AT = $1\,(t)^3$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{1}{128} (7 + 3s + 2s^2) (7 + 7s + 2s^2) (-1 + s)^2$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", (1 + v[2] v[5]) (1 + v[1] v[3] v[4] v[6])
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6]

Eigenvalues

R: [1. I, −1. I, 1., −1., 1., −1.]

B: [−1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 2

$$\text{PROTO–M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 6

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{12}$ ($v[1]v[2] + v[1]v[3] + v[1]v[4] + v[1]v[5] + 4v[1]v[6] + v[2]v[3] + v[2]v[4] + 4v[2]v[5] + v[2]v[6] + 4v[3]v[4] + v[3]v[5] + v[3]v[6] + v[4]v[5] + v[4]v[6] + v[5]v[6]$)
degree 3 : $\frac{1}{8}$ ($3v[1]v[2]v[3] + 3v[1]v[2]v[4] + 2v[1]v[2]v[5] + 2v[1]v[2]v[6] + 2v[1]v[3]v[4] + 3v[1]v[3]v[5] + 2v[1]v[3]v[6] + 3v[1]v[4]v[5] + 2v[1]v[4]v[6] + 2v[1]v[5]v[6] + 2v[2]v[3]v[4] + 2v[2]v[3]v[5] + 3v[2]v[3]v[6] + 2v[2]v[4]v[5] + 3v[2]v[4]v[6] + 2v[2]v[5]v[6] + 2v[3]v[4]v[5] + 2v[3]v[4]v[6] + 3v[3]v[5]v[6] + 3v[4]v[5]v[6]$)
degree 4 : $\frac{1}{12}$ ($v[1]v[2]v[3]v[4] + v[1]v[2]v[3]v[5] + v[1]v[2]v[3]v[6] + v[1]v[2]v[4]v[5] + v[1]v[2]v[4]v[6] + 4v[1]v[2]v[5]v[6] + v[1]v[3]v[4]v[5] + 4v[1]v[3]v[4]v[6] + v[1]v[3]v[5]v[6] + v[1]v[4]v[5]v[6] + 4v[2]v[3]v[4]v[5] + v[2]v[3]v[4]v[6] + v[2]v[3]v[5]v[6] + v[2]v[4]v[5]v[6] + v[3]v[4]v[5]v[6]$)
degree 5 : $\frac{1}{6}$ ($v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[6] + v[1]v[2]v[3]v[5]v[6] + v[1]v[2]v[4]v[5]v[6] + v[1]v[3]v[4]v[5]v[6] + v[2]v[3]v[4]v[5]v[6]$)
degree 6 : 1 ($v[3]$) ($v[4]$) ($v[1]$) ($v[2]$) ($v[6]$) ($v[5]$)

Group spectrum $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

KERNEL STRUCTURE

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$\pi 6 = [1]$$

supp $\pi 6$ = {1}

$$u6 = [1]$$

supp u6 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 5, 6, 1, 4, 3]

B-BLOCKS,

[6, 1, 2, 5, 3, 4]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N - check: true

$b_1 = \{4\}$
$b_2 = \{1\}$
$b_3 = \{5\}$
$b_4 = \{6\}$
$b_5 = \{3\}$
$b_6 = \{2\}$

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & h[2] & h[2] & h[2] & h[3] \\ h[2] & h[1] & h[2] & h[2] & h[3] & h[2] \\ h[2] & h[2] & h[1] & h[3] & h[2] & h[2] \\ h[2] & h[2] & h[3] & h[1] & h[2] & h[2] \\ h[2] & h[3] & h[2] & h[2] & h[1] & h[2] \\ h[3] & h[2] & h[2] & h[2] & h[2] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: $11 \oplus 2/0$

$$\text{CLB} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 5}, {1, 3, 4, 6}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 4, 5, 6}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4}, {1}, {5}, {6}, {3}, {2}}

1, "range", [1, 2, 3, 4, 5, 6], [[6, 5, 4, 3, 2, 1], [6, 5, 3, 4, 2, 1], [6, 4, 5, 2, 3, 1], [6, 4, 2, 5, 3, 1], [6, 3, 5, 2, 4, 1], [6, 3, 2, 5, 4, 1], [6, 2, 4, 3, 5, 1], [6, 2, 3, 4, 5, 1], [5, 6, 4, 3, 1, 2], [5, 6, 3, 4, 1, 2], [5, 4, 6, 1, 3, 2], [5, 4, 1, 6, 3, 2], [5, 3, 6, 1, 4, 2], [5, 3, 1, 6, 4, 2], [5, 1, 4, 3, 6, 2], [5, 1, 3, 4, 6, 2], [4, 6, 5, 2, 1, 3], [4, 6, 2, 5, 1, 3], [4, 5, 6, 1, 2, 3], [4, 5, 1, 6, 2, 3], [4, 2, 6, 1, 5, 3], [4, 2, 1, 6, 5, 3], [4, 1, 5, 2, 6, 3], [4, 1, 2, 5, 6, 3], [3, 6, 5, 2, 1, 4], [3, 6, 2, 5, 1, 4], [3, 5, 6, 1, 2, 4], [3, 5, 1, 6, 2, 4], [3, 2, 6, 1, 5, 4], [3, 2, 1, 6, 5, 4], [3, 1, 5, 2, 6, 4], [3, 1, 2, 5, 6, 4], [2, 6, 4, 3, 1, 5], [2, 6, 3, 4, 1, 5], [2, 4, 6, 1, 3, 5], [2, 4, 1, 6, 3, 5], [2, 3, 6, 1, 4, 5], [2, 3, 1, 6, 4, 5], [2, 1, 4, 3, 6, 5], [2, 1, 3, 4, 6, 5], [1, 5, 4, 3, 2, 6], [1, 5, 3, 4, 2, 6], [1, 4, 5, 2, 3, 6], [1, 4, 2, 5, 3, 6], [1, 3, 5, 2, 4, 6], [1, 3, 2, 5, 4, 6], [1, 2, 4, 3, 5, 6], [1, 2, 3, 4, 5, 6]]

"group has", 48, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

g_1 =   [[1, 6], [2, 5], [3, 4]]

g_2 =   [[1, 6], [2, 5]]

g_3 =   [[1, 6], [2, 4], [3, 5]]

g_4 =   [[1, 6], [2, 4, 5, 3]]

g_5 =   [[1, 6], [2, 3, 5, 4]]

linear dimension, 14

"Symmetric?", true

Is Z in Vec(K)? true
$(8h[3] - 16h[1] \;\; 8h[1] \;\; -4h[2] \;\; 2h[2] \;\; 2h[2] \;\; 8h[1] \;\; -4h[2] \;\; 2h[2] \;\; 2h[2] \;\; 2h[2] \;\; 2h[2] \;\; 2h[2] \;\; 2h[2] \;\; 8h[$

"Basis for Z(G)"

1, "coeff", 8

Z[1] = $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 2

Z[2] = $\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$

3, "coeff", 8

$$Z[\,3\,] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 4. & -2. & -2. & 0 & 0 & 0 \\ 1. & -1. & 1. & -1. & 1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 15t^5 + 27t^6 + 38t^7 + 60t^8 + 84t^9 + 122t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}

KERNEL HIERARCHY

$\pi 6 = \begin{pmatrix} 1 \end{pmatrix}$

{1}

$u6 = \begin{pmatrix} 1 \end{pmatrix}$

{1}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 5 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u5 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

picheck $\begin{pmatrix} 5 & 5 & 5 & 5 & 5 & 5 \end{pmatrix}$

$\pi 4 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

$u4 = \begin{pmatrix} \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} \end{pmatrix}$

picheck $(20\ \ 20\ \ 20\ \ 20\ \ 20\ \ 20)$

$\pi 3 = (6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6)$

$u3 = \left(\frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\right)$

picheck $(60\ \ 60\ \ 60\ \ 60\ \ 60\ \ 60)$

$\pi 2 = (24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24)$

$u2 = \left(\frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\right)$

picheck $(120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120)$

$\pi 1 = (120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120)$

$u1 = \left(\frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\right)$

picheck $(120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120)$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 5 & 4 & 4 & 4 & 4 & 4 \\ 4 & 5 & 4 & 4 & 4 & 4 \\ 4 & 4 & 5 & 4 & 4 & 4 \\ 4 & 4 & 4 & 5 & 4 & 4 \\ 4 & 4 & 4 & 4 & 5 & 4 \\ 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta}$ = [0, 0, 0, 0, 0, 0]

ker $N_C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$ $\begin{pmatrix} s+t & s+t & s+t & s+t & s+t & s+t \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & t & 0 \\ 0 & 0 & t & s & 0 \\ -s & -s & -s & -s & -s+t \\ -t & -t & -t & -t & -t+s \\ s & t & 0 & 0 & 0 \\ t & s & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & t & 0 & 0 \\ 0 & 0 & t & s & 0 & 0 \\ 0 & 0 & 0 & 0 & t & s \\ 0 & 0 & 0 & 0 & s & t \\ t & s & 0 & 0 & 0 & 0 \\ s & t & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$CNM = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 1T + 24\Omega$$

$\Omega \left(\frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{2}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\right)$

$T\ (1\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1)$

"IS NM in Vec(K)?", true

NM $(5\ 4\ 4\ 13\ 4\ 4\ 5\ 4\ 4\ 4\ 4\ 4\ 4\ 5)$

"IS MN in Vec(K)?", true

MN $(5\ 4\ 4\ 13\ 4\ 4\ 5\ 4\ 4\ 4\ 4\ 4\ 4\ 5)$

$$\boldsymbol{\tau} = 6/1 \text{ , rank} = 6 \text{ , ratio} = 1/1 \text{ , } n^2 / r = 6/1$$

$$\boldsymbol{\tau}' = 30/1 \text{ , } r' = 5/6 \text{ , } \boldsymbol{\tau} / n^2 = 1/6$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\text{max r} = 6/1 \text{ , } r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 6\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 48

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 1 vs no. of idems 1

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1.]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 1.200000000, 1.200000000, 1.200000000, 1.200000000, 1.200000000]

NullSpace M_C

{[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0]}

NullSpace N_C

{[1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 6.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1], [−1, 0, 0, 1, 0, 0], [−1, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 1, 0]}

NullSpace N_0

{}

Eigenvalues M

[5., −1., −1., −1., −1., −1.]

Eigenvalues N

[5., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

==

{3, 6}

R: [4, 5, 1, 6, 3, 2]
B: [5, 4, 6, 1, 2, 3]

TRACE TWO = 2

det AT = $-1\,(\,t\,)^{3}$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{1}{128}\,(\,7 + 3s + 2s^{2}\,)\,(\,7 + 7s + 2s^{2}\,)\,(\,-1 + s\,)^{2}$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6]
"B CYCLES", (1 + v[1] v[2] v[4] v[5]) (1 + v[3] v[6])

Eigenvalues

R: [−1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

B: [1. I, −1. I, 1., −1., 1., −1.]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 6

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)

degree 2: $\frac{1}{3}$ ($4v[1]v[2] + v[1]v[3] + v[1]v[4] + v[1]v[5] + v[1]v[6] + v[2]v[3] + v[2]v[4] + v[2]v[5] + v[2]v[6] + v[3]v[4] + v[3]v[5] + 4v[3]v[6] + 4v[4]v[5] + v[4]v[6] + v[5]v[6]$)

degree 3 : $\frac{1}{12}$ ($2v[1]v[2]v[3] + 2v[1]v[2]v[4] + 2v[1]v[2]v[5] + 2v[1]v[2]v[6] + 3v[1]v[3]v[4] + 3v[1]v[3]v[5] + 2v[1]v[3]v[6] + 2v[1]v[4]v[5] + 3v[1]v[4]v[6] + 3v[1]v[5]v[6] + 3v[2]v[3]v[4] + 3v[2]v[3]v[5] + 2v[2]v[3]v[6] + 2v[2]v[4]v[5] + 3v[2]v[4]v[6] + 3v[2]v[5]v[6] + 2v[3]v[4]v[5] + 2v[3]v[4]v[6] + 2v[3]v[5]v[6] + 2v[4]v[5]v[6]$)

degree 4 : $\frac{1}{12}$ ($v[1]v[2]v[3]v[4] + v[1]v[2]v[3]v[5] + 4v[1]v[2]v[3]v[6] + 4v[1]v[2]v[4]v[5] + v[1]v[2]v[4]v[6] + v[1]v[2]v[5]v[6] + v[1]v[3]v[4]v[5] + v[1]v[3]v[4]v[6] + v[1]v[3]v[5]v[6] + v[1]v[4]v[5]v[6] + v[2]v[3]v[4]v[5] + v[2]v[3]v[4]v[6] + v[2]v[3]v[5]v[6] + v[2]v[4]v[5]v[6] + 4v[3]v[4]v[5]v[6]$)

degree 5 : $\frac{1}{6}$ ($v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[6] + v[1]v[2]v[3]v[5]v[6] + v[1]v[2]v[4]v[5]v[6] + v[1]v[3]v[4]v[5]v[6] + v[2]v[3]v[4]v[5]v[6]$)

degree 6 : 1 ($v[3]$) ($v[4]$) ($v[1]$) ($v[2]$) ($v[6]$) ($v[5]$)

Group spectrum $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

KERNEL STRUCTURE

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$\pi 6 = [1]$$

supp $\pi 6$ = {1}

$$u6 = [1]$$

supp u6 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 5, 6, 1, 3, 4]

B–BLOCKS,

[6, 1, 2, 5, 4, 3]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

$b_1 = \{4\}$
$b_2 = \{1\}$
$b_3 = \{5\}$
$b_4 = \{6\}$
$b_5 = \{3\}$
$b_6 = \{2\}$

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & h[3] & h[3] & h[3] & h[3] \\ h[2] & h[1] & h[3] & h[3] & h[3] & h[3] \\ h[3] & h[3] & h[1] & h[3] & h[3] & h[2] \\ h[3] & h[3] & h[3] & h[1] & h[2] & h[3] \\ h[3] & h[3] & h[3] & h[2] & h[1] & h[3] \\ h[3] & h[3] & h[2] & h[3] & h[3] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: $11 \oplus 2/0$

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4, 5, 6}}, true

Ω_B in Vec(K)? , {{1, 2, 4, 5}, {3, 6}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4}, {1}, {5}, {6}, {3}, {2}}

1, "range", [1, 2, 3, 4, 5, 6], [[6, 3, 5, 2, 1, 4], [6, 3, 5, 1, 2, 4], [6, 3, 4, 2, 1, 5], [6, 3, 4, 1, 2, 5], [6, 3, 2, 5, 4, 1], [6, 3, 2, 4, 5, 1], [6, 3, 1, 5, 4, 2], [6, 3, 1, 4, 5, 2], [5, 4, 6, 2, 1, 3], [5, 4, 6, 1, 2, 3], [5, 4, 3, 2, 1, 6], [5, 4, 3, 1, 2, 6], [5, 4, 2, 6, 3, 1], [5, 4, 2, 3, 6, 1], [5, 4, 1, 6, 3, 2], [5, 4, 1, 3, 6, 2], [4, 5, 6, 2, 1, 3], [4, 5, 6, 1, 2, 3], [4, 5, 3, 2, 1, 6], [4, 5, 3, 1, 2, 6], [4, 5, 2, 6, 3, 1], [4, 5, 2, 3, 6, 1], [4, 5, 1, 6, 3, 2], [4, 5, 1, 3, 6, 2], [3, 6, 5, 2, 1, 4], [3, 6, 5, 1, 2, 4], [3, 6, 4, 2, 1, 5], [3, 6, 4, 1, 2, 5], [3, 6, 2, 5, 4, 1], [3, 6, 2, 4, 5, 1], [3, 6, 1, 5, 4, 2], [3, 6, 1, 4, 5, 2], [2, 1, 6, 5, 4, 3], [2, 1, 6, 4, 5, 3], [2, 1, 5, 6, 3, 4], [2, 1, 5, 3, 6, 4], [2, 1, 4, 6, 3, 5], [2, 1, 4, 3, 6, 5], [2, 1, 3, 5, 4, 6], [2, 1, 3, 4, 5, 6], [1, 2, 6, 5, 4, 3], [1, 2, 6, 4, 5, 3], [1, 2, 5, 6, 3, 4], [1, 2, 5, 3, 6, 4], [1, 2, 4, 6, 3, 5], [1, 2, 4, 3, 6, 5], [1, 2, 3, 5, 4, 6], [1, 2, 3, 4, 5, 6]]

"group has", 48, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$

g_1 =   [[1, 6, 4, 2, 3, 5]]

g_2 =   [[1, 6, 4], [2, 3, 5]]

g_3 =   [[1, 6, 5], [2, 3, 4]]

g_4 =   [[1, 6, 5, 2, 3, 4]]

g_5 =   [[1, 6], [2, 3], [4, 5]]

linear dimension, 14

"Symmetric?", true

Is Z in Vec(K)? true
$(-2h[3] \;\; 2h[3] \;\; 2h[3] \;\; -2h[3]-8h[1] \;\; 8h[1] \;\; 2h[3] \;\; -2h[3]-8h[1] \;\; 8h[1] \;\; 2h[3] \;\; 2h[3] \;\; 2h[3] \;\; 2h[3] \;\; 8h$

"Basis for Z(G)"

1, "coeff", 8

Z[1] = $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 8

Z[2] = $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$

3, "coeff", 2

$$Z[3] = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. & 1. & -1. \\ 4. & -2. & -2. & 0 & 0 & 0 \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 15t^5 + 27t^6 + 38t^7 + 60t^8 + 84t^9 + 122t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}

KERNEL HIERARCHY

$\pi 6 = (1)$

{1}

$u6 = (1)$

{1}

picheck $(1 \; 1 \; 1 \; 1 \; 1 \; 1)$

$\pi = \left(\frac{1}{6} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{6}\right)$

$\pi 5 = (1 \; 1 \; 1 \; 1 \; 1 \; 1)$

$u5 = \left(\frac{1}{6} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{6}\right)$

picheck $(5 \; 5 \; 5 \; 5 \; 5 \; 5)$

$\pi 4 = (2 \; 2 \; 2 \; 2 \; 2 \; 2 \; 2 \; 2 \; 2 \; 2 \; 2 \; 2 \; 2 \; 2 \; 2)$

$u4 = \left(\frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18} \; \frac{1}{18}\right)$

picheck (20 20 20 20 20 20)

$\pi 3 = (6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6)$

$u3 = \left(\frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\right)$

picheck (60 60 60 60 60 60)

$\pi 2 = (24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24)$

$u2 = \left(\frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\right)$

picheck (120 120 120 120 120 120)

$\pi 1 = (120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120)$

$u1 = \left(\frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\right)$

picheck (120 120 120 120 120 120)

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 5 & 4 & 4 & 4 & 4 & 4 \\ 4 & 5 & 4 & 4 & 4 & 4 \\ 4 & 4 & 5 & 4 & 4 & 4 \\ 4 & 4 & 4 & 5 & 4 & 4 \\ 4 & 4 & 4 & 4 & 5 & 4 \\ 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta}$ = [0, 0, 0, 0, 0, 0]

ker $N_C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} s+t & s+t & s+t & s+t & s+t & s+t \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & s & t \\ 0 & 0 & 0 & t & s \\ -s+t & -s & -s & -s & -s \\ -t+s & -t & -t & -t & -t \\ 0 & t & s & 0 & 0 \\ 0 & s & t & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & 0 & 0 & 0 & s \\ 0 & s & 0 & 0 & 0 & t \\ t & 0 & s & 0 & 0 & 0 \\ s & 0 & t & 0 & 0 & 0 \\ 0 & 0 & 0 & t & s & 0 \\ 0 & 0 & 0 & s & t & 0 \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 1T + 24\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

"IS NM in Vec(K)?", true

$\text{NM} \begin{pmatrix} 5 & 4 & 4 & 12 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$

"IS MN in Vec(K)?", true

$\text{MN} \begin{pmatrix} 5 & 4 & 4 & 12 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$

$$\boldsymbol{\tau} = 6/1 \text{ , rank} = 6 \text{ , ratio} = 1/1 \text{ , } n^2 / r = 6/1$$

$$\boldsymbol{\tau}' = 30/1 \text{ , } r' = 5/6 \text{ , } \boldsymbol{\tau} / n^2 = 1/6$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\text{max r} = 6/1 \text{ , } r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 6\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 48

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 1 vs no. of idems 1

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$M_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{5}{6} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&1&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&1&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&1&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&1&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.200000000, 1.200000000, 1.200000000, 1.200000000, 1.200000000]

NullSpace M_C

{[1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0]}

NullSpace N_C

{[1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 6.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 1], [−1, 0, 1, 0, 0, 0], [−1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0]}

NullSpace N_0

{}

Eigenvalues M

[5., −1., −1., −1., −1., −1.]

Eigenvalues N

[5., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

==

{4, 5}

R: [4, 5, 6, 1, 2, 3]
B: [5, 4, 1, 6, 3, 2]

TRACE TWO = 4

det AT = $-1\,(t)^3$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{1}{128}\,(7+2s)^2\,(-1+s)^4$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", (1 + v[3] v[6]) (1 + v[2] v[5]) (1 + v[1] v[4])
"B CYCLES", (1 + v[2] v[4] v[6]) (1 + v[1] v[3] v[5])

Eigenvalues

R: [1., −1., 1., −1., 1., −1.]

B: [1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 4

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 6

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)

degree 2: $\frac{1}{3}$ ($2v[1]v[2] + v[1]v[3] + 2v[1]v[4] + v[1]v[5] + 2v[1]v[6] + 2v[2]v[3] + v[2]v[4] + 2v[2]v[5] + v[2]v[6] + 2v[3]v[4] + v[3]v[5] + 2v[3]v[6] + 2v[4]v[5] + v[4]v[6] + 2v[5]v[6]$)

degree 3 : $\frac{1}{6}$ ($v[1]v[2]v[3] + v[1]v[2]v[4] + v[1]v[2]v[5] + v[1]v[2]v[6] + v[1]v[3]v[4] + 3v[1]v[3]v[5] + v[1]v[3]v[6] + v[1]v[4]v[5] + v[1]v[4]v[6] + v[1]v[5]v[6] + v[2]v[3]v[4] + v[2]v[3]v[5] + v[2]v[3]v[6] + v[2]v[4]v[5] + 3v[2]v[4]v[6] + v[2]v[5]v[6] + v[3]v[4]v[5] + v[3]v[4]v[6] + v[3]v[5]v[6] + v[4]v[5]v[6]$)

degree 4 : $\frac{1}{3}$ ($2v[1]v[2]v[3]v[4] + v[1]v[2]v[3]v[5] + 2v[1]v[2]v[3]v[6] + 2v[1]v[2]v[4]v[5] + v[1]v[2]v[4]v[6] + 2v[1]v[2]v[5]v[6] + v[1]v[3]v[4]v[5] + 2v[1]v[3]v[4]v[6] + v[1]v[3]v[5]v[6] + 2v[1]v[4]v[5]v[6] + 2v[2]v[3]v[4]v[5] + v[2]v[3]v[4]v[6] + 2v[2]v[3]v[5]v[6] + v[2]v[4]v[5]v[6] + 2v[3]v[4]v[5]v[6]$)

degree 5 : $\frac{1}{6}$ ($v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[6] + v[1]v[2]v[3]v[5]v[6] + v[1]v[2]v[4]v[5]v[6] + v[1]v[3]v[4]v[5]v[6] + v[2]v[3]v[4]v[5]v[6]$)

degree 6 : 1 ($v[3]$) ($v[4]$) ($v[1]$) ($v[2]$) ($v[6]$) ($v[5]$)

Group spectrum $1 + t + 4t^2 + 4t^3 + 4t^4 + t^5 + t^6$

KERNEL STRUCTURE

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$\pi 6 = [1]$$

supp π6 = {1}

$$u6 = [1]$$

supp u6 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\alpha = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 1, 6, 5, 4, 3]

B−BLOCKS,

[6, 5, 2, 1, 3, 4]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{4\}$
$b_2 = \{1\}$
$b_3 = \{5\}$
$b_4 = \{6\}$
$b_5 = \{3\}$
$b_6 = \{2\}$

dim(span of partition vectors), rank(N_0), rank(N):   6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[3] & h[6] & h[4] & h[2] & h[5] \\ h[3] & h[1] & h[4] & h[6] & h[5] & h[2] \\ h[2] & h[4] & h[1] & h[5] & h[6] & h[3] \\ h[4] & h[2] & h[5] & h[1] & h[3] & h[6] \\ h[6] & h[5] & h[2] & h[3] & h[1] & h[4] \\ h[5] & h[6] & h[3] & h[2] & h[4] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 5, Shape: 3 ⊕ 2/0

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4}, {3, 6}, {2, 5}}, true

Ω_B in Vec(K)? , {{1, 3, 5}, {2, 4, 6}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I−V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4}, {1}, {5}, {6}, {3}, {2}}

1, "range", [1, 2, 3, 4, 5, 6], [[6, 3, 2, 5, 4, 1], [5, 4, 1, 6, 3, 2], [4, 5, 6, 1, 2, 3], [3, 6, 5, 2, 1, 4], [2, 1, 4, 3, 6, 5], [1, 2, 3, 4, 5, 6]]

"group has", 6, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 6], [2, 3], [4, 5]]

$g_2 =$ [[1, 5, 3], [2, 4, 6]]

$g_3 =$ [[1, 4], [2, 5], [3, 6]]

$g_4 =$ [[1, 3, 5], [2, 6, 4]]

$g_5 =$ [[1, 2], [3, 4], [5, 6]]

linear dimension, 6

"Symmetric?", false

Is Z in Vec(K)? false

$$\left(\frac{||(h[2]+h[4]+h[6])||}{||3||} \quad \frac{||(h[3]+h[5])||}{||2||} \quad \frac{||(h[2]+h[4]+h[6])||}{||3||} \quad \frac{||(h[3]+h[5])||}{||2||} \quad \frac{||(h[2]+h[4]+h[6])||}{||3||} \quad h[1] \right)$$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$

3, "coeff", 1

$$Z[\,3\,] = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

4, "coeff", 1

$$Z[\,4\,] = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

5, "coeff", 1

$$Z[\,5\,] = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

6, "coeff", 1

$$Z[\,6\,] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
1, 5, true
1, 6, true
2, 3, false
2, 4, false
2, 5, false
2, 6, false
3, 4, false
3, 5, true
3, 6, false
4, 5, false

4, 6, false
5, 6, false

EIGS =

$$\begin{pmatrix} 1. & 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. & 1. \\ 1. & -0.5000000000 + 0.8660254040I & -0.5000000000 - 0.8660254040I & 1. & -0.5000000000 + 0.866 \\ 1. & -1. & 1. & -1. & 1. \\ 1. & -0.5000000000 + 0.8660254040I & -0.5000000000 - 0.8660254040I & 1. & -0.5000000000 + 0.866 \\ 1. & -1. & 1. & -1. & 1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 6 \\ 3 & 0 & 3 & 0 & 3 & 15 \\ 0 & 2 & 0 & 2 & 0 & 20 \\ 3 & 0 & 3 & 0 & 3 & 15 \\ 0 & 0 & 0 & 0 & 0 & 6 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 4t^2 + 4t^3 + 4t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 5t^2 + 10t^3 + 24t^4 + 42t^5 + 83t^6 + 132t^7 + 222t^8 + 335t^9 + 511 t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}

KERNEL HIERARCHY

$\pi 6 = (1)$

{1}

$u6 = (1)$

{1}

picheck $(1 \ 1 \ 1 \ 1 \ 1 \ 1)$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 5 = (1 \ 1 \ 1 \ 1 \ 1 \ 1)$

$u5 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

picheck $(5 \;\; 5 \;\; 5 \;\; 5 \;\; 5 \;\; 5)$

$\pi 4 = (2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2)$

$u4 = \left(\frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18} \;\; \frac{1}{18}\right)$

picheck $(20 \;\; 20 \;\; 20 \;\; 20 \;\; 20 \;\; 20)$

$\pi 3 = (6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6 \;\; 6)$

$u3 = \left(\frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36} \;\; \frac{1}{36}\right)$

picheck $(60 \;\; 60 \;\; 60 \;\; 60 \;\; 60 \;\; 60)$

$\pi 2 = (24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24)$

$u2 = \left(\frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54} \;\; \frac{1}{54}\right)$

picheck $(120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120)$

$\pi 1 = (120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120)$

$u1 = \left(\frac{5}{324} \;\; \frac{5}{324} \;\; \frac{5}{324} \;\; \frac{5}{324} \;\; \frac{5}{324} \;\; \frac{5}{324}\right)$

picheck $(120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120)$

Column Projections

$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

Row Projections

$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 5 & 4 & 4 & 4 & 4 & 4 \\ 4 & 5 & 4 & 4 & 4 & 4 \\ 4 & 4 & 5 & 4 & 4 & 4 \\ 4 & 4 & 4 & 5 & 4 & 4 \\ 4 & 4 & 4 & 4 & 5 & 4 \\ 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0]$

$\ker N_C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} t+s & t+s & t+s & t+s & t+s & t+s \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t & 0 & 0 & s & 0 \\ s & 0 & 0 & t & 0 \\ -t & -t & -t & -t & -t+s \\ -s & -s & -s & -s & -s+t \\ 0 & s & t & 0 & 0 \\ 0 & t & s & 0 & 0 \end{pmatrix}$$ RB checks

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & t & s & 0 \\ 0 & 0 & 0 & s & t & 0 \\ t & 0 & s & 0 & 0 & 0 \\ s & 0 & t & 0 & 0 & 0 \\ 0 & t & 0 & 0 & 0 & s \\ 0 & s & 0 & 0 & 0 & t \end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 1T + 24\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

"IS NM in Vec(K)?", true

NM $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$

$$\boldsymbol{\tau} = 6/1 \text{ , rank} = 6 \text{ , ratio} = 1/1 \text{ , } n^2 / r = 6/1$$

$$\boldsymbol{\tau}' = 30/1 \text{ , } r' = 5/6 \text{ , } \boldsymbol{\tau} / n^2 = 1/6$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\text{max r} = 6/1 \text{ , } r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 6\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 6
out of total no. of elements equal to 6

dim span idems 1 vs no. of idems 1

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.200000000, 1.200000000, 1.200000000, 1.200000000, 1.200000000]

NullSpace M_C

{[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0]}

NullSpace N_c

{[1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 6.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[1, 0, 0, 0, 0, −1], [0, 1, 0, 0, 0, −1], [0, 0, 1, 0, 0, −1], [0, 0, 0, 1, 0, −1], [0, 0, 0, 0, 1, −1]}

NullSpace N_0

{}

Eigenvalues M

[5., −1., −1., −1., −1., −1.]

Eigenvalues N

[5., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix}0&0&0&1&0&0\\0&0&1&0&0&0\\0&1&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&1&0\end{pmatrix} \quad \begin{pmatrix}0&0&1&0&1&0\\0&0&0&1&0&1\\1&0&0&0&1&0\\0&1&0&0&0&1\\1&0&1&0&0&0\\0&1&0&1&0&0\end{pmatrix} \quad \begin{pmatrix}0&1&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&1&0\\0&0&0&1&0&0\\0&0&1&0&0&0\end{pmatrix} \quad \begin{pmatrix}0&0&0&0&0&1\\0&0&0&0&1&0\\0&0&0&1&0&0\\0&0&1&0&0&0\\0&1&0&0&0&0\\1&0&0&0&0&0\end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix}$$

1, 3 : commutator = $\begin{pmatrix} 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \end{pmatrix}$

1, 4 : commutator = $\begin{pmatrix} 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \end{pmatrix}$

2, 3 : commutator = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

2, 4 : commutator = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

3, 4 : commutator = $\begin{pmatrix} 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \end{pmatrix}$

===

{4, 6}

R: [4, 5, 6, 1, 3, 2]
B: [5, 4, 1, 6, 2, 3]

TRACE TWO = 2

det AT = $1\,(t)^3$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{1}{128}(-1+s)^2(7+7s+2s^2)(7+3s+2s^2)$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", (1 + v[1] v[4]) (1 + v[2] v[3] v[5] v[6])
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6]

Eigenvalues

R: [1. I, −1. I, 1., −1., 1., −1.]

B: [−1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 2

$$PROTO\!-\!M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 6

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)

degree 2: $\frac{1}{12}$ ($v[1]v[2] + v[1]v[3] + 4v[1]v[4] + v[1]v[5] + v[1]v[6] + 4v[2]v[3] + v[2]v[4] + v[2]v[5] + v[2]v[6] + v[3]v[4] + v[3]v[5] + v[3]v[6] + v[4]v[5] + v[4]v[6] + 4v[5]v[6]$)

degree 3 : $\frac{1}{12}$ ($2v[1]v[2]v[3] + 2v[1]v[2]v[4] + 3v[1]v[2]v[5] + 3v[1]v[2]v[6] + 2v[1]v[3]v[4] + 3v[1]v[3]v[5] + 3v[1]v[3]v[6] + 2v[1]v[4]v[5] + 2v[1]v[4]v[6] + 2v[1]v[5]v[6] + 2v[2]v[3]v[4] + 2v[2]v[3]v[5] + 2v[2]v[3]v[6] + 3v[2]v[4]v[5] + 3v[2]v[4]v[6] + 2v[2]v[5]v[6] + 3v[3]v[4]v[5] + 3v[3]v[4]v[6] + 2v[3]v[5]v[6] + 2v[4]v[5]v[6]$)

degree 4 : $\frac{1}{3}$ ($4v[1]v[2]v[3]v[4] + v[1]v[2]v[3]v[5] + v[1]v[2]v[3]v[6] + v[1]v[2]v[4]v[5] + v[1]v[2]v[4]v[6] + v[1]v[2]v[5]v[6] + v[1]v[3]v[4]v[5] + v[1]v[3]v[4]v[6] + v[1]v[3]v[5]v[6] + 4v[1]v[4]v[5]v[6] + v[2]v[3]v[4]v[5] + v[2]v[3]v[4]v[6] + 4v[2]v[3]v[5]v[6] + v[2]v[4]v[5]v[6] + v[3]v[4]v[5]v[6]$)

degree 5 : $\frac{1}{6}$ ($v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[6] + v[1]v[2]v[3]v[5]v[6] + v[1]v[2]v[4]v[5]v[6] + v[1]v[3]v[4]v[5]v[6] + v[2]v[3]v[4]v[5]v[6]$)

degree 6 : 1 ($v[3]$) ($v[4]$) ($v[1]$) ($v[2]$) ($v[6]$) ($v[5]$)

Group spectrum $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

KERNEL STRUCTURE

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$\pi 6 = [1]$$

supp $\pi 6$ = {1}

$$u6 = [1]$$

supp u6 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 1, 6, 5, 3, 4]

B–BLOCKS,

[6, 5, 2, 1, 4, 3]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

$b_1 = \{4\}$
$b_2 = \{1\}$
$b_3 = \{5\}$
$b_4 = \{6\}$
$b_5 = \{3\}$
$b_6 = \{2\}$

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[3] & h[1] & h[1] & h[2] & h[1] & h[1] \\ h[1] & h[3] & h[2] & h[1] & h[1] & h[1] \\ h[1] & h[2] & h[3] & h[1] & h[1] & h[1] \\ h[2] & h[1] & h[1] & h[3] & h[1] & h[1] \\ h[1] & h[1] & h[1] & h[1] & h[3] & h[2] \\ h[1] & h[1] & h[1] & h[1] & h[2] & h[3] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: $11 \oplus 2/0$

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4}, {2, 3, 5, 6}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 4, 5, 6}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4}, {1}, {5}, {6}, {3}, {2}}

1, "range", [1, 2, 3, 4, 5, 6], [[6, 4, 1, 5, 3, 2], [6, 4, 1, 5, 2, 3], [6, 3, 2, 5, 4, 1], [6, 3, 2, 5, 1, 4], [6, 2, 3, 5, 4, 1], [6, 2, 3, 5, 1, 4], [6, 1, 4, 5, 3, 2], [6, 1, 4, 5, 2, 3], [5, 4, 1, 6, 3, 2], [5, 4, 1, 6, 2, 3], [5, 3, 2, 6, 4, 1], [5, 3, 2, 6, 1, 4], [5, 2, 3, 6, 4, 1], [5, 2, 3, 6, 1, 4], [5, 1, 4, 6, 3, 2], [5, 1, 4, 6, 2, 3], [4, 6, 5, 1, 3, 2], [4, 6, 5, 1, 2, 3], [4, 5, 6, 1, 3, 2], [4, 5, 6, 1, 2, 3], [4, 3, 2, 1, 6, 5], [4, 3, 2, 1, 5, 6], [4, 2, 3, 1, 6, 5], [4, 2, 3, 1, 5, 6], [3, 6, 5, 2, 4, 1], [3, 6, 5, 2, 1, 4], [3, 5, 6, 2, 4, 1], [3, 5, 6, 2, 1, 4], [3, 4, 1, 2, 6, 5], [3, 4, 1, 2, 5, 6], [3, 1, 4, 2, 6, 5], [3, 1, 4, 2, 5, 6], [2, 6, 5, 3, 4, 1], [2, 6, 5, 3, 1, 4], [2, 5, 6, 3, 4, 1], [2, 5, 6, 3, 1, 4], [2, 4, 1, 3, 6, 5], [2, 4, 1, 3, 5, 6], [2, 1, 4, 3, 6, 5], [2, 1, 4, 3, 5, 6], [1, 6, 5, 4, 3, 2], [1, 6, 5, 4, 2, 3], [1, 5, 6, 4, 3, 2], [1, 5, 6, 4, 2, 3], [1, 3, 2, 4, 6, 5], [1, 3, 2, 4, 5, 6], [1, 2, 3, 4, 6, 5], [1, 2, 3, 4, 5, 6]]

"group has", 48, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 6, 2, 4, 5, 3]]

$g_2 =$ [[1, 6, 3], [2, 4, 5]]

$g_3 =$ [[1, 6], [2, 3], [4, 5]]

$g_4 =$ [[1, 6, 4, 5], [2, 3]]

$g_5 =$ [[1, 6], [4, 5]]

linear dimension, 14

"Symmetric?", true

Is Z in Vec(K)? true
$\left(-2h[2] \quad 2h[2] \quad -8h[1]-2h[2] \quad 2h[2] \quad 8h[1] \quad 2h[2] \quad 2h[2] \quad -8h[1]-2h[2] \quad 2h[2] \quad 8h[3] \quad 8h[1] \quad 2h[2] \quad 2\right.$

"Basis for Z(G)"

1, "coeff", 8

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 2

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$

3, "coeff", 8

$$Z[3] = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 4. & -2. & -2. & 0 & 0 & 0 \\ 1. & -1. & 1. & -1. & 1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 15t^5 + 27t^6 + 38t^7 + 60t^8 + 84t^9 + 122t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}

KERNEL HIERARCHY

$\pi 6 = (1)$

{1}

$u6 = (1)$

{1}

picheck $(1 \ 1 \ 1 \ 1 \ 1 \ 1)$

$\pi = \left(\frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6}\right)$

$\pi 5 = (1 \ 1 \ 1 \ 1 \ 1 \ 1)$

$u5 = \left(\frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6}\right)$

picheck $(5 \ 5 \ 5 \ 5 \ 5 \ 5)$

$\pi 4 = (2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2)$

$u4 = \left(\frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18} \ \frac{1}{18}\right)$

picheck $(20\ \ 20\ \ 20\ \ 20\ \ 20\ \ 20)$

$\pi 3 = (6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6)$

$u3 = \left(\frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\right)$

picheck $(60\ \ 60\ \ 60\ \ 60\ \ 60\ \ 60)$

$\pi 2 = (24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24)$

$u2 = \left(\frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\right)$

picheck $(120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120)$

$\pi 1 = (120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120)$

$u1 = \left(\frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\ \ \frac{5}{324}\right)$

picheck $(120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120)$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks

$$\mathrm{T} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathrm{NM} = \begin{pmatrix} 5 & 4 & 4 & 4 & 4 & 4 \\ 4 & 5 & 4 & 4 & 4 & 4 \\ 4 & 4 & 5 & 4 & 4 & 4 \\ 4 & 4 & 4 & 5 & 4 & 4 \\ 4 & 4 & 4 & 4 & 5 & 4 \\ 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta}$ = [0, 0, 0, 0, 0, 0]

$\ker N_C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} t+s & t+s & t+s & t+s & t+s & t+s \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -t & -t & -t & -t & s-t \\ -s & -s & -s & -s & t-s \\ 0 & s & t & 0 & 0 \\ 0 & t & s & 0 & 0 \\ t & 0 & 0 & s & 0 \\ s & 0 & 0 & t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} t & 0 & 0 & 0 & 0 & s \\ s & 0 & 0 & 0 & 0 & t \\ 0 & s & t & 0 & 0 & 0 \\ 0 & t & s & 0 & 0 & 0 \\ 0 & 0 & 0 & t & s & 0 \\ 0 & 0 & 0 & s & t & 0 \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \ \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \ \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$CNM = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 1T + 24\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

"IS NM in Vec(K)?", true

$NM \begin{pmatrix} 5 & 4 & 4 & 4 & 9 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$

"IS MN in Vec(K)?", true

$MN \begin{pmatrix} 5 & 4 & 4 & 4 & 9 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$

$\boldsymbol{\tau} = 6/1$, rank = 6 , ratio = 1/1 , $n^2 / r = 6/1$

$\boldsymbol{\tau}' = 30/1$, $r' = 5/6$, $\boldsymbol{\tau} / n^2 = 1/6$

$p^2 = 1/6$, min $\boldsymbol{\tau} = 6/1$, $\boldsymbol{\tau}$–check is positive? 0/1

max r = 6/1 , r–check is positive? 0/1

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 6\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 48

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 1 vs no. of idems 1

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$M_C = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{5}{6} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&1&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&1&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&1&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&1&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.200000000, 1.200000000, 1.200000000, 1.200000000, 1.200000000]

NullSpace M_C

{[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0]}

NullSpace N_C

{[1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 6.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 1, 0, 0, 0, 0], [−1, 0, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

NullSpace N_0

{}

Eigenvalues M

[5., −1., −1., −1., −1., −1.]

Eigenvalues N

[5., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

==

{2, 3, 6}

R: [4, 4, 1, 6, 3, 2]
B: [5, 5, 6, 1, 2, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{512}(-7+s)(-1+s)(7+s^2)$

RANK of R is 5

R ranking is 3, "vs", 5

RBAR ranking 1, "vs", 3

RANK of B is 5

B ranking is 2, "vs", 5

BBAR ranking 1, "vs", 4

"R CYCLES", 1 + v[2] v[4] v[6]
"B CYCLES", (1 + v[3] v[6]) (1 + v[2] v[5])

Eigenvalues

R: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [1., −1., 1., −1., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0]}

NullSpace of R^*

{[−1, 1, 0, 0, 0, 0]}

NullSpace of B^*

{[1, −1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO−M} = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 3

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3   "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)

degree 2: $\frac{1}{12}$ ($v[1]v[3] + v[1]v[4] + v[1]v[5] + v[1]v[6] + v[2]v[3] + v[2]v[4] + v[2]v[5] + v[2]v[6] + v[3]v[4] + v[3]v[5] + v[4]v[6] + v[5]v[6]$)

degree 3 : $\frac{1}{8}$ ($v[3] + v[6]$) ($v[1] + v[2]$) ($v[4] + v[5]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{4, 5}, {1, 2}, {3, 6}}

"RG1" = {2, 5, 6}

"RG2" = {2, 4, 6}

"RG3" = {2, 3, 5}

"RG4" = {2, 3, 4}

"RG5" = {1, 5, 6}

"RG6" = {1, 4, 6}

"RG7" = {1, 3, 5}

"RG8" = {1, 3, 4}

$$\pi 3 = [0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0]$$

supp $\pi 3$ = {5, 6, 9, 10, 11, 12, 15, 16}

$$u3 = [0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0]$$

supp u3 = {5, 6, 9, 10, 11, 12, 15, 16}

Action of R on ranges, [[4], [2], [8], [6], [4], [2], [8], [6]]
Action of B on ranges, [[3], [7], [1], [5], [3], [7], [1], [5]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 1]

B-BLOCKS,

[2, 1, 3]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$b_1 = \{4, 5\}$
$b_2 = \{1, 2\}$
$b_3 = \{3, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 17, Shape: 3 ⊕ 14/12

$$\mathrm{CLB} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4, 6}}, true

Ω_B in Vec(K)? , {{3, 6}, {2, 5}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4, 5}, {1, 2}, {3, 6}}

1, "range", [2, 5, 6], [[6, 6, 5, 2, 2, 5], [6, 6, 2, 5, 5, 2], [5, 5, 6, 2, 2, 6], [5, 5, 2, 6, 6, 2], [2, 2, 6, 5, 5, 6], [2, 2, 5, 6, 6, 5]]

2, "range", [2, 4, 6], [[6, 6, 4, 2, 2, 4], [6, 6, 2, 4, 4, 2], [4, 4, 6, 2, 2, 6], [4, 4, 2, 6, 6, 2], [2, 2, 6, 4, 4, 6], [2, 2, 4, 6, 6, 4]]

3, "range", [2, 3, 5], [[5, 5, 3, 2, 2, 3], [5, 5, 2, 3, 3, 2], [3, 3, 5, 2, 2, 5], [3, 3, 2, 5, 5, 2], [2, 2, 5, 3, 3, 5], [2, 2, 3, 5, 5, 3]]

4, "range", [2, 3, 4], [[4, 4, 3, 2, 2, 3], [4, 4, 2, 3, 3, 2], [3, 3, 4, 2, 2, 4], [3, 3, 2, 4, 4, 2], [2, 2, 4, 3, 3, 4], [2, 2, 3, 4, 4, 3]]

5, "range", [1, 5, 6], [[6, 6, 5, 1, 1, 5], [6, 6, 1, 5, 5, 1], [5, 5, 6, 1, 1, 6], [5, 5, 1, 6, 6, 1], [1, 1, 6, 5, 5, 6], [1, 1, 5, 6, 6, 5]]

6, "range", [1, 4, 6], [[6, 6, 4, 1, 1, 4], [6, 6, 1, 4, 4, 1], [4, 4, 6, 1, 1, 6], [4, 4, 1, 6, 6, 1], [1, 1, 6, 4, 4, 6], [1, 1, 4, 6, 6, 4]]

7, "range", [1, 3, 5], [[5, 5, 3, 1, 1, 3], [5, 5, 1, 3, 3, 1], [3, 3, 5, 1, 1, 5], [3, 3, 1, 5, 5, 1], [1, 1, 5, 3, 3, 5], [1, 1, 3, 5, 5, 3]]

8, "range", [1, 3, 4], [[4, 4, 3, 1, 1, 3], [4, 4, 1, 3, 3, 1], [3, 3, 4, 1, 1, 4], [3, 3, 1, 4, 4, 1], [1, 1, 4, 3, 3, 4], [1, 1, 3, 4, 4, 3]] "group has", 6, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 3, 2]]

$g_2 =$ [[1, 3]]

$g_3 =$ [[1, 2]]

$g_4 =$ [[1, 2, 3]]

$g_5 =$ []

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & h[2] & 2h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

Z[1] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {[1, 3, 6], 7}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {[1, 5, 6], 10}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {[2, 5, 6], 16}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {[3, 5, 6], 19}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

{5, 6, 9, 10, 11, 12, 15, 16}

$u3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

{5, 6, 9, 10, 11, 12, 15, 16}

picheck $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 4 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 0 & 2 & 2 \end{pmatrix}$

$u2 = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$P_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 4 & 2 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 4 & 2 & 2 & 4 \\ 2 & 2 & 2 & 4 & 4 & 2 \\ 2 & 2 & 2 & 4 & 4 & 2 \\ 2 & 2 & 4 & 2 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 0, 0, 1, -1, 0]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} t & -t & -s & 0 & 0 & s \\ s & -s & -t & 0 & 0 & t \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & -1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s+t \\ 0 & s+t \\ -s+t & -s \\ -t+s & -t \\ -t+s & -t \\ -s+t & -s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s+t & 0 \\ 0 & s+t & 0 \\ t & 0 & s \\ s & 0 & t \\ s & 0 & t \\ t & 0 & s \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 0 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 & 1 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 4T + 12\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

"IS NM in Vec(K)?", true

$NM \begin{pmatrix} 4 & 2 & 2 & 6 & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 4 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 4 & 2 & 2 & 6 & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 4 \end{pmatrix}$

$$\boldsymbol{\tau} = 12/1 \text{ , rank } = 3 \text{ , ratio } = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 6/1$$

$$\text{max r} = 6/1 \text{ , } r\text{-check is positive? } 1/2$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 4 vs no. of idems 8

"PT1" = {{4, 5}, {1, 2}, {3, 6}}

"RG1" = {2, 5, 6}

"RG2" = {2, 4, 6}

"RG3" = {2, 3, 5}

"RG4" = {2, 3, 4}

"RG5" = {1, 5, 6}

"RG6" = {1, 4, 6}

"RG7" = {1, 3, 5}

"RG8" = {1, 3, 4}

$$M_c = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C

[1., 2., 2., 0., 0., 0.]

Eigenvalues M_C–scaled

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C–scaled

[1.200000000, 2.400000000, 2.400000000, 0., 0., 0.]

NullSpace M_C

{[1, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 1], [0, 0, 0, 1, 1, 0]}

NullSpace N_C

{[−1, 1, 0, 0, 0, 0], [0, 0, 0, −1, 1, 0], [0, 0, −1, 0, 0, 1]}

Eigenvalues M_0

[6., 0., 0., 2., 2., 2.]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[−1, −1, 1, 0, 0, 1], [−1, −1, 0, 1, 1, 0]}

NullSpace N_0

{[−1, 1, 0, 0, 0, 0], [0, 0, 0, −1, 1, 0], [0, 0, −1, 0, 0, 1]}

Eigenvalues M

[4., −2., −2., 0., 0., 0.]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{[0, 0, 0, −1, 1, 0], [0, 0, −1, 0, 0, 1], [−1, 1, 0, 0, 0, 0]}

NullSpace N

{[−1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 1], [0, 0, 0, −1, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

==

20, [1, −1, 1, −1, −1, 1]

==

{2, 4, 5}

R: [4, 4, 6, 1, 2, 3]
B: [5, 5, 1, 6, 3, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{512}(-1+s)(7+s^2)(-7+s)$

RANK of R is 5

R ranking is 2, "vs", 5

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[3] v[6]) (1 + v[1] v[4])
"B CYCLES", 1 + v[1] v[3] v[5]

Eigenvalues

R: [1., −1., 1., −1., 0., 0.]

B: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0]}

NullSpace of R^*

{[−1, 1, 0, 0, 0, 0]}

NullSpace of B^*

{[−1, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 3

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}(v[1]+v[2]+v[3]+v[4]+v[5]+v[6])$

degree 2: $\frac{1}{12}$ (v[1]v[3] + v[1]v[4] + v[1]v[5] + v[1]v[6] + v[2]v[3] + v[2]v[4] + v[2]v[5] + v[2]v[6] + v[3]v[
4] + v[3]v[5] + v[4]v[6] + v[5]v[6])
degree 3 : $\frac{1}{8}$ (v[3] + v[6]) (v[1] + v[2]) (v[4] + v[5])

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{4, 5}, {1, 2}, {3, 6}}

"RG1" = {2, 5, 6}

"RG2" = {2, 4, 6}

"RG3" = {2, 3, 5}

"RG4" = {2, 3, 4}

"RG5" = {1, 5, 6}

"RG6" = {1, 4, 6}

"RG7" = {1, 3, 5}

"RG8" = {1, 3, 4}

$$\pi 3 = [0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0]$$

supp π3 = {5, 6, 9, 10, 11, 12, 15, 16}

$$u3 = [0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0]$$

supp u3 = {5, 6, 9, 10, 11, 12, 15, 16}

Action of R on ranges, [[4], [8], [2], [6], [4], [8], [2], [6]]
Action of B on ranges, [[3], [1], [7], [5], [3], [1], [7], [5]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 1, 3]

B−BLOCKS,

[2, 3, 1]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

b_1 = {4, 5}
b_2 = {1, 2}
b_3 = {3, 6}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 17, Shape: 3 ⊕ 14/12

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4}, {3, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 5}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \left(\frac{1}{3}\ \ 0\ \ \frac{1}{6}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{6}\right)$ vs $\left(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\right)$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \left(\frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\right)$ vs $\left(\frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\right)$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4, 5}, {1, 2}, {3, 6}}

1, "range", [2, 5, 6], [[6, 6, 5, 2, 2, 5], [6, 6, 2, 5, 5, 2], [5, 5, 6, 2, 2, 6], [5, 5, 2, 6, 6, 2], [2, 2, 6, 5, 5, 6], [2, 2, 5, 6, 6, 5]]

2, "range", [2, 4, 6], [[6, 6, 4, 2, 2, 4], [6, 6, 2, 4, 4, 2], [4, 4, 6, 2, 2, 6], [4, 4, 2, 6, 6, 2], [2, 2, 6, 4, 4, 6], [2, 2, 4, 6, 6, 4]]

3, "range", [2, 3, 5], [[5, 5, 3, 2, 2, 3], [5, 5, 2, 3, 3, 2], [3, 3, 5, 2, 2, 5], [3, 3, 2, 5, 5, 2], [2, 2, 5, 3, 3, 5], [2, 2, 3, 5, 5, 3]]

4, "range", [2, 3, 4], [[4, 4, 3, 2, 2, 3], [4, 4, 2, 3, 3, 2], [3, 3, 4, 2, 2, 4], [3, 3, 2, 4, 4, 2], [2, 2, 4, 3, 3, 4], [2, 2, 3, 4, 4, 3]]

5, "range", [1, 5, 6], [[6, 6, 5, 1, 1, 5], [6, 6, 1, 5, 5, 1], [5, 5, 6, 1, 1, 6], [5, 5, 1, 6, 6, 1], [1, 1, 6, 5, 5, 6], [1, 1, 5, 6, 6, 5]]

6, "range", [1, 4, 6], [[6, 6, 4, 1, 1, 4], [6, 6, 1, 4, 4, 1], [4, 4, 6, 1, 1, 6], [4, 4, 1, 6, 6, 1], [1, 1, 6, 4, 4, 6], [1, 1, 4, 6, 6, 4]]

7, "range", [1, 3, 5], [[5, 5, 3, 1, 1, 3], [5, 5, 1, 3, 3, 1], [3, 3, 5, 1, 1, 5], [3, 3, 1, 5, 5, 1], [1, 1, 5, 3, 3, 5], [1, 1, 3, 5, 5, 3]]

8, "range", [1, 3, 4], [[4, 4, 3, 1, 1, 3], [4, 4, 1, 3, 3, 1], [3, 3, 4, 1, 1, 4], [3, 3, 1, 4, 4, 1], [1, 1, 4, 3, 3, 4], [1, 1, 3, 4, 4, 3]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 3, 2]]

$g_2 =$ [[1, 3]]

$g_3 =$ [[1, 2]]

$g_4 =$ [[1, 2, 3]]

$g_5 =$ []

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\left(h[2]\ \ 0\ \ 0\ \ h[2]\ \ 2\,h[1]\right)$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {[1, 3, 6], 7}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {[1, 5, 6], 10}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {[2, 5, 6], 16}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {[3, 5, 6], 19}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

{5, 6, 9, 10, 11, 12, 15, 16}

$u3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

{5, 6, 9, 10, 11, 12, 15, 16}

picheck $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 4 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 0 & 2 & 2 \end{pmatrix}$

$u2 = \left(0 \ \ \tfrac{1}{3} \ \ \tfrac{1}{3} \ \ \tfrac{1}{3} \ \ \tfrac{1}{3} \ \ \tfrac{1}{3} \ \ \tfrac{1}{3} \ \ \tfrac{1}{3} \ \ \tfrac{1}{3} \ \ \tfrac{1}{3} \ \ \tfrac{1}{3} \ \ 0 \ \ 0 \ \ \tfrac{1}{3} \ \ \tfrac{1}{3}\right)$

picheck $(8 \ \ 8 \ \ 8 \ \ 8 \ \ 8 \ \ 8)$

$\pi 1 = (8 \ \ 8 \ \ 8 \ \ 8 \ \ 8 \ \ 8)$

$u1 = \left(\tfrac{2}{9} \ \ \tfrac{2}{9} \ \ \tfrac{2}{9} \ \ \tfrac{2}{9} \ \ \tfrac{2}{9} \ \ \tfrac{2}{9}\right)$

picheck $(8 \ \ 8 \ \ 8 \ \ 8 \ \ 8 \ \ 8)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem–checks} \quad \text{N0–checks}$$

$$P_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem–checks} \quad \text{N0–checks}$$

$$P_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem–checks} \quad \text{N0–checks}$$

$$P_4 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem–checks} \quad \text{N0–checks}$$

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 4 & 2 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 4 & 2 & 2 & 4 \\ 2 & 2 & 2 & 4 & 4 & 2 \\ 2 & 2 & 2 & 4 & 4 & 2 \\ 2 & 2 & 4 & 2 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 0, 0, 1, -1, 0]$

$$\ker N_C = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ s & -s & -t & 0 & 0 & t \\ t & -t & -s & 0 & 0 & s \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & -1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s+t \\ 0 & s+t \\ -t+s & -t \\ -s+t & -s \\ -s+t & -s \\ -t+s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & s+t \\ 0 & 0 & s+t \\ t & s & 0 \\ s & t & 0 \\ s & t & 0 \\ t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \; RR^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 0 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 & 1 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 4T + 12\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

"IS NM in Vec(K)?", true

$\text{NM} \begin{pmatrix} 4 & 2 & 2 & 6 & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 4 \end{pmatrix}$

"IS MN in Vec(K)?", true

$\text{MN} \begin{pmatrix} 4 & 2 & 2 & 6 & 2 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 4 \end{pmatrix}$

$$\boldsymbol{\tau} = 12/1 \text{ , rank} = 3 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 6/1$$

$$\text{max r} = 6/1 \text{ , } r\text{–check is positive? } 1/2$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 4 vs no. of idems 8

"PT1" = {{4, 5}, {1, 2}, {3, 6}}

"RG1" = {2, 5, 6}

"RG2" = {2, 4, 6}

"RG3" = {2, 3, 5}

"RG4" = {2, 3, 4}

"RG5" = {1, 5, 6}

"RG6" = {1, 4, 6}

"RG7" = {1, 3, 5}

"RG8" = {1, 3, 4}

$$M_C = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C

[1., 2., 2., 0., 0., 0.]

Eigenvalues M_C−scaled

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C−scaled

[1.200000000, 2.400000000, 2.400000000, 0., 0., 0.]

NullSpace M_C

{[0, 0, 1, 0, 0, 1], [1, 1, 0, 0, 0, 0], [0, 0, 0, 1, 1, 0]}

NullSpace N_C

{[−1, 1, 0, 0, 0, 0], [0, 0, 0, −1, 1, 0], [0, 0, −1, 0, 0, 1]}

Eigenvalues M_0

[6., 0., 0., 2., 2., 2.]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[−1, −1, 0, 1, 1, 0], [−1, −1, 1, 0, 0, 1]}

NullSpace N_0

{[0, 0, −1, 0, 0, 1], [0, 0, 0, −1, 1, 0], [−1, 1, 0, 0, 0, 0]}

Eigenvalues M

[4., −2., −2., 0., 0., 0.]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{[0, 0, 0, 1, −1, 0], [0, 0, 1, 0, 0, −1], [1, −1, 0, 0, 0, 0]}

NullSpace N

{[0, 0, 0, 1, −1, 0], [1, −1, 0, 0, 0, 0], [0, 0, 1, 0, 0, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

==

{2, 5, 6}

R: [4, 4, 6, 6, 2, 2]
B: [5, 5, 1, 1, 3, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 3

Level 2 det = $\frac{-1}{512}(-7+s)^2(1+s)^3(-1+s)$

RANK of R is 3

R ranking is 1, "vs", 3

RBAR ranking 1, "vs", 3

RANK of B is 3

B ranking is 1, "vs", 3

BBAR ranking 1, "vs", 3

"R CYCLES", 1 + v[2] v[4] v[6]

"B CYCLES", 1 + v[1] v[3] v[5]

Eigenvalues

R: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0]}

NullSpace of R^*

{[0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 1, −1], [1, −1, 0, 0, 0, 0]}

NullSpace of B^*

{[−1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, −1], [0, 0, −1, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 \\ 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 \\ 2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{6}$ ($v[1]v[3] + v[1]v[5] + v[2]v[4] + v[2]v[6] + v[3]v[5] + v[4]v[6]$)
degree 3 : $\frac{1}{2}$ ($v[1]v[3]v[5] + v[2]v[4]v[6]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 2}, {5, 6}, {3, 4}}

"RG1" = {2, 4, 6}

"RG2" = {1, 3, 5}

$$\pi 3 = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]$$

supp $\pi 3$ = {6, 15}

$$u3 = [0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0]$$

supp u3 = {6, 7, 8, 9, 12, 13, 14, 15}

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 1]

B-BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

b_1 = {1, 2}
b_2 = {5, 6}
b_3 = {3, 4}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & h[3] & 0 & h[2] & 0 \\ 0 & h[1] & 0 & h[3] & 0 & h[2] \\ h[2] & 0 & h[1] & 0 & h[3] & 0 \\ 0 & h[2] & 0 & h[1] & 0 & h[3] \\ h[3] & 0 & h[2] & 0 & h[1] & 0 \\ 0 & h[3] & 0 & h[2] & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 5, Shape: $0 \oplus 5/3$

$$\text{CLB} = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 5}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2}, {5, 6}, {3, 4}}

1, "range", [2, 4, 6], [[6, 6, 2, 2, 4, 4], [4, 4, 6, 6, 2, 2], [2, 2, 4, 4, 6, 6]]

2, "range", [1, 3, 5], [[5, 5, 1, 1, 3, 3], [3, 3, 5, 5, 1, 1], [1, 1, 3, 3, 5, 5]]

"group has", 3, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

g_1 = [[1, 3, 2]]

g_2 = [[1, 2, 3]]

g_3 = []

linear dimension, 3

"Symmetric?", false

Is Z in Vec(K)? true
$\begin{pmatrix} h[3] & h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

3, "coeff", 1

Z[3] = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix}1 & 1 & 1\\0 & 0 & 3\\0 & 0 & 3\\1 & 1 & 1\end{pmatrix}$$

Group spectrum: $1+t+t^2+t^3$

Molien Series to order 10: $1+t+2t^2+4t^3+5t^4+7t^5+10t^6+12t^7+15t^8+19t^9+22t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {[1, 3, 6], 7}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {[1, 5, 6], 10}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {[2, 5, 6], 16}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {[3, 5, 6], 19}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix}0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$

{6, 15}

$u3 = \begin{pmatrix}0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$

{6, 7, 8, 9, 12, 13, 14, 15}

picheck $\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

$\pi = \begin{pmatrix}\frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6}\end{pmatrix}$

$\pi 2 = \begin{pmatrix}0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0\end{pmatrix}$

$u2 = \begin{pmatrix}0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0\end{pmatrix}$

picheck $\begin{pmatrix}2 & 2 & 2 & 2 & 2 & 2\end{pmatrix}$

$\pi 1 = \begin{pmatrix}2 & 2 & 2 & 2 & 2 & 2\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9}\end{pmatrix}$

picheck $\begin{pmatrix}2 & 2 & 2 & 2 & 2 & 2\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$ idem–checks

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 4 & 2 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 4 \\ 2 & 2 & 2 & 2 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, -1, 1, -1, 1]$

$$\ker N_c = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $(1 \;\; 1 \;\; 1)$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -t & -s & -t & -s \\ -t & -s & -t & -s \\ 0 & 0 & t & s \\ 0 & 0 & t & s \\ t & s & 0 & 0 \\ t & s & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & 0 & t \\ s & 0 & 0 & 0 & t \\ t & t+s & -t & t & -t \\ t & t+s & -t & t & -t \\ 0 & 0 & t & s & 0 \\ 0 & 0 & t & s & 0 \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = (2 \;\; 2 \;\; 0 \;\; 2 \;\; 0)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 3, "vs", 3

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 12\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

"IS NM in Vec(K)?", true

NM $\begin{pmatrix} 2 & 2 & 2 & 2 & 4 & 4 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 2 & 2 & 2 & 2 & 4 & 4 \end{pmatrix}$

$$\boldsymbol{\tau} = 12/1 \text{ , rank } = 3 \text{ , ratio } = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 6/1$$

$$\text{max r} = 6/1 \text{ , } r\text{–check is positive? } 1/2$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 3

KERNEL HAS LINEAR DIMENSION 6
out of total no. of elements equal to 6

dim span idems 2 vs no. of idems 2

"PT1" = {{1, 2}, {5, 6}, {3, 4}}

"RG1" = {2, 4, 6}

"RG2" = {1, 3, 5}

$$M_C = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 6.]

Eigenvalues N_C

[1., 2., 2., 0., 0., 0.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 6.]

Eigenvalues N_C–scaled

[1.200000000, 2.400000000, 2.400000000, 0., 0., 0.]

NullSpace M_C

{[−1, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 1], [1, 0, 0, 1, 0, 0], [1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0]}

NullSpace N_C

{[0, 0, 0, 0, 1, −1], [0, 0, 1, −1, 0, 0], [−1, 1, 0, 0, 0, 0]}

Eigenvalues M_0

[6., 6., 0., 0., 0., 0.]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[0, 0, −1, 0, 1, 0], [0, 0, 0, −1, 0, 1], [0, 1, 0, −1, 0, 0], [1, 0, −1, 0, 0, 0]}

NullSpace N_0

{[0, 0, 0, 0, −1, 1], [0, 0, −1, 1, 0, 0], [−1, 1, 0, 0, 0, 0]}

Eigenvalues M

[4., 4., −2., −2., −2., −2.]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[0, 0, −1, 1, 0, 0], [0, 0, 0, 0, −1, 1], [−1, 1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

===

{3, 4, 5}

R: [4, 5, 1, 1, 2, 3]
B: [5, 4, 6, 6, 3, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{512}(-1+s)(-7+s)(7+s^2)$

RANK of R is 5

R ranking is 2, "vs", 5

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[1] v[4]) (1 + v[2] v[5])
"B CYCLES", 1 + v[2] v[4] v[6]

Eigenvalues

R: [1., -1., 1., -1., 0., 0.]

B: [0., 0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 0, 1]}

NullSpace of B

{[1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, -1, 1, 0, 0]}

NullSpace of B^*

{[0, 0, -1, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 3

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}$ (v[1] + v[2] + v[3] + v[4] + v[5] + v[6])
degree 2: $\frac{1}{12}$ (v[1]v[2] + v[1]v[3] + v[1]v[4] + v[1]v[5] + v[2]v[3] + v[2]v[4] + v[2]v[6] + v[3]v[5] + v[3]v[
6] + v[4]v[5] + v[4]v[6] + v[5]v[6])
degree 3 : $\frac{1}{8}$ (v[3] + v[4]) (v[1] + v[6]) (v[2] + v[5])

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {3, 4}}

"RG1" = {4, 5, 6}

"RG2" = {3, 5, 6}

"RG3" = {2, 4, 6}

"RG4" = {2, 3, 6}

"RG5" = {1, 4, 5}

"RG6" = {1, 3, 5}

"RG7" = {1, 2, 4}

"RG8" = {1, 2, 3}

$$\pi 3 = [1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1]$$

supp π3 = {1, 2, 6, 8, 13, 15, 19, 20}

$$u3 = [1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1]$$

supp u3 = {1, 2, 6, 8, 13, 15, 19, 20}

Action of R on ranges, [[8], [8], [6], [6], [7], [7], [5], [5]]
Action of B on ranges, [[4], [4], [3], [3], [2], [2], [1], [1]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[3, 2, 1]

B–BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1 = \{1, 6\}$
$b_2 = \{2, 5\}$
$b_3 = \{3, 4\}$

dim(span of partition vectors), rank(N_0), rank(N):   3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 17, Shape: 3 ⊕ 14/12

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4}, {2, 5}}, true

Ω_B in Vec(K)? , {{2, 4, 6}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {3, 4}}

1, "range", [4, 5, 6], [[6, 5, 4, 4, 5, 6], [6, 4, 5, 5, 4, 6], [5, 6, 4, 4, 6, 5], [5, 4, 6, 6, 4, 5], [4, 6, 5, 5, 6, 4], [4, 5, 6, 6, 5, 4]]

2, "range", [3, 5, 6], [[6, 5, 3, 3, 5, 6], [6, 3, 5, 5, 3, 6], [5, 6, 3, 3, 6, 5], [5, 3, 6, 6, 3, 5], [3, 6, 5, 5, 6, 3], [3, 5, 6, 6, 5, 3]]

3, "range", [2, 4, 6], [[6, 4, 2, 2, 4, 6], [6, 2, 4, 4, 2, 6], [4, 6, 2, 2, 6, 4], [4, 2, 6, 6, 2, 4], [2, 6, 4, 4, 6, 2], [2, 4, 6, 6, 4, 2]]

4, "range", [2, 3, 6], [[6, 3, 2, 2, 3, 6], [6, 2, 3, 3, 2, 6], [3, 6, 2, 2, 6, 3], [3, 2, 6, 6, 2, 3], [2, 6, 3, 3, 6, 2], [2, 3, 6, 6, 3, 2]]

5, "range", [1, 4, 5], [[5, 4, 1, 1, 4, 5], [5, 1, 4, 4, 1, 5], [4, 5, 1, 1, 5, 4], [4, 1, 5, 5, 1, 4], [1, 5, 4, 4, 5, 1], [1, 4, 5, 5, 4, 1]]

6, "range", [1, 3, 5], [[5, 3, 1, 1, 3, 5], [5, 1, 3, 3, 1, 5], [3, 5, 1, 1, 5, 3], [3, 1, 5, 5, 1, 3], [1, 5, 3, 3, 5, 1], [1, 3, 5, 5, 3, 1]]

7, "range", [1, 2, 4], [[4, 2, 1, 1, 2, 4], [4, 1, 2, 2, 1, 4], [2, 4, 1, 1, 4, 2], [2, 1, 4, 4, 1, 2], [1, 4, 2, 2, 4, 1], [1, 2, 4, 4, 2, 1]]

8, "range", [1, 2, 3], [[3, 2, 1, 1, 2, 3], [3, 1, 2, 2, 1, 3], [2, 3, 1, 1, 3, 2], [2, 1, 3, 3, 1, 2], [1, 3, 2, 2, 3, 1], [1, 2, 3, 3, 2, 1]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

$g_3 =$ [[2, 3]]

$g_4 =$ [[1, 3, 2]]

$g_5 =$ [[1, 2, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} 2h[1] & 0 & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {[1, 3, 6], 7}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {[1, 5, 6], 10}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {[2, 5, 6], 16}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {[3, 5, 6], 19}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

{1, 2, 6, 8, 13, 15, 19, 20}

$u3 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

{1, 2, 6, 8, 13, 15, 19, 20}

picheck $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 4 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 2 & 2 & 2 & 2 & 0 & 2 & 2 & 0 & 2 & 0 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 2 & 2 & 2 & 2 & 4 \\ 2 & 4 & 2 & 2 & 4 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 4 & 2 & 2 & 4 & 2 \\ 4 & 2 & 2 & 2 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 0, 0, 0, 0, -1]$

$$\ker N_c = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & s & -s & -t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -s & -t & t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 0 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \\ -1 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -s & -s+t \\ -t & -t+s \\ s+t & 0 \\ s+t & 0 \\ -t & -t+s \\ -s & -s+t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} s & 0 & t \\ t & 0 & s \\ 0 & s+t & 0 \\ 0 & s+t & 0 \\ t & 0 & s \\ s & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 3

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 1 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 12\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$

"IS NM in Vec(K)?", true

$\mathrm{NM} \begin{pmatrix} 4 & 2 & 2 & 8 & 2 & 2 & 4 & 2 & 4 & 2 & 2 & 2 & 2 & 4 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 4 & 2 & 2 & 8 & 2 & 2 & 4 & 2 & 4 & 2 & 2 & 2 & 2 & 4 \end{pmatrix}$

$$\boldsymbol{\tau} = 12/1 \text{ , rank } = 3 \text{ , ratio } = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 6/1$$

$$\text{max r} = 6/1 \text{ , } r\text{-check is positive? } 1/2$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 4 vs no. of idems 8

"PT1" = {{1, 6}, {2, 5}, {3, 4}}

"RG1" = {4, 5, 6}

"RG2" = {3, 5, 6}

"RG3" = {2, 4, 6}

"RG4" = {2, 3, 6}

"RG5" = {1, 4, 5}

"RG6" = {1, 3, 5}

"RG7" = {1, 2, 4}

"RG8" = {1, 2, 3}

$$M_c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 \\ \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} \\ 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C

[1., 2., 2., 0., 0., 0.]

Eigenvalues M_C–scaled

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C–scaled

[1.200000000, 2.400000000, 2.400000000, 0., 0., 0.]

NullSpace M_C

{[0, 0, 1, 1, 0, 0], [1, 0, 0, 0, 0, 1], [0, 1, 0, 0, 1, 0]}

NullSpace N_C

{[0, 0, 1, −1, 0, 0], [0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[6., 0., 0., 2., 2., 2.]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[0, 1, −1, −1, 1, 0], [1, 0, −1, −1, 0, 1]}

NullSpace N_0

{[−1, 0, 0, 0, 0, 1], [0, 0, −1, 1, 0, 0], [0, −1, 0, 0, 1, 0]}

Eigenvalues M

[4., −2., −2., 0., 0., 0.]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{[0, 0, 1, −1, 0, 0], [0, 1, 0, 0, −1, 0], [1, 0, 0, 0, 0, −1]}

NullSpace N

{[0, 0, 1, −1, 0, 0], [1, 0, 0, 0, 0, −1], [0, 1, 0, 0, −1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{4, 5, 6}

R: [4, 5, 6, 1, 2, 2]
B: [5, 4, 1, 6, 3, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{1}{512}(-7+s)(-1+s)(7+s^2)$

RANK of R is 5

R ranking is 2, "vs", 5

RBAR ranking 1, "vs", 4

RANK of B is 5

B ranking is 3, "vs", 5

BBAR ranking 1, "vs", 3

"R CYCLES", (1 + v[1] v[4]) (1 + v[2] v[5])
"B CYCLES", 1 + v[1] v[3] v[5]

Eigenvalues

R: [1., −1., 1., −1., 0., 0.]

B: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 1, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, −1, 1]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 3

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}\,(\, v[1] + v[2] + v[3] + v[4] + v[5] + v[6]\,)$
degree 2: $\frac{1}{12}\,(\, v[1]v[2] + v[1]v[3] + v[1]v[5] + v[1]v[6] + v[2]v[4] + v[2]v[5] + v[2]v[6] + v[3]v[4] + v[3]v[5] + v[3]v[6] + v[4]v[5] + v[4]v[6]\,)$
degree 3 : $\frac{1}{8}\,(\, v[1] + v[4]\,)\,(\, v[2] + v[3]\,)\,(\, v[5] + v[6]\,)$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 4}, {2, 3}, {5, 6}}

"RG1" = {3, 4, 6}

"RG2" = {3, 4, 5}

"RG3" = {2, 4, 6}

"RG4" = {2, 4, 5}

"RG5" = {1, 3, 6}

"RG6" = {1, 3, 5}

"RG7" = {1, 2, 6}

"RG8" = {1, 2, 5}

$$\pi 3 = [0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]$$

supp $\pi 3$ = {3, 4, 6, 7, 14, 15, 17, 18}

$$u3 = [0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]$$

supp u3 = {3, 4, 6, 7, 14, 15, 17, 18}

Action of R on ranges, [[7], [7], [8], [8], [3], [3], [4], [4]]
Action of B on ranges, [[5], [5], [1], [1], [6], [6], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 3, 2]

B-BLOCKS,

[2, 3, 1]

with invariant measure, [1, 1, 1]

N by blocks, N − check: true

$b_1 = \{1, 4\}$
$b_2 = \{2, 3\}$
$b_3 = \{5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 17, Shape: 3 ⊕ 14/12

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4}, {2, 5}}, true

Ω_B in Vec(K)? , {{1, 3, 5}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 4}, {2, 3}, {5, 6}}

1, "range", [3, 4, 6], [[6, 4, 4, 6, 3, 3], [6, 3, 3, 6, 4, 4], [4, 6, 6, 4, 3, 3], [4, 3, 3, 4, 6, 6], [3, 6, 6, 3, 4, 4], [3, 4, 4, 3, 6, 6]]

2, "range", [3, 4, 5], [[5, 4, 4, 5, 3, 3], [5, 3, 3, 5, 4, 4], [4, 5, 5, 4, 3, 3], [4, 3, 3, 4, 5, 5], [3, 5, 5, 3, 4, 4], [3, 4, 4, 3, 5, 5]]

3, "range", [2, 4, 6], [[6, 4, 4, 6, 2, 2], [6, 2, 2, 6, 4, 4], [4, 6, 6, 4, 2, 2], [4, 2, 2, 4, 6, 6], [2, 6, 6, 2, 4, 4], [2, 4, 4, 2, 6, 6]]

4, "range", [2, 4, 5], [[5, 4, 4, 5, 2, 2], [5, 2, 2, 5, 4, 4], [4, 5, 5, 4, 2, 2], [4, 2, 2, 4, 5, 5], [2, 5, 5, 2, 4, 4], [2, 4, 4, 2, 5, 5]]

5, "range", [1, 3, 6], [[6, 3, 3, 6, 1, 1], [6, 1, 1, 6, 3, 3], [3, 6, 6, 3, 1, 1], [3, 1, 1, 3, 6, 6], [1, 6, 6, 1, 3, 3], [1, 3, 3, 1, 6, 6]]

6, "range", [1, 3, 5], [[5, 3, 3, 5, 1, 1], [5, 1, 1, 5, 3, 3], [3, 5, 5, 3, 1, 1], [3, 1, 1, 3, 5, 5], [1, 5, 5, 1, 3, 3], [1, 3, 3, 1, 5, 5]]

7, "range", [1, 2, 6], [[6, 2, 2, 6, 1, 1], [6, 1, 1, 6, 2, 2], [2, 6, 6, 2, 1, 1], [2, 1, 1, 2, 6, 6], [1, 6, 6, 1, 2, 2], [1, 2, 2, 1, 6, 6]]

8, "range", [1, 2, 5], [[5, 2, 2, 5, 1, 1], [5, 1, 1, 5, 2, 2], [2, 5, 5, 2, 1, 1], [2, 1, 1, 2, 5, 5], [1, 5, 5, 1, 2, 2], [1, 2, 2, 1, 5, 5]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 3]]

$g_2 =$ [[2, 3]]

$g_3 =$ [[1, 3]]

$g_4 =$ []

$g_5 =$ [[1, 3, 2]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 0 & 0 & 2h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n−choose−rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {[1, 3, 6], 7}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {[1, 5, 6], 10}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {[2, 5, 6], 16}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {[3, 5, 6], 19}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$$\pi 3 = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

{3, 4, 6, 7, 14, 15, 17, 18}

$$u3 = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

{3, 4, 6, 7, 14, 15, 17, 18}

picheck $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 4 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 2 = \begin{pmatrix} 2 & 2 & 0 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 \end{pmatrix}$$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$

picheck $\begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ idem-checks N0-checks

$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 2 & 2 & 4 & 2 & 2 \\ 2 & 4 & 4 & 2 & 2 & 2 \\ 2 & 4 & 4 & 2 & 2 & 2 \\ 4 & 2 & 2 & 4 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 4 \\ 2 & 2 & 2 & 2 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 1, -1, 0, 0, 0]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ s & 0 & 0 & -s & -t & t \\ t & 0 & 0 & -t & -s & s \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 0 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \\ -1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} t & s \\ s & t \\ s & t \\ t & s \\ -s-t & -s-t \\ -s-t & -s-t \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t & s & 0 \\ s & t & 0 \\ s & t & 0 \\ t & s & 0 \\ 0 & 0 & s+t \\ 0 & 0 & s+t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 2 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 1 & 1 & 0 & 1 & 1 \\ 1 & 2 & 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 0 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 4T + 12\Omega$$

$$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$T \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

"IS NM in Vec(K)?", true

$$NM \begin{pmatrix} 4 & 2 & 2 & 2 & 8 & 4 & 4 & 2 & 2 & 2 & 4 & 2 & 2 & 4 \end{pmatrix}$$

"IS MN in Vec(K)?", true

$$MN \begin{pmatrix} 4 & 2 & 2 & 2 & 8 & 4 & 4 & 2 & 2 & 2 & 4 & 2 & 2 & 4 \end{pmatrix}$$

$\boldsymbol{\tau} = 12/1$, rank = 3 , ratio = 4/1 , $n^2 / r = 12/1$

$\boldsymbol{\tau}' = 24/1$, $r' = 2/3$, $\boldsymbol{\tau} / n^2 = 1/3$

$p^2 = 1/6$, min $\boldsymbol{\tau} = 6/1$, $\boldsymbol{\tau}$–check is positive? 6/1

max r = 6/1 , r–check is positive? 1/2

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 1, partitions and, 8, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 48

dim span idems 4 vs no. of idems 8

"PT1" = {{1, 4}, {2, 3}, {5, 6}}

"RG1" = {3, 4, 6}

"RG2" = {3, 4, 5}

"RG3" = {2, 4, 6}

"RG4" = {2, 4, 5}

"RG5" = {1, 3, 6}

"RG6" = {1, 3, 5}

"RG7" = {1, 2, 6}

"RG8" = {1, 2, 5}

$$M_C = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C

[1., 2., 2., 0., 0., 0.]

Eigenvalues M_C−scaled

[2., 2., 2., 0., 0., 0.]

Eigenvalues N_C−scaled

[1.200000000, 2.400000000, 2.400000000, 0., 0., 0.]

NullSpace M_C

{[0, 1, 1, 0, 0, 0], [0, 0, 0, 0, 1, 1], [1, 0, 0, 1, 0, 0]}

NullSpace N_C

{[−1, 0, 0, 1, 0, 0], [0, 0, 0, 0, −1, 1], [0, −1, 1, 0, 0, 0]}

Eigenvalues M_0

[6., 0., 0., 2., 2., 2.]

Eigenvalues N_0

[2., 2., 2., 0., 0., 0.]

NullSpace M_0

{[1, 0, 0, 1, −1, −1], [0, 1, 1, 0, −1, −1]}

NullSpace N_0

{[0, 0, 0, 0, 1, −1], [−1, 0, 0, 1, 0, 0], [0, 1, −1, 0, 0, 0]}

Eigenvalues M

[4., −2., −2., 0., 0., 0.]

Eigenvalues N

[4., −2., −2., 0., 0., 0.]

NullSpace M

{[1, 0, 0, −1, 0, 0], [0, 0, 0, 0, 1, −1], [0, 1, −1, 0, 0, 0]}

NullSpace N

{[1, 0, 0, −1, 0, 0], [0, 0, 0, 0, 1, −1], [0, 1, −1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

==

{2, 3, 4, 5}

R: [4, 4, 1, 1, 2, 3]
B: [5, 5, 6, 6, 3, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{512} (-1 + s) (1 + s) (7 + 2 s + s^2)^2$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[1] v[4]

"B CYCLES", 1 + v[2] v[3] v[5] v[6]

Eigenvalues

R: [0., 0., 0., 0., 1., −1.]

B: [0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 1, −1, 0, 0], [1, −1, 0, 0, 0, 0]}

NullSpace of B^*

{[0, 0, 1, −1, 0, 0], [1, −1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 1 & \frac{2}{3} & \frac{1}{3} \\ 1 & 1 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 1 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 1 \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)

degree 2: $\frac{1}{3}$ ($v[1]v[4] + v[2]v[3] + v[5]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 6}, {3, 4, 5}}

"PT2" = {{1, 2, 5}, {3, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 3}

"RG3" = {1, 4}

$$\pi 2 = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1]$$

supp $\pi 2$ = {3, 6, 15}

$$u2 = [0, 3, 3, 2, 1, 3, 3, 2, 1, 0, 1, 2, 1, 2, 3]$$

supp u2 = {2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15}

Action of R on ranges, [[2], [3], [3]]
Action of B on ranges, [[2], [1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 1, 2, 1]

B–BLOCKS,

[4, 3, 1, 2]

with invariant measure, [2, 2, 1, 1]

N by blocks, N – check: true

b_1 = {1, 2, 6}
b_2 = {3, 4, 5}
b_3 = {1, 2, 5}
b_4 = {3, 4, 6}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[2] & h[1] & 0 & 0 & 0 \\ 0 & h[1] & h[2] & 0 & 0 & 0 \\ h[1] & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: 3 ⊕ 7/5

$$CLB = \begin{pmatrix} 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4}}, true

Ω_B in Vec(K)? , {{2, 3, 5, 6}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 6}, {3, 4, 5}}

1, "range", [5, 6], [[6, 6, 5, 5, 5, 6], [5, 5, 6, 6, 6, 5]]

2, "range", [2, 3], [[3, 3, 2, 2, 2, 3], [2, 2, 3, 3, 3, 2]]

3, "range", [1, 4], [[4, 4, 1, 1, 1, 4], [1, 1, 4, 4, 4, 1]]

2, "partition", {{1, 2, 5}, {3, 4, 6}}

1, "range", [5, 6], [[6, 6, 5, 5, 6, 5], [5, 5, 6, 6, 5, 6]]

2, "range", [2, 3], [[3, 3, 2, 2, 3, 2], [2, 2, 3, 3, 2, 3]]

3, "range", [1, 4], [[4, 4, 1, 1, 4, 1], [1, 1, 4, 4, 1, 4]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

{3, 6, 15}

$u2 = \begin{pmatrix} 0 & 3 & 3 & 2 & 1 & 3 & 3 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 3 \end{pmatrix}$

{2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$ idem−checks

$$PP_2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$ idem−checks

$$T = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{9} & \frac{2}{9} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{9} & \frac{1}{9} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{1}{3} & 0 \\ \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 3 & 3 & 0 & 0 & 1 & 2 \\ 3 & 3 & 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 3 & 2 & 1 \\ 0 & 0 & 3 & 3 & 2 & 1 \\ 1 & 1 & 2 & 2 & 3 & 0 \\ 2 & 2 & 1 & 1 & 0 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 0, 0, 1, -1, -1]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -s & s+t & s+t & -s & -t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} t & 0 & -s \\ t & 0 & -s \\ -t & 0 & s \\ -t & 0 & s \\ 0 & s-t & 0 \\ 0 & t-s & 0 \end{pmatrix}$$ RB checks

$$\ker M_C = \begin{pmatrix} -1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{pmatrix} \quad \begin{pmatrix} s & 0 & t & 0 \\ s & 0 & t & 0 \\ -s & s+t & -t & s+t \\ -s & s+t & -t & s+t \\ 0 & s & 0 & t \\ 0 & t & 0 & s \end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 3 & 0 & 3 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 1 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\ Omega} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 3&0&0&3&0&0\\0&3&3&0&0&0\\0&3&3&0&0&0\\3&0&0&3&0&0\\0&0&0&0&3&3\\0&0&0&0&3&3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&1&0&0&\frac{1}{3}&\frac{2}{3}\\ 1&1&0&0&\frac{1}{3}&\frac{2}{3}\\ 0&0&1&1&\frac{2}{3}&\frac{1}{3}\\ 0&0&1&1&\frac{2}{3}&\frac{1}{3}\\ \frac{1}{3}&\frac{1}{3}&\frac{2}{3}&\frac{2}{3}&1&0\\ \frac{2}{3}&\frac{2}{3}&\frac{1}{3}&\frac{1}{3}&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 9T + 0\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} \frac{1}{3} & \frac{1}{9} & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

"IS NM in Vec(K)?", true

$\mathrm{NM} \begin{pmatrix} 3 & 1 & 1 & 2 & 1 & 0 & 0 & 3 & 3 \end{pmatrix}$

"IS MN in Vec(K)?", true

$\mathrm{MN} \begin{pmatrix} 3 & 1 & 1 & 2 & 1 & 0 & 0 & 3 & 3 \end{pmatrix}$

$$\boldsymbol{\tau} = 18/1 \text{ , rank} = 2 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 12/1$$

$$\text{max r} = 6/1 \text{ , } r\text{–check is positive? } 2/3$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 3, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 9
out of total no. of elements equal to 12

dim span idems 6 vs no. of idems 6

"PT1" = {{1, 2, 6}, {3, 4, 5}}

"PT2" = {{1, 2, 5}, {3, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 3}

"RG3" = {1, 4}

$$M_C = \begin{pmatrix} 2 & -1 & -1 & 2 & -1 & -1 \\ -1 & 2 & 2 & -1 & -1 & -1 \\ -1 & 2 & 2 & -1 & -1 & -1 \\ 2 & -1 & -1 & 2 & -1 & -1 \\ -1 & -1 & -1 & -1 & 2 & 2 \\ -1 & -1 & -1 & -1 & 2 & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} \\ \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{2} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & \frac{1}{2} & \frac{5}{6} & \frac{-1}{6} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ 1 & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & 1 \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{1}{5} & \frac{3}{5} \\ 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{1}{5} & \frac{3}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{3}{5} & \frac{1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{3}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{3}{5} & \frac{3}{5} & 1 & \frac{-1}{5} \\ \frac{3}{5} & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} & \frac{-1}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[6., 6., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 2., 2.187184271, 0.8128157289]

Eigenvalues M_C–scaled

[3., 3., 0., 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 0., 0., 2.400000000, 2.624621125, 0.9753788748]

NullSpace M_C

{[1, 1, 0, 0, 1, 0], [0, −1, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0], [1, 1, 0, 0, 0, 1]}

NullSpace N_C

{[−1, 0, −1, 0, 1, 1], [−1, 1, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0]}

Eigenvalues M_0

[6., 6., 6., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[0, −1, 1, 0, 0, 0], [0, 0, 0, 0, −1, 1], [−1, 0, 0, 1, 0, 0]}

NullSpace N_0

{[−1, 1, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0], [−1, 0, −1, 0, 1, 1]}

Eigenvalues M

[3., −3., 3., −3., 3., −3.]

Eigenvalues N

[0., 0., 0., 3., −0.8128157289, −2.187184271]

NullSpace M

{}

NullSpace N

{[0, 1, 0, 1, −1, −1], [0, 1, 1, 0, −1, −1], [1, −1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 3 & 3 & 2 & 1 \\ 0 & 0 & 3 & 3 & 2 & 1 \\ 3 & 3 & 0 & 0 & 1 & 2 \\ 3 & 3 & 0 & 0 & 1 & 2 \\ 2 & 2 & 1 & 1 & 0 & 3 \\ 1 & 1 & 2 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 3, 4, 6}

R: [4, 4, 1, 1, 3, 2]
B: [5, 5, 6, 6, 2, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{512}(1+s)(-7+s^2)^2(-1+s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[1] v[4]
"B CYCLES", (1 + v[2] v[5]) (1 + v[3] v[6])

Eigenvalues

R: [0., 0., 0., 0., 1., −1.]

B: [1., −1., 1., −1., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, −1, 1, 0, 0], [−1, 1, 0, 0, 0, 0]}

NullSpace of B^*

{[0, 0, −1, 1, 0, 0], [−1, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{3}$ ($v[1]v[4] + v[2]v[3] + v[5]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 5}, {3, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 3}

"RG3" = {1, 4}

$$\pi 2 = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1]$$

supp $\pi 2$ = {3, 6, 15}

$$u2 = [0, 1, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1]$$

supp u2 = {2, 3, 5, 6, 7, 9, 11, 13, 15}

Action of R on ranges, [[2], [3], [3]]
Action of B on ranges, [[2], [1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 1]

B–BLOCKS,

[1, 2]

with invariant measure, [1, 1]

N by blocks, N – check: true

$b_1 = \{1, 2, 5\}$
$b_2 = \{3, 4, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[2] & h[1] & 0 & 0 & 0 \\ 0 & h[1] & h[2] & 0 & 0 & 0 \\ h[1] & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 7, Shape: $0 \oplus 7/5$

$$\text{CLB} = \begin{pmatrix} -1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4}}, true

Ω_B in Vec(K)? , {{3, 6}, {2, 5}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega–checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 5}, {3, 4, 6}}

1, "range", [5, 6], [[6, 6, 5, 5, 6, 5], [5, 5, 6, 6, 5, 6]]

2, "range", [2, 3], [[3, 3, 2, 2, 3, 2], [2, 2, 3, 3, 2, 3]]

3, "range", [1, 4], [[4, 4, 1, 1, 4, 1], [1, 1, 4, 4, 1, 4]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\,2\,] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

{3, 6, 15}

$u2 = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$

{2, 3, 5, 6, 7, 9, 11, 13, 15}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$ idem−checks

$$T = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 3 & 3 & 0 & 0 & 3 & 0 \\ 3 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 & 3 \\ 0 & 0 & 3 & 3 & 0 & 3 \\ 3 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 3 & 0 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [1, 0, 0, 1, −1, −1]

$$\ker N_C = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -t & -s & s & t & 0 \\ -s & s & t & 0 & 0 & -t \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -t & -s & s & t & 0 \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & 0 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} s & 0 & t \\ s & 0 & t \\ -s & 0 & -t \\ -s & 0 & -t \\ 0 & -t+s & 0 \\ 0 & -s+t & 0 \end{pmatrix}$$ RB checks

$$\ker M_C = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} s+t & s & -s & 0 \\ s+t & s & -s & 0 \\ 0 & t & s & 0 \\ 0 & t & s & 0 \\ s & s & 0 & -s+t \\ t & t & 0 & -t+s \end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 3 & 3 & 0 & 0 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 3&0&0&3&0&0\\ 0&3&3&0&0&0\\ 0&3&3&0&0&0\\ 3&0&0&3&0&0\\ 0&0&0&0&3&3\\ 0&0&0&0&3&3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&1&0&0&1&0\\ 1&1&0&0&1&0\\ 0&0&1&1&0&1\\ 0&0&1&1&0&1\\ 1&1&0&0&1&0\\ 0&0&1&1&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 9T + 0\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

"IS NM in Vec(K)?", true

NM $\begin{pmatrix} 0 & 3 & 0 & 0 & 3 & 3 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 0 & 3 & 0 & 0 & 3 & 3 \end{pmatrix}$

$$\boldsymbol{\tau} = 18/1 \text{ , rank} = 2 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 12/1$$

$$\text{max r} = 6/1 \text{ , } r\text{–check is positive? } 2/3$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 6
out of total no. of elements equal to 6

dim span idems 3 vs no. of idems 3

"PT1" = {{1, 2, 5}, {3, 4, 6}}

"RG1" = {5, 6}

"RG2" = {2, 3}

"RG3" = {1, 4}

$$M_C = \begin{pmatrix} 2 & -1 & -1 & 2 & -1 & -1 \\ -1 & 2 & 2 & -1 & -1 & -1 \\ -1 & 2 & 2 & -1 & -1 & -1 \\ 2 & -1 & -1 & 2 & -1 & -1 \\ -1 & -1 & -1 & -1 & 2 & 2 \\ -1 & -1 & -1 & -1 & 2 & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ 1 & \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & 1 \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} \\ 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & 1 \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & 1 \\ 1 & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[6., 6., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 3., 2.]

Eigenvalues M_C–scaled

[3., 3., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 3.600000000, 2.400000000]

NullSpace M_C

{[−1, 0, 0, 1, 0, 0], [0, −1, 1, 0, 0, 0], [1, 1, 0, 0, 0, 1], [1, 1, 0, 0, 1, 0]}

NullSpace N_c

{[0, 0, −1, 0, 0, 1], [−1, 0, 0, 0, 1, 0], [−1, 1, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0]}

Eigenvalues M_0

[6., 6., 6., 0., 0., 0.]

Eigenvalues N_0

[3., 3., 0., 0., 0., 0.]

NullSpace M_0

{[0, 1, −1, 0, 0, 0], [−1, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, −1]}

NullSpace N_0

{[0, 0, 0, −1, 0, 1], [0, 1, 0, 0, −1, 0], [0, 0, 1, −1, 0, 0], [1, 0, 0, 0, −1, 0]}

Eigenvalues M

[3., −3., 3., −3., 3., −3.]

Eigenvalues N

[0., 0., 0., 0., 3., −3.]

NullSpace M

{}

NullSpace N

{[0, 0, 0, 1, 0, −1], [0, 0, 1, 0, 0, −1], [0, 1, 0, 0, −1, 0], [1, 0, 0, 0, −1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

==

{3, 4, 5, 6}

R: [4, 5, 1, 1, 2, 2]
B: [5, 4, 6, 6, 3, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{512}\,(-1+s)\,(1+s)\,(-7+s^2)^2$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 2

"R CYCLES", (1 + v[1] v[4]) (1 + v[2] v[5])
"B CYCLES", 1 + v[3] v[6]

Eigenvalues

R: [1., −1., 1., −1., 0., 0.]

B: [0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 1, −1, 0, 0], [0, 0, 0, 0, −1, 1]}

NullSpace of B^*

{[0, 0, −1, 1, 0, 0], [0, 0, 0, 0, −1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{3}$ ($v[1]v[2] + v[3]v[6] + v[4]v[5]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 4}, {2, 5, 6}}

"RG1" = {4, 5}

"RG2" = {3, 6}

"RG3" = {1, 2}

$$\pi 2 = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0]$$

supp $\pi 2$ = {1, 12, 13}

$$u2 = [1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0]$$

supp u2 = {1, 4, 5, 6, 7, 11, 12, 13, 14}

Action of R on ranges, [[3], [3], [1]]
Action of B on ranges, [[2], [2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2]

B-BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N – check: true

$b_1 = \{1, 3, 4\}$
$b_2 = \{2, 5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & 0 & 0 & 0 & 0 \\ h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & h[2] \\ 0 & 0 & 0 & h[1] & h[2] & 0 \\ 0 & 0 & 0 & h[2] & h[1] & 0 \\ 0 & 0 & h[2] & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 7, Shape: 0 ⊕ 7/5

$$\text{CLB} = \begin{pmatrix} -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4}, {2, 5}}, true

Ω_B in Vec(K)? , {{3, 6}}, true

$$V = \begin{pmatrix} \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{2} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 4}, {2, 5, 6}}

1, "range", [4, 5], [[5, 4, 5, 5, 4, 4], [4, 5, 4, 4, 5, 5]]

2, "range", [3, 6], [[6, 3, 6, 6, 3, 3], [3, 6, 3, 3, 6, 6]]

3, "range", [1, 2], [[2, 1, 2, 2, 1, 1], [1, 2, 1, 1, 2, 2]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$

{1, 12, 13}

$u2 = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$

{1, 4, 5, 6, 7, 11, 12, 13, 14}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$ idem–checks

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 3 & 0 & 3 & 3 & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 3 \\ 3 & 0 & 3 & 3 & 0 & 0 \\ 3 & 0 & 3 & 3 & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 3 \\ 0 & 3 & 0 & 0 & 3 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [1, 1, -1, 0, 0, -1]

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & s & t & -t & -s & 0 \\ 0 & s & t & -t & -s & 0 \\ -s & 0 & 0 & s & t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & -1 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -s+t \\ 0 & 0 & -t+s \\ -s & t & 0 \\ -s & t & 0 \\ s & -t & 0 \\ s & -t & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} t & s & 0 & 0 \\ s & t & 0 & 0 \\ t & t & -t & s \\ t & t & -t & s \\ s & s & t & -s \\ s & s & t & -s \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 3 & 3 & 0 & 0 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 3&3&0&0&0&0\\3&3&0&0&0&0\\0&0&3&0&0&3\\0&0&0&3&3&0\\0&0&0&3&3&0\\0&0&3&0&0&3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&1&1&0&0\\0&1&0&0&1&1\\1&0&1&1&0&0\\1&0&1&1&0&0\\0&1&0&0&1&1\\0&1&0&0&1&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 9T + 0\Omega$$

$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$

"IS NM in Vec(K)?", true

NM $\begin{pmatrix} 0 & 0 & 3 & 3 & 0 & 3 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 0 & 0 & 3 & 3 & 0 & 3 \end{pmatrix}$

$$\boldsymbol{\tau} = 18/1 \text{ , rank} = 2 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 12/1$$

$$\text{max r} = 6/1 \text{ , } r\text{-check is positive? } 2/3$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 6
out of total no. of elements equal to 6

dim span idems 3 vs no. of idems 3

"PT1" = {{1, 3, 4}, {2, 5, 6}}

"RG1" = {4, 5}

"RG2" = {3, 6}

"RG3" = {1, 2}

$$M_C = \begin{pmatrix} 2 & 2 & -1 & -1 & -1 & -1 \\ 2 & 2 & -1 & -1 & -1 & -1 \\ -1 & -1 & 2 & -1 & -1 & 2 \\ -1 & -1 & -1 & 2 & 2 & -1 \\ -1 & -1 & -1 & 2 & 2 & -1 \\ -1 & -1 & 2 & -1 & -1 & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ 1 & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & 1 \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & 1 & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & 1 & 1 & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \\ 1 & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ 1 & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \\ \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[6., 6., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 3., 2.]

Eigenvalues M_C–scaled

[3., 3., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 3.600000000, 2.400000000]

NullSpace M_C

{[1, 0, 0, 1, 0, 1], [0, 0, 0, −1, 1, 0], [−1, 1, 0, 0, 0, 0], [1, 0, 1, 1, 0, 0]}

NullSpace N_c

{[−1, 0, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0], [0, −1, 0, 0, 0, 1]}

Eigenvalues M_0

[6., 6., 6., 0., 0., 0.]

Eigenvalues N_0

[3., 3., 0., 0., 0., 0.]

NullSpace M_0

{[−1, 1, 0, 0, 0, 0], [0, 0, 0, −1, 1, 0], [0, 0, −1, 0, 0, 1]}

NullSpace N_0

{[−1, 0, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0], [0, −1, 0, 0, 0, 1]}

Eigenvalues M

[3., −3., 3., −3., 3., −3.]

Eigenvalues N

[0., 0., 0., 0., 3., −3.]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 0, 1, 0], [0, −1, 0, 0, 0, 1], [−1, 0, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$