T - Run

[4, 5, 1, 6, 3, 1], [2, 4, 5, 3, 1, 4]

$\tilde{\pi}$ = [6, 3, 5, 6, 4, 3]
$\boldsymbol{\delta}$ = [3, 1, 2, 3, 2, 1]

POSSIBLE RANKS

1 x 27
3 x 9

BASE DETERMINANT 161757/1048576, .1542634964

NullSpace of Δ

{1, 2, 3, 4, 5, 6}

Nullspace of A

` det(A) = ` 1/32

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[4] v[6] + v[3] v[5]

Degree 3
v[1] v[3] v[4] + v[1] v[2] v[5] + v[1] v[4] v[6]

Degree 4
v[3] v[4] v[5] v[6] + v[1] v[2] v[3] v[4] + v[1] v[2] v[3] v[5] + v[1] v[3] v[4] v[5] + v[1] v[2] v[4] v[6]

Degree 5
v[1] v[2] v[3] v[4] v[5] + v[1] v[3] v[4] v[5] v[6] + v[1] v[2] v[4] v[5] v[6]

Degree 6
2 v[1] v[2] v[3] v[4] v[5] v[6]

===

20, [1, −1, 1, −1, −1, 1]

===

{4, 5, 6}

R: [4, 5, 1, 3, 1, 4]
B: [2, 4, 5, 6, 3, 1]

TRACE TWO = 1

det AT = $\frac{-3}{32}(-1+t)^3(1+3t^2)$

$$AT = \begin{pmatrix} 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{9}{33554432}(-5176224 - 2926950s - 216943s^2 + 274969s^3 + 75916s^4 + 40880s^5 - 8842s^6 - 8842s^7 - 1972s^8 - 738s^9 - 63s^{10} + 9s^{11})(-1+s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 1, "vs", 3

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", 1 + v[1] v[3] v[4]
"B CYCLES", (1 + v[3] v[5]) (1 + v[1] v[2] v[4] v[6])

Eigenvalues

R: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [1. I, −1. I, 1., −1., 1., −1.]

NullSpace of R

{[0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]}

NullSpace of B

{}

NullSpace of R^*

{[−1, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0]}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 16 & 24 & 8 & 0 \\ 0 & 0 & 4 & 0 & 8 & 12 \\ 16 & 4 & 0 & 16 & 0 & 4 \\ 24 & 0 & 16 & 0 & 8 & 0 \\ 8 & 8 & 0 & 8 & 0 & 8 \\ 0 & 12 & 4 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{2}{3} & 1 & 1 & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & 1 & \frac{1}{3} & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ 1 & 1 & 0 & 1 & 0 & 1 \\ \frac{1}{3} & 1 & 1 & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{2}{9}\,(6v[1] + 3v[2] + 5v[3] + 6v[4] + 4v[5] + 3v[6])$

degree 2: $\frac{4}{27}\,(4v[1]v[3] + 6v[1]v[4] + 2v[1]v[5] + v[2]v[3] + 2v[2]v[5] + 3v[2]v[6] + 4v[3]v[4] + v[3]v[6] + 2v[4]v[5] + 2v[5]v[6])$

degree 3 : $\frac{4}{9}\,(4v[1]v[3]v[4] + 2v[1]v[4]v[5] + v[2]v[3]v[6] + 2v[2]v[5]v[6])$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{3, 5}, {1, 6}, {2, 4}}

"PT2" = {{3, 5}, {1, 2}, {4, 6}}

"RG1" = {2, 5, 6}

"RG2" = {2, 3, 6}

"RG3" = {1, 4, 5}

"RG4" = {1, 3, 4}

$$\pi 3 = [0, 0, 0, 0, 4, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0]$$

supp $\pi 3$ = {5, 8, 13, 16}

$$u3 = [2, 0, 2, 0, 3, 0, 1, 3, 0, 1, 1, 0, 3, 1, 0, 3, 0, 2, 0, 2]$$

supp u3 = {1, 3, 5, 7, 8, 10, 11, 13, 14, 16, 18, 20}

Action of R on ranges, [[3], [3], [4], [4]]
Action of B on ranges, [[4], [3], [2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{2}{9} & \frac{1}{9} & \frac{2}{9} & \frac{4}{9} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[4, 1, 1, 3, 3]

B–BLOCKS,

[1, 3, 5, 2, 4]

with invariant measure, [3, 1, 2, 2, 1]

N by blocks, N – check: true

$b_1 = \{3, 5\}$
$b_2 = \{1, 2\}$
$b_3 = \{1, 6\}$
$b_4 = \{2, 4\}$
$b_5 = \{4, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 4, 4, 4

LIE STRUCTURE

Dimension of Lie algebra: 18, Shape: $8 \oplus 10/8$

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3, 4}}, true

Ω_B in Vec(K)? , {{3, 5}, {1, 2, 4, 6}}, true

$$V = \begin{pmatrix} \frac{1}{27} & \frac{-13}{27} & \frac{7}{81} & \frac{10}{27} & \frac{-16}{81} & \frac{5}{27} \\ \frac{4}{27} & \frac{2}{27} & \frac{1}{81} & \frac{-14}{27} & \frac{44}{81} & \frac{-7}{27} \\ \frac{2}{9} & \frac{1}{9} & \frac{-4}{27} & \frac{2}{9} & \frac{-14}{27} & \frac{1}{9} \\ \frac{2}{27} & \frac{1}{27} & \frac{41}{81} & \frac{-7}{27} & \frac{22}{81} & \frac{-17}{27} \\ \frac{2}{9} & \frac{1}{9} & \frac{-13}{27} & \frac{2}{9} & \frac{-5}{27} & \frac{1}{9} \\ \frac{-10}{27} & \frac{-5}{27} & \frac{11}{81} & \frac{8}{27} & \frac{-2}{81} & \frac{4}{27} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 5}, {1, 6}, {2, 4}}

1, "range", [2, 5, 6], [[6, 5, 2, 5, 2, 6], [6, 2, 5, 2, 5, 6], [5, 6, 2, 6, 2, 5], [5, 2, 6, 2, 6, 5], [2, 6, 5, 6, 5, 2], [2, 5, 6, 5, 6, 2]]

2, "range", [2, 3, 6], [[6, 3, 2, 3, 2, 6], [6, 2, 3, 2, 3, 6], [3, 6, 2, 6, 2, 3], [3, 2, 6, 2, 6, 3], [2, 6, 3, 6, 3, 2], [2, 3, 6, 3, 6, 2]]

3, "range", [1, 4, 5], [[5, 4, 1, 4, 1, 5], [5, 1, 4, 1, 4, 5], [4, 5, 1, 5, 1, 4], [4, 1, 5, 1, 5, 4], [1, 5, 4, 5, 4, 1], [1, 4, 5, 4, 5, 1]]

4, "range", [1, 3, 4], [[4, 3, 1, 3, 1, 4], [4, 1, 3, 1, 3, 4], [3, 4, 1, 4, 1, 3], [3, 1, 4, 1, 4, 3], [1, 4, 3, 4, 3, 1], [1, 3, 4, 3, 4, 1]]

2, "partition", {{3, 5}, {1, 2}, {4, 6}}

1, "range", [2, 5, 6], [[6, 6, 5, 2, 5, 2], [6, 6, 2, 5, 2, 5], [5, 5, 6, 2, 6, 2], [5, 5, 2, 6, 2, 6], [2, 2, 6, 5, 6, 5], [2, 2, 5, 6, 5, 6]]

2, "range", [2, 3, 6], [[6, 6, 3, 2, 3, 2], [6, 6, 2, 3, 2, 3], [3, 3, 6, 2, 6, 2], [3, 3, 2, 6, 2, 6], [2, 2, 6, 3, 6, 3], [2, 2, 3, 6, 3, 6]]

3, "range", [1, 4, 5], [[5, 5, 4, 1, 4, 1], [5, 5, 1, 4, 1, 4], [4, 4, 5, 1, 5, 1], [4, 4, 1, 5, 1, 5], [1, 1, 5, 4, 5, 4], [1, 1, 4, 5, 4, 5]]

4, "range", [1, 3, 4], [[4, 4, 3, 1, 3, 1], [4, 4, 1, 3, 1, 3], [3, 3, 4, 1, 4, 1], [3, 3, 1, 4, 1, 4], [1, 1, 4, 3, 4, 3], [1, 1, 3, 4, 3, 4]]

"group has", 6, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

$g_3 =$ [[1, 3, 2]]

$g_4 =$ [[2, 3]]

$g_5 =$ [[1, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 2h[1] - h[2] & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

Z[1] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 4 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \end{pmatrix}$

{5, 8, 13, 16}

$u3 = \begin{pmatrix} 2 & 0 & 2 & 0 & 3 & 0 & 1 & 3 & 0 & 1 & 1 & 0 & 3 & 1 & 0 & 3 & 0 & 2 & 0 & 2 \end{pmatrix}$

{1, 3, 5, 7, 8, 10, 11, 13, 14, 16, 18, 20}

picheck $\begin{pmatrix} 6 & 3 & 5 & 6 & 4 & 3 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{2}{9} & \frac{1}{9} & \frac{5}{27} & \frac{2}{9} & \frac{4}{27} & \frac{1}{9} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 0 & 4 & 6 & 2 & 0 & 1 & 0 & 2 & 3 & 4 & 0 & 1 & 2 & 0 & 2 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{2}{3} & 1 & 1 & 1 & \frac{1}{3} & 1 & \frac{1}{3} & 1 & 1 & 1 & 0 & 1 & 1 & \frac{2}{3} & 1 \end{pmatrix}$

picheck $\begin{pmatrix} 12 & 6 & 10 & 12 & 8 & 6 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 12 & 6 & 10 & 12 & 8 & 6 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 12 & 6 & 10 & 12 & 8 & 6 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{2}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 \\ \frac{2}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{2}{3} & \frac{1}{9} & 0 & 0 & 0 & \frac{2}{9} \\ \frac{2}{9} & \frac{1}{3} & 0 & \frac{4}{9} & 0 & 0 \\ 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 \\ 0 & \frac{2}{9} & 0 & \frac{2}{3} & 0 & \frac{1}{9} \\ 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 \\ \frac{4}{9} & 0 & 0 & \frac{2}{9} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{16}{3} & \frac{16}{9} & \frac{20}{9} & \frac{8}{3} & \frac{16}{9} & \frac{20}{9} \\ \frac{32}{9} & \frac{8}{3} & \frac{20}{9} & \frac{40}{9} & \frac{16}{9} & \frac{4}{3} \\ \frac{8}{3} & \frac{4}{3} & \frac{40}{9} & \frac{8}{3} & \frac{32}{9} & \frac{4}{3} \\ \frac{8}{3} & \frac{20}{9} & \frac{20}{9} & \frac{16}{3} & \frac{16}{9} & \frac{16}{9} \\ \frac{8}{3} & \frac{4}{3} & \frac{40}{9} & \frac{8}{3} & \frac{32}{9} & \frac{4}{3} \\ \frac{40}{9} & \frac{4}{3} & \frac{20}{9} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [3, -3, 1, 3, -1, -3]$

$$\ker N_c = \begin{pmatrix} 1 & -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -t & t & s & -t & -s & t \\ 0 & 0 & t & 0 & -t & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 3 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} -s & t & -s \\ -t & 0 & -t+s \\ s & 0 & t \\ 0 & -t & -t+s \\ s & 0 & t \\ t-s & 0 & -s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & t & 0 \\ t & 0 & 0 & s \\ 0 & s & 0 & t \\ t & t & -t & s \\ 0 & s & 0 & t \\ s & t & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = \begin{pmatrix} 2 & 2 & 0 & 2 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ \frac{2}{3} & 1 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 0 & 1 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 6, "vs", 3

$$\mathrm{CNM} = \begin{pmatrix} 0 & \frac{-4}{3} & \frac{-4}{9} & 0 & \frac{-8}{9} & \frac{-4}{3} \\ \frac{4}{3} & 0 & \frac{8}{9} & \frac{4}{3} & \frac{4}{9} & 0 \\ \frac{4}{9} & \frac{-8}{9} & 0 & \frac{4}{9} & \frac{-4}{9} & \frac{-8}{9} \\ 0 & \frac{-4}{3} & \frac{-4}{9} & 0 & \frac{-8}{9} & \frac{-4}{3} \\ \frac{8}{9} & \frac{-4}{9} & \frac{4}{9} & \frac{8}{9} & 0 & \frac{-4}{9} \\ \frac{4}{3} & 0 & \frac{8}{9} & \frac{4}{3} & \frac{4}{9} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{-1}{9} & 0 & 0 & 0 & \frac{-2}{9} \\ \frac{1}{9} & 0 & 0 & \frac{2}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{9} & 0 \\ 0 & \frac{-2}{9} & 0 & 0 & 0 & \frac{-1}{9} \\ 0 & 0 & \frac{1}{9} & 0 & 0 & 0 \\ \frac{2}{9} & 0 & 0 & \frac{1}{9} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{-1}{9} & \frac{-1}{27} & 0 & \frac{-2}{27} & \frac{-1}{9} \\ \frac{1}{9} & 0 & \frac{2}{27} & \frac{1}{9} & \frac{1}{27} & 0 \\ \frac{1}{27} & \frac{-2}{27} & 0 & \frac{1}{27} & \frac{-1}{27} & \frac{-2}{27} \\ 0 & \frac{-1}{9} & \frac{-1}{27} & 0 & \frac{-2}{27} & \frac{-1}{9} \\ \frac{2}{27} & \frac{-1}{27} & \frac{1}{27} & \frac{2}{27} & 0 & \frac{-1}{27} \\ \frac{1}{9} & 0 & \frac{2}{27} & \frac{1}{9} & \frac{1}{27} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{8}{3} & \frac{8}{9} & 0 \\ 0 & \frac{4}{3} & \frac{4}{9} & 0 & \frac{8}{9} & \frac{4}{3} \\ \frac{16}{9} & \frac{4}{9} & \frac{20}{9} & \frac{16}{9} & 0 & \frac{4}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{8}{3} & \frac{8}{9} & 0 \\ \frac{8}{9} & \frac{8}{9} & 0 & \frac{8}{9} & \frac{16}{9} & \frac{8}{9} \\ 0 & \frac{4}{3} & \frac{4}{9} & 0 & \frac{8}{9} & \frac{4}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & 1 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 12\Omega$$

$$\Omega \left(\frac{5}{27} \;\; 0 \;\; \frac{5}{27} \;\; \frac{2}{9} \;\; \frac{5}{27} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{4}{27} \;\; \frac{2}{9} \;\; \frac{5}{27} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{9} \;\; \frac{4}{27} \;\; \frac{2}{9} \;\; \frac{5}{27} \;\; \frac{1}{9} \;\; \frac{2}{9} \right)$$

$$T \left(0 \;\; 0 \;\; \frac{7}{9} \;\; 0 \;\; \frac{5}{9} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{4}{9} \;\; 0 \;\; \frac{1}{3} \;\; \frac{2}{9} \;\; \frac{2}{9} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{9} \;\; \frac{2}{3} \right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \left(\frac{20}{9} \;\; 0 \;\; \frac{16}{3} \;\; \frac{8}{3} \;\; \frac{40}{9} \;\; \frac{4}{3} \;\; \frac{8}{3} \;\; \frac{16}{9} \;\; \frac{40}{9} \;\; \frac{20}{9} \;\; \frac{8}{3} \;\; \frac{32}{9} \;\; \frac{20}{9} \;\; \frac{16}{9} \;\; \frac{8}{3} \;\; \frac{20}{9} \;\; \frac{16}{9} \;\; \frac{16}{3} \right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN}\left(2 \quad \frac{-1}{3} \quad \frac{41}{9} \quad 2 \quad 4 \quad 2 \quad 2 \quad 2 \quad \frac{29}{9} \quad 2 \quad \frac{11}{3} \quad \frac{25}{9} \quad \frac{31}{9} \quad 2 \quad \frac{5}{3} \quad 2 \quad \frac{23}{9} \quad \frac{13}{3}\right)$$

$$\boldsymbol{\tau} = 12/1 \text{ , rank } = 3 \text{ , ratio } = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 131/729 \text{ , min } \boldsymbol{\tau} = 524/81 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 448/81$$

$$\text{max r} = 729/131 \text{ , } r\text{-check is positive? } 112/243$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 48

dim span idems 5 vs no. of idems 8

"PT1" = {{3, 5}, {1, 6}, {2, 4}}

"PT2" = {{3, 5}, {1, 2}, {4, 6}}

"RG1" = {2, 5, 6}

"RG2" = {2, 3, 6}

"RG3" = {1, 4, 5}

"RG4" = {1, 3, 4}

$$M_c = \begin{pmatrix} \frac{8}{9} & \frac{-8}{9} & \frac{8}{27} & \frac{8}{9} & \frac{-8}{27} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{8}{9} & \frac{-8}{27} & \frac{-8}{9} & \frac{8}{27} & \frac{8}{9} \\ \frac{8}{27} & \frac{-8}{27} & \frac{80}{81} & \frac{8}{27} & \frac{-80}{81} & \frac{-8}{27} \\ \frac{8}{9} & \frac{-8}{9} & \frac{8}{27} & \frac{8}{9} & \frac{-8}{27} & \frac{-8}{9} \\ \frac{-8}{27} & \frac{8}{27} & \frac{-80}{81} & \frac{-8}{27} & \frac{80}{81} & \frac{8}{27} \\ \frac{-8}{9} & \frac{8}{9} & \frac{-8}{27} & \frac{-8}{9} & \frac{8}{27} & \frac{8}{9} \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{598}{729} & \frac{112}{729} & \frac{-131}{729} & \frac{-131}{729} & \frac{-131}{729} & \frac{355}{729} \\ \frac{112}{729} & \frac{598}{729} & \frac{-131}{729} & \frac{355}{729} & \frac{-131}{729} & \frac{-131}{729} \\ \frac{-131}{729} & \frac{-131}{729} & \frac{598}{729} & \frac{-131}{729} & \frac{598}{729} & \frac{-131}{729} \\ \frac{-131}{729} & \frac{355}{729} & \frac{-131}{729} & \frac{598}{729} & \frac{-131}{729} & \frac{112}{729} \\ \frac{-131}{729} & \frac{-131}{729} & \frac{598}{729} & \frac{-131}{729} & \frac{598}{729} & \frac{-131}{729} \\ \frac{355}{729} & \frac{-131}{729} & \frac{-131}{729} & \frac{112}{729} & \frac{-131}{729} & \frac{598}{729} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & -1 & \frac{1}{3} & 1 & \frac{-1}{3} & -1 \\ -1 & 1 & \frac{-1}{3} & -1 & \frac{1}{3} & 1 \\ \frac{3}{10} & \frac{-3}{10} & 1 & \frac{3}{10} & -1 & \frac{-3}{10} \\ 1 & -1 & \frac{1}{3} & 1 & \frac{-1}{3} & -1 \\ \frac{-3}{10} & \frac{3}{10} & -1 & \frac{-3}{10} & 1 & \frac{3}{10} \\ -1 & 1 & \frac{-1}{3} & -1 & \frac{1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{56}{299} & \frac{-131}{598} & \frac{-131}{598} & \frac{-131}{598} & \frac{355}{598} \\ \frac{56}{299} & 1 & \frac{-131}{598} & \frac{355}{598} & \frac{-131}{598} & \frac{-131}{598} \\ \frac{-131}{598} & \frac{-131}{598} & 1 & \frac{-131}{598} & 1 & \frac{-131}{598} \\ \frac{-131}{598} & \frac{355}{598} & \frac{-131}{598} & 1 & \frac{-131}{598} & \frac{56}{299} \\ \frac{-131}{598} & \frac{-131}{598} & 1 & \frac{-131}{598} & 1 & \frac{-131}{598} \\ \frac{355}{598} & \frac{-131}{598} & \frac{-131}{598} & \frac{56}{299} & \frac{-131}{598} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 3.917225066, 1.613639132]

Eigenvalues N_C

[0., 0., 2., 0.6666666667, 1.333333333, 0.9218106996]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 4.341640786, 1.658359214]

Eigenvalues N_C–scaled

[0., 0., 0.8127090301, 1.123745819, 1.625418060, 2.438127090]

NullSpace M_C

{[0, 0, 0, 1, 0, 1], [1, 0, 0, 0, 0, 1], [0, 0, 1, 0, 1, 0], [0, 1, 0, 0, 0, −1]}

NullSpace N_C

{[0, 0, −1, 0, 1, 0], [1, −1, 0, 1, 0, −1]}

Eigenvalues M_0

[0., 0., 0., 7.043268479, 1.607968479, 3.348763041]

Eigenvalues N_0

[1.333333333, 0.6666666667, 0., 0., 2., 2.]

NullSpace M_0

{[−1, 0, 1, 0, 1, −1], [0, 1, 0, 0, 0, −1], [−1, 0, 0, 1, 0, 0]}

NullSpace N_0

{[−1, 1, 0, −1, 0, 1], [0, 0, 1, 0, −1, 0]}

Eigenvalues M

[−1.333333333, −2.666666667, 1.833039588, 4.567463350, −0.3230364042, −2.077466534]

Eigenvalues N

[0., 0., −2., −0.6666666667, 4., −1.333333333]

NullSpace M

{}

NullSpace N

{[1, −1, 0, 1, 0, −1], [0, 0, −1, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 3 & 3 & 3 & 1 \\ 2 & 0 & 3 & 1 & 3 & 3 \\ 3 & 3 & 0 & 3 & 0 & 3 \\ 3 & 1 & 3 & 0 & 3 & 2 \\ 3 & 3 & 0 & 3 & 0 & 3 \\ 1 & 3 & 3 & 2 & 3 & 0 \end{pmatrix}$$