T-Run

[4, 4, 4, 7, 7, 7, 1, 1, 1], [2, 9, 5, 8, 3, 8, 5, 6, 2]

$\tilde{\pi}$ = [3, 2, 1, 3, 2, 1, 3, 2, 1]
$\boldsymbol{\delta}$ = [3, 2, 1, 3, 2, 1, 3, 2, 1]

POSSIBLE RANKS

1 x 18
2 x 9
3 x 6

BASE DETERMINANT 2151937075/68719476736, .3131480589e-1

NullSpace of Δ

{3, 7, 8}, {1, 2, 4, 5, 6, 9}

Nullspace of A

[{2, 5, 6, 9},{1, 4}] `,` [{3, 8},{7}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[2] v[9] + v[6] v[8] + v[5] v[7] + v[3] v[5]

Degree 3
v[1] v[2] v[9] + v[1] v[4] v[7] + v[1] v[4] v[8]

Degree 4
v[5] v[6] v[7] v[8] + v[2] v[6] v[9] v[8] + v[1] v[2] v[4] v[7] + v[3] v[5] v[6] v[8] + v[1] v[2] v[4] v[8] + v[2] v[3] v[5] v[9] + v[3] v[4] v[5] v[7] + v[2] v[5] v[7] v[9]

Degree 5
v[1] v[2] v[5] v[7] v[9] + v[1] v[3] v[4] v[5] v[7] + 2 v[1] v[4] v[6] v[7] v[8] + v[1] v[2] v[4] v[7] v[9] + v[1] v[2] v[3] v[5] v[9] + v[1] v[2] v[6] v[9] v[8] + v[1] v[2] v[4] v[9] v[8] + v[1] v[4] v[5] v[7] v[8] + v[1] v[3] v[4] v[5] v[8]

Degree 6
v[2] v[3] v[4] v[5] v[7] v[9] + v[1] v[2] v[3] v[4] v[5] v[8] + 2 v[3] v[4] v[5] v[6] v[7] v[8] + v[1] v[2] v[4] v[5] v[7] v[8] + v[1] v[2] v[3] v[4] v[5] v[7] + v[2] v[5] v[6] v[7] v[9] v[8] + v[2] v[3] v[5] v[6] v[9] v[8] + 2 v[1] v[2] v[4] v[6] v[7] v[8]

Degree 7
2 v[1] v[3] v[4] v[5] v[6] v[7] v[8] + v[1] v[2] v[3] v[4] v[5] v[9] v[8] + v[1] v[2] v[4] v[5] v[7] v[9] v[8] + v[1] v[2] v[3] v[5] v[6] v[9] v[8] + v[1] v[2] v[5] v[6] v[7] v[9] v[8] + 2 v[1] v[2] v[3] v[4] v[5] v[7] v[9] + 2 v[1] v[2] v[4] v[6] v[7] v[9] v[8]

Degree 8

2 v[2] v[3] v[4] v[5] v[6] v[7] v[9] v[8] + 2 v[1] v[2] v[3] v[4] v[5] v[6] v[7] v[8]

Degree 9
4 v[1] v[2] v[3] v[4] v[5] v[6] v[7] v[9] v[8]

==

{}

R: [4, 4, 4, 7, 7, 7, 1, 1, 1]
B: [2, 9, 5, 8, 3, 8, 5, 6, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 7

Level 2 det = $\frac{-13}{68719476736}(1+s)(2151937075 - 2919349200s + 1379486034s^2 - 225303388s^3 - 50253534s^4 + 60365832s^5 - 37156446s^6 + 14568332s^7 - 2724528s^8 - 156512s^9 + 279270s^{10} - 146692s^{11} + 53454s^{12} - 12120s^{13} + 1702s^{14} - 172s^{15} + 13s^{16})(-1+s)$

RANK of R is 3

R ranking is 1, "vs", 3

RBAR ranking 1, "vs", 3

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", 1 + v[1] v[4] v[7]
"B CYCLES", (1 + v[6] v[8]) (1 + v[3] v[5]) (1 + v[2] v[9])

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

B: [1., −1., 1., −1., 1., −1., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, 0, 0, −1, 1, 0], [−1, 0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, −1, 0, 1], [−1, 1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0, 0], [0, 0, 0, 1, −1, 0, 0, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 1, 0, −1, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0, −1], [0, 0, 1, 0, 0, 0, −1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&0&9&0&0&9&0&0 \\ 0&0&0&0&6&0&0&6&0 \\ 0&0&0&0&0&3&0&0&3 \\ 9&0&0&0&0&0&9&0&0 \\ 0&6&0&0&0&0&0&6&0 \\ 0&0&3&0&0&0&0&0&3 \\ 9&0&0&9&0&0&0&0&0 \\ 0&6&0&0&6&0&0&0&0 \\ 0&0&3&0&0&3&0&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&\frac{1}{2}&\frac{1}{2}&1&\frac{2}{3}&1&1&\frac{5}{6}&\frac{1}{2} \\ \frac{1}{2}&0&\frac{1}{3}&\frac{2}{3}&1&\frac{2}{3}&\frac{5}{6}&1&1 \\ \frac{1}{2}&\frac{1}{3}&0&1&1&1&\frac{1}{2}&\frac{2}{3}&1 \\ 1&\frac{2}{3}&1&0&\frac{1}{3}&0&1&1&1 \\ \frac{2}{3}&1&1&\frac{1}{3}&0&\frac{1}{3}&1&1&\frac{2}{3} \\ 1&\frac{2}{3}&1&0&\frac{1}{3}&0&1&1&1 \\ 1&\frac{5}{6}&\frac{1}{2}&1&1&1&0&\frac{1}{6}&\frac{1}{2} \\ \frac{5}{6}&1&\frac{2}{3}&1&1&1&\frac{1}{6}&0&\frac{1}{3} \\ \frac{1}{2}&1&1&1&\frac{2}{3}&1&\frac{1}{2}&\frac{1}{3}&0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 9

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 1 "Trace mark", 1, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}$ ($3v[1] + 2v[2] + v[3] + 3v[4] + 2v[5] + v[6] + 3v[7] + 2v[8] + v[9]$)
degree 2: $\frac{1}{6}$ ($3v[1]v[4] + 3v[1]v[7] + 2v[2]v[5] + 2v[2]v[8] + v[3]v[6] + v[3]v[9] + 3v[4]v[7] + 2v[5]v[8] + v[6]v[9]$)
degree 3 : $\frac{1}{2}$ ($3v[1]v[4]v[7] + 2v[2]v[5]v[8] + v[3]v[6]v[9]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 5, 9}, {2, 4, 6}, {3, 7, 8}}

"PT2" = {{1, 8, 9}, {2, 3, 7}, {4, 5, 6}}

"PT3" = {{7, 8, 9}, {4, 5, 6}, {1, 2, 3}}

"RG1" = {3, 6, 9}

"RG2" = {2, 5, 8}

"RG3" = {1, 4, 7}

$\pi 3$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 3$ = {16, 42, 61}

$u3$ = [2, 1, 1, 1, 2, 2, 0, 3, 1, 3, 0, 0, 0, 0, 0, 6, 5, 3, 0, 4, 3, 3, 6, 5, 3, 0, 0, 0, 0, 2, 0, 0, 0, 2, 0, 0, 3, 4, 4, 0, 5,
6, 4, 3, 4, 4, 0, 2, 2, 2, 0, 3, 4, 6, 2, 3, 4, 4, 3, 4, 6, 0, 0, 0, 0, 2, 2, 0, 0, 0, 0, 1, 3, 2, 2, 2, 0, 1, 1, 0, 1, 3, 2, 0]

supp u3 = {1, 2, 3, 4, 5, 6, 8, 9, 10, 16, 17, 18, 20, 21, 22, 23, 24, 25, 30, 34, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 66, 67, 72, 73, 74, 75, 76, 78, 79, 81, 82, 83}

Action of R on ranges, [[3], [3], [3]]
Action of B on ranges, [[2], [1], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{pmatrix}$

RPARTS [3, 3, 3]

BPARTS [2, 1, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[4, 8, 7, 7, 4, 7, 8, 4]

B-BLOCKS,

[6, 5, 7, 2, 2, 1, 3, 1]

with invariant measure, [2, 2, 2, 3, 1, 1, 4, 3]

N by blocks, N - check: true

b_1 = {1, 5, 9}
b_2 = {2, 4, 6}
b_3 = {3, 7, 8}
b_4 = {7, 8, 9}
b_5 = {1, 8, 9}
b_6 = {2, 3, 7}
b_7 = {4, 5, 6}
b_8 = {1, 2, 3}

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & h[1] & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & h[1] & 0 \\ 0 & 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & h[1] \\ h[1] & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 \\ 0 & 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & h[1] \\ h[1] & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 & 0 \\ 0 & h[1] & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 \\ 0 & 0 & h[1] & 0 & 0 & h[1] & 0 & 0 & h[2] \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4, 7}}, true

Ω_B in Vec(K)? , {{6, 8}, {2, 9}, {3, 5}}, true

$$V = \begin{pmatrix} \frac{1}{30} & \frac{-29}{45} & \frac{7}{90} & \frac{7}{30} & \frac{7}{45} & \frac{7}{90} & \frac{7}{30} & \frac{7}{45} & \frac{-29}{90} \\ \frac{-1}{10} & \frac{-2}{5} & \frac{1}{10} & \frac{3}{10} & \frac{1}{5} & \frac{1}{10} & \frac{3}{10} & \frac{1}{5} & \frac{-7}{10} \\ \frac{2}{15} & \frac{4}{45} & \frac{-16}{45} & \frac{13}{30} & \frac{-32}{45} & \frac{13}{90} & \frac{-1}{15} & \frac{13}{45} & \frac{2}{45} \\ \frac{-1}{15} & \frac{-2}{45} & \frac{8}{45} & \frac{1}{30} & \frac{16}{45} & \frac{-29}{90} & \frac{8}{15} & \frac{-29}{45} & \frac{-1}{45} \\ \frac{1}{5} & \frac{2}{15} & \frac{-8}{15} & \frac{-1}{10} & \frac{-1}{15} & \frac{-1}{30} & \frac{2}{5} & \frac{-1}{15} & \frac{1}{15} \\ \frac{-1}{15} & \frac{-2}{45} & \frac{8}{45} & \frac{1}{30} & \frac{16}{45} & \frac{-29}{90} & \frac{8}{15} & \frac{-29}{45} & \frac{-1}{45} \\ \frac{8}{15} & \frac{16}{45} & \frac{-19}{45} & \frac{7}{30} & \frac{-38}{45} & \frac{7}{90} & \frac{-4}{15} & \frac{7}{45} & \frac{8}{45} \\ \frac{2}{5} & \frac{4}{15} & \frac{-1}{15} & \frac{3}{10} & \frac{-2}{15} & \frac{-17}{30} & \frac{-1}{5} & \frac{-2}{15} & \frac{2}{15} \\ \frac{13}{30} & \frac{-17}{45} & \frac{1}{90} & \frac{1}{30} & \frac{1}{45} & \frac{1}{90} & \frac{1}{30} & \frac{1}{45} & \frac{-17}{90} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 5, 9}, {2, 4, 6}, {3, 7, 8}}

1, "range", [3, 6, 9], [[9, 6, 3, 6, 9, 6, 3, 3, 9], [9, 3, 6, 3, 9, 3, 6, 6, 9], [6, 9, 3, 9, 6, 9, 3, 3, 6], [6, 3, 9, 3, 6, 3, 9, 9, 6], [3, 9, 6, 9, 3, 9, 6, 6, 3], [3, 6, 9, 6, 3, 6, 9, 9, 3]]

2, "range", [2, 5, 8], [[8, 5, 2, 5, 8, 5, 2, 2, 8], [8, 2, 5, 2, 8, 2, 5, 5, 8], [5, 8, 2, 8, 5, 8, 2, 2, 5], [5, 2, 8, 2, 5, 2, 8, 8, 5], [2, 8, 5, 8, 2, 8, 5, 5, 2], [2, 5, 8, 5, 2, 5, 8, 8, 2]]

3, "range", [1, 4, 7], [[7, 4, 1, 4, 7, 4, 1, 1, 7], [7, 1, 4, 1, 7, 1, 4, 4, 7], [4, 7, 1, 7, 4, 7, 1, 1, 4], [4, 1, 7, 1, 4, 1, 7, 7, 4], [1, 7, 4, 7, 1, 7, 4, 4, 1], [1, 4, 7, 4, 1, 4, 7, 7, 1]]

2, "partition", {{1, 8, 9}, {2, 3, 7}, {4, 5, 6}}

1, "range", [3, 6, 9], [[9, 6, 6, 3, 3, 3, 6, 9, 9], [9, 3, 3, 6, 6, 6, 3, 9, 9], [6, 9, 9, 3, 3, 3, 9, 6, 6], [6, 3, 3, 9, 9, 9, 3, 6, 6], [3, 9, 9, 6, 6, 6, 9, 3, 3], [3, 6, 6, 9, 9, 9, 6, 3, 3]]

2, "range", [2, 5, 8], [[8, 5, 5, 2, 2, 2, 5, 8, 8], [8, 2, 2, 5, 5, 5, 2, 8, 8], [5, 8, 8, 2, 2, 2, 8, 5, 5], [5, 2, 2, 8, 8, 8, 2, 5, 5], [2, 8, 8, 5, 5, 5, 8, 2, 2], [2, 5, 5, 8, 8, 8, 5, 2, 2]]

3, "range", [1, 4, 7], [[7, 4, 4, 1, 1, 1, 4, 7, 7], [7, 1, 1, 4, 4, 4, 1, 7, 7], [4, 7, 7, 1, 1, 1, 7, 4, 4], [4, 1, 1, 7, 7, 7, 1, 4, 4], [1, 7, 7, 4, 4, 4, 7, 1, 1], [1, 4, 4, 7, 7, 7, 4, 1, 1]]

3, "partition", {{7, 8, 9}, {4, 5, 6}, {1, 2, 3}}

1, "range", [3, 6, 9], [[9, 9, 9, 6, 6, 6, 3, 3, 3], [9, 9, 9, 3, 3, 3, 6, 6, 6], [6, 6, 6, 9, 9, 9, 3, 3, 3], [6, 6, 6, 3, 3, 3, 9, 9, 9], [3, 3, 3, 9, 9, 9, 6, 6, 6], [3, 3, 3, 6, 6, 6, 9, 9, 9]]

2, "range", [2, 5, 8], [[8, 8, 8, 5, 5, 5, 2, 2, 2], [8, 8, 8, 2, 2, 2, 5, 5, 5], [5, 5, 5, 8, 8, 8, 2, 2, 2], [5, 5, 5, 2, 2, 2, 8, 8, 8], [2, 2, 2, 8, 8, 8, 5, 5, 5], [2, 2, 2, 5, 5, 5, 8, 8, 8]]

3, "range", [1, 4, 7], [[7, 7, 7, 4, 4, 4, 1, 1, 1], [7, 7, 7, 1, 1, 1, 4, 4, 4], [4, 4, 4, 7, 7, 7, 1, 1, 1], [4, 4, 4, 1, 1, 1, 7, 7, 7], [1, 1, 1, 7, 7, 7, 4, 4, 4], [1, 1, 1, 4, 4, 4, 7, 7, 7]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

g_1 = []

g_2 = [[1, 2]]

g_3 = [[2, 3]]

g_4 = [[1, 3, 2]]

g_5 = [[1, 2, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} 2h[1] & 0 & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

Z[1] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 2, 9]}, {8, [1, 3, 4]}, {9, [1, 3, 5]}, {10, [1, 3, 6]}, {11, [1, 3, 7]}, {12, [1, 3, 8]}, {13, [1, 3, 9]}, {14, [1, 4, 5]}, {15, [1, 4, 6]}, {16, [1, 4, 7]}, {17, [1, 4, 8]}, {18, [1, 4, 9]}, {19, [1, 5, 6]}, {20, [1, 5, 7]}, {21, [1, 5, 8]}, {22, [1, 5, 9]}, {23, [1, 6, 7]}, {24, [1, 6, 8]}, {25, [1, 6, 9]}, {26, [1, 7, 8]}, {27, [1, 7, 9]}, {28, [1, 8, 9]}, {29, [2, 3, 4]}, {30, [2, 3, 5]}, {31, [2, 3, 6]}, {32, [2, 3, 7]}, {33, [2, 3, 8]}, {34, [2, 3, 9]}, {35, [2, 4, 5]}, {36, [2, 4, 6]}, {37, [2, 4, 7]}, {38, [2, 4, 8]}, {39, [2, 4, 9]}, {40, [2, 5, 6]}, {41, [2, 5, 7]}, {42, [2, 5, 8]}, {43, [2, 5, 9]}, {44, [2, 6, 7]}, {45, [2, 6, 8]}, {46, [2, 6, 9]}, {47, [2, 7, 8]}, {48, [2, 7, 9]}, {49, [2, 8, 9]}, {50, [3, 4, 5]}, {51, [3, 4, 6]}, {52, [3, 4, 7]}, {53, [3, 4, 8]}, {54, [3, 4, 9]}, {55, [3, 5, 6]}, {56, [3, 5, 7]}, {57, [3, 5, 8]}, {58, [3, 5, 9]}, {59, [3, 6, 7]}, {60, [3, 6, 8]}, {61, [3, 6, 9]}, {62, [3, 7, 8]}, {63, [3, 7, 9]}, {64, [3, 8, 9]}, {65, [4, 5, 6]}, {66, [4, 5, 7]}, {67, [4, 5, 8]}, {68, [4, 5, 9]}, {69, [4, 6, 7]}, {70, [4, 6, 8]}, {71, [4, 6, 9]}, {72, [4, 7, 8]}, {73, [4, 7, 9]}, {74, [4, 8, 9]}, {75, [5, 6, 7]}, {76, [5, 6, 8]}, {77, [5, 6, 9]}, {78, [5, 7, 8]}, {79, [5, 7, 9]}, {80, [5, 8, 9]}, {81, [6, 7, 8]}, {82, [6, 7, 9]}, {83, [6, 8, 9]}, {84, [7, 8, 9]}

KERNEL HIERARCHY

$\pi 3 =$
$(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 3\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$

{16, 42, 61}

$u3 =$
$(2\ 1\ 1\ 1\ 2\ 2\ 0\ 3\ 1\ 3\ 0\ 0\ 0\ 0\ 0\ 6\ 5\ 3\ 0\ 4\ 3\ 3\ 6\ 5\ 3\ 0\ 0\ 0\ 0\ 2\ 0\ 0\ 0\ 2\ 0\ 0\ 3$

{1, 2, 3, 4, 5, 6, 8, 9, 10, 16, 17, 18, 20, 21, 22, 23, 24, 25, 30, 34, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 66, 67, 72, 73, 74, 75, 76, 78, 79, 81, 82, 83}

picheck $(3\ 2\ 1\ 3\ 2\ 1\ 3\ 2\ 1)$

$\pi = \left(\frac{1}{6}\ \frac{1}{9}\ \frac{1}{18}\ \frac{1}{6}\ \frac{1}{9}\ \frac{1}{18}\ \frac{1}{6}\ \frac{1}{9}\ \frac{1}{18}\right)$

$\pi 2 =$
$(0\ 0\ 3\ 0\ 0\ 3\ 0\ 0\ 0\ 0\ 2\ 0\ 0\ 2\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 3\ 0\ 0\ 0\ 0\ 2\ 0\ 0\ 0\ 1\ 0\ 0\ 0)$

$u2 =$
$\left(1\ 1\ 2\ \frac{4}{3}\ 2\ 2\ \frac{5}{3}\ 1\ \frac{2}{3}\ \frac{4}{3}\ 2\ \frac{4}{3}\ \frac{5}{3}\ 2\ 2\ 2\ 2\ 2\ 1\ \frac{4}{3}\ 2\ \frac{2}{3}\ 0\ 2\ 2\ 2\ \frac{2}{3}\ 2\ 2\ \frac{4}{3}\ 2\ 2\ 2\ \frac{1}{3}\ 1\ \frac{2}{3}\right)$

picheck $(6\ 4\ 2\ 6\ 4\ 2\ 6\ 4\ 2)$

$\pi 1 = (6\ 4\ 2\ 6\ 4\ 2\ 6\ 4\ 2)$

$u1 = \left(\frac{4}{3}\ \frac{4}{3}\ \frac{4}{3}\ \frac{4}{3}\ \frac{4}{3}\ \frac{4}{3}\ \frac{4}{3}\ \frac{4}{3}\ \frac{4}{3}\right)$

picheck $(6\ 4\ 2\ 6\ 4\ 2\ 6\ 4\ 2)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & \frac{1}{12} & 0 & \frac{1}{9} & 0 & 0 & \frac{1}{18} & \frac{1}{12} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{18} & \frac{1}{12} & 0 & 0 \\ \frac{1}{4} & \frac{2}{9} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{9} & 0 \\ 0 & \frac{1}{9} & 0 & \frac{1}{2} & \frac{2}{9} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{9} & 0 & 0 & \frac{1}{18} \\ 0 & \frac{1}{9} & 0 & \frac{1}{2} & \frac{2}{9} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{18} & \frac{1}{12} & 0 & 0 & 0 & \frac{1}{2} & \frac{5}{18} & \frac{1}{12} \\ \frac{1}{12} & 0 & \frac{1}{18} & 0 & 0 & 0 & \frac{5}{12} & \frac{1}{3} & \frac{1}{9} \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{9} & 0 & \frac{1}{4} & \frac{2}{9} & \frac{1}{6} \end{pmatrix} \quad NM = \begin{pmatrix} 9 & \frac{9}{2} & \frac{9}{4} & \frac{9}{2} & 4 & \frac{3}{2} & \frac{9}{2} & \frac{7}{2} & \frac{9}{4} \\ \frac{27}{4} & 6 & \frac{5}{2} & 6 & 3 & 2 & \frac{21}{4} & 3 & \frac{3}{2} \\ \frac{27}{4} & 5 & 3 & \frac{9}{2} & 3 & \frac{3}{2} & \frac{27}{4} & 4 & \frac{3}{2} \\ \frac{9}{2} & 4 & \frac{3}{2} & 9 & 5 & 3 & \frac{9}{2} & 3 & \frac{3}{2} \\ 6 & 3 & \frac{3}{2} & \frac{15}{2} & 6 & \frac{5}{2} & \frac{9}{2} & 3 & 2 \\ \frac{9}{2} & 4 & \frac{3}{2} & 9 & 5 & 3 & \frac{9}{2} & 3 & \frac{3}{2} \\ \frac{9}{2} & \frac{7}{2} & \frac{9}{4} & \frac{9}{2} & 3 & \frac{3}{2} & 9 & \frac{11}{2} & \frac{9}{4} \\ \frac{21}{4} & 3 & 2 & \frac{9}{2} & 3 & \frac{3}{2} & \frac{33}{4} & 6 & \frac{5}{2} \\ \frac{27}{4} & 3 & \frac{3}{2} & \frac{9}{2} & 4 & \frac{3}{2} & \frac{27}{4} & 5 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\Delta = [3, -2, -1, 3, -2, -1, 3, -2, -1]$

$$\ker N_C = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -t & t & 0 & -t & t & 0 & -t & t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\Delta$ via ker NC $\begin{pmatrix} 3 & -2 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} 0 & s & 0 & t & 0 & 0 \\ 0 & s & 0 & 0 & -t & -t \\ t & s & 0 & 0 & 0 & 0 \\ -t & -s & -s & -t & 0 & 0 \\ 0 & -s & -s & 0 & t & 0 \\ -t & -s & -s & -t & 0 & 0 \\ t & 0 & s & 0 & 0 & 0 \\ 0 & 0 & s & 0 & 0 & t \\ 0 & 0 & s & t & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} -1 & 1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & t & 0 & 0 & s & 0 & 0 \\ 0 & 0 & t & 0 & s & 0 & 0 \\ 0 & 0 & 0 & 0 & s & 0 & t \\ s & 0 & 0 & t & 0 & 0 & 0 \\ s & t & -t & t & 0 & -t & t \\ s & 0 & 0 & t & 0 & 0 & 0 \\ -s & s & 0 & s & -s & 0 & s+t \\ -s & s & 0 & s & -s & t & s \\ -s & s+t & 0 & s & -s & 0 & s \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 3 & 0 & 3 & 0 & 0 & 3 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 1 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 1 & 0 & 0 & 0 & 1 & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 0 & 1 & \frac{1}{3} & 1 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 1 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & 1 & \frac{1}{3} & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 1 & 0 & 0 & 0 & 1 & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{2}{3} & 1 & \frac{1}{3} \\ 1 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 \\ \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} \\ 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 \\ 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 \\ \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} \\ 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 \\ 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 \\ \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} \\ 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 \end{pmatrix} \quad \text{Skew } T = \begin{pmatrix} 0 & \frac{-1}{12} & \frac{-1}{6} & 0 & \frac{-1}{18} & 0 & 0 & \frac{-1}{36} & \frac{-1}{6} \\ \frac{1}{12} & 0 & \frac{-1}{9} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & 0 \\ \frac{1}{6} & \frac{1}{9} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & 0 & 0 & \frac{-1}{9} & \frac{-1}{3} & 0 & 0 & 0 \\ \frac{1}{18} & 0 & 0 & \frac{1}{9} & 0 & \frac{-1}{9} & 0 & 0 & \frac{-1}{18} \\ 0 & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{9} & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{36} & \frac{-1}{6} & 0 & 0 & 0 & 0 & \frac{-5}{36} & \frac{-1}{6} \\ \frac{1}{36} & 0 & \frac{-1}{18} & 0 & 0 & 0 & \frac{5}{36} & 0 & \frac{-1}{9} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{1}{18} & 0 & \frac{1}{6} & \frac{1}{9} & 0 \end{pmatrix} \quad \text{Skew}$$

$$\text{Omega} = \begin{pmatrix} 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} \\ \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} \\ \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 \\ \frac{1}{2} & \frac{2}{3} & 1 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 1 & \frac{2}{3} & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 1 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 1 & \frac{2}{3} & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 & 1 & \frac{5}{6} & \frac{1}{2} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{5}{6} & 1 & \frac{2}{3} \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} & \frac{2}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 9T + 27\Omega$$

Ω

$$\begin{pmatrix} 0 & 0 & 0 & \frac{1}{18} & \frac{1}{9} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{18} & \frac{1}{9} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{18} & \frac{1}{9} & \frac{1}{6} & \frac{1}{18} & \frac{1}{9} & \frac{1}{6} & \frac{1}{18} & \frac{1}{9} & \frac{1}{6} & \frac{1}{18} & \frac{1}{9} & \frac{1}{6} & \frac{1}{18} & \frac{1}{9} & \frac{1}{6} & \frac{1}{18} & \frac{1}{9} & \frac{1}{6} \end{pmatrix}$$

T

$$\left(0 \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{5}{12} \;\; \frac{1}{12} \;\; \frac{1}{18} \;\; \frac{1}{3} \;\; 0 \;\; \frac{1}{9} \;\; 0 \;\; 0 \;\; \frac{-1}{9} \;\; \frac{-1}{6} \;\; \frac{1}{6} \;\; \frac{2}{9} \;\; \frac{1}{4} \;\; \frac{1}{18} \;\; 0 \;\; \frac{1}{6} \;\; \frac{1}{9} \;\; \frac{1}{3} \;\; \frac{1}{4} \;\; \frac{1}{12} \;\; \frac{1}{18} \;\; 0 \;\; 0 \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{2} \right.$$

"IS NM in Vec(K)?", true

NM

$$\begin{pmatrix} 0 & 3 & 0 & \frac{5}{2} & 5 & \frac{33}{4} & \frac{3}{4} & \frac{1}{2} & 3 & \frac{3}{2} & 4 & \frac{9}{2} & 0 & -1 & \frac{-3}{2} & 3 & 5 & \frac{27}{4} & 2 & 3 & 6 & \frac{5}{2} & 6 & \frac{27}{4} & \frac{9}{4} & \frac{7}{2} & \frac{9}{2} & \frac{3}{2} & 4 & \frac{9}{2} & \frac{9}{4} & \frac{9}{2} & 9 \end{pmatrix}$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{3}{4}\ 3\ 0\ 5\ 5\ \frac{11}{2}\ \frac{3}{4}\ \frac{1}{2}\ \frac{11}{4}\ 3\ 4\ 3\ \frac{-3}{4}\ \frac{-5}{4}\ -1\ \frac{21}{4}\ \frac{9}{2}\ \frac{9}{2}\ 4\ 3\ 4\ 5\ 6\ \frac{9}{2}\ \frac{9}{2}\ 3\ \frac{9}{4}\ 3\ \frac{17}{4}\ 3\ \frac{9}{2}\ \frac{19}{4}\ \ldots\right.$$

$$\boldsymbol{\tau} = 27/1 \text{ , rank} = 3 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 27/1$$

$$\boldsymbol{\tau}' = 54/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 7/54 \text{ , min } \boldsymbol{\tau} = 21/2 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 33/2$$

$$\text{max r} = 54/7 \text{ , } r\text{-check is positive? } 11/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 27\Omega$$

There are, 3, partitions and, 3, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 33
out of total no. of elements equal to 54

dim span idems 9 vs no. of idems 9

"PT1" = {{1, 5, 9}, {2, 4, 6}, {3, 7, 8}}

"PT2" = {{1, 8, 9}, {2, 3, 7}, {4, 5, 6}}

"PT3" = {{7, 8, 9}, {4, 5, 6}, {1, 2, 3}}

"RG1" = {3, 6, 9}

"RG2" = {2, 5, 8}

"RG3" = {1, 4, 7}

$$M_c = \begin{pmatrix} \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} \\ \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} \\ \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} \\ \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} \\ \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} \\ \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} \\ \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} \\ \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} \\ \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{47}{54} & \frac{10}{27} & \frac{10}{27} & \frac{-7}{54} & \frac{11}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{1}{27} & \frac{10}{27} \\ \frac{10}{27} & \frac{47}{54} & \frac{29}{54} & \frac{11}{54} & \frac{-7}{54} & \frac{11}{54} & \frac{1}{27} & \frac{-7}{54} & \frac{-7}{54} \\ \frac{10}{27} & \frac{29}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{10}{27} & \frac{11}{54} & \frac{-7}{54} \\ \frac{-7}{54} & \frac{11}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{29}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} \\ \frac{11}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{29}{54} & \frac{47}{54} & \frac{29}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{11}{54} \\ \frac{-7}{54} & \frac{11}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{29}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} \\ \frac{-7}{54} & \frac{1}{27} & \frac{10}{27} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{19}{27} & \frac{10}{27} \\ \frac{1}{27} & \frac{-7}{54} & \frac{11}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{19}{27} & \frac{47}{54} & \frac{29}{54} \\ \frac{10}{27} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{11}{54} & \frac{-7}{54} & \frac{10}{27} & \frac{29}{54} & \frac{47}{54} \end{pmatrix}$$

$$M_c\text{-scaled} = \begin{pmatrix} 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\ \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} \\ \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 \\ 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\ \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} \\ \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 \\ 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\ \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} \\ \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 \end{pmatrix} \quad N_c\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{20}{47} & \frac{20}{47} & \frac{-7}{47} & \frac{11}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{2}{47} & \frac{20}{47} \\ \frac{20}{47} & 1 & \frac{29}{47} & \frac{11}{47} & \frac{-7}{47} & \frac{11}{47} & \frac{2}{47} & \frac{-7}{47} & \frac{-7}{47} \\ \frac{20}{47} & \frac{29}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{20}{47} & \frac{11}{47} & \frac{-7}{47} \\ \frac{-7}{47} & \frac{11}{47} & \frac{-7}{47} & 1 & \frac{29}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} \\ \frac{11}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{29}{47} & 1 & \frac{29}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{11}{47} \\ \frac{-7}{47} & \frac{11}{47} & \frac{-7}{47} & 1 & \frac{29}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} \\ \frac{-7}{47} & \frac{2}{47} & \frac{20}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & 1 & \frac{38}{47} & \frac{20}{47} \\ \frac{2}{47} & \frac{-7}{47} & \frac{11}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{38}{47} & 1 & \frac{29}{47} \\ \frac{20}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{11}{47} & \frac{-7}{47} & \frac{20}{47} & \frac{29}{47} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 10.95416346, 5.545836544]

Eigenvalues N_C

[0., 0., 0., 1.833333333, 0.2002036259, 0.4493916271, 1.191909004, 1.656493898, 2.502001845]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 5.170820393, 3.829179607]

Eigenvalues N_C–scaled

[0., 0., 0., 2.106382979, 0.2300211871, 0.5163222948, 1.369427366, 1.903205755, 2.874640418]

NullSpace M_C

{[−1, 0, 0, 1, 0, 0, 0, 0, 0], [1, 1, 1, 0, 0, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 0, 1, 0, 0], [1, 1, 0, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0, 0], [1, 1, 0, 0, 0, 1, 0, 0, 0]}

NullSpace N_C

{[−1, 1, 0, −1, 1, 0, −1, 1, 0], [−1, 0, 1, 0, 0, 0, −1, 0, 1], [0, 0, 0, −1, 0, 1, 0, 0, 0]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 9., 4.500000000, 13.50000000]

Eigenvalues N_0

[0., 0., 0., 3., 0.2002036259, 0.4493916271, 1.191909004, 1.656493898, 2.502001845]

NullSpace M_0

{[0, 0, 0, 0, 0, 1, 0, 0, −1], [−1, 0, 0, 0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0, 0, −1, 0], [0, 0, 1, 0, 0, 0, 0, 0, −1], [−1, 0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, −1, 0]}

NullSpace N_0

{[−1, 1, 0, 0, 1, −1, −1, 1, 0], [−1, 0, 1, 0, 0, 0, −1, 0, 1], [0, 0, 0, 1, 0, −1, 0, 0, 0]}

Eigenvalues M

[3., 6., 9., −3., −1.500000000, −4.500000000, −3., −1.500000000, −4.500000000]

Eigenvalues N

[0., 0., 0., 6., −0.2002036259, −0.4493916271, −1.191909004, −1.656493898, −2.502001845]

NullSpace M

{}

NullSpace N

{[0, 0, 0, −1, 0, 1, 0, 0, 0], [−1, 1, 0, −1, 1, 0, −1, 1, 0], [−1, 0, 1, 0, 0, 0, −1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 3 & 6 & 4 & 6 & 6 & 5 & 3 \\ 3 & 0 & 2 & 4 & 6 & 4 & 5 & 6 & 6 \\ 3 & 2 & 0 & 6 & 6 & 6 & 3 & 4 & 6 \\ 6 & 4 & 6 & 0 & 2 & 0 & 6 & 6 & 6 \\ 4 & 6 & 6 & 2 & 0 & 2 & 6 & 6 & 4 \\ 6 & 4 & 6 & 0 & 2 & 0 & 6 & 6 & 6 \\ 6 & 5 & 3 & 6 & 6 & 6 & 0 & 1 & 3 \\ 5 & 6 & 4 & 6 & 6 & 6 & 1 & 0 & 2 \\ 3 & 6 & 6 & 6 & 4 & 6 & 3 & 2 & 0 \end{pmatrix}$$

==

20, [1, 1, −1, 1, 1, 1, −1, 1, 1]
==

40, [1, −1, −1, 1, 1, −1, 1, 1, 1]

===

{2, 5, 8}

R: [4, 9, 4, 7, 3, 7, 1, 6, 1]
B: [2, 4, 5, 8, 7, 8, 5, 1, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 7

Level 2 det = $\frac{-25}{34359738368}(-1+s)(2151937075 + 3015496600s + 2210295050s^2 + 1094937238s^3 + 198995974s^4 - 123494394s^5 - 89983086s^6 - 17010978s^7 - 2752068s^8 - 1792258s^9 - 1659554s^{10} - 465118s^{11} - 42486s^{12} + 6322s^{13} + 10758s^{14} + 3050s^{15} + 1025s^{16} + 50s^{17})$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 3

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", 1 + v[1] v[4] v[7]
"B CYCLES", (1 + v[5] v[7]) (1 + v[1] v[2] v[4] v[8])

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [1. I, −1. I, 1., −1., 1., −1., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, 0, 0, −1, 0, 1], [0, 0, 0, 1, 0, −1, 0, 0, 0], [1, 0, −1, 0, 0, 0, 0, 0, 0]}

NullSpace of B^*

{[0, 0, −1, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, −1, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 9 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3 \\ 9 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 \\ 9 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{4}{5} & \frac{1}{2} & 1 & \frac{4}{5} & 1 & 1 & \frac{2}{5} & \frac{1}{2} \\ \frac{4}{5} & 0 & \frac{17}{20} & \frac{3}{5} & 1 & \frac{3}{5} & \frac{3}{5} & 1 & \frac{11}{20} \\ \frac{1}{2} & \frac{17}{20} & 0 & 1 & \frac{3}{4} & 1 & \frac{1}{2} & \frac{2}{5} & 1 \\ 1 & \frac{3}{5} & 1 & 0 & \frac{7}{10} & 0 & 1 & \frac{7}{10} & 1 \\ \frac{4}{5} & 1 & \frac{3}{4} & \frac{7}{10} & 0 & \frac{7}{10} & \frac{1}{2} & 1 & \frac{11}{20} \\ 1 & \frac{3}{5} & 1 & 0 & \frac{7}{10} & 0 & 1 & \frac{7}{10} & 1 \\ 1 & \frac{3}{5} & \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 & \frac{9}{10} & \frac{1}{2} \\ \frac{2}{5} & 1 & \frac{2}{5} & \frac{7}{10} & 1 & \frac{7}{10} & \frac{9}{10} & 0 & \frac{9}{10} \\ \frac{1}{2} & \frac{11}{20} & 1 & 1 & \frac{11}{20} & 1 & \frac{1}{2} & \frac{9}{10} & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 9

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}$ ($3v[1] + 2v[2] + v[3] + 3v[4] + 2v[5] + v[6] + 3v[7] + 2v[8] + v[9]$)

degree 2: $\frac{1}{6}$ ($3v[1]v[4] + 3v[1]v[7] + 2v[2]v[5] + 2v[2]v[8] + v[3]v[6] + v[3]v[9] + 3v[4]v[7] + 2v[5]v[8] + v[6]v[9]$)

degree 3 : $\frac{1}{2}$ ($3v[1]v[4]v[7] + 2v[2]v[5]v[8] + v[3]v[6]v[9]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{3, 7, 8}, {4, 5, 6}, {1, 2, 9}}

"PT2" = {{3, 5, 7}, {1, 2, 9}, {4, 6, 8}}

"PT3" = {{1, 5, 9}, {2, 4, 6}, {3, 7, 8}}

"PT4" = {{3, 5, 7}, {2, 4, 6}, {1, 8, 9}}

"PT5" = {{1, 5, 9}, {2, 3, 7}, {4, 6, 8}}

"PT6" = {{1, 8, 9}, {2, 3, 7}, {4, 5, 6}}

"PT7" = {{1, 3, 8}, {4, 5, 6}, {2, 7, 9}}

"PT8" = {{1, 3, 8}, {2, 4, 6}, {5, 7, 9}}

"RG1" = {3, 6, 9}

"RG2" = {2, 5, 8}

"RG3" = {1, 4, 7}

$\pi 3$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 3$ = {16, 42, 61}

$u3$ = [3, 8, 12, 8, 8, 4, 5, 10, 1, 10, 0, 6, 0, 10, 0, 20, 2, 10, 10, 6, 4, 5, 20, 2, 10, 6, 0, 0, 9, 12, 9, 5, 5, 8, 6, 0, 4, 6, 3, 6, 2, 20, 2, 4, 6, 3, 10, 3, 9, 9, 0, 10, 2, 20, 9, 5, 3, 6, 10, 2, 20, 0, 0, 6, 0, 4, 8, 5, 0, 0, 0, 12, 10, 12, 4, 8, 5, 8, 1, 9, 12, 10, 12, 6]

supp u3 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84}

Action of R on ranges, [[3], [1], [3]]
Action of B on ranges, [[2], [3], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{pmatrix}$

RPARTS [7, 7, 7, 7, 7, 7, 8, 8]

BPARTS [6, 4, 1, 2, 3, 5, 5, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{30} & \frac{1}{6} & \frac{1}{15} & \frac{1}{12} & \frac{2}{15} & \frac{1}{60} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[8, 9, B, 2, 8, B, 8, 2, 4, B, 4, 2]

B–BLOCKS,

[1, C, 5, A, 8, C, 3, 7, 1, 6, 3, 4]

with invariant measure, [5, 10, 4, 8, 2, 2, 3, 6, 5, 4, 5, 6]

N by blocks, N – check: true

b_1 = {3, 5, 7}
b_2 = {1, 3, 8}
b_3 = {1, 5, 9}
b_4 = {2, 4, 6}
b_5 = {3, 7, 8}
b_6 = {1, 8, 9}
b_7 = {2, 3, 7}
b_8 = {4, 5, 6}
b_9 = {5, 7, 9}
b_{10} = {1, 2, 9}
b_{11} = {2, 7, 9}
b_{12} = {4, 6, 8}

dim(span of partition vectors), rank(N_0), rank(N):   6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & h[2] & 0 & 0 & h[2] & 0 & 0 \\ 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & h[2] & 0 \\ 0 & 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & h[2] \\ h[2] & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 & 0 \\ 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 \\ 0 & 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & h[2] \\ h[2] & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[2] & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 \\ 0 & 0 & h[2] & 0 & 0 & h[2] & 0 & 0 & h[1] \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4, 7}}, true

Ω_B in Vec(K)? , {{1, 2, 4, 8}, {5, 7}}, true

$$V = \begin{pmatrix} \frac{1}{30} & \frac{-29}{45} & \frac{7}{90} & \frac{7}{30} & \frac{7}{45} & \frac{7}{90} & \frac{7}{30} & \frac{7}{45} & \frac{-29}{90} \\ \frac{1}{10} & \frac{2}{5} & \frac{-1}{10} & \frac{-3}{10} & \frac{-1}{5} & \frac{-1}{10} & \frac{-3}{10} & \frac{-1}{5} & \frac{7}{10} \\ \frac{2}{15} & \frac{4}{45} & \frac{-16}{45} & \frac{13}{30} & \frac{-32}{45} & \frac{13}{90} & \frac{-1}{15} & \frac{13}{45} & \frac{2}{45} \\ \frac{-1}{15} & \frac{-2}{45} & \frac{8}{45} & \frac{1}{30} & \frac{16}{45} & \frac{-29}{90} & \frac{8}{15} & \frac{-29}{45} & \frac{-1}{45} \\ \frac{-1}{5} & \frac{-2}{15} & \frac{8}{15} & \frac{1}{10} & \frac{1}{15} & \frac{1}{30} & \frac{-2}{5} & \frac{1}{15} & \frac{-1}{15} \\ \frac{-1}{15} & \frac{-2}{45} & \frac{8}{45} & \frac{1}{30} & \frac{16}{45} & \frac{-29}{90} & \frac{8}{15} & \frac{-29}{45} & \frac{-1}{45} \\ \frac{8}{15} & \frac{16}{45} & \frac{-19}{45} & \frac{7}{30} & \frac{-38}{45} & \frac{7}{90} & \frac{-4}{15} & \frac{7}{45} & \frac{8}{45} \\ \frac{-2}{5} & \frac{-4}{15} & \frac{1}{15} & \frac{-3}{10} & \frac{2}{15} & \frac{17}{30} & \frac{1}{5} & \frac{2}{15} & \frac{-2}{15} \\ \frac{13}{30} & \frac{-17}{45} & \frac{1}{90} & \frac{1}{30} & \frac{1}{45} & \frac{1}{90} & \frac{1}{30} & \frac{1}{45} & \frac{-17}{90} \end{pmatrix}$$

Omega–checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 7, 8}, {4, 5, 6}, {1, 2, 9}}

1, "range", [3, 6, 9], [[9, 9, 6, 3, 3, 3, 6, 6, 9], [9, 9, 3, 6, 6, 6, 3, 3, 9], [6, 6, 9, 3, 3, 3, 9, 9, 6], [6, 6, 3, 9, 9, 9, 3, 3, 6], [3, 3, 9, 6, 6, 6, 9, 9, 3], [3, 3, 6, 9, 9, 9, 6, 6, 3]]

2, "range", [2, 5, 8], [[8, 8, 5, 2, 2, 2, 5, 5, 8], [8, 8, 2, 5, 5, 5, 2, 2, 8], [5, 5, 8, 2, 2, 2, 8, 8, 5], [5, 5, 2, 8, 8, 8, 2, 2, 5], [2, 2, 8, 5, 5, 5, 8, 8, 2], [2, 2, 5, 8, 8, 8, 5, 5, 2]]

3, "range", [1, 4, 7], [[7, 7, 4, 1, 1, 1, 4, 4, 7], [7, 7, 1, 4, 4, 4, 1, 1, 7], [4, 4, 7, 1, 1, 1, 7, 7, 4], [4, 4, 1, 7, 7, 7, 1, 1, 4], [1, 1, 7, 4, 4, 4, 7, 7, 1], [1, 1, 4, 7, 7, 7, 4, 4, 1]]

2, "partition", {{3, 5, 7}, {1, 2, 9}, {4, 6, 8}}

1, "range", [3, 6, 9], [[9, 9, 6, 3, 6, 3, 6, 3, 9], [9, 9, 3, 6, 3, 6, 3, 6, 9], [6, 6, 9, 3, 9, 3, 9, 3, 6], [6, 6, 3, 9, 3, 9, 3, 9, 6], [3, 3, 9, 6, 9, 6, 9, 6, 3], [3, 3, 6, 9, 6, 9, 6, 9, 3]]

2, "range", [2, 5, 8], [[8, 8, 5, 2, 5, 2, 5, 2, 8], [8, 8, 2, 5, 2, 5, 2, 5, 8], [5, 5, 8, 2, 8, 2, 8, 2, 5], [5, 5, 2, 8, 2, 8, 2, 8, 5], [2, 2, 8, 5, 8, 5, 8, 5, 2], [2, 2, 5, 8, 5, 8, 5, 8, 2]]

3, "range", [1, 4, 7], [[7, 7, 4, 1, 4, 1, 4, 1, 7], [7, 7, 1, 4, 1, 4, 1, 4, 7], [4, 4, 7, 1, 7, 1, 7, 1, 4], [4, 4, 1, 7, 1, 7, 1, 7, 4], [1, 1, 7, 4, 7, 4, 7, 4, 1], [1, 1, 4, 7, 4, 7, 4, 7, 1]]

3, "partition", {{1, 5, 9}, {2, 4, 6}, {3, 7, 8}}

1, "range", [3, 6, 9], [[9, 6, 3, 6, 9, 6, 3, 3, 9], [9, 3, 6, 3, 9, 3, 6, 6, 9], [6, 9, 3, 9, 6, 9, 3, 3, 6], [6, 3, 9, 3, 6, 3, 9, 9, 6], [3, 9, 6, 9, 3, 9, 6, 6, 3], [3, 6, 9, 6, 3, 6, 9, 9, 3]]

2, "range", [2, 5, 8], [[8, 5, 2, 5, 8, 5, 2, 2, 8], [8, 2, 5, 2, 8, 2, 5, 5, 8], [5, 8, 2, 8, 5, 8, 2, 2, 5], [5, 2, 8, 2, 5, 2, 8, 8, 5], [2, 8, 5, 8, 2, 8, 5, 5, 2], [2, 5, 8, 5, 2, 5, 8, 8, 2]]

3, "range", [1, 4, 7], [[7, 4, 1, 4, 7, 4, 1, 1, 7], [7, 1, 4, 1, 7, 1, 4, 4, 7], [4, 7, 1, 7, 4, 7, 1, 1, 4], [4, 1, 7, 1, 4, 1, 7, 7, 4], [1, 7, 4, 7, 1, 7, 4, 4, 1], [1, 4, 7, 4, 1, 4, 7, 7, 1]]

4, "partition", {{3, 5, 7}, {2, 4, 6}, {1, 8, 9}}

1, "range", [3, 6, 9], [[9, 6, 3, 6, 3, 6, 3, 9, 9], [9, 3, 6, 3, 6, 3, 6, 9, 9], [6, 9, 3, 9, 3, 9, 3, 6, 6], [6, 3, 9, 3, 9, 3, 9, 6, 6], [3, 9, 6, 9, 6, 9, 6, 3, 3], [3, 6, 9, 6, 9, 6, 9, 3, 3]]

2, "range", [2, 5, 8], [[8, 5, 2, 5, 2, 5, 2, 8, 8], [8, 2, 5, 2, 5, 2, 5, 8, 8], [5, 8, 2, 8, 2, 8, 2, 5, 5], [5, 2, 8, 2, 8, 2, 8, 5, 5], [2, 8, 5, 8, 5, 8, 5, 2, 2], [2, 5, 8, 5, 8, 5, 8, 2, 2]]

3, "range", [1, 4, 7], [[7, 4, 1, 4, 1, 4, 1, 7, 7], [7, 1, 4, 1, 4, 1, 4, 7, 7], [4, 7, 1, 7, 1, 7, 1, 4, 4], [4, 1, 7, 1, 7, 1, 7, 4, 4], [1, 7, 4, 7, 4, 7, 4, 1, 1], [1, 4, 7, 4, 7, 4, 7, 1, 1]]

5, "partition", {{1, 5, 9}, {2, 3, 7}, {4, 6, 8}}

1, "range", [3, 6, 9], [[9, 6, 6, 3, 9, 3, 6, 3, 9], [9, 3, 3, 6, 9, 6, 3, 6, 9], [6, 9, 9, 3, 6, 3, 9, 3, 6], [6, 3, 3, 9, 6, 9, 3, 9, 6], [3, 9, 9, 6, 3, 6, 9, 6, 3], [3, 6, 6, 9, 3, 9, 6, 9, 3]]

2, "range", [2, 5, 8], [[8, 5, 5, 2, 8, 2, 5, 2, 8], [8, 2, 2, 5, 8, 5, 2, 5, 8], [5, 8, 8, 2, 5, 2, 8, 2, 5], [5, 2, 2, 8, 5, 8, 2, 8, 5], [2, 8, 8, 5, 2, 5, 8, 5, 2], [2, 5, 5, 8, 2, 8, 5, 8, 2]]

3, "range", [1, 4, 7], [[7, 4, 4, 1, 7, 1, 4, 1, 7], [7, 1, 1, 4, 7, 4, 1, 4, 7], [4, 7, 7, 1, 4, 1, 7, 1, 4], [4, 1, 1, 7, 4, 7, 1, 7, 4], [1, 7, 7, 4, 1, 4, 7, 4, 1], [1, 4, 4, 7, 1, 7, 4, 7, 1]]

6, "partition", {{1, 8, 9}, {2, 3, 7}, {4, 5, 6}}

1, "range", [3, 6, 9], [[9, 6, 6, 3, 3, 3, 6, 9, 9], [9, 3, 3, 6, 6, 6, 3, 9, 9], [6, 9, 9, 3, 3, 3, 9, 6, 6], [6, 3, 3, 9, 9, 9, 3, 6, 6], [3, 9, 9, 6, 6, 6, 9, 3, 3], [3, 6, 6, 9, 9, 9, 6, 3, 3]]

2, "range", [2, 5, 8], [[8, 5, 5, 2, 2, 2, 5, 8, 8], [8, 2, 2, 5, 5, 5, 2, 8, 8], [5, 8, 8, 2, 2, 2, 8, 5, 5], [5, 2, 2, 8, 8, 8, 2, 5, 5], [2, 8, 8, 5, 5, 5, 8, 2, 2], [2, 5, 5, 8, 8, 8, 5, 2, 2]]

3, "range", [1, 4, 7], [[7, 4, 4, 1, 1, 1, 4, 7, 7], [7, 1, 1, 4, 4, 4, 1, 7, 7], [4, 7, 7, 1, 1, 1, 7, 4, 4], [4, 1, 1, 7, 7, 7, 1, 4, 4], [1, 7, 7, 4, 4, 4, 7, 1, 1], [1, 4, 4, 7, 7, 7, 4, 1, 1]]

7, "partition", {{1, 3, 8}, {4, 5, 6}, {2, 7, 9}}

1, "range", [3, 6, 9], [[9, 6, 9, 3, 3, 3, 6, 9, 6], [9, 3, 9, 6, 6, 6, 3, 9, 3], [6, 9, 6, 3, 3, 3, 9, 6, 9], [6, 3, 6, 9, 9, 9, 3, 6, 3], [3, 9, 3, 6, 6, 6, 9, 3, 9], [3, 6, 3, 9, 9, 9, 6, 3, 6]]

2, "range", [2, 5, 8], [[8, 5, 8, 2, 2, 2, 5, 8, 5], [8, 2, 8, 5, 5, 5, 2, 8, 2], [5, 8, 5, 2, 2, 2, 8, 5, 8], [5, 2, 5, 8, 8, 8, 2, 5, 2], [2, 8, 2, 5, 5, 5, 8, 2, 8], [2, 5, 2, 8, 8, 8, 5, 2, 5]]

3, "range", [1, 4, 7], [[7, 4, 7, 1, 1, 1, 4, 7, 4], [7, 1, 7, 4, 4, 4, 1, 7, 1], [4, 7, 4, 1, 1, 1, 7, 4, 7], [4, 1, 4, 7, 7, 7, 1, 4, 1], [1, 7, 1, 4, 4, 4, 7, 1, 7], [1, 4, 1, 7, 7, 7, 4, 1, 4]]

8, "partition", {{1, 3, 8}, {2, 4, 6}, {5, 7, 9}}

1, "range", [3, 6, 9], [[9, 6, 9, 6, 3, 6, 3, 9, 3], [9, 3, 9, 3, 6, 3, 6, 9, 6], [6, 9, 6, 9, 3, 9, 3, 6, 3], [6, 3, 6, 3, 9, 3, 9, 6, 9], [3, 9, 3, 9, 6, 9, 6, 3, 6], [3, 6, 3, 6, 9, 6, 9, 3, 9]]

2, "range", [2, 5, 8], [[8, 5, 8, 5, 2, 5, 2, 8, 2], [8, 2, 8, 2, 5, 2, 5, 8, 5], [5, 8, 5, 8, 2, 8, 2, 5, 2], [5, 2, 5, 2, 8, 2, 8, 5, 8], [2, 8, 2, 8, 5, 8, 5, 2, 5], [2, 5, 2, 5, 8, 5, 8, 2, 8]]

3, "range", [1, 4, 7], [[7, 4, 7, 4, 1, 4, 1, 7, 1], [7, 1, 7, 1, 4, 1, 4, 7, 4], [4, 7, 4, 7, 1, 7, 1, 4, 1], [4, 1, 4, 1, 7, 1, 7, 4, 7], [1, 7, 1, 7, 4, 7, 4, 1, 4], [1, 4, 1, 4, 7, 4, 7, 1, 7]]

"group has", 6, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

$g_3 =$ [[1, 3, 2]]

$g_4 =$ [[2, 3]]

$g_5 =$ [[1, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 2h[1]-h[2] & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 2, 9]}, {8, [1, 3, 4]}, {9, [1, 3, 5]}, {10, [1, 3, 6]}, {11, [1, 3, 7]}, {12, [1, 3, 8]}, {13, [1, 3, 9]}, {14, [1, 4, 5]}, {15, [1, 4, 6]}, {16, [1, 4, 7]}, {17, [1, 4, 8]}, {18, [1, 4, 9]}, {19, [1, 5, 6]}, {20, [1, 5, 7]}, {21, [1, 5, 8]}, {22, [1, 5, 9]}, {23, [1, 6, 7]}, {24, [1, 6, 8]}, {25, [1, 6, 9]}, {26, [1, 7, 8]}, {27, [1, 7, 9]}, {28, [1, 8, 9]}, {29, [2, 3, 4]}, {30, [2, 3, 5]}, {31, [2, 3, 6]}, {32, [2, 3, 7]}, {33, [2, 3, 8]}, {34, [2, 3, 9]}, {35, [2, 4, 5]}, {36, [2, 4, 6]}, {37, [2, 4, 7]}, {38, [2, 4, 8]}, {39, [2, 4, 9]}, {40, [2, 5, 6]}, {41, [2, 5, 7]}, {42, [2, 5, 8]}, {43, [2, 5, 9]}, {44, [2, 6, 7]}, {45, [2, 6, 8]}, {46, [2, 6, 9]}, {47, [2, 7, 8]}, {48, [2, 7, 9]}, {49, [2, 8, 9]}, {50, [3, 4, 5]}, {51, [3, 4, 6]}, {52, [3, 4, 7]}, {53, [3, 4, 8]}, {54, [3, 4, 9]}, {55, [3, 5, 6]}, {56, [3, 5, 7]}, {57, [3, 5, 8]}, {58, [3, 5, 9]}, {59, [3, 6, 7]}, {60, [3, 6, 8]}, {61, [3, 6, 9]}, {62, [3, 7, 8]}, {63, [3, 7, 9]}, {64, [3, 8, 9]}, {65, [4, 5, 6]}, {66, [4, 5, 7]}, {67, [4, 5, 8]}, {68, [4, 5, 9]}, {69, [4, 6, 7]}, {70, [4, 6, 8]}, {71, [4, 6, 9]}, {72, [4, 7, 8]}, {73, [4, 7, 9]}, {74, [4, 8, 9]}, {75, [5, 6, 7]}, {76, [5, 6, 8]}, {77, [5, 6, 9]}, {78, [5, 7, 8]}, {79, [5, 7, 9]}, {80, [5, 8, 9]}, {81, [6, 7, 8]}, {82, [6, 7, 9]}, {83, [6, 8, 9]}, {84, [7, 8, 9]}

KERNEL HIERARCHY

$\pi 3 =$
$\left(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 3\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right.$

{16, 42, 61}

$u3 =$
$\left(3\ 8\ 12\ 8\ 8\ 4\ 5\ 10\ 1\ 10\ 0\ 6\ 0\ 10\ 0\ 20\ 2\ 10\ 10\ 6\ 4\ 5\ 20\ 2\ 10\ 6\ 0\ 0\ 9\ 12\ 9\ 5\ 5\right.$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84}

picheck $\left(3\ 2\ 1\ 3\ 2\ 1\ 3\ 2\ 1\right)$

$\pi = \left(\frac{1}{6}\ \frac{1}{9}\ \frac{1}{18}\ \frac{1}{6}\ \frac{1}{9}\ \frac{1}{18}\ \frac{1}{6}\ \frac{1}{9}\ \frac{1}{18}\right)$

$\pi 2 =$
$\left(0\ 0\ 3\ 0\ 0\ 3\ 0\ 0\ 0\ 0\ 2\ 0\ 0\ 2\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 3\ 0\ 0\ 0\ 0\ 2\ 0\ 0\ 0\ 1\ 0\ 0\ 0\right)$

$u2 =$
$\left(\frac{16}{3}\ \frac{10}{3}\ \frac{20}{3}\ \frac{16}{3}\ \frac{20}{3}\ \frac{20}{3}\ \frac{8}{3}\ \frac{10}{3}\ \frac{17}{3}\ 4\ \frac{20}{3}\ 4\ 4\ \frac{20}{3}\ \frac{11}{3}\ \frac{20}{3}\ 5\ \frac{20}{3}\ \frac{10}{3}\ \frac{8}{3}\ \frac{20}{3}\ \frac{14}{3}\ 0\ \frac{20}{3}\ \frac{14}{3}\ \frac{20}{3}\ \frac{14}{3}\ \frac{10}{3}\right.$

picheck $\left(6\ 4\ 2\ 6\ 4\ 2\ 6\ 4\ 2\right)$

$\pi 1 = \left(6\ 4\ 2\ 6\ 4\ 2\ 6\ 4\ 2\right)$

$u1 = \left(\frac{40}{9}\ \frac{40}{9}\ \frac{40}{9}\ \frac{40}{9}\ \frac{40}{9}\ \frac{40}{9}\ \frac{40}{9}\ \frac{40}{9}\ \frac{40}{9}\right)$

picheck $\left(6\ 4\ 2\ 6\ 4\ 2\ 6\ 4\ 2\right)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{3}{20} & 0 & 0 & \frac{2}{5} & 0 & 0 & \frac{9}{20} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{3}{10} & 0 & 0 & \frac{9}{20} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{3}{5} & 0 & 0 & \frac{3}{10} & 0 & 0 & \frac{1}{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & 0 & \frac{3}{5} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{20} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{3}{5} & 0 \\ 0 & \frac{2}{5} & 0 & 0 & \frac{3}{10} & 0 & 0 & \frac{3}{10} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{5} & 0 & 0 & \frac{3}{10} & 0 & 0 & \frac{3}{10} & 0 \\ 0 & \frac{2}{5} & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{9}{20} & 0 & 0 & \frac{9}{20} & 0 & 0 & \frac{1}{10} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{5} & 0 & 0 & \frac{2}{5} & 0 & 0 & \frac{2}{5} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{5} & 0 & 0 & \frac{3}{10} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{3}{5} & 0 & 0 & \frac{3}{10} & 0 & 0 & \frac{1}{10} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_7 = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} \\ \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} \\ \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_8 = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} \\ \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{15} & \frac{1}{12} & 0 & \frac{1}{15} & 0 & 0 & \frac{1}{5} & \frac{1}{12} \\ \frac{1}{10} & \frac{1}{3} & \frac{1}{40} & \frac{1}{5} & 0 & \frac{1}{15} & \frac{1}{5} & 0 & \frac{3}{40} \\ \frac{1}{4} & \frac{1}{20} & \frac{1}{6} & 0 & \frac{1}{12} & 0 & \frac{1}{4} & \frac{1}{5} & 0 \\ 0 & \frac{2}{15} & 0 & \frac{1}{2} & \frac{1}{10} & \frac{1}{6} & 0 & \frac{1}{10} & 0 \\ \frac{1}{10} & 0 & \frac{1}{24} & \frac{3}{20} & \frac{1}{3} & \frac{1}{20} & \frac{1}{4} & 0 & \frac{3}{40} \\ 0 & \frac{2}{15} & 0 & \frac{1}{2} & \frac{1}{10} & \frac{1}{6} & 0 & \frac{1}{10} & 0 \\ 0 & \frac{2}{15} & \frac{1}{12} & 0 & \frac{1}{6} & 0 & \frac{1}{2} & \frac{1}{30} & \frac{1}{12} \\ \frac{3}{10} & 0 & \frac{1}{10} & \frac{3}{20} & 0 & \frac{1}{20} & \frac{1}{20} & \frac{1}{3} & \frac{1}{60} \\ \frac{1}{4} & \frac{3}{20} & 0 & 0 & \frac{3}{20} & 0 & \frac{1}{4} & \frac{1}{30} & \frac{1}{6} \end{pmatrix} \quad NM = \begin{pmatrix} 9 & \frac{18}{5} & \frac{9}{4} & \frac{9}{2} & \frac{18}{5} & \frac{3}{2} & \frac{9}{2} & \frac{24}{5} & \frac{9}{4} \\ \frac{27}{5} & 6 & \frac{69}{40} & \frac{63}{10} & 3 & \frac{21}{10} & \frac{63}{10} & 3 & \frac{87}{40} \\ \frac{27}{4} & \frac{69}{20} & 3 & \frac{9}{2} & \frac{15}{4} & \frac{3}{2} & \frac{27}{4} & \frac{24}{5} & \frac{3}{2} \\ \frac{9}{2} & \frac{21}{5} & \frac{3}{2} & 9 & \frac{39}{10} & 3 & \frac{9}{2} & \frac{39}{10} & \frac{3}{2} \\ \frac{27}{5} & 3 & \frac{15}{8} & \frac{117}{20} & 6 & \frac{39}{20} & \frac{27}{4} & 3 & \frac{87}{40} \\ \frac{9}{2} & \frac{21}{5} & \frac{3}{2} & 9 & \frac{39}{10} & 3 & \frac{9}{2} & \frac{39}{10} & \frac{3}{2} \\ \frac{9}{2} & \frac{21}{5} & \frac{9}{4} & \frac{9}{2} & \frac{9}{2} & \frac{3}{2} & 9 & \frac{33}{10} & \frac{9}{4} \\ \frac{36}{5} & 3 & \frac{12}{5} & \frac{117}{20} & 3 & \frac{39}{20} & \frac{99}{20} & 6 & \frac{33}{20} \\ \frac{27}{4} & \frac{87}{20} & \frac{3}{2} & \frac{9}{2} & \frac{87}{20} & \frac{3}{2} & \frac{27}{4} & \frac{33}{10} & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -2, 1, 1, -2, 1, 1, -2, 1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & -1 & 1 & 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s+t & -t & s & -s+t & -t & s & -s+t & -t & s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -2 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -s & -t & 0 & -s & -t & 0 \\ -t & 0 & 0 & -t & 0 & s \\ -s & t & 0 & -s & 0 & 0 \\ 0 & 0 & 0 & s & t & 0 \\ 0 & 0 & -s & t & 0 & -s \\ 0 & 0 & 0 & s & t & 0 \\ s & t & 0 & 0 & 0 & 0 \\ t & 0 & s & 0 & 0 & 0 \\ s & -t & 0 & 0 & -t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & 0 & 0 & t & 0 \\ t & 0 & 0 & 0 & s & 0 & 0 \\ s & t & 0 & 0 & 0 & 0 & 0 \\ -s & s & s+t & -s & 0 & s & 0 \\ -t & t & t & -t & 0 & t & s \\ -s & s & s+t & -s & 0 & s & 0 \\ 0 & t & 0 & s & 0 & 0 & 0 \\ 0 & s & s & t & -s & s & -s \\ 0 & 0 & 0 & s & 0 & t & 0 \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 3 & 3 & 0 & 0 & 3 & 0 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 1 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 1 & \frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 1 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & \frac{2}{5} & 0 & 0 & \frac{2}{5} & 0 & 0 & \frac{1}{5} & 1 \\ \frac{2}{5} & 1 & \frac{3}{10} & \frac{3}{10} & 0 & \frac{3}{10} & \frac{3}{10} & 0 & \frac{2}{5} \\ 0 & \frac{3}{10} & 1 & 0 & \frac{1}{2} & 0 & 1 & \frac{1}{5} & 0 \\ 0 & \frac{3}{10} & 0 & 1 & \frac{1}{10} & 1 & 0 & \frac{3}{5} & 0 \\ \frac{2}{5} & 0 & \frac{1}{2} & \frac{1}{10} & 1 & \frac{1}{10} & \frac{1}{2} & 0 & \frac{2}{5} \\ 0 & \frac{3}{10} & 0 & 1 & \frac{1}{10} & 1 & 0 & \frac{3}{5} & 0 \\ 0 & \frac{3}{10} & 1 & 0 & \frac{1}{2} & 0 & 1 & \frac{1}{5} & 0 \\ \frac{1}{5} & 0 & \frac{1}{5} & \frac{3}{5} & 0 & \frac{3}{5} & \frac{1}{5} & 1 & \frac{1}{5} \\ 1 & \frac{2}{5} & 0 & 0 & \frac{2}{5} & 0 & 0 & \frac{1}{5} & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 3

$$CNM = \begin{pmatrix} 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 \\ \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} \\ 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 \\ 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 \\ \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} \\ 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 \\ 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 \\ \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} \\ 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{-1}{30} & \frac{-1}{6} & 0 & \frac{-1}{30} & 0 & 0 & \frac{-1}{10} & \frac{-1}{6} \\ \frac{1}{30} & 0 & \frac{-1}{40} & \frac{1}{15} & 0 & \frac{-1}{15} & \frac{1}{15} & 0 & \frac{-3}{40} \\ \frac{1}{6} & \frac{1}{40} & 0 & 0 & \frac{1}{24} & 0 & \frac{1}{6} & \frac{1}{10} & 0 \\ 0 & \frac{-1}{15} & 0 & 0 & \frac{-1}{20} & \frac{-1}{3} & 0 & \frac{-1}{20} & 0 \\ \frac{1}{30} & 0 & \frac{-1}{24} & \frac{1}{20} & 0 & \frac{-1}{20} & \frac{1}{12} & 0 & \frac{-3}{40} \\ 0 & \frac{1}{15} & 0 & \frac{1}{3} & \frac{1}{20} & 0 & 0 & \frac{1}{20} & 0 \\ 0 & \frac{-1}{15} & \frac{-1}{6} & 0 & \frac{-1}{12} & 0 & 0 & \frac{-1}{60} & \frac{-1}{6} \\ \frac{1}{10} & 0 & \frac{-1}{10} & \frac{1}{20} & 0 & \frac{-1}{20} & \frac{1}{60} & 0 & \frac{-1}{60} \\ \frac{1}{6} & \frac{3}{40} & 0 & 0 & \frac{3}{40} & 0 & \frac{1}{6} & \frac{1}{60} & 0 \end{pmatrix} \quad \text{Skew}$$

$$\text{Omega} = \begin{pmatrix} 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} \\ \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} \\ \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{5} & \frac{1}{2} & 0 & \frac{1}{5} & 0 & 0 & \frac{3}{5} & \frac{1}{2} \\ \frac{1}{5} & 1 & \frac{3}{20} & \frac{2}{5} & 0 & \frac{2}{5} & \frac{2}{5} & 0 & \frac{9}{20} \\ \frac{1}{2} & \frac{3}{20} & 1 & 0 & \frac{1}{4} & 0 & \frac{1}{2} & \frac{3}{5} & 0 \\ 0 & \frac{2}{5} & 0 & 1 & \frac{3}{10} & 1 & 0 & \frac{3}{10} & 0 \\ \frac{1}{5} & 0 & \frac{1}{4} & \frac{3}{10} & 1 & \frac{3}{10} & \frac{1}{2} & 0 & \frac{9}{20} \\ 0 & \frac{2}{5} & 0 & 1 & \frac{3}{10} & 1 & 0 & \frac{3}{10} & 0 \\ 0 & \frac{2}{5} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 1 & \frac{1}{10} & \frac{1}{2} \\ \frac{3}{5} & 0 & \frac{3}{5} & \frac{3}{10} & 0 & \frac{3}{10} & \frac{1}{10} & 1 & \frac{1}{10} \\ \frac{1}{2} & \frac{9}{20} & 0 & 0 & \frac{9}{20} & 0 & \frac{1}{2} & \frac{1}{10} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 27\Omega$$

Ω

$\left(\frac{1}{6}\ \frac{1}{3}\ \frac{1}{2}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ 0\ 0\ 0\ \frac{1}{9}\ \frac{2}{9}\ \frac{1}{3}\ \frac{1}{9}\ \frac{2}{9}\ \frac{1}{3}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\right)$

T

$\left(\frac{19}{120}\ \frac{1}{15}\ \frac{7}{10}\ \frac{1}{24}\ 0\ \frac{1}{10}\ \frac{1}{12}\ \frac{-1}{30}\ \frac{1}{2}\ \frac{3}{20}\ \frac{-1}{60}\ \frac{9}{20}\ \frac{3}{20}\ \frac{-1}{15}\ \frac{9}{20}\ \frac{1}{6}\ \frac{1}{20}\ \frac{1}{4}\ \frac{1}{15}\ 0\ \frac{1}{5}\ \frac{1}{40}\ \frac{1}{3}\ \frac{1}{10}\ \frac{1}{12}\ \frac{1}{5}\ 0\ 0\right.$

"IS NM in Vec(K)?", true

NM

$\left(\frac{237}{40}\ \frac{48}{5}\ \frac{99}{5}\ \frac{15}{8}\ 3\ \frac{27}{5}\ \frac{3}{4}\ \frac{-3}{10}\ \frac{9}{2}\ \frac{87}{20}\ \frac{117}{20}\ \frac{261}{20}\ \frac{87}{20}\ \frac{27}{5}\ \frac{261}{20}\ 3\ \frac{69}{20}\ \frac{27}{4}\ \frac{21}{10}\ 3\ \frac{63}{10}\ \frac{69}{40}\ 6\ \frac{27}{5}\ \frac{9}{4}\ \frac{24}{5}\ \frac{9}{2}\right.$

"IS MN in Vec(K)?", false

MN

$\left(\frac{111}{10}\ \frac{369}{40}\ \frac{147}{10}\ \frac{15}{4}\ 3\ \frac{18}{5}\ \frac{3}{2}\ \frac{-9}{20}\ \frac{15}{4}\ \frac{159}{20}\ \frac{57}{10}\ \frac{189}{20}\ \frac{159}{20}\ \frac{27}{5}\ \frac{189}{20}\ \frac{21}{4}\ \frac{147}{40}\ \frac{9}{2}\ \frac{21}{5}\ 3\ \frac{21}{5}\ \frac{69}{20}\ 6\ \frac{18}{5}\ \frac{9}{2}\right.$

$\boldsymbol{\tau}$ = 27/1 , rank = 3 , ratio = 9/1 , n^2 / r = 27/1

$\boldsymbol{\tau}'$ = 54/1 , r' = 2/3 , $\boldsymbol{\tau} / n^2$ = 1/3

p^2 = 7/54 , min $\boldsymbol{\tau}$ = 21/2 , $\boldsymbol{\tau}$–check is positive? 33/2

max r = 54/7 , r–check is positive? 11/18

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 27\Omega$$

There are, 8, partitions and, 3, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 33
out of total no. of elements equal to 144

dim span idems 19 vs no. of idems 24

"PT1" = {{3, 7, 8}, {4, 5, 6}, {1, 2, 9}}

"PT2" = {{3, 5, 7}, {1, 2, 9}, {4, 6, 8}}

"PT3" = {{1, 5, 9}, {2, 4, 6}, {3, 7, 8}}

"PT4" = {{3, 5, 7}, {2, 4, 6}, {1, 8, 9}}

"PT5" = {{1, 5, 9}, {2, 3, 7}, {4, 6, 8}}

"PT6" = {{1, 8, 9}, {2, 3, 7}, {4, 5, 6}}

"PT7" = {{1, 3, 8}, {4, 5, 6}, {2, 7, 9}}

"PT8" = {{1, 3, 8}, {2, 4, 6}, {5, 7, 9}}

"RG1" = {3, 6, 9}

"RG2" = {2, 5, 8}

"RG3" = {1, 4, 7}

$$M_C = \begin{pmatrix} \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} \\ \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} \\ \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} \\ \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} \\ \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} \\ \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} \\ \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} \\ \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} \\ \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{47}{54} & \frac{19}{270} & \frac{10}{27} & \frac{-7}{54} & \frac{19}{270} & \frac{-7}{54} & \frac{-7}{54} & \frac{127}{270} & \frac{10}{27} \\ \frac{19}{270} & \frac{47}{54} & \frac{11}{540} & \frac{73}{270} & \frac{-7}{54} & \frac{73}{270} & \frac{73}{270} & \frac{-7}{54} & \frac{173}{540} \\ \frac{10}{27} & \frac{11}{540} & \frac{47}{54} & \frac{-7}{54} & \frac{13}{108} & \frac{-7}{54} & \frac{10}{27} & \frac{127}{270} & \frac{-7}{54} \\ \frac{-7}{54} & \frac{73}{270} & \frac{-7}{54} & \frac{47}{54} & \frac{23}{135} & \frac{47}{54} & \frac{-7}{54} & \frac{23}{135} & \frac{-7}{54} \\ \frac{19}{270} & \frac{-7}{54} & \frac{13}{108} & \frac{23}{135} & \frac{47}{54} & \frac{23}{135} & \frac{10}{27} & \frac{-7}{54} & \frac{173}{540} \\ \frac{-7}{54} & \frac{73}{270} & \frac{-7}{54} & \frac{47}{54} & \frac{23}{135} & \frac{47}{54} & \frac{-7}{54} & \frac{23}{135} & \frac{-7}{54} \\ \frac{-7}{54} & \frac{73}{270} & \frac{10}{27} & \frac{-7}{54} & \frac{10}{27} & \frac{-7}{54} & \frac{47}{54} & \frac{-4}{135} & \frac{10}{27} \\ \frac{127}{270} & \frac{-7}{54} & \frac{127}{270} & \frac{23}{135} & \frac{-7}{54} & \frac{23}{135} & \frac{-4}{135} & \frac{47}{54} & \frac{-4}{135} \\ \frac{10}{27} & \frac{173}{540} & \frac{-7}{54} & \frac{-7}{54} & \frac{173}{540} & \frac{-7}{54} & \frac{10}{27} & \frac{-4}{135} & \frac{47}{54} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\ \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} \\ \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 \\ 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\ \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} \\ \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 \\ 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\ \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} \\ \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{19}{235} & \frac{20}{47} & \frac{-7}{47} & \frac{19}{235} & \frac{-7}{47} & \frac{-7}{47} & \frac{127}{235} & \frac{20}{47} \\ \frac{19}{235} & 1 & \frac{11}{470} & \frac{73}{235} & \frac{-7}{47} & \frac{73}{235} & \frac{73}{235} & \frac{-7}{47} & \frac{173}{470} \\ \frac{20}{47} & \frac{11}{470} & 1 & \frac{-7}{47} & \frac{13}{94} & \frac{-7}{47} & \frac{20}{47} & \frac{127}{235} & \frac{-7}{47} \\ \frac{-7}{47} & \frac{73}{235} & \frac{-7}{47} & 1 & \frac{46}{235} & 1 & \frac{-7}{47} & \frac{46}{235} & \frac{-7}{47} \\ \frac{19}{235} & \frac{-7}{47} & \frac{13}{94} & \frac{46}{235} & 1 & \frac{46}{235} & \frac{20}{47} & \frac{-7}{47} & \frac{173}{470} \\ \frac{-7}{47} & \frac{73}{235} & \frac{-7}{47} & 1 & \frac{46}{235} & 1 & \frac{-7}{47} & \frac{46}{235} & \frac{-7}{47} \\ \frac{-7}{47} & \frac{73}{235} & \frac{20}{47} & \frac{-7}{47} & \frac{20}{47} & \frac{-7}{47} & 1 & \frac{-8}{235} & \frac{20}{47} \\ \frac{127}{235} & \frac{-7}{47} & \frac{127}{235} & \frac{46}{235} & \frac{-7}{47} & \frac{46}{235} & \frac{-8}{235} & 1 & \frac{-8}{235} \\ \frac{20}{47} & \frac{173}{470} & \frac{-7}{47} & \frac{-7}{47} & \frac{173}{470} & \frac{-7}{47} & \frac{20}{47} & \frac{-8}{235} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 10.95416346, 5.545836544]

Eigenvalues N_C

[0., 0., 0., 1.833333333, 0.3860524124, 0.9219980442, 1.063150398, 1.606429918, 2.022369227]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 5.170820393, 3.829179607]

Eigenvalues N_C–scaled

[0., 0., 0., 2.106382979, 0.4435495801, 1.059316902, 1.221491947, 1.845685438, 2.323573154]

NullSpace M_C

{[-1, 0, 0, 1, 0, 0, 0, 0, 0], [-1, 0, 0, 0, 0, 0, 1, 0, 0], [0, 0, -1, 0, 0, 1, 0, 0, 0], [1, 0, 1, 0, 1, 0, 0, 0, 0], [1, 0, 1, 0, 0, 0, 0, 1, 0], [1, 1, 1, 0, 0, 0, 0, 0, 0], [0, 0, -1, 0, 0, 0, 0, 0, 1]}

NullSpace N_C

{[0, 0, 0, 1, 0, -1, 0, 0, 0], [-1, 0, 1, 0, 0, 0, -1, 0, 1], [-1, 1, 0, 0, 1, -1, -1, 1, 0]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 9., 4.500000000, 13.50000000]

Eigenvalues N_0

[0., 0., 0., 3., 0.3860524124, 0.9219980442, 1.063150398, 1.606429918, 2.022369227]

NullSpace M_0

{[0, 0, -1, 0, 0, 1, 0, 0, 0], [0, -1, 0, 0, 0, 0, 0, 1, 0], [0, 0, -1, 0, 0, 0, 0, 0, 1], [-1, 0, 0, 1, 0, 0, 0, 0, 0], [0, -1, 0, 0, 1, 0, 0, 0, 0], [-1, 0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace N_0

{[0, 0, 0, -1, 0, 1, 0, 0, 0], [-1, 1, 0, -1, 1, 0, -1, 1, 0], [-1, 0, 1, 0, 0, 0, -1, 0, 1]}

Eigenvalues M

[3., 6., 9., -3., -1.500000000, -4.500000000, -3., -1.500000000, -4.500000000]

Eigenvalues N

[0., 0., 0., 6., -0.3860524124, -0.9219980442, -1.063150398, -1.606429918, -2.022369227]

NullSpace M

{}

NullSpace N

{[0, 0, 0, -1, 0, 1, 0, 0, 0], [1, 0, -1, 0, 0, 0, 1, 0, -1], [0, 1, -1, -1, 1, 0, 0, 1, -1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 16 & 10 & 20 & 16 & 20 & 20 & 8 & 10 \\ 16 & 0 & 17 & 12 & 20 & 12 & 12 & 20 & 11 \\ 10 & 17 & 0 & 20 & 15 & 20 & 10 & 8 & 20 \\ 20 & 12 & 20 & 0 & 14 & 0 & 20 & 14 & 20 \\ 16 & 20 & 15 & 14 & 0 & 14 & 10 & 20 & 11 \\ 20 & 12 & 20 & 0 & 14 & 0 & 20 & 14 & 20 \\ 20 & 12 & 10 & 20 & 10 & 20 & 0 & 18 & 10 \\ 8 & 20 & 8 & 14 & 20 & 14 & 18 & 0 & 18 \\ 10 & 11 & 20 & 20 & 11 & 20 & 10 & 18 & 0 \end{pmatrix}$$

==

60, [1, 1, -1, -1, 1, -1, 1, 1, 1]

==

{3, 6, 9}

R: [4, 4, 5, 7, 7, 8, 1, 1, 2]
B: [2, 9, 4, 8, 3, 7, 5, 6, 1]

TRACE TWO = 1

det AT = $\frac{1}{16}(-1+t)^3(1+t^2)(t)^2$

$$AT = \begin{pmatrix} 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 9

Level 2 det = $\frac{1}{2199023255552}(-1+s)(-275447945600 - 375215182400s + 22431106792s^2 + 264331374652s^3 + 87473625556s^4 - 69818225208s^5 - 38804089703s^6 + 10738855781s^7 + 9028083582s^8 - 1165605838s^9 - 1513918486s^{10} + 47019266s^{11} + 175916058s^{12} + 6840730s^{13} - 12614900s^{14} - 755772s^{15} + 541894s^{16} + 25750s^{17} + 4734s^{18} + 4466s^{19} - 2486s^{20} - 1002s^{21} + 11s^{22} + 39s^{23} + 4s^{24})$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 3

RANK of B is 9

B ranking is 3, "vs", 9

BBAR ranking 3, "vs", 9

"R CYCLES", 1 + v[1] v[4] v[7]
"B CYCLES", (1 + v[1] v[2] v[9]) (1 + v[3] v[4] v[5] v[6] v[7] v[8])

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [−1., 0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 −

0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{}

NullSpace of R^*

{[1, −1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1, 0]}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 9 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 3 \\ 9 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 3 \\ 9 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{186}{535} & \frac{56}{107} & 1 & \frac{414}{535} & \frac{96}{107} & 1 & \frac{94}{107} & \frac{62}{107} \\ \frac{186}{535} & 0 & \frac{293}{535} & \frac{434}{535} & 1 & \frac{81}{107} & \frac{90}{107} & 1 & \frac{372}{535} \\ \frac{56}{107} & \frac{293}{535} & 0 & \frac{104}{107} & \frac{105}{107} & 1 & \frac{54}{107} & \frac{252}{535} & 1 \\ 1 & \frac{434}{535} & \frac{104}{107} & 0 & \frac{126}{535} & \frac{18}{107} & 1 & \frac{102}{107} & \frac{92}{107} \\ \frac{414}{535} & 1 & \frac{105}{107} & \frac{126}{535} & 0 & \frac{36}{107} & \frac{106}{107} & 1 & \frac{73}{107} \\ \frac{96}{107} & \frac{81}{107} & 1 & \frac{18}{107} & \frac{36}{107} & 0 & \frac{100}{107} & \frac{97}{107} & 1 \\ 1 & \frac{90}{107} & \frac{54}{107} & 1 & \frac{106}{107} & \frac{100}{107} & 0 & \frac{18}{107} & \frac{60}{107} \\ \frac{94}{107} & 1 & \frac{252}{535} & \frac{102}{107} & 1 & \frac{97}{107} & \frac{18}{107} & 0 & \frac{333}{535} \\ \frac{62}{107} & \frac{372}{535} & 1 & \frac{92}{107} & \frac{73}{107} & 1 & \frac{60}{107} & \frac{333}{535} & 0 \end{pmatrix}$$

"RANK of N is ", 7, "RANK of M is ", 9

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}$ ($3v[1] + 2v[2] + v[3] + 3v[4] + 2v[5] + v[6] + 3v[7] + 2v[8] + v[9]$)
degree 2: $\frac{1}{6}$ ($3v[1]v[4] + 3v[1]v[7] + 2v[2]v[5] + 2v[2]v[8] + v[3]v[6] + v[3]v[9] + 3v[4]v[7] + 2v[5]v[8] + v[6]$
$v[9]$)
degree 3 : $\frac{1}{2}$ ($3v[1]v[4]v[7] + 2v[2]v[5]v[8] + v[3]v[6]v[9]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{3, 7, 8}, {4, 5, 6}, {1, 2, 9}}

"PT2" = {{3, 5, 7}, {1, 2, 9}, {4, 6, 8}}

"PT3" = {{3, 4, 8}, {5, 6, 7}, {1, 2, 9}}

"PT4" = {{6, 7, 8}, {1, 2, 9}, {3, 4, 5}}

"PT5" = {{1, 5, 9}, {2, 4, 6}, {3, 7, 8}}

"PT6" = {{3, 5, 7}, {2, 4, 6}, {1, 8, 9}}

"PT7" = {{1, 5, 9}, {2, 3, 7}, {4, 6, 8}}

"PT8" = {{1, 8, 9}, {2, 3, 7}, {4, 5, 6}}

"PT9" = {{3, 7, 8}, {2, 4, 9}, {1, 5, 6}}

"PT10" = {{3, 4, 8}, {2, 7, 9}, {1, 5, 6}}

"PT11" = {{3, 7, 8}, {1, 2, 6}, {4, 5, 9}}

"PT12" = {{7, 8, 9}, {1, 2, 6}, {3, 4, 5}}

"PT13" = {{1, 3, 8}, {5, 6, 7}, {2, 4, 9}}

"PT14" = {{1, 3, 8}, {4, 5, 6}, {2, 7, 9}}

"PT15" = {{1, 2, 3}, {6, 7, 8}, {4, 5, 9}}

"PT16" = {{7, 8, 9}, {4, 5, 6}, {1, 2, 3}}

"RG1" = {3, 6, 9}

"RG2" = {2, 5, 8}

"RG3" = {1, 4, 7}

$\pi 3$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 3$ = {16, 42, 61}

$u3$ = [323, 255, 195, 235, 303, 363, 0, 795, 447, 675, 45, 70, 165, 15, 105, 1605, 1335, 705, 110, 1227, 1047, 685, 1335, 1095, 765, 75, 225, 245, 531, 849, 489, 25, 30, 390, 75, 125, 1047, 1227, 646, 150, 1335, 1605, 606, 855, 1065, 726, 15, 470, 510, 343, 225, 765, 686, 1335, 510, 785, 726, 1065, 705, 606, 1605, 0, 105, 150, 0, 363, 303, 35, 165, 145, 45, 195, 675, 699, 430, 390, 30, 255, 375, 489, 215, 795, 849, 55]

supp u3 = {1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84}

Action of R on ranges, [[2], [3], [3]]
Action of B on ranges, [[3], [1], [2]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{pmatrix}$$

RPARTS [10, C, C, 10, 1, 4, B, F, 1, B, 10, 10, 4, F, 10, 10]

BPARTS [1, 3, 4, 2, E, D, A, 9, 10, C, 8, 6, F, B, 7, 5]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{39}{214} & \frac{1}{535} & \frac{1}{1070} & \frac{2}{535} & \frac{89}{535} & \frac{7}{1605} & \frac{47}{1605} & \frac{17}{535} & \frac{17}{1070} & \frac{47}{3210} & \frac{34}{535} & \frac{14}{1605} & \frac{7}{3210} & \frac{89}{1070} & \frac{94}{1605} & \frac{178}{535} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[F, 12, E, 6, 10, C, 12, C, 10, E, 14, D, 8, C, 8, 8, 14, D, 6, D, F]

B-BLOCKS,

[12, 7, 14, 11, 3, C, E, 5, D, 9, 13, 6, 4, 15, A, 10, F, 2, 8, B, 1]

with invariant measure, [25, 10, 137, 314, 274, 687, 5, 548, 29, 58, 98, 1011, 628, 100, 116, 303, 157, 20, 49, 196, 50]

N by blocks, N - check: true

b_1 = {3, 4, 8}
b_2 = {3, 5, 7}
b_3 = {1, 3, 8}
b_4 = {1, 5, 9}
b_5 = {2, 4, 6}
b_6 = {3, 7, 8}
b_7 = {5, 6, 7}
b_8 = {7, 8, 9}
b_9 = {2, 4, 9}
b_{10} = {1, 8, 9}
b_{11} = {2, 3, 7}
b_{12} = {4, 5, 6}
b_{13} = {1, 2, 3}
b_{14} = {6, 7, 8}
b_{15} = {1, 2, 6}
b_{16} = {1, 2, 9}
b_{17} = {2, 7, 9}
b_{18} = {3, 4, 5}
b_{19} = {1, 5, 6}
b_{20} = {4, 5, 9}
b_{21} = {4, 6, 8}

dim(span of partition vectors), rank(N_0), rank(N): 7, 7, 7

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & h[1] & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & h[1] & 0 \\ 0 & 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & h[1] \\ h[1] & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 \\ 0 & 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & h[1] \\ h[1] & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 & 0 \\ 0 & h[1] & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 \\ 0 & 0 & h[1] & 0 & 0 & h[1] & 0 & 0 & h[2] \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 4, 7}}, true

Ω_B in Vec(K)? , {{3, 4, 5, 6, 7, 8}, {1, 2, 9}}, true

$$V = \begin{pmatrix} \frac{1}{30} & \frac{-29}{45} & \frac{7}{90} & \frac{7}{30} & \frac{7}{45} & \frac{7}{90} & \frac{7}{30} & \frac{7}{45} & \frac{-29}{90} \\ \frac{-1}{10} & \frac{-2}{5} & \frac{1}{10} & \frac{3}{10} & \frac{1}{5} & \frac{1}{10} & \frac{3}{10} & \frac{1}{5} & \frac{-7}{10} \\ \frac{-2}{15} & \frac{-4}{45} & \frac{16}{45} & \frac{-13}{30} & \frac{32}{45} & \frac{-13}{90} & \frac{1}{15} & \frac{-13}{45} & \frac{-2}{45} \\ \frac{-1}{15} & \frac{-2}{45} & \frac{8}{45} & \frac{1}{30} & \frac{16}{45} & \frac{-29}{90} & \frac{8}{15} & \frac{-29}{45} & \frac{-1}{45} \\ \frac{1}{5} & \frac{2}{15} & \frac{-8}{15} & \frac{-1}{10} & \frac{-1}{15} & \frac{-1}{30} & \frac{2}{5} & \frac{-1}{15} & \frac{1}{15} \\ \frac{1}{15} & \frac{2}{45} & \frac{-8}{45} & \frac{-1}{30} & \frac{-16}{45} & \frac{29}{90} & \frac{-8}{15} & \frac{29}{45} & \frac{1}{45} \\ \frac{8}{15} & \frac{16}{45} & \frac{-19}{45} & \frac{7}{30} & \frac{-38}{45} & \frac{7}{90} & \frac{-4}{15} & \frac{7}{45} & \frac{8}{45} \\ \frac{2}{5} & \frac{4}{15} & \frac{-1}{15} & \frac{3}{10} & \frac{-2}{15} & \frac{-17}{30} & \frac{-1}{5} & \frac{-2}{15} & \frac{2}{15} \\ \frac{-13}{30} & \frac{17}{45} & \frac{-1}{90} & \frac{-1}{30} & \frac{-1}{45} & \frac{-1}{90} & \frac{-1}{30} & \frac{-1}{45} & \frac{17}{90} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 7, 8}, {4, 5, 6}, {1, 2, 9}}

1, "range", [3, 6, 9], [[9, 9, 6, 3, 3, 3, 6, 6, 9], [9, 9, 3, 6, 6, 6, 3, 3, 9], [6, 6, 9, 3, 3, 3, 9, 9, 6], [6, 6, 3, 9, 9, 9, 3, 3, 6], [3, 3, 9, 6, 6, 6, 9, 9, 3], [3, 3, 6, 9, 9, 9, 6, 6, 3]]

2, "range", [2, 5, 8], [[8, 8, 5, 2, 2, 2, 5, 5, 8], [8, 8, 2, 5, 5, 5, 2, 2, 8], [5, 5, 8, 2, 2, 2, 8, 8, 5], [5, 5, 2, 8, 8, 8, 2, 2, 5], [2, 2, 8, 5, 5, 5, 8, 8, 2], [2, 2, 5, 8, 8, 8, 5, 5, 2]]

3, "range", [1, 4, 7], [[7, 7, 4, 1, 1, 1, 4, 4, 7], [7, 7, 1, 4, 4, 4, 1, 1, 7], [4, 4, 7, 1, 1, 1, 7, 7, 4], [4, 4, 1, 7, 7, 7, 1, 1, 4], [1, 1, 7, 4, 4, 4, 7, 7, 1], [1, 1, 4, 7, 7, 7, 4, 4, 1]]

2, "partition", {{3, 5, 7}, {1, 2, 9}, {4, 6, 8}}

1, "range", [3, 6, 9], [[9, 9, 6, 3, 6, 3, 6, 3, 9], [9, 9, 3, 6, 3, 6, 3, 6, 9], [6, 6, 9, 3, 9, 3, 9, 3, 6], [6, 6, 3, 9, 3, 9, 3, 9, 6], [3, 3, 9, 6, 9, 6, 9, 6, 3], [3, 3, 6, 9, 6, 9, 6, 9, 3]]

2, "range", [2, 5, 8], [[8, 8, 5, 2, 5, 2, 5, 2, 8], [8, 8, 2, 5, 2, 5, 2, 5, 8], [5, 5, 8, 2, 8, 2, 8, 2, 5], [5, 5, 2, 8, 2, 8, 2, 8, 5], [2, 2, 8, 5, 8, 5, 8, 5, 2], [2, 2, 5, 8, 5, 8, 5, 8, 2]]

3, "range", [1, 4, 7], [[7, 7, 4, 1, 4, 1, 4, 1, 7], [7, 7, 1, 4, 1, 4, 1, 4, 7], [4, 4, 7, 1, 7, 1, 7, 1, 4], [4, 4, 1, 7, 1, 7, 1, 7, 4], [1, 1, 7, 4, 7, 4, 7, 4, 1], [1, 1, 4, 7, 4, 7, 4, 7, 1]]

3, "partition", {{3, 4, 8}, {5, 6, 7}, {1, 2, 9}}

1, "range", [3, 6, 9], [[9, 9, 6, 6, 3, 3, 3, 6, 9], [9, 9, 3, 3, 6, 6, 6, 3, 9], [6, 6, 9, 9, 3, 3, 3, 9, 6], [6, 6, 3, 3, 9, 9, 9, 3, 6], [3, 3, 9, 9, 6, 6, 6, 9, 3], [3, 3, 6, 6, 9, 9, 9, 6, 3]]

2, "range", [2, 5, 8], [[8, 8, 5, 5, 2, 2, 2, 5, 8], [8, 8, 2, 2, 5, 5, 5, 2, 8], [5, 5, 8, 8, 2, 2, 2, 8, 5], [5, 5, 2, 2, 8, 8, 8, 2, 5], [2, 2, 8, 8, 5, 5, 5, 8, 2], [2, 2, 5, 5, 8, 8, 8, 5, 2]]

3, "range", [1, 4, 7], [[7, 7, 4, 4, 1, 1, 1, 4, 7], [7, 7, 1, 1, 4, 4, 4, 1, 7], [4, 4, 7, 7, 1, 1, 1, 7, 4], [4, 4, 1, 1, 7, 7, 7, 1, 4], [1, 1, 7, 7, 4, 4, 4, 7, 1], [1, 1, 4, 4, 7, 7, 7, 4, 1]]

4, "partition", {{6, 7, 8}, {1, 2, 9}, {3, 4, 5}}

1, "range", [3, 6, 9], [[9, 9, 6, 6, 6, 3, 3, 3, 9], [9, 9, 3, 3, 3, 6, 6, 6, 9], [6, 6, 9, 9, 9, 3, 3, 3, 6], [6, 6, 3, 3, 3, 9, 9, 9, 6], [3, 3, 9, 9, 9, 6, 6, 6, 3], [3, 3, 6, 6, 6, 9, 9, 9, 3]]

2, "range", [2, 5, 8], [[8, 8, 5, 5, 5, 2, 2, 2, 8], [8, 8, 2, 2, 2, 5, 5, 5, 8], [5, 5, 8, 8, 8, 2, 2, 2, 5], [5, 5, 2, 2, 2, 8, 8, 8, 5], [2, 2, 8, 8, 8, 5, 5, 5, 2], [2, 2, 5, 5, 5, 8, 8, 8, 2]]

3, "range", [1, 4, 7], [[7, 7, 4, 4, 4, 1, 1, 1, 7], [7, 7, 1, 1, 1, 4, 4, 4, 7], [4, 4, 7, 7, 7, 1, 1, 1, 4], [4, 4, 1, 1, 1, 7, 7, 7, 4], [1, 1, 7, 7, 7, 4, 4, 4, 1], [1, 1, 4, 4, 4, 7, 7, 7, 1]]

5, "partition", {{1, 5, 9}, {2, 4, 6}, {3, 7, 8}}

1, "range", [3, 6, 9], [[9, 6, 3, 6, 9, 6, 3, 3, 9], [9, 3, 6, 3, 9, 3, 6, 6, 9], [6, 9, 3, 9, 6, 9, 3, 3, 6], [6, 3, 9, 3, 6, 3, 9, 9, 6], [3, 9, 6, 9, 3, 9, 6, 6, 3], [3, 6, 9, 6, 3, 6, 9, 9, 3]]

2, "range", [2, 5, 8], [[8, 5, 2, 5, 8, 5, 2, 2, 8], [8, 2, 5, 2, 8, 2, 5, 5, 8], [5, 8, 2, 8, 5, 8, 2, 2, 5], [5, 2, 8, 2, 5, 2, 8, 8, 5], [2, 8, 5, 8, 2, 8, 5, 5, 2], [2, 5, 8, 5, 2, 5, 8, 8, 2]]

3, "range", [1, 4, 7], [[7, 4, 1, 4, 7, 4, 1, 1, 7], [7, 1, 4, 1, 7, 1, 4, 4, 7], [4, 7, 1, 7, 4, 7, 1, 1, 4], [4, 1, 7, 1, 4, 1, 7, 7, 4], [1, 7, 4, 7, 1, 7, 4, 4, 1], [1, 4, 7, 4, 1, 4, 7, 7, 1]]

6, "partition", {{3, 5, 7}, {2, 4, 6}, {1, 8, 9}}

1, "range", [3, 6, 9], [[9, 6, 3, 6, 3, 6, 3, 9, 9], [9, 3, 6, 3, 6, 3, 6, 9, 9], [6, 9, 3, 9, 3, 9, 3, 6, 6], [6, 3, 9, 3, 9, 3, 9, 6, 6], [3, 9, 6, 9, 6, 9, 6, 3, 3], [3, 6, 9, 6, 9, 6, 9, 3, 3]]

2, "range", [2, 5, 8], [[8, 5, 2, 5, 2, 5, 2, 8, 8], [8, 2, 5, 2, 5, 2, 5, 8, 8], [5, 8, 2, 8, 2, 8, 2, 5, 5], [5, 2, 8, 2, 8, 2, 8, 5, 5], [2, 8, 5, 8, 5, 8, 5, 2, 2], [2, 5, 8, 5, 8, 5, 8, 2, 2]]

3, "range", [1, 4, 7], [[7, 4, 1, 4, 1, 4, 1, 7, 7], [7, 1, 4, 1, 4, 1, 4, 7, 7], [4, 7, 1, 7, 1, 7, 1, 4, 4], [4, 1, 7, 1, 7, 1, 7, 4, 4], [1, 7, 4, 7, 4, 7, 4, 1, 1], [1, 4, 7, 4, 7, 4, 7, 1, 1]]

7, "partition", {{1, 5, 9}, {2, 3, 7}, {4, 6, 8}}

1, "range", [3, 6, 9], [[9, 6, 6, 3, 9, 3, 6, 3, 9], [9, 3, 3, 6, 9, 6, 3, 6, 9], [6, 9, 9, 3, 6, 3, 9, 3, 6], [6, 3, 3, 9, 6, 9, 3, 9, 6], [3, 9, 9, 6, 3, 6, 9, 6, 3], [3, 6, 6, 9, 3, 9, 6, 9, 3]]

2, "range", [2, 5, 8], [[8, 5, 5, 2, 8, 2, 5, 2, 8], [8, 2, 2, 5, 8, 5, 2, 5, 8], [5, 8, 8, 2, 5, 2, 8, 2, 5], [5, 2, 2, 8, 5, 8, 2, 8, 5], [2, 8, 8, 5, 2, 5, 8, 5, 2], [2, 5, 5, 8, 2, 8, 5, 8, 2]]

3, "range", [1, 4, 7], [[7, 4, 4, 1, 7, 1, 4, 1, 7], [7, 1, 1, 4, 7, 4, 1, 4, 7], [4, 7, 7, 1, 4, 1, 7, 1, 4], [4, 1, 1, 7, 4, 7, 1, 7, 4], [1, 7, 7, 4, 1, 4, 7, 4, 1], [1, 4, 4, 7, 1, 7, 4, 7, 1]]

8, "partition", {{1, 8, 9}, {2, 3, 7}, {4, 5, 6}}

1, "range", [3, 6, 9], [[9, 6, 6, 3, 3, 3, 6, 9, 9], [9, 3, 3, 6, 6, 6, 3, 9, 9], [6, 9, 9, 3, 3, 3, 9, 6, 6], [6, 3, 3, 9, 9, 9, 3, 6, 6], [3, 9, 9, 6, 6, 6, 9, 3, 3], [3, 6, 6, 9, 9, 9, 6, 3, 3]]

2, "range", [2, 5, 8], [[8, 5, 5, 2, 2, 2, 5, 8, 8], [8, 2, 2, 5, 5, 5, 2, 8, 8], [5, 8, 8, 2, 2, 2, 8, 5, 5], [5, 2, 2, 8, 8, 8, 2, 5, 5], [2, 8, 8, 5, 5, 5, 8, 2, 2], [2, 5, 5, 8, 8, 8, 5, 2, 2]]

3, "range", [1, 4, 7], [[7, 4, 4, 1, 1, 1, 4, 7, 7], [7, 1, 1, 4, 4, 4, 1, 7, 7], [4, 7, 7, 1, 1, 1, 7, 4, 4], [4, 1, 1, 7, 7, 7, 1, 4, 4], [1, 7, 7, 4, 4, 4, 7, 1, 1], [1, 4, 4, 7, 7, 7, 4, 1, 1]]

9, "partition", {{3, 7, 8}, {2, 4, 9}, {1, 5, 6}}

1, "range", [3, 6, 9], [[9, 6, 3, 6, 9, 9, 3, 3, 6], [9, 3, 6, 3, 9, 9, 6, 6, 3], [6, 9, 3, 9, 6, 6, 3, 3, 9], [6, 3, 9, 3, 6, 6, 9, 9, 3], [3, 9, 6, 9, 3, 3, 6, 6, 9], [3, 6, 9, 6, 3, 3, 9, 9, 6]]

2, "range", [2, 5, 8], [[8, 5, 2, 5, 8, 8, 2, 2, 5], [8, 2, 5, 2, 8, 8, 5, 5, 2], [5, 8, 2, 8, 5, 5, 2, 2, 8], [5, 2, 8, 2, 5, 5, 8, 8, 2], [2, 8, 5, 8, 2, 2, 5, 5, 8], [2, 5, 8, 5, 2, 2, 8, 8, 5]]

3, "range", [1, 4, 7], [[7, 4, 1, 4, 7, 7, 1, 1, 4], [7, 1, 4, 1, 7, 7, 4, 4, 1], [4, 7, 1, 7, 4, 4, 1, 1, 7], [4, 1, 7, 1, 4, 4, 7, 7, 1], [1, 7, 4, 7, 1, 1, 4, 4, 7], [1, 4, 7, 4, 1, 1, 7, 7, 4]]

10, "partition", {{3, 4, 8}, {2, 7, 9}, {1, 5, 6}}

1, "range", [3, 6, 9], [[9, 6, 3, 3, 9, 9, 6, 3, 6], [9, 3, 6, 6, 9, 9, 3, 6, 3], [6, 9, 3, 3, 6, 6, 9, 3, 9], [6, 3, 9, 9, 6, 6, 3, 9, 3], [3, 9, 6, 6, 3, 3, 9, 6, 9], [3, 6, 9, 9, 3, 3, 6, 9, 6]]

2, "range", [2, 5, 8], [[8, 5, 2, 2, 8, 8, 5, 2, 5], [8, 2, 5, 5, 8, 8, 2, 5, 2], [5, 8, 2, 2, 5, 5, 8, 2, 8], [5, 2, 8, 8, 5, 5, 2, 8, 2], [2, 8, 5, 5, 2, 2, 8, 5, 8], [2, 5, 8, 8, 2, 2, 5, 8, 5]]

3, "range", [1, 4, 7], [[7, 4, 1, 1, 7, 7, 4, 1, 4], [7, 1, 4, 4, 7, 7, 1, 4, 1], [4, 7, 1, 1, 4, 4, 7, 1, 7], [4, 1, 7, 7, 4, 4, 1, 7, 1], [1, 7, 4, 4, 1, 1, 7, 4, 7], [1, 4, 7, 7, 1, 1, 4, 7, 4]]

11, "partition", {{3, 7, 8}, {1, 2, 6}, {4, 5, 9}}

1, "range", [3, 6, 9], [[9, 9, 6, 3, 3, 9, 6, 6, 3], [9, 9, 3, 6, 6, 9, 3, 3, 6], [6, 6, 9, 3, 3, 6, 9, 9, 3], [6, 6, 3, 9, 9, 6, 3, 3, 9], [3, 3, 9, 6, 6, 3, 9, 9, 6], [3, 3, 6, 9, 9, 3, 6, 6, 9]]

2, "range", [2, 5, 8], [[8, 8, 5, 2, 2, 8, 5, 5, 2], [8, 8, 2, 5, 5, 8, 2, 2, 5], [5, 5, 8, 2, 2, 5, 8, 8, 2], [5, 5, 2, 8, 8, 5, 2, 2, 8], [2, 2, 8, 5, 5, 2, 8, 8, 5], [2, 2, 5, 8, 8, 2, 5, 5, 8]]

3, "range", [1, 4, 7], [[7, 7, 4, 1, 1, 7, 4, 4, 1], [7, 7, 1, 4, 4, 7, 1, 1, 4], [4, 4, 7, 1, 1, 4, 7, 7, 1], [4, 4, 1, 7, 7, 4, 1, 1, 7], [1, 1, 7, 4, 4, 1, 7, 7, 4], [1, 1, 4, 7, 7, 1, 4, 4, 7]]

12, "partition", {{7, 8, 9}, {1, 2, 6}, {3, 4, 5}}

1, "range", [3, 6, 9], [[9, 9, 6, 6, 6, 9, 3, 3, 3], [9, 9, 3, 3, 3, 9, 6, 6, 6], [6, 6, 9, 9, 9, 6, 3, 3, 3], [6, 6, 3, 3, 3, 6, 9, 9, 9], [3, 3, 9, 9, 9, 3, 6, 6, 6], [3, 3, 6, 6, 6, 3, 9, 9, 9]]

2, "range", [2, 5, 8], [[8, 8, 5, 5, 5, 8, 2, 2, 2], [8, 8, 2, 2, 2, 8, 5, 5, 5], [5, 5, 8, 8, 8, 5, 2, 2, 2], [5, 5, 2, 2, 2, 5, 8, 8, 8], [2, 2, 8, 8, 8, 2, 5, 5, 5], [2, 2, 5, 5, 5, 2, 8, 8, 8]]

3, "range", [1, 4, 7], [[7, 7, 4, 4, 4, 7, 1, 1, 1], [7, 7, 1, 1, 1, 7, 4, 4, 4], [4, 4, 7, 7, 7, 4, 1, 1, 1], [4, 4, 1, 1, 1, 4, 7, 7, 7], [1, 1, 7, 7, 7, 1, 4, 4, 4], [1, 1, 4, 4, 4, 1, 7, 7, 7]]

13, "partition", {{1, 3, 8}, {5, 6, 7}, {2, 4, 9}}

1, "range", [3, 6, 9], [[9, 6, 9, 6, 3, 3, 3, 9, 6], [9, 3, 9, 3, 6, 6, 6, 9, 3], [6, 9, 6, 9, 3, 3, 3, 6, 9], [6, 3, 6, 3, 9, 9, 9, 6, 3], [3, 9, 3, 9, 6, 6, 6, 3, 9], [3, 6, 3, 6, 9, 9, 9, 3, 6]]

2, "range", [2, 5, 8], [[8, 5, 8, 5, 2, 2, 2, 8, 5], [8, 2, 8, 2, 5, 5, 5, 8, 2], [5, 8, 5, 8, 2, 2, 2, 5, 8], [5, 2, 5, 2, 8, 8, 8, 5, 2], [2, 8, 2, 8, 5, 5, 5, 2, 8], [2, 5, 2, 5, 8, 8, 8, 2, 5]]

3, "range", [1, 4, 7], [[7, 4, 7, 4, 1, 1, 1, 7, 4], [7, 1, 7, 1, 4, 4, 4, 7, 1], [4, 7, 4, 7, 1, 1, 1, 4, 7], [4, 1, 4, 1, 7, 7, 7, 4, 1], [1, 7, 1, 7, 4, 4, 4, 1, 7], [1, 4, 1, 4, 7, 7, 7, 1, 4]]

14, "partition", {{1, 3, 8}, {4, 5, 6}, {2, 7, 9}}

1, "range", [3, 6, 9], [[9, 6, 9, 3, 3, 3, 6, 9, 6], [9, 3, 9, 6, 6, 6, 3, 9, 3], [6, 9, 6, 3, 3, 3, 9, 6, 9], [6, 3, 6, 9, 9, 9, 3, 6, 3], [3, 9, 3, 6, 6, 6, 9, 3, 9], [3, 6, 3, 9, 9, 9, 6, 3, 6]]

2, "range", [2, 5, 8], [[8, 5, 8, 2, 2, 2, 5, 8, 5], [8, 2, 8, 5, 5, 5, 2, 8, 2], [5, 8, 5, 2, 2, 2, 8, 5, 8], [5, 2, 5, 8, 8, 8, 2, 5, 2], [2, 8, 2, 5, 5, 5, 8, 2, 8], [2, 5, 2, 8, 8, 8, 5, 2, 5]]

3, "range", [1, 4, 7], [[7, 4, 7, 1, 1, 1, 4, 7, 4], [7, 1, 7, 4, 4, 4, 1, 7, 1], [4, 7, 4, 1, 1, 1, 7, 4, 7], [4, 1, 4, 7, 7, 7, 1, 4, 1], [1, 7, 1, 4, 4, 4, 7, 1, 7], [1, 4, 1, 7, 7, 7, 4, 1, 4]]

15, "partition", {{1, 2, 3}, {6, 7, 8}, {4, 5, 9}}

1, "range", [3, 6, 9], [[9, 9, 9, 6, 6, 3, 3, 3, 6], [9, 9, 9, 3, 3, 6, 6, 6, 3], [6, 6, 6, 9, 9, 3, 3, 3, 9], [6, 6, 6, 3, 3, 9, 9, 9, 3], [3, 3, 3, 9, 9, 6, 6, 6, 9], [3, 3, 3, 6, 6, 9, 9, 9, 6]]

2, "range", [2, 5, 8], [[8, 8, 8, 5, 5, 2, 2, 2, 5], [8, 8, 8, 2, 2, 5, 5, 5, 2], [5, 5, 5, 8, 8, 2, 2, 2, 8], [5, 5, 5, 2, 2, 8, 8, 8, 2], [2, 2, 2, 8, 8, 5, 5, 5, 8], [2, 2, 2, 5, 5, 8, 8, 8, 5]]

3, "range", [1, 4, 7], [[7, 7, 7, 4, 4, 1, 1, 1, 4], [7, 7, 7, 1, 1, 4, 4, 4, 1], [4, 4, 4, 7, 7, 1, 1, 1, 7], [4, 4, 4, 1, 1, 7, 7, 7, 1], [1, 1, 1, 7, 7, 4, 4, 4, 7], [1, 1, 1, 4, 4, 7, 7, 7, 4]]

16, "partition", {{7, 8, 9}, {4, 5, 6}, {1, 2, 3}}

1, "range", [3, 6, 9], [[9, 9, 9, 6, 6, 6, 3, 3, 3], [9, 9, 9, 3, 3, 3, 6, 6, 6], [6, 6, 6, 9, 9, 9, 3, 3, 3], [6, 6, 6, 3, 3, 3, 9, 9, 9], [3, 3, 3, 9, 9, 9, 6, 6, 6], [3, 3, 3, 6, 6, 6, 9, 9, 9]]

2, "range", [2, 5, 8], [[8, 8, 8, 5, 5, 5, 2, 2, 2], [8, 8, 8, 2, 2, 2, 5, 5, 5], [5, 5, 5, 8, 8, 8, 2, 2, 2], [5, 5, 5, 2, 2, 2, 8, 8, 8], [2, 2, 2, 8, 8, 8, 5, 5, 5], [2, 2, 2, 5, 5, 5, 8, 8, 8]]

3, "range", [1, 4, 7], [[7, 7, 7, 4, 4, 4, 1, 1, 1], [7, 7, 7, 1, 1, 1, 4, 4, 4], [4, 4, 4, 7, 7, 7, 1, 1, 1], [4, 4, 4, 1, 1, 1, 7, 7, 7], [1, 1, 1, 7, 7, 7, 4, 4, 4], [1, 1, 1, 4, 4, 4, 7, 7, 7]]

"group has", 6, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

g_1 =   [[1, 2]]

g_2 =   []

g_3 =   [[1, 3, 2]]

g_4 =   [[2, 3]]

g_5 =   [[1, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 2h[1]-h[2] & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 2, 7]}, {6, [1, 2, 8]}, {7, [1, 2, 9]}, {8, [1, 3, 4]}, {9, [1, 3, 5]}, {10, [1, 3, 6]}, {11, [1, 3, 7]}, {12, [1, 3, 8]}, {13, [1, 3, 9]}, {14, [1, 4, 5]}, {15, [1, 4, 6]}, {16, [1, 4, 7]}, {17, [1, 4, 8]}, {18, [1, 4, 9]}, {19, [1, 5, 6]}, {20, [1, 5, 7]}, {21, [1, 5, 8]}, {22, [1, 5, 9]}, {23, [1, 6, 7]}, {24, [1, 6, 8]}, {25, [1, 6, 9]}, {26, [1, 7, 8]}, {27, [1, 7, 9]}, {28, [1, 8, 9]}, {29, [2, 3, 4]}, {30, [2, 3, 5]}, {31, [2, 3, 6]}, {32, [2, 3, 7]}, {33, [2, 3, 8]}, {34, [2, 3, 9]}, {35, [2, 4, 5]}, {36, [2, 4, 6]}, {37, [2, 4, 7]}, {38, [2, 4, 8]}, {39, [2, 4, 9]}, {40, [2, 5, 6]}, {41, [2, 5, 7]}, {42, [2, 5, 8]}, {43, [2, 5, 9]}, {44, [2, 6, 7]}, {45, [2, 6, 8]}, {46, [2, 6, 9]}, {47, [2, 7, 8]}, {48, [2, 7, 9]}, {49, [2, 8, 9]}, {50, [3, 4, 5]}, {51, [3, 4, 6]}, {52, [3, 4, 7]}, {53, [3, 4, 8]}, {54, [3, 4, 9]}, {55, [3, 5, 6]}, {56, [3, 5, 7]}, {57, [3, 5, 8]}, {58, [3, 5, 9]}, {59, [3, 6, 7]}, {60, [3, 6, 8]}, {61, [3, 6, 9]}, {62, [3, 7, 8]}, {63, [3, 7, 9]}, {64, [3, 8, 9]}, {65, [4, 5, 6]}, {66, [4, 5, 7]}, {67, [4, 5, 8]}, {68, [4, 5, 9]}, {69, [4, 6, 7]}, {70, [4, 6, 8]}, {71, [4, 6, 9]}, {72, [4, 7, 8]}, {73, [4, 7, 9]}, {74, [4, 8, 9]}, {75, [5, 6, 7]}, {76, [5, 6, 8]}, {77, [5, 6, 9]}, {78, [5, 7, 8]}, {79, [5, 7, 9]}, {80, [5, 8, 9]}, {81, [6, 7, 8]}, {82, [6, 7, 9]}, {83, [6, 8, 9]}, {84, [7, 8, 9]}

KERNEL HIERARCHY

$\pi3$ =
(0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

{16, 42, 61}

$u3$ =
(323 255 195 235 303 363 0 795 447 675 45 70 165 15 105 1605 1335 705 110 1227 1

{1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84}

picheck $\begin{pmatrix} 3 & 2 & 1 & 3 & 2 & 1 & 3 & 2 & 1 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & \frac{1}{9} & \frac{1}{18} \end{pmatrix}$$

$\pi 2$ =
(0 0 3 0 0 3 0 0 0 0 2 0 0 2 0 0 0 1 0 0 1 0 0 3 0 0 0 0 2 0 0 0 1 0 0 0)

$u2$ =
(186 280 535 414 480 535 470 310 293 434 535 405 450 535 372 520 525 535 270 25

picheck $\begin{pmatrix} 6 & 4 & 2 & 6 & 4 & 2 & 6 & 4 & 2 \end{pmatrix}$

$$\pi 1 = \begin{pmatrix} 6 & 4 & 2 & 6 & 4 & 2 & 6 & 4 & 2 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{1070}{3} & \frac{1070}{3} & \frac{1070}{3} & \frac{1070}{3} & \frac{1070}{3} & \frac{1070}{3} & \frac{1070}{3} & \frac{1070}{3} & \frac{1070}{3} \end{pmatrix}$$

picheck $\begin{pmatrix} 6 & 4 & 2 & 6 & 4 & 2 & 6 & 4 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{51}{107} & 0 & 0 & \frac{11}{107} & 0 & 0 & \frac{45}{107} \\ 0 & 0 & \frac{242}{535} & 0 & 0 & \frac{26}{107} & 0 & 0 & \frac{163}{535} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{107} & 0 & 0 & \frac{89}{107} & 0 & 0 & \frac{15}{107} \\ 0 & 0 & \frac{2}{107} & 0 & 0 & \frac{71}{107} & 0 & 0 & \frac{34}{107} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{53}{107} & 0 & 0 & \frac{7}{107} & 0 & 0 & \frac{47}{107} \\ 0 & 0 & \frac{283}{535} & 0 & 0 & \frac{10}{107} & 0 & 0 & \frac{202}{535} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & \frac{349}{535} & 0 & 0 & \frac{121}{535} & 0 & 0 & \frac{13}{107} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{242}{535} & 0 & 0 & \frac{2}{107} & 0 & 0 & \frac{283}{535} & 0 \\ 0 & \frac{101}{535} & 0 & 0 & \frac{409}{535} & 0 & 0 & \frac{5}{107} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{26}{107} & 0 & 0 & \frac{71}{107} & 0 & 0 & \frac{10}{107} & 0 \\ 0 & \frac{17}{107} & 0 & 0 & \frac{1}{107} & 0 & 0 & \frac{89}{107} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{163}{535} & 0 & 0 & \frac{34}{107} & 0 & 0 & \frac{202}{535} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{349}{535} & 0 & 0 & \frac{101}{535} & 0 & 0 & \frac{17}{107} & 0 & 0 \\ \frac{51}{107} & 0 & 0 & \frac{3}{107} & 0 & 0 & \frac{53}{107} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{121}{535} & 0 & 0 & \frac{409}{535} & 0 & 0 & \frac{1}{107} & 0 & 0 \\ \frac{11}{107} & 0 & 0 & \frac{89}{107} & 0 & 0 & \frac{7}{107} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{13}{107} & 0 & 0 & \frac{5}{107} & 0 & 0 & \frac{89}{107} & 0 & 0 \\ \frac{45}{107} & 0 & 0 & \frac{15}{107} & 0 & 0 & \frac{47}{107} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_7 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_8 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_9 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_{10} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_{11} = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_{12} = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_{13} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_{14} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} \\ \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} \\ \frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_{15} = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_{16} = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & \frac{349}{1605} & \frac{17}{214} & 0 & \frac{121}{1605} & \frac{11}{642} & 0 & \frac{13}{321} & \frac{15}{214} \\ \frac{349}{1070} & \frac{1}{3} & \frac{121}{1605} & \frac{101}{1070} & 0 & \frac{13}{321} & \frac{17}{214} & 0 & \frac{163}{3210} \\ \frac{51}{214} & \frac{242}{1605} & \frac{1}{6} & \frac{3}{214} & \frac{2}{321} & 0 & \frac{53}{214} & \frac{283}{1605} & 0 \\ 0 & \frac{101}{1605} & \frac{1}{214} & \frac{1}{2} & \frac{409}{1605} & \frac{89}{642} & 0 & \frac{5}{321} & \frac{5}{214} \\ \frac{121}{1070} & 0 & \frac{1}{321} & \frac{409}{1070} & \frac{1}{3} & \frac{71}{642} & \frac{1}{214} & 0 & \frac{17}{321} \\ \frac{11}{214} & \frac{26}{321} & 0 & \frac{89}{214} & \frac{71}{321} & \frac{1}{6} & \frac{7}{214} & \frac{10}{321} & 0 \\ 0 & \frac{17}{321} & \frac{53}{642} & 0 & \frac{1}{321} & \frac{7}{642} & \frac{1}{2} & \frac{89}{321} & \frac{47}{642} \\ \frac{13}{214} & 0 & \frac{283}{3210} & \frac{5}{214} & 0 & \frac{5}{321} & \frac{89}{214} & \frac{1}{3} & \frac{101}{1605} \\ \frac{45}{214} & \frac{163}{1605} & 0 & \frac{15}{214} & \frac{34}{321} & 0 & \frac{47}{214} & \frac{202}{1605} & \frac{1}{6} \end{pmatrix} \quad NM =$$

$$\begin{pmatrix} 9 & \frac{2652}{535} & \frac{237}{107} & \frac{9}{2} & \frac{1968}{535} & \frac{177}{107} & \frac{9}{2} & \frac{360}{107} & \frac{228}{107} \\ \frac{3978}{535} & 6 & \frac{2331}{1070} & \frac{2862}{535} & 3 & \frac{399}{214} & \frac{558}{107} & 3 & \frac{1047}{535} \\ \frac{711}{107} & \frac{2331}{535} & 3 & \frac{495}{107} & \frac{327}{107} & \frac{3}{2} & \frac{720}{107} & \frac{2454}{535} & \frac{3}{2} \\ \frac{9}{2} & \frac{1908}{535} & \frac{165}{107} & 9 & \frac{2832}{535} & \frac{294}{107} & \frac{9}{2} & \frac{336}{107} & \frac{183}{107} \\ \frac{2952}{535} & 3 & \frac{327}{214} & \frac{4248}{535} & 6 & \frac{267}{107} & \frac{486}{107} & 3 & \frac{423}{214} \\ \frac{531}{107} & \frac{399}{107} & \frac{3}{2} & \frac{882}{107} & \frac{534}{107} & 3 & \frac{513}{107} & \frac{351}{107} & \frac{3}{2} \\ \frac{9}{2} & \frac{372}{107} & \frac{240}{107} & \frac{9}{2} & \frac{324}{107} & \frac{171}{107} & 9 & \frac{588}{107} & \frac{231}{107} \\ \frac{540}{107} & 3 & \frac{1227}{535} & \frac{504}{107} & 3 & \frac{351}{214} & \frac{882}{107} & 6 & \frac{2211}{1070} \\ \frac{684}{107} & \frac{2094}{535} & \frac{3}{2} & \frac{549}{107} & \frac{423}{107} & \frac{3}{2} & \frac{693}{107} & \frac{2211}{535} & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [2, −1, −1, 2, −1, −1, 2, −1, −1]

ker $N_C = \begin{pmatrix} -1 & 1 & 0 & -1 & 1 & 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & -t & t & 0 & -t & t & 0 & -t & t \\ -s+t & -t+s & 0 & -s+t & -t+s & 0 & -s+t & -t+s & 0 \end{pmatrix}$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & s & 0 & 0 & t & 0 \\ t & s & 0 & 0 & 0 & 0 \\ 0 & t & -s & 0 & -s & 0 \\ 0 & -s & t & -s & 0 & 0 \\ 0 & -s & 0 & -s & 0 & t \\ 0 & -t & s & -t & 0 & 0 \\ 0 & 0 & -t & s & -t & 0 \\ -t & 0 & 0 & s & 0 & -t \\ 0 & 0 & 0 & t & s & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 1 & 1 & -1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & t & 0 & s & 0 & 0 & 0 \\ t & 0 & 0 & s & 0 & 0 & 0 \\ 0 & 0 & s & t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s & t \\ -t & t & t & 0 & -t & s & t \\ 0 & 0 & 0 & 0 & 0 & t & s \\ 0 & s & s+t & -s & 0 & -s & s \\ 0 & s & s & -s & t & -s & s \\ 0 & s+t & t & -t & 0 & -t & t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 3 & 3 & 0 & 0 & 0 & 3 \end{pmatrix}$

$$R N_0 R^* = \begin{pmatrix} 1 & 1 & \frac{409}{535} & 0 & 0 & \frac{5}{107} & 0 & 0 & \frac{101}{535} \\ 1 & 1 & \frac{409}{535} & 0 & 0 & \frac{5}{107} & 0 & 0 & \frac{101}{535} \\ \frac{409}{535} & \frac{409}{535} & 1 & \frac{1}{107} & \frac{1}{107} & 0 & \frac{121}{535} & \frac{121}{535} & 0 \\ 0 & 0 & \frac{1}{107} & 1 & 1 & \frac{89}{107} & 0 & 0 & \frac{17}{107} \\ 0 & 0 & \frac{1}{107} & 1 & 1 & \frac{89}{107} & 0 & 0 & \frac{17}{107} \\ \frac{5}{107} & \frac{5}{107} & 0 & \frac{89}{107} & \frac{89}{107} & 1 & \frac{13}{107} & \frac{13}{107} & 0 \\ 0 & 0 & \frac{121}{535} & 0 & 0 & \frac{13}{107} & 1 & 1 & \frac{349}{535} \\ 0 & 0 & \frac{121}{535} & 0 & 0 & \frac{13}{107} & 1 & 1 & \frac{349}{535} \\ \frac{101}{535} & \frac{101}{535} & 0 & \frac{17}{107} & \frac{17}{107} & 0 & \frac{349}{535} & \frac{349}{535} & 1 \end{pmatrix}, \quad R R^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & \frac{163}{535} & \frac{101}{535} & 0 & \frac{242}{535} & \frac{17}{107} & 0 & \frac{26}{107} & \frac{349}{535} \\ \frac{163}{535} & 1 & \frac{15}{107} & \frac{202}{535} & 0 & \frac{47}{107} & \frac{34}{107} & 0 & \frac{45}{107} \\ \frac{101}{535} & \frac{15}{107} & 1 & \frac{5}{107} & \frac{3}{107} & 0 & \frac{409}{535} & \frac{89}{107} & 0 \\ 0 & \frac{202}{535} & \frac{5}{107} & 1 & \frac{283}{535} & \frac{89}{107} & 0 & \frac{10}{107} & \frac{13}{107} \\ \frac{242}{535} & 0 & \frac{3}{107} & \frac{283}{535} & 1 & \frac{53}{107} & \frac{2}{107} & 0 & \frac{51}{107} \\ \frac{17}{107} & \frac{47}{107} & 0 & \frac{89}{107} & \frac{53}{107} & 1 & \frac{1}{107} & \frac{7}{107} & 0 \\ 0 & \frac{34}{107} & \frac{409}{535} & 0 & \frac{2}{107} & \frac{1}{107} & 1 & \frac{71}{107} & \frac{121}{535} \\ \frac{26}{107} & 0 & \frac{89}{107} & \frac{10}{107} & 0 & \frac{7}{107} & \frac{71}{107} & 1 & \frac{11}{107} \\ \frac{349}{535} & \frac{45}{107} & 0 & \frac{13}{107} & \frac{51}{107} & 0 & \frac{121}{535} & \frac{11}{107} & 1 \end{pmatrix}, \quad B B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 9, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 \\ \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} \\ 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 \\ 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 \\ \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} \\ 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 \\ 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 & 0 & \frac{-3}{2} & -3 \\ \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} & \frac{3}{2} & 0 & \frac{-3}{2} \\ 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 & 3 & \frac{3}{2} & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & \frac{-349}{3210} & \frac{-17}{107} & 0 & \frac{-121}{3210} & \frac{-11}{321} & 0 & \frac{-13}{642} & \frac{-15}{107} \\ \frac{349}{3210} & 0 & \frac{-121}{1605} & \frac{101}{3210} & 0 & \frac{-13}{321} & \frac{17}{642} & 0 & \frac{-163}{3210} \\ \frac{17}{107} & \frac{121}{1605} & 0 & \frac{1}{107} & \frac{1}{321} & 0 & \frac{53}{321} & \frac{283}{3210} & 0 \\ 0 & \frac{-101}{3210} & \frac{-1}{107} & 0 & \frac{-409}{3210} & \frac{-89}{321} & 0 & \frac{-5}{642} & \frac{-5}{107} \\ \frac{121}{3210} & 0 & \frac{-1}{321} & \frac{409}{3210} & 0 & \frac{-71}{642} & \frac{1}{642} & 0 & \frac{-17}{321} \\ \frac{11}{321} & \frac{13}{321} & 0 & \frac{89}{321} & \frac{71}{642} & 0 & \frac{7}{321} & \frac{5}{321} & 0 \\ 0 & \frac{-17}{642} & \frac{-53}{321} & 0 & \frac{-1}{642} & \frac{-7}{321} & 0 & \frac{-89}{642} & \frac{-47}{321} \\ \frac{13}{642} & 0 & \frac{-283}{3210} & \frac{5}{642} & 0 & \frac{-5}{321} & \frac{89}{642} & 0 & \frac{-101}{1605} \\ \frac{15}{107} & \frac{163}{3210} & 0 & \frac{5}{107} & \frac{17}{321} & 0 & \frac{47}{321} & \frac{101}{1605} & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} \\ \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} \\ \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 & \frac{9}{2} & 0 & 0 \\ 0 & 3 & 0 & 0 & 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{349}{535} & \frac{51}{107} & 0 & \frac{121}{535} & \frac{11}{107} & 0 & \frac{13}{107} & \frac{45}{107} \\ \frac{349}{535} & 1 & \frac{242}{535} & \frac{101}{535} & 0 & \frac{26}{107} & \frac{17}{107} & 0 & \frac{163}{535} \\ \frac{51}{107} & \frac{242}{535} & 1 & \frac{3}{107} & \frac{2}{107} & 0 & \frac{53}{107} & \frac{283}{535} & 0 \\ 0 & \frac{101}{535} & \frac{3}{107} & 1 & \frac{409}{535} & \frac{89}{107} & 0 & \frac{5}{107} & \frac{15}{107} \\ \frac{121}{535} & 0 & \frac{2}{107} & \frac{409}{535} & 1 & \frac{71}{107} & \frac{1}{107} & 0 & \frac{34}{107} \\ \frac{11}{107} & \frac{26}{107} & 0 & \frac{89}{107} & \frac{71}{107} & 1 & \frac{7}{107} & \frac{10}{107} & 0 \\ 0 & \frac{17}{107} & \frac{53}{107} & 0 & \frac{1}{107} & \frac{7}{107} & 1 & \frac{89}{107} & \frac{47}{107} \\ \frac{13}{107} & 0 & \frac{283}{535} & \frac{5}{107} & 0 & \frac{10}{107} & \frac{89}{107} & 1 & \frac{202}{535} \\ \frac{45}{107} & \frac{163}{535} & 0 & \frac{15}{107} & \frac{34}{107} & 0 & \frac{47}{107} & \frac{202}{535} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 9T + 27\Omega$$

Ω

$$\left(\frac{1}{9}\ \frac{2}{9}\ \frac{1}{3}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ 0\ 0\ 0\ 0\ 0\ 0\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\right.$$

T

$$\left(\frac{31}{321}\ \frac{86}{321}\ \frac{103}{214}\ 0\ \frac{26}{321}\ \frac{11}{214}\ \frac{14}{321}\ \frac{94}{321}\ \frac{53}{107}\ \frac{-1}{642}\ \frac{-101}{1605}\ \frac{121}{1070}\ \frac{6}{107}\ \frac{108}{535}\ \frac{1}{2}\ \frac{1}{214}\ \frac{101}{1605}\ 0\ 0\ \frac{2}{321}\ \frac{3}{214}\ \frac{1}{6}\right.$$

"IS NM in Vec(K)?", true

NM

$$\left(\frac{414}{107}\ \frac{900}{107}\ \frac{2853}{214}\ \frac{3}{2}\ \frac{399}{107}\ \frac{531}{107}\ \frac{405}{214}\ \frac{603}{107}\ \frac{1917}{214}\ \frac{-3}{214}\ \frac{-303}{535}\ \frac{1089}{1070}\ \frac{54}{107}\ \frac{972}{535}\ \frac{9}{2}\ \frac{165}{107}\ \frac{1908}{535}\ \frac{9}{2}\ \frac{3}{2}\ \frac{327}{107}\right.$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{814}{107}\ \frac{946}{107}\ \frac{2021}{214}\ \frac{7}{2}\ \frac{827}{214}\ \frac{391}{107}\ \frac{391}{107}\ \frac{637}{107}\ \frac{1385}{214}\ \frac{46}{107}\ \frac{-187}{535}\ \frac{1261}{1070}\ \frac{162}{107}\ \frac{1458}{535}\ \frac{9}{2}\ \frac{281}{107}\ \frac{1792}{535}\ \frac{5}{2}\ \frac{7}{2}\ \frac{755}{214}\right.$$

$$\boldsymbol{\tau} = 27/1 \text{ , rank} = 3 \text{ , ratio} = 9/1 \text{ , } n^2 / r = 27/1$$

$$\boldsymbol{\tau}' = 54/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 7/54 \text{ , min } \boldsymbol{\tau} = 21/2 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 33/2$$

$$\text{max r} = 54/7 \text{ , } r\text{-check is positive? } 11/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 27\Omega$$

There are, 16, partitions and, 3, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 39
out of total no. of elements equal to 288

dim span idems 27 vs no. of idems 48

"PT1" = {{3, 7, 8}, {4, 5, 6}, {1, 2, 9}}

"PT2" = {{3, 5, 7}, {1, 2, 9}, {4, 6, 8}}

"PT3" = {{3, 4, 8}, {5, 6, 7}, {1, 2, 9}}

"PT4" = {{6, 7, 8}, {1, 2, 9}, {3, 4, 5}}

"PT5" = {{1, 5, 9}, {2, 4, 6}, {3, 7, 8}}

"PT6" = {{3, 5, 7}, {2, 4, 6}, {1, 8, 9}}

"PT7" = {{1, 5, 9}, {2, 3, 7}, {4, 6, 8}}

"PT8" = {{1, 8, 9}, {2, 3, 7}, {4, 5, 6}}

"PT9" = {{3, 7, 8}, {2, 4, 9}, {1, 5, 6}}

"PT10" = {{3, 4, 8}, {2, 7, 9}, {1, 5, 6}}

"PT11" = {{3, 7, 8}, {1, 2, 6}, {4, 5, 9}}

"PT12" = {{7, 8, 9}, {1, 2, 6}, {3, 4, 5}}

"PT13" = {{1, 3, 8}, {5, 6, 7}, {2, 4, 9}}

"PT14" = {{1, 3, 8}, {4, 5, 6}, {2, 7, 9}}

"PT15" = {{1, 2, 3}, {6, 7, 8}, {4, 5, 9}}

"PT16" = {{7, 8, 9}, {4, 5, 6}, {1, 2, 3}}

"RG1" = {3, 6, 9}

"RG2" = {2, 5, 8}

"RG3" = {1, 4, 7}

$$M_C = \begin{pmatrix} \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} \\ \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} \\ \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} \\ \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} \\ \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} \\ \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} \\ \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{9}{4} & \frac{-3}{2} & \frac{-3}{4} \\ \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} & \frac{-3}{2} & 2 & \frac{-1}{2} \\ \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{4} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix}
\frac{47}{54} & \frac{15101}{28890} & \frac{2005}{5778} & \frac{-7}{54} & \frac{2789}{28890} & \frac{-155}{5778} & \frac{-7}{54} & \frac{-47}{5778} & \frac{1681}{5778} \\
\frac{15101}{28890} & \frac{47}{54} & \frac{9323}{28890} & \frac{1709}{28890} & \frac{-7}{54} & \frac{655}{5778} & \frac{169}{5778} & \frac{-7}{54} & \frac{5057}{28890} \\
\frac{2005}{5778} & \frac{9323}{28890} & \frac{47}{54} & \frac{-587}{5778} & \frac{-641}{5778} & \frac{-7}{54} & \frac{2113}{5778} & \frac{11537}{28890} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{1709}{28890} & \frac{-587}{5778} & \frac{47}{54} & \frac{18341}{28890} & \frac{4057}{5778} & \frac{-7}{54} & \frac{-479}{5778} & \frac{61}{5778} \\
\frac{2789}{28890} & \frac{-7}{54} & \frac{-641}{5778} & \frac{18341}{28890} & \frac{47}{54} & \frac{3085}{5778} & \frac{-695}{5778} & \frac{-7}{54} & \frac{1087}{5778} \\
\frac{-155}{5778} & \frac{655}{5778} & \frac{-7}{54} & \frac{4057}{5778} & \frac{3085}{5778} & \frac{47}{54} & \frac{-371}{5778} & \frac{-209}{5778} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{169}{5778} & \frac{2113}{5778} & \frac{-7}{54} & \frac{-695}{5778} & \frac{-371}{5778} & \frac{47}{54} & \frac{4057}{5778} & \frac{1789}{5778} \\
\frac{-47}{5778} & \frac{-7}{54} & \frac{11537}{28890} & \frac{-479}{5778} & \frac{-7}{54} & \frac{-209}{5778} & \frac{4057}{5778} & \frac{47}{54} & \frac{7163}{28890} \\
\frac{1681}{5778} & \frac{5057}{28890} & \frac{-7}{54} & \frac{61}{5778} & \frac{1087}{5778} & \frac{-7}{54} & \frac{1789}{5778} & \frac{7163}{28890} & \frac{47}{54}
\end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix}
1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\
\frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} \\
\frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 \\
1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\
\frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} \\
\frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 \\
1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\
\frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} & \frac{-3}{4} & 1 & \frac{-1}{4} \\
\frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1 & \frac{-3}{5} & \frac{-2}{5} & 1
\end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix}
1 & \frac{15101}{25145} & \frac{2005}{5029} & \frac{-7}{47} & \frac{2789}{25145} & \frac{-155}{5029} & \frac{-7}{47} & \frac{-1}{107} & \frac{1681}{5029} \\
\frac{15101}{25145} & 1 & \frac{9323}{25145} & \frac{1709}{25145} & \frac{-7}{47} & \frac{655}{5029} & \frac{169}{5029} & \frac{-7}{47} & \frac{5057}{25145} \\
\frac{2005}{5029} & \frac{9323}{25145} & 1 & \frac{-587}{5029} & \frac{-641}{5029} & \frac{-7}{47} & \frac{2113}{5029} & \frac{11537}{25145} & \frac{-7}{47} \\
\frac{-7}{47} & \frac{1709}{25145} & \frac{-587}{5029} & 1 & \frac{18341}{25145} & \frac{4057}{5029} & \frac{-7}{47} & \frac{-479}{5029} & \frac{61}{5029} \\
\frac{2789}{25145} & \frac{-7}{47} & \frac{-641}{5029} & \frac{18341}{25145} & 1 & \frac{3085}{5029} & \frac{-695}{5029} & \frac{-7}{47} & \frac{1087}{5029} \\
\frac{-155}{5029} & \frac{655}{5029} & \frac{-7}{47} & \frac{4057}{5029} & \frac{3085}{5029} & 1 & \frac{-371}{5029} & \frac{-209}{5029} & \frac{-7}{47} \\
\frac{-7}{47} & \frac{169}{5029} & \frac{2113}{5029} & \frac{-7}{47} & \frac{-695}{5029} & \frac{-371}{5029} & 1 & \frac{4057}{5029} & \frac{1789}{5029} \\
\frac{-1}{107} & \frac{-7}{47} & \frac{11537}{25145} & \frac{-479}{5029} & \frac{-7}{47} & \frac{-209}{5029} & \frac{4057}{5029} & 1 & \frac{7163}{25145} \\
\frac{1681}{5029} & \frac{5057}{25145} & \frac{-7}{47} & \frac{61}{5029} & \frac{1087}{5029} & \frac{-7}{47} & \frac{1789}{5029} & \frac{7163}{25145} & 1
\end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 10.95416346, 5.545836544]

Eigenvalues N_C

[0., 0., 1.833333333, 0.1834570064, 0.2994612445, 0.5737448721, 1.042324551, 1.556005848, 2.345006479]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 5.170820393, 3.829179607]

Eigenvalues N_C–scaled

[0., 0., 2.106382979, 0.2107803903, 0.3440618555, 0.6591962362, 1.197564379, 1.787751400, 2.694262763]

NullSpace M_C

{[1, 0, 1, 0, 0, 0, 0, 1, 0], [1, 0, 1, 0, 1, 0, 0, 0, 0], [1, 1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 0, 1], [0, 0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0, 0]}

NullSpace N_C

{[−1, 0, 1, −1, 0, 1, −1, 0, 1], [−1, 1, 0, −1, 1, 0, −1, 1, 0]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 9., 4.500000000, 13.50000000]

Eigenvalues N_0

[0., 0., 3., 0.1834570064, 0.2994612445, 0.5737448721, 1.042324551, 1.556005848, 2.345006479]

NullSpace M_0

{[0, 0, −1, 0, 0, 0, 0, 0, 1], [1, 0, 0, −1, 0, 0, 0, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 1, 0, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1, 0]}

NullSpace N_0

{[−1, 0, 1, −1, 0, 1, −1, 0, 1], [−1, 1, 0, −1, 1, 0, −1, 1, 0]}

Eigenvalues M

[3., 6., 9., −3., −1.500000000, −4.500000000, −3., −1.500000000, −4.500000000]

Eigenvalues N

[0., 0., 6., −0.1834570064, −0.2994612445, −0.5737448721, −1.042324551, −1.556005848, −2.345006479]

NullSpace M

{}

NullSpace N

{[−1, 0, 1, −1, 0, 1, −1, 0, 1], [−1, 1, 0, −1, 1, 0, −1, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 186 & 280 & 535 & 414 & 480 & 535 & 470 & 310 \\ 186 & 0 & 293 & 434 & 535 & 405 & 450 & 535 & 372 \\ 280 & 293 & 0 & 520 & 525 & 535 & 270 & 252 & 535 \\ 535 & 434 & 520 & 0 & 126 & 90 & 535 & 510 & 460 \\ 414 & 535 & 525 & 126 & 0 & 180 & 530 & 535 & 365 \\ 480 & 405 & 535 & 90 & 180 & 0 & 500 & 485 & 535 \\ 535 & 450 & 270 & 535 & 530 & 500 & 0 & 90 & 300 \\ 470 & 535 & 252 & 510 & 535 & 485 & 90 & 0 & 333 \\ 310 & 372 & 535 & 460 & 365 & 535 & 300 & 333 & 0 \end{pmatrix}$$

==

80, [1, 1, 1, −1, 1, −1, 1, 1, −1]
==

100, [1, −1, −1, 1, −1, 1, −1, 1, 1]

===

{2, 4, 7, 9}

R: [4, 9, 4, 8, 7, 7, 5, 1, 2]
B: [2, 4, 5, 7, 3, 8, 1, 6, 1]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 7, "vs", 8

Level 2 det = $\frac{1}{2199023255552}(-1+s)(-137723972800 + 64927469600s + 122287437820s^2 - 65469249366s^3 - 35094544264s^4 + 19357202379s^5 + 7065193172s^6 - 3096147653s^7 - 1692049858s^8 + 967225608s^9 + 54980652s^{10} - 222994932s^{11} + 41718020s^{12} + 27205542s^{13} - 3064996s^{14} - 3957258s^{15} + 297568s^{16} + 313040s^{17} - 22840s^{18} - 12494s^{19} - 604s^{20} + 919s^{21} - 32s^{22} - 25s^{23} + 2s^{24})$

RANK of R is 7

R ranking is 4, "vs", 7

RBAR ranking 4, "vs", 7

RANK of B is 8

B ranking is 4, "vs", 8

BBAR ranking 4, "vs", 8

"R CYCLES", (1 + v[5] v[7]) (1 + v[2] v[9]) (1 + v[1] v[4] v[8])
"B CYCLES", (1 + v[1] v[2] v[4] v[7]) (1 + v[6] v[8]) (1 + v[3] v[5])

Eigenvalues

R: [−0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 0., 0., −1., −1., 1., 1., 1.]

B: [0., 1. I, −1. I, 1., −1., 1., −1., 1., −1.]

NullSpace of R

{[0, 0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of R^*

{[1, 0, −1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, −1, 0, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, 0, 0, 1, 0, −1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 284 & 0 & 0 & 208 & 174 & 382 & 0 & 92 \\ 284 & 0 & 104 & 188 & 0 & 0 & 0 & 184 & 0 \\ 0 & 104 & 0 & 0 & 0 & 114 & 162 & 0 & 0 \\ 0 & 188 & 0 & 0 & 324 & 92 & 342 & 0 & 194 \\ 208 & 0 & 0 & 324 & 0 & 0 & 0 & 228 & 0 \\ 174 & 0 & 114 & 92 & 0 & 0 & 0 & 0 & 0 \\ 382 & 0 & 162 & 342 & 0 & 0 & 0 & 254 & 0 \\ 0 & 184 & 0 & 0 & 228 & 0 & 254 & 0 & 94 \\ 92 & 0 & 0 & 194 & 0 & 0 & 0 & 94 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($3v[1] + 2v[2] + v[3] + 3v[4] + 2v[5] + v[6] + 3v[7] + 2v[8] + v[9]$)

degree 2: $\frac{71}{855}$ ($142v[1]v[2] + 104v[1]v[5] + 87v[1]v[6] + 191v[1]v[7] + 46v[1]v[9] + 52v[2]v[3] + 94v[2]v[4] + 92v[2]v[8] + 57v[3]v[6] + 81v[3]v[7] + 162v[4]v[5] + 46v[4]v[6] + 171v[4]v[7] + 97v[4]v[9] + 114v[5]v[8] + 127v[7]v[8] + 47v[8]v[9]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7, 9}}

"RG1" = {8, 9}

"RG2" = {7, 8}

"RG3" = {5, 8}

"RG4" = {2, 8}

"RG5" = {4, 9}

"RG6" = {4, 7}

"RG7" = {4, 6}

"RG8" = {4, 5}

"RG9" = {2, 4}

"RG10" = {3, 7}

"RG11" = {3, 6}

"RG12" = {2, 3}

"RG13" = {1, 9}

"RG14" = {1, 7}

"RG15" = {1, 6}

"RG16" = {1, 5}

"RG17" = {1, 2}

$\pi 2$ = [142, 0, 0, 104, 87, 191, 0, 46, 52, 94, 0, 0, 0, 92, 0, 0, 0, 57, 81, 0, 0, 162, 46, 171, 0, 97, 0, 0, 114, 0, 0, 0, 0, 127, 0, 47]

supp $\pi 2$ = {1, 4, 5, 6, 8, 9, 10, 14, 18, 19, 22, 23, 24, 26, 29, 34, 36}

$u2$ = [1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1]

supp u2 = {1, 4, 5, 6, 8, 9, 10, 14, 17, 18, 19, 21, 22, 23, 24, 26, 29, 32, 34, 36}

Action of R on ranges, [[17], [16], [14], [13], [4], [3], [2], [2], [1], [8], [6], [5], [9], [8], [6], [6], [5]]
Action of B on ranges, [[15], [15], [11], [7], [14], [14], [2], [10], [6], [16], [3], [8], [17], [17], [4], [12], [9]]

$$\boldsymbol{\beta} = \left(\frac{47}{1710} \;\; \frac{127}{1710} \;\; \frac{1}{15} \;\; \frac{46}{855} \;\; \frac{97}{1710} \;\; \frac{1}{10} \;\; \frac{23}{855} \;\; \frac{9}{95} \;\; \frac{47}{855} \;\; \frac{9}{190} \;\; \frac{1}{30} \;\; \frac{26}{855} \;\; \frac{23}{855} \;\; \frac{191}{1710} \;\; \frac{29}{570} \;\; \frac{52}{855} \;\; \frac{71}{855}\right)$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[1, 2]

B–BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N – check: true

b_1 = {1, 3, 4, 8}
b_2 = {2, 5, 6, 7, 9}

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 9}, {1, 4, 8}, {5, 7}}, false

Ω_B in Vec(K)? , {{6, 8}, {1, 2, 4, 7}, {3, 5}}, false

$$V = \begin{pmatrix} \frac{1}{30} & \frac{-29}{45} & \frac{7}{90} & \frac{7}{30} & \frac{7}{45} & \frac{7}{90} & \frac{7}{30} & \frac{7}{45} & \frac{-29}{90} \\ \frac{1}{10} & \frac{2}{5} & \frac{-1}{10} & \frac{-3}{10} & \frac{-1}{5} & \frac{-1}{10} & \frac{-3}{10} & \frac{-1}{5} & \frac{7}{10} \\ \frac{2}{15} & \frac{4}{45} & \frac{-16}{45} & \frac{13}{30} & \frac{-32}{45} & \frac{13}{90} & \frac{-1}{15} & \frac{13}{45} & \frac{2}{45} \\ \frac{1}{15} & \frac{2}{45} & \frac{-8}{45} & \frac{-1}{30} & \frac{-16}{45} & \frac{29}{90} & \frac{-8}{15} & \frac{29}{45} & \frac{1}{45} \\ \frac{1}{5} & \frac{2}{15} & \frac{-8}{15} & \frac{-1}{10} & \frac{-1}{15} & \frac{-1}{30} & \frac{2}{5} & \frac{-1}{15} & \frac{1}{15} \\ \frac{-1}{15} & \frac{-2}{45} & \frac{8}{45} & \frac{1}{30} & \frac{16}{45} & \frac{-29}{90} & \frac{8}{15} & \frac{-29}{45} & \frac{-1}{45} \\ \frac{-8}{15} & \frac{-16}{45} & \frac{19}{45} & \frac{-7}{30} & \frac{38}{45} & \frac{-7}{90} & \frac{4}{15} & \frac{-7}{45} & \frac{-8}{45} \\ \frac{2}{5} & \frac{4}{15} & \frac{-1}{15} & \frac{3}{10} & \frac{-2}{15} & \frac{-17}{30} & \frac{-1}{5} & \frac{-2}{15} & \frac{2}{15} \\ \frac{-13}{30} & \frac{17}{45} & \frac{-1}{90} & \frac{-1}{30} & \frac{-1}{45} & \frac{-1}{90} & \frac{-1}{30} & \frac{-1}{45} & \frac{17}{90} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{4}{27} & \frac{1}{9} & 0 & \frac{4}{27} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{4}{27} & \frac{1}{9} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{5}{36} & \frac{5}{36} & \frac{1}{9} & \frac{5}{36} & \frac{1}{9} & \frac{1}{9} & \frac{5}{36} & \frac{1}{9} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 4, 8}, {2, 5, 6, 7, 9}}

1, "range", [8, 9], [[9, 8, 9, 9, 8, 8, 8, 9, 8], [8, 9, 8, 8, 9, 9, 9, 8, 9]]

2, "range", [7, 8], [[8, 7, 8, 8, 7, 7, 7, 8, 7], [7, 8, 7, 7, 8, 8, 8, 7, 8]]

3, "range", [5, 8], [[8, 5, 8, 8, 5, 5, 5, 8, 5], [5, 8, 5, 5, 8, 8, 8, 5, 8]]

4, "range", [2, 8], [[8, 2, 8, 8, 2, 2, 2, 8, 2], [2, 8, 2, 2, 8, 8, 8, 2, 8]]

5, "range", [4, 9], [[9, 4, 9, 9, 4, 4, 4, 9, 4], [4, 9, 4, 4, 9, 9, 9, 4, 9]]

6, "range", [4, 7], [[7, 4, 7, 7, 4, 4, 4, 7, 4], [4, 7, 4, 4, 7, 7, 7, 4, 7]]

7, "range", [4, 6], [[6, 4, 6, 6, 4, 4, 4, 6, 4], [4, 6, 4, 4, 6, 6, 6, 4, 6]]

8, "range", [4, 5], [[5, 4, 5, 5, 4, 4, 4, 5, 4], [4, 5, 4, 4, 5, 5, 5, 4, 5]]

9, "range", [2, 4], [[4, 2, 4, 4, 2, 2, 2, 4, 2], [2, 4, 2, 2, 4, 4, 4, 2, 4]]

10, "range", [3, 7], [[7, 3, 7, 7, 3, 3, 3, 7, 3], [3, 7, 3, 3, 7, 7, 7, 3, 7]]

11, "range", [3, 6], [[6, 3, 6, 6, 3, 3, 3, 6, 3], [3, 6, 3, 3, 6, 6, 6, 3, 6]]

12, "range", [2, 3], [[3, 2, 3, 3, 2, 2, 2, 3, 2], [2, 3, 2, 2, 3, 3, 3, 2, 3]]

13, "range", [1, 9], [[9, 1, 9, 9, 1, 1, 1, 9, 1], [1, 9, 1, 1, 9, 9, 9, 1, 9]]

14, "range", [1, 7], [[7, 1, 7, 7, 1, 1, 1, 7, 1], [1, 7, 1, 1, 7, 7, 7, 1, 7]]

15, "range", [1, 6], [[6, 1, 6, 6, 1, 1, 1, 6, 1], [1, 6, 1, 1, 6, 6, 6, 1, 6]]

16, "range", [1, 5], [[5, 1, 5, 5, 1, 1, 1, 5, 1], [1, 5, 1, 1, 5, 5, 5, 1, 5]]

17, "range", [1, 2], [[2, 1, 2, 2, 1, 1, 1, 2, 1], [1, 2, 1, 1, 2, 2, 2, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

g_1 =   [[1, 2]]

g_2 =   []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [1, 9]}, {9, [2, 3]}, {10, [2, 4]}, {11, [2, 5]}, {12, [2, 6]}, {13, [2, 7]}, {14, [2, 8]}, {15, [2, 9]}, {16, [3, 4]}, {17, [3, 5]}, {18, [3, 6]}, {19, [3, 7]}, {20, [3, 8]}, {21, [3, 9]}, {22, [4, 5]}, {23, [4, 6]}, {24, [4, 7]}, {25, [4, 8]}, {26, [4, 9]}, {27, [5, 6]}, {28, [5, 7]}, {29, [5, 8]}, {30, [5, 9]}, {31, [6, 7]}, {32, [6, 8]}, {33, [6, 9]}, {34, [7, 8]}, {35, [7, 9]}, {36, [8, 9]}

KERNEL HIERARCHY

$\pi 2$ =
(142 0 0 104 87 191 0 46 52 94 0 0 0 92 0 0 0 57 81 0 0 162 46 171 0 97 0 0 114

{1, 4, 5, 6, 8, 9, 10, 14, 18, 19, 22, 23, 24, 26, 29, 34, 36}

$u2$ =
$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$

{1, 4, 5, 6, 8, 9, 10, 14, 17, 18, 19, 21, 22, 23, 24, 26, 29, 32, 34, 36}

picheck $\begin{pmatrix} 570 & 380 & 190 & 570 & 380 & 190 & 570 & 380 & 190 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & \frac{1}{9} & \frac{1}{18} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 570 & 380 & 190 & 570 & 380 & 190 & 570 & 380 & 190 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 570 & 380 & 190 & 570 & 380 & 190 & 570 & 380 & 190 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \end{pmatrix}$$ idem-checks N0-checks

$$P_9 = \begin{pmatrix} 0&0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{10} = \begin{pmatrix} 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{11} = \begin{pmatrix} 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{12} = \begin{pmatrix} 0&0&1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{15} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{16} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{17} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{9} & 0 \\ 0 & \frac{2}{9} & 0 & 0 & \frac{2}{9} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} \\ \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{9} & 0 \\ \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{9} & 0 \\ 0 & \frac{2}{9} & 0 & 0 & \frac{2}{9} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} \\ 0 & \frac{2}{9} & 0 & 0 & \frac{2}{9} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} \\ 0 & \frac{2}{9} & 0 & 0 & \frac{2}{9} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} \\ \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{9} & 0 \\ 0 & \frac{2}{9} & 0 & 0 & \frac{2}{9} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{9} & 0 \\ 0 & \frac{2}{9} & 0 & 0 & \frac{2}{9} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} \\ \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{9} & 0 \\ \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{9} & 0 \\ 0 & \frac{2}{9} & 0 & 0 & \frac{2}{9} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} \\ 0 & \frac{2}{9} & 0 & 0 & \frac{2}{9} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} \\ 0 & \frac{2}{9} & 0 & 0 & \frac{2}{9} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} \\ \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{9} & 0 \\ 0 & \frac{2}{9} & 0 & 0 & \frac{2}{9} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{20}{3} & 0 & \frac{20}{9} & \frac{20}{3} & 0 & 0 & 0 & \frac{40}{9} & 0 \\ 0 & \frac{40}{9} & 0 & 0 & \frac{40}{9} & \frac{20}{9} & \frac{20}{3} & 0 & \frac{20}{9} \\ \frac{20}{3} & 0 & \frac{20}{9} & \frac{20}{3} & 0 & 0 & 0 & \frac{40}{9} & 0 \\ \frac{20}{3} & 0 & \frac{20}{9} & \frac{20}{3} & 0 & 0 & 0 & \frac{40}{9} & 0 \\ 0 & \frac{40}{9} & 0 & 0 & \frac{40}{9} & \frac{20}{9} & \frac{20}{3} & 0 & \frac{20}{9} \\ 0 & \frac{40}{9} & 0 & 0 & \frac{40}{9} & \frac{20}{9} & \frac{20}{3} & 0 & \frac{20}{9} \\ 0 & \frac{40}{9} & 0 & 0 & \frac{40}{9} & \frac{20}{9} & \frac{20}{3} & 0 & \frac{20}{9} \\ \frac{20}{3} & 0 & \frac{20}{9} & \frac{20}{3} & 0 & 0 & 0 & \frac{40}{9} & 0 \\ 0 & \frac{40}{9} & 0 & 0 & \frac{40}{9} & \frac{20}{9} & \frac{20}{3} & 0 & \frac{20}{9} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, -1, 1, 1, -1, 0, 1, 1]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -t & 0 & 0 & t & 0 & 0 & 0 & 0 \\ 0 & -t & 0 & -s & 0 & 0 & t & s & 0 \\ 0 & 0 & t & -t & 0 & 0 & s & 0 & -s \\ 0 & 0 & 0 & -t & 0 & 0 & s & t & -s \\ t & 0 & 0 & -t & s & 0 & 0 & 0 & -s \\ s & -t & 0 & -s & 0 & t & 0 & 0 & 0 \\ t & s & 0 & -t & 0 & 0 & 0 & 0 & -s \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & 1 & -1 & 0 & 1 & 1 \end{pmatrix}$

M0 is invertible. det= 7915130081940690625/533378347616448

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} t+s \\ t+s \\ t+s \\ t+s \\ t+s \\ t+s \\ t+s \\ t+s \\ t+s \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = (9)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 7, 8, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & \frac{-20}{9} & \frac{-41}{9} & 0 & \frac{-20}{9} & \frac{-40}{9} & 0 & \frac{-41}{18} & \frac{-40}{9} \\ \frac{20}{9} & 0 & \frac{-20}{9} & \frac{20}{9} & 0 & \frac{-41}{18} & \frac{41}{18} & 0 & \frac{-41}{18} \\ \frac{41}{9} & \frac{20}{9} & 0 & \frac{41}{9} & \frac{20}{9} & 0 & \frac{40}{9} & \frac{41}{18} & 0 \\ 0 & \frac{-20}{9} & \frac{-41}{9} & 0 & \frac{-20}{9} & \frac{-40}{9} & 0 & \frac{-41}{18} & \frac{-40}{9} \\ \frac{20}{9} & 0 & \frac{-20}{9} & \frac{20}{9} & 0 & \frac{-41}{18} & \frac{41}{18} & 0 & \frac{-41}{18} \\ \frac{40}{9} & \frac{41}{18} & 0 & \frac{40}{9} & \frac{41}{18} & 0 & \frac{41}{9} & \frac{20}{9} & 0 \\ 0 & \frac{-41}{18} & \frac{-40}{9} & 0 & \frac{-41}{18} & \frac{-41}{9} & 0 & \frac{-20}{9} & \frac{-41}{9} \\ \frac{41}{18} & 0 & \frac{-41}{18} & \frac{41}{18} & 0 & \frac{-20}{9} & \frac{20}{9} & 0 & \frac{-20}{9} \\ \frac{40}{9} & \frac{41}{18} & 0 & \frac{40}{9} & \frac{41}{18} & 0 & \frac{41}{9} & \frac{20}{9} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{-2}{9} & 0 & 0 & 0 & 0 & \frac{-1}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{9} & \frac{1}{9} & 0 & \frac{-1}{9} \\ \frac{2}{9} & 0 & 0 & \frac{2}{9} & 0 & 0 & 0 & \frac{1}{9} & 0 \\ 0 & 0 & \frac{-2}{9} & 0 & 0 & 0 & 0 & \frac{-1}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{9} & \frac{1}{9} & 0 & \frac{-1}{9} \\ 0 & \frac{1}{9} & 0 & 0 & \frac{1}{9} & 0 & \frac{2}{9} & 0 & 0 \\ 0 & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-2}{9} & 0 & 0 & \frac{-2}{9} \\ \frac{1}{9} & 0 & \frac{-1}{9} & \frac{1}{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{9} & 0 & 0 & \frac{1}{9} & 0 & \frac{2}{9} & 0 & 0 \end{pmatrix}$$

$$\text{Skew Omega} = \begin{pmatrix} 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{41}{6} & \frac{284}{171} & 0 & 0 & \frac{208}{171} & \frac{58}{57} & \frac{382}{171} & 0 & \frac{92}{171} \\ \frac{284}{171} & \frac{41}{9} & \frac{104}{171} & \frac{188}{171} & 0 & 0 & 0 & \frac{184}{171} & 0 \\ 0 & \frac{104}{171} & \frac{41}{18} & 0 & 0 & \frac{2}{3} & \frac{18}{19} & 0 & 0 \\ 0 & \frac{188}{171} & 0 & \frac{41}{6} & \frac{36}{19} & \frac{92}{171} & 2 & 0 & \frac{194}{171} \\ \frac{208}{171} & 0 & 0 & \frac{36}{19} & \frac{41}{9} & 0 & 0 & \frac{4}{3} & 0 \\ \frac{58}{57} & 0 & \frac{2}{3} & \frac{92}{171} & 0 & \frac{41}{18} & 0 & 0 & 0 \\ \frac{382}{171} & 0 & \frac{18}{19} & 2 & 0 & 0 & \frac{41}{6} & \frac{254}{171} & 0 \\ 0 & \frac{184}{171} & 0 & 0 & \frac{4}{3} & 0 & \frac{254}{171} & \frac{41}{9} & \frac{94}{171} \\ \frac{92}{171} & 0 & 0 & \frac{194}{171} & 0 & 0 & 0 & \frac{94}{171} & \frac{41}{18} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 20T + 0\Omega$$

$\Omega\left(\frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\right)$

$T\left(\frac{1}{3}\ \frac{1}{9}\ \frac{2}{9}\ 0\ 0\ \frac{2}{9}\ 0\ 0\ \frac{2}{9}\ 0\ 0\ 0\ \frac{1}{3}\ \frac{1}{9}\ 0\ \frac{1}{3}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM}\left(\frac{20}{3}\ \frac{20}{9}\ \frac{40}{9}\ 0\ 0\ \frac{40}{9}\ 0\ 0\ \frac{40}{9}\ 0\ 0\ 0\ \frac{20}{3}\ \frac{20}{9}\ 0\ \frac{20}{3}\right)$

"IS MN in Vec(K)?", false

$\mathrm{MN}\left(4\ 4\ 4\ 0\ 0\ 4\ 0\ 0\ 5\ 0\ 0\ 0\ 5\ 5\ 0\ 5\right)$

$$\boldsymbol{\tau} = 41/1 \text{ , rank} = 2 \text{ , ratio} = 41/2 \text{ , } n^2 / r = 81/2$$

$$\boldsymbol{\tau}' = 40/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 41/81$$

$$p^2 = 7/54 \text{ , min } \boldsymbol{\tau} = 21/2 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 61/2$$

$$\text{max r} = 54/7 \text{ , } r\text{-check is positive? } 20/27$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{2} T + 40\Omega$$

There are, 1, partitions and, 17, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 16
out of total no. of elements equal to 34

dim span idems 8 vs no. of idems 17

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7, 9}}

"RG1" = {8, 9}

"RG2" = {7, 8}

"RG3" = {5, 8}

"RG4" = {2, 8}

"RG5" = {4, 9}

"RG6" = {4, 7}

"RG7" = {4, 6}

"RG8" = {4, 5}

"RG9" = {2, 4}

"RG10" = {3, 7}

"RG11" = {3, 6}

"RG12" = {2, 3}

"RG13" = {1, 9}

"RG14" = {1, 7}

"RG15" = {1, 6}

"RG16" = {1, 5}

"RG17" = {1, 2}

$$M_c = \begin{pmatrix} \frac{55}{12} & \frac{55}{342} & \frac{-3}{4} & \frac{-9}{4} & \frac{-97}{342} & \frac{61}{228} & \frac{-11}{684} & \frac{-3}{2} & \frac{-145}{684} \\ \frac{55}{342} & \frac{32}{9} & \frac{37}{342} & \frac{-137}{342} & -1 & \frac{-1}{2} & \frac{-3}{2} & \frac{13}{171} & \frac{-1}{2} \\ \frac{-3}{4} & \frac{37}{342} & \frac{73}{36} & \frac{-3}{4} & \frac{-1}{2} & \frac{5}{12} & \frac{15}{76} & \frac{-1}{2} & \frac{-1}{4} \\ \frac{-9}{4} & \frac{-137}{342} & \frac{-3}{4} & \frac{55}{12} & \frac{15}{38} & \frac{-145}{684} & \frac{-1}{4} & \frac{-3}{2} & \frac{263}{684} \\ \frac{-97}{342} & -1 & \frac{-1}{2} & \frac{15}{38} & \frac{32}{9} & \frac{-1}{2} & \frac{-3}{2} & \frac{1}{3} & \frac{-1}{2} \\ \frac{61}{228} & \frac{-1}{2} & \frac{5}{12} & \frac{-145}{684} & \frac{-1}{2} & \frac{73}{36} & \frac{-3}{4} & \frac{-1}{2} & \frac{-1}{4} \\ \frac{-11}{684} & \frac{-3}{2} & \frac{15}{76} & \frac{-1}{4} & \frac{-3}{2} & \frac{-3}{4} & \frac{55}{12} & \frac{-5}{342} & \frac{-3}{4} \\ \frac{-3}{2} & \frac{13}{171} & \frac{-1}{2} & \frac{-3}{2} & \frac{1}{3} & \frac{-1}{2} & \frac{-5}{342} & \frac{32}{9} & \frac{17}{342} \\ \frac{-145}{684} & \frac{-1}{2} & \frac{-1}{4} & \frac{263}{684} & \frac{-1}{2} & \frac{-1}{4} & \frac{-3}{4} & \frac{17}{342} & \frac{73}{36} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix}
\frac{47}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{47}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{47}{54} \\
\frac{47}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} \\
\frac{47}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{47}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{47}{54} \\
\frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{47}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{47}{54} \\
\frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{47}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{47}{54} \\
\frac{47}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{47}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{47}{54}
\end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix}
1 & \frac{2}{57} & \frac{-9}{55} & \frac{-27}{55} & \frac{-194}{3135} & \frac{61}{1045} & \frac{-1}{285} & \frac{-18}{55} & \frac{-29}{627} \\
\frac{55}{1216} & 1 & \frac{37}{1216} & \frac{-137}{1216} & \frac{-9}{32} & \frac{-9}{64} & \frac{-27}{64} & \frac{13}{608} & \frac{-9}{64} \\
\frac{-27}{73} & \frac{74}{1387} & 1 & \frac{-27}{73} & \frac{-18}{73} & \frac{15}{73} & \frac{135}{1387} & \frac{-18}{73} & \frac{-9}{73} \\
\frac{-27}{55} & \frac{-274}{3135} & \frac{-9}{55} & 1 & \frac{18}{209} & \frac{-29}{627} & \frac{-3}{55} & \frac{-18}{55} & \frac{263}{3135} \\
\frac{-97}{1216} & \frac{-9}{32} & \frac{-9}{64} & \frac{135}{1216} & 1 & \frac{-9}{64} & \frac{-27}{64} & \frac{3}{32} & \frac{-9}{64} \\
\frac{183}{1387} & \frac{-18}{73} & \frac{15}{73} & \frac{-145}{1387} & \frac{-18}{73} & 1 & \frac{-27}{73} & \frac{-18}{73} & \frac{-9}{73} \\
\frac{-1}{285} & \frac{-18}{55} & \frac{9}{209} & \frac{-3}{55} & \frac{-18}{55} & \frac{-9}{55} & 1 & \frac{-2}{627} & \frac{-9}{55} \\
\frac{-27}{64} & \frac{13}{608} & \frac{-9}{64} & \frac{-27}{64} & \frac{3}{32} & \frac{-9}{64} & \frac{-5}{1216} & 1 & \frac{17}{1216} \\
\frac{-145}{1387} & \frac{-18}{73} & \frac{-9}{73} & \frac{263}{1387} & \frac{-18}{73} & \frac{-9}{73} & \frac{-27}{73} & \frac{34}{1387} & 1
\end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix}
1 & \frac{-7}{47} & 1 & 1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & 1 & \frac{-7}{47} \\
\frac{-7}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & 1 & 1 & 1 & \frac{-7}{47} & 1 \\
1 & \frac{-7}{47} & 1 & 1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & 1 & \frac{-7}{47} \\
1 & \frac{-7}{47} & 1 & 1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & 1 & \frac{-7}{47} \\
\frac{-7}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & 1 & 1 & 1 & \frac{-7}{47} & 1 \\
\frac{-7}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & 1 & 1 & 1 & \frac{-7}{47} & 1 \\
\frac{-7}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & 1 & 1 & 1 & \frac{-7}{47} & 1 \\
1 & \frac{-7}{47} & 1 & 1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & 1 & \frac{-7}{47} \\
\frac{-7}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & 1 & 1 & 1 & \frac{-7}{47} & 1
\end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{12} & \frac{-1}{18} & \frac{1}{36} & \frac{1}{12} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{12} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{12} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{12} & \frac{1}{18} & \frac{1}{36} & \frac{1}{12} & \frac{-1}{18} & \frac{1}{36} \\ \frac{1}{12} & \frac{-1}{18} & \frac{1}{36} & \frac{1}{12} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{12} & \frac{1}{18} & \frac{-1}{36} \\ \frac{1}{12} & \frac{-1}{18} & \frac{1}{36} & \frac{1}{12} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{12} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{12} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{12} & \frac{1}{18} & \frac{1}{36} & \frac{1}{12} & \frac{-1}{18} & \frac{1}{36} \\ \frac{-1}{12} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{12} & \frac{1}{18} & \frac{1}{36} & \frac{1}{12} & \frac{-1}{18} & \frac{1}{36} \\ \frac{-1}{12} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{12} & \frac{1}{18} & \frac{1}{36} & \frac{1}{12} & \frac{-1}{18} & \frac{1}{36} \\ \frac{1}{12} & \frac{-1}{18} & \frac{1}{36} & \frac{1}{12} & \frac{-1}{18} & \frac{-1}{36} & \frac{-1}{12} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{12} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{12} & \frac{1}{18} & \frac{1}{36} & \frac{1}{12} & \frac{-1}{18} & \frac{1}{36} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{-1}{12} \\ \frac{-1}{18} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{18} & \frac{1}{18} \\ \frac{1}{36} & \frac{-1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{36} & \frac{-1}{36} \\ \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{-1}{12} \\ \frac{-1}{18} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{18} & \frac{1}{18} \\ \frac{-1}{36} & \frac{1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{-1}{36} & \frac{1}{36} \\ \frac{-1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} \\ \frac{1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{1}{36} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{-1}{36} & \frac{1}{36} \end{pmatrix}$$

$$\text{commutator} = \begin{pmatrix} 0 & \frac{-1}{36} & \frac{1}{18} & 0 & \frac{-1}{36} & \frac{-1}{18} & 0 & \frac{1}{36} & \frac{-1}{18} \\ \frac{1}{36} & 0 & \frac{-1}{36} & \frac{1}{36} & 0 & \frac{1}{36} & \frac{-1}{36} & 0 & \frac{1}{36} \\ \frac{-1}{18} & \frac{1}{36} & 0 & \frac{-1}{18} & \frac{1}{36} & 0 & \frac{1}{18} & \frac{-1}{36} & 0 \\ 0 & \frac{-1}{36} & \frac{1}{18} & 0 & \frac{-1}{36} & \frac{-1}{18} & 0 & \frac{1}{36} & \frac{-1}{18} \\ \frac{1}{36} & 0 & \frac{-1}{36} & \frac{1}{36} & 0 & \frac{1}{36} & \frac{-1}{36} & 0 & \frac{1}{36} \\ \frac{1}{18} & \frac{-1}{36} & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & \frac{-1}{18} & \frac{1}{36} & 0 \\ 0 & \frac{1}{36} & \frac{-1}{18} & 0 & \frac{1}{36} & \frac{1}{18} & 0 & \frac{-1}{36} & \frac{1}{18} \\ \frac{-1}{36} & 0 & \frac{1}{36} & \frac{-1}{36} & 0 & \frac{-1}{36} & \frac{1}{36} & 0 & \frac{-1}{36} \\ \frac{1}{18} & \frac{-1}{36} & 0 & \frac{1}{18} & \frac{-1}{36} & 0 & \frac{-1}{18} & \frac{1}{36} & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0.1118793035, 1.745290388, 2.723735798, 3.079326871, 4.526287211, 5.264164101, 5.961708558, 7.087607770]

Eigenvalues N_C

[0., 0., 0., 0., 0., 0., 0., 4.641554909, 3.191778425]

Eigenvalues M_C–scaled

[0., 0.03281340121, 0.7675812720, 1.102901475, 1.208154342, 1.313527735, 1.422784477, 1.452468140, 1.699769157]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 0., 0., 0., 5.332850320, 3.667149680]

NullSpace M_C

{[1, 1, 1, 1, 1, 1, 1, 1, 1]}

NullSpace N_c

{[0, 0, 1, 0, 0, 0, 0, −1, 0], [0, 0, 0, 0, 0, −1, 0, 0, 1], [0, 0, 0, 1, 0, 0, 0, −1, 0], [0, 0, 0, 0, 1, −1, 0, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, −1, 0], [0, 1, 0, 0, 0, −1, 0, 0, 0]}

Eigenvalues M_0

[0.1104838232, 1.719841449, 2.607000483, 2.793490989, 4.490954921, 5.168818519, 5.819856907, 7.083691964, 11.20586095]

Eigenvalues N_0

[0., 0., 0., 0., 0., 0., 0., 5., 4.]

NullSpace M_0

{}

NullSpace N_0

{[−1, 0, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 0, 1]}

Eigenvalues M

[0., 0.1322291293, 0.4540714571, 1.186228425, 5.143306206]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 0., 4.472135954, −4.472135954]

NullSpace M

/[−10527 25083 850 −19001]\ { [0, ------, 0, 0, -----, 1, ----, 0, ------] } \[4772 9544 1193 4772]/

NullSpace N

{[0, 0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, −1, 0, 0, 1], [1, 0, 0, 0, 0, 0, 0, −1, 0], [0, 1, 0, 0, 0, −1, 0, 0, 0], [0, 0, 0, 0, 1, −1, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, −1, 0], [0, 0, 0, 1, 0, 0, 0, −1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

==

120, [1, −1, 1, 1, −1, −1, 1, −1, 1]
==

140, [1, 1, −1, 1, −1, −1, 1, −1, 1]
==

160, [1, 1, 1, 1, −1, −1, −1, 1, −1]
==

180, [1, −1, −1, 1, 1, −1, −1, −1, 1]
==

200, [1, 1, −1, −1, −1, −1, 1, −1, 1]
==

220, [1, −1, −1, −1, −1, −1, −1, 1, 1]

==

{2, 3, 5, 6, 8, 9}

R: [4, 9, 5, 7, 3, 8, 1, 6, 2]
B: [2, 4, 4, 8, 7, 7, 5, 1, 1]

TRACE TWO = 2

det AT = $\frac{-1}{16}(1+t^2)(t)^2(1+t)^3$

$$AT = \begin{pmatrix} 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 9

Level 2 det = $\frac{1}{2199023255552}(-1+s)^2(275447945600 - 56854516800s - 179876593320s^2 + 16038723988s^3 + 38027670700s^4 + 7250860988s^5 - 7667567913s^6 - 888049096s^7 + 1607512502s^8 - 249803268s^9 - 111755582s^{10} - 17585324s^{11} - 5260738s^{12} + 14753980s^{13} - 595976s^{14} - 1011836s^{15} + 15794s^{16} + 91844s^{17} - 35530s^{18} + 1368s^{19} + 2094s^{20} - 248s^{21} - 31s^{22} + 4s^{23})$

RANK of R is 9

R ranking is 2, "vs", 9

RBAR ranking 2, "vs", 9

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", (1 + v[6] v[8]) (1 + v[3] v[5]) (1 + v[2] v[9]) (1 + v[1] v[4] v[7])
"B CYCLES", (1 + v[5] v[7]) (1 + v[1] v[2] v[4] v[8])

Eigenvalues

R: [−0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, −1., −1., −1., 1., 1., 1., 1.]

B: [1. I, −1. I, 1., −1., 1., −1., 0., 0., 0.]

NullSpace of R

{}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{}

NullSpace of B^*

{[0, −1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, −1, 1]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 4 & 15 & 8 & 4 & 15 & 8 & 4 \\ 8 & 0 & 0 & 8 & 10 & 0 & 8 & 10 & 0 \\ 4 & 0 & 0 & 4 & 0 & 5 & 4 & 0 & 5 \\ 15 & 8 & 4 & 0 & 8 & 4 & 15 & 8 & 4 \\ 8 & 10 & 0 & 8 & 0 & 0 & 8 & 10 & 0 \\ 4 & 0 & 5 & 4 & 0 & 0 & 4 & 0 & 5 \\ 15 & 8 & 4 & 15 & 8 & 4 & 0 & 8 & 4 \\ 8 & 10 & 0 & 8 & 10 & 0 & 8 & 0 & 0 \\ 4 & 0 & 5 & 4 & 0 & 5 & 4 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & \frac{1}{3} & 1 & 1 & \frac{2}{3} & 1 & 1 & 1 \\ 1 & \frac{1}{3} & 0 & 1 & 1 & 1 & 1 & \frac{2}{3} & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & \frac{1}{3} & 1 & 1 & \frac{2}{3} \\ 1 & \frac{2}{3} & 1 & 1 & \frac{1}{3} & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & \frac{2}{3} & 1 & 1 & 1 & 1 & 0 & \frac{1}{3} \\ 1 & 1 & 1 & 1 & \frac{2}{3} & 1 & 1 & \frac{1}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 9

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{6}$ ($3v[1] + 2v[2] + v[3] + 3v[4] + 2v[5] + v[6] + 3v[7] + 2v[8] + v[9]$)

degree 2: $\frac{2}{27}$ ($2v[3]v[7] + 2v[6]v[7] + 4v[1]v[5] + 4v[5]v[7] + 2v[7]v[9] + 3v[6]v[9] + 2v[3]v[4] + 6v[2]v[8] + 2v[1]v[6] + 3v[3]v[6] + 4v[7]v[8] + 4v[4]v[5] + 3v[3]v[9] + 4v[4]v[8] + 6v[5]v[8] + 4v[2]v[7] + 2v[1]v[9] + 4v[1]v[2] + 9v[4]v[7] + 4v[2]v[4] + 2v[4]v[9] + 4v[1]v[8] + 9v[1]v[7] + 2v[1]v[3] + 2v[4]v[6] + 9v[1]v[4] + 6v[2]v[5]$)

degree 3 : $\frac{1}{27}$ ($v[1]v[4]v[6] + v[1]v[3]v[7] + v[4]v[6]v[9] + v[4]v[7]v[9] + v[1]v[7]v[9] + 2v[4]v[5]v[7] + v[3]v[6]v[7] + 2v[4]v[7]v[8] + 2v[1]v[2]v[8] + v[6]v[7]v[9] + v[1]v[6]v[9] + 2v[2]v[7]v[8] + v[1]v[4]v[9] + 2v[4]v[5]v[8] + 2v[1]v[5]v[7] + 2v[2]v[4]v[8] + v[3]v[4]v[9] + 2v[1]v[4]v[8] + 2v[1]v[4]v[5] + 27v[1]v[4]v[7] + 2v[1]v[5]v[8]$

$+ v[3]v[4]v[7] + 2v[1]v[2]v[4] + 2v[2]v[5]v[7] + 2v[5]v[7]v[8] + 2v[1]v[2]v[5] + v[1]v[3]v[6] + v[1]v[3]v[4] + v[1]v[6]v[7] + 18v[2]v[5]v[8] + 2v[1]v[2]v[7] + v[3]v[7]v[9] + v[4]v[6]v[7] + 9v[3]v[6]v[9] + v[1]v[3]v[9] + 2v[2]v[4]v[7] + 2v[1]v[7]v[8] + 2v[2]v[4]v[5] + v[3]v[4]v[6])$

degree 4 : $\frac{2}{27}(4v[1]v[2]v[7]v[8] + 3v[1]v[4]v[7]v[9] + 4v[1]v[4]v[5]v[8] + 2v[1]v[3]v[7]v[9] + 3v[3]v[4]v[6]v[9] + 3v[1]v[3]v[4]v[7] + 2v[3]v[4]v[6]v[7] + 6v[1]v[4]v[5]v[7] + 6v[1]v[2]v[5]v[8] + 3v[1]v[4]v[6]v[7] + 4v[2]v[4]v[5]v[7] + 2v[1]v[3]v[4]v[9] + 6v[1]v[4]v[7]v[8] + 2v[1]v[3]v[4]v[6] + 6v[1]v[2]v[4]v[7] + 2v[3]v[4]v[7]v[9] + 4v[1]v[2]v[5]v[7] + 4v[1]v[2]v[4]v[8] + 4v[2]v[4]v[7]v[8] + 2v[1]v[6]v[7]v[9] + 4v[4]v[5]v[7]v[8] + 6v[2]v[4]v[5]v[8] + 6v[2]v[5]v[7]v[8] + 3v[1]v[3]v[6]v[9] + 2v[4]v[6]v[7]v[9] + 2v[1]v[3]v[6]v[7] + 3v[3]v[6]v[7]v[9] + 4v[1]v[2]v[4]v[5] + 2v[1]v[4]v[6]v[9] + 4v[1]v[5]v[7]v[8])$

degree 5 : $\frac{1}{9}(2v[1]v[2]v[4]v[5]v[7] + 2v[1]v[2]v[4]v[5]v[8] + 2v[1]v[2]v[4]v[7]v[8] + 2v[1]v[2]v[5]v[7]v[8] + v[1]v[3]v[4]v[6]v[7] + v[1]v[3]v[4]v[6]v[9] + v[1]v[3]v[4]v[7]v[9] + v[1]v[3]v[6]v[7]v[9] + 2v[1]v[4]v[5]v[7]v[8] + v[1]v[4]v[6]v[7]v[9] + 2v[2]v[4]v[5]v[7]v[8] + v[3]v[4]v[6]v[7]v[9])$

degree 6 : $\frac{2}{3}(v[4])(v[7])(2v[2]v[5]v[8] + v[3]v[6]v[9])(v[1])$

Group spectrum $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

KERNEL STRUCTURE

"PT1" = {{1}, {5, 9}, {3, 8}, {4}, {7}, {2, 6}}

"PT2" = {{1}, {2, 3}, {4}, {7}, {5, 6}, {8, 9}}

"RG1" = {1, 3, 4, 6, 7, 9}

"RG2" = {1, 2, 4, 5, 7, 8}

π6 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp π6 = {24, 43}

u6 = [0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 2, 0, 2, 2, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 2, 2, 0, 2, 3, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp u6 = {2, 9, 24, 25, 27, 28, 30, 36, 39, 40, 42, 43, 51, 55}

Action of R on ranges, [[2], [1]]
Action of B on ranges, [[2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[6, 4, 7, 2, 1, 5, 3, 9, 8]

B-BLOCKS,

[8, 6, 5, 1, 4, 7, 6, 5, 1]

with invariant measure, [3, 1, 1, 2, 3, 3, 2, 2, 1]

N by blocks, N – check: true

b_1 = {1}
b_2 = {5, 9}
b_3 = {3, 8}
b_4 = {2, 3}
b_5 = {4}
b_6 = {7}
b_7 = {5, 6}
b_8 = {8, 9}
b_9 = {2, 6}

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & h[1] & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & h[1] & 0 \\ 0 & 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & h[1] \\ h[1] & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 \\ 0 & 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & h[1] \\ h[1] & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 & 0 \\ 0 & h[1] & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 \\ 0 & 0 & h[1] & 0 & 0 & h[1] & 0 & 0 & h[2] \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{6, 8}, {1, 4, 7}, {2, 9}, {3, 5}}, true

Ω_B in Vec(K)? , {{1, 2, 4, 8}, {5, 7}}, true

$$V = \begin{pmatrix} \frac{1}{30} & \frac{-29}{45} & \frac{7}{90} & \frac{7}{30} & \frac{7}{45} & \frac{7}{90} & \frac{7}{30} & \frac{7}{45} & \frac{-29}{90} \\ \frac{1}{10} & \frac{2}{5} & \frac{-1}{10} & \frac{-3}{10} & \frac{-1}{5} & \frac{-1}{10} & \frac{-3}{10} & \frac{-1}{5} & \frac{7}{10} \\ \frac{-2}{15} & \frac{-4}{45} & \frac{16}{45} & \frac{-13}{30} & \frac{32}{45} & \frac{-13}{90} & \frac{1}{15} & \frac{-13}{45} & \frac{-2}{45} \\ \frac{-1}{15} & \frac{-2}{45} & \frac{8}{45} & \frac{1}{30} & \frac{16}{45} & \frac{-29}{90} & \frac{8}{15} & \frac{-29}{45} & \frac{-1}{45} \\ \frac{-1}{5} & \frac{-2}{15} & \frac{8}{15} & \frac{1}{10} & \frac{1}{15} & \frac{1}{30} & \frac{-2}{5} & \frac{1}{15} & \frac{-1}{15} \\ \frac{1}{15} & \frac{2}{45} & \frac{-8}{45} & \frac{-1}{30} & \frac{-16}{45} & \frac{29}{90} & \frac{-8}{15} & \frac{29}{45} & \frac{1}{45} \\ \frac{8}{15} & \frac{16}{45} & \frac{-19}{45} & \frac{7}{30} & \frac{-38}{45} & \frac{7}{90} & \frac{-4}{15} & \frac{7}{45} & \frac{8}{45} \\ \frac{-2}{5} & \frac{-4}{15} & \frac{1}{15} & \frac{-3}{10} & \frac{2}{15} & \frac{17}{30} & \frac{1}{5} & \frac{2}{15} & \frac{-2}{15} \\ \frac{-13}{30} & \frac{17}{45} & \frac{-1}{90} & \frac{-1}{30} & \frac{-1}{45} & \frac{-1}{90} & \frac{-1}{30} & \frac{-1}{45} & \frac{17}{90} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \left(\frac{1}{9} \;\; \frac{1}{9} \;\; \frac{1}{9} \;\; \frac{1}{9} \;\; \frac{1}{9} \;\; \frac{1}{9} \;\; \frac{1}{9} \;\; \frac{1}{9} \;\; \frac{1}{9}\right)$ vs $(0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0)$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \left(\frac{1}{6} \;\; \frac{1}{6} \;\; 0 \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; 0 \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; 0\right)$ vs $(0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0)$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1}, {5, 9}, {3, 8}, {4}, {7}, {2, 6}}

1, "range", [1, 3, 4, 6, 7, 9], [[9, 7, 4, 6, 1, 7, 3, 4, 1], [9, 7, 1, 3, 4, 7, 6, 1, 4], [9, 4, 7, 3, 1, 4, 6, 7, 1], [9, 4, 1, 6, 7, 4, 3, 1, 7], [9, 1, 7, 6, 4, 1, 3, 7, 4], [9, 1, 4, 3, 7, 1, 6, 4, 7], [7, 9, 6, 1, 3, 9, 4, 6, 3], [7, 9, 3, 4, 6, 9, 1, 3, 6], [7, 6, 9, 4, 3, 6, 1, 9, 3], [7, 6, 3, 1, 9, 6, 4, 3, 9], [7, 3, 9, 1, 6, 3, 4, 9, 6], [7, 3, 6, 4, 9, 3, 1, 6, 9], [6, 7, 4, 3, 1, 7, 9, 4, 1], [6, 7, 1, 9, 4, 7, 3, 1, 4], [6, 4, 7, 9, 1, 4, 3, 7, 1], [6, 4, 1, 3, 7, 4, 9, 1, 7], [6, 1, 7, 3, 4, 1, 9, 7, 4], [6, 1, 4, 9, 7, 1, 3, 4, 7], [4, 9, 6, 7, 3, 9, 1, 6, 3], [4, 9, 3, 1, 6, 9, 7, 3, 6], [4, 6, 9, 1, 3, 6, 7, 9, 3], [4, 6, 3, 7, 9, 6, 1, 3, 9], [4, 3, 9, 7, 6, 3, 1, 9, 6], [4, 3, 6, 1, 9, 3, 7, 6, 9], [3, 7, 4, 9, 1, 7, 6, 4, 1], [3, 7, 1, 6, 4, 7, 9, 1, 4], [3, 4, 7, 6, 1, 4, 9, 7, 1], [3, 4, 1, 9, 7, 4, 6, 1, 7], [3, 1, 7, 9, 4, 1, 6, 7, 4], [3, 1, 4, 6, 7, 1, 9, 4, 7], [1, 9, 6, 4, 3, 9, 7, 6, 3], [1, 9, 3, 7, 6, 9, 4, 3, 6], [1, 6, 9, 7, 3, 6, 4, 9, 3], [1, 6, 3, 4, 9, 6, 7, 3, 9], [1, 3, 9, 4, 6, 3, 7, 9, 6], [1, 3, 6, 7, 9, 3, 4, 6, 9]]

2, "range", [1, 2, 4, 5, 7, 8], [[8, 7, 4, 5, 1, 7, 2, 4, 1], [8, 7, 1, 2, 4, 7, 5, 1, 4], [8, 4, 7, 2, 1, 4, 5, 7, 1], [8, 4, 1, 5, 7, 4, 2, 1, 7], [8, 1, 7, 5, 4, 1, 2, 7, 4], [8, 1, 4, 2, 7, 1, 5, 4, 7], [7, 8, 5, 1, 2, 8, 4, 5, 2], [7, 8, 2, 4, 5, 8, 1, 2, 5], [7, 5, 8, 4, 2, 5, 1, 8, 2], [7, 5, 2, 1, 8, 5, 4, 2, 8], [7, 2, 8, 1, 5, 2, 4, 8, 5], [7, 2, 5, 4, 8, 2, 1, 5, 8], [5, 7, 4, 2, 1, 7, 8, 4, 1], [5, 7, 1, 8, 4, 7, 2, 1, 4], [5, 4, 7, 8, 1, 4, 2, 7, 1], [5, 4, 1, 2, 7, 4, 8, 1, 7], [5, 1, 7, 2, 4, 1, 8, 7, 4], [5, 1, 4, 8, 7, 1, 2, 4, 7], [4, 8, 5, 7, 2, 8, 1, 5, 2], [4, 8, 2, 1, 5, 8, 7, 2, 5], [4, 5, 8, 1, 2, 5, 7, 8, 2], [4, 5, 2, 7, 8, 5, 1, 2, 8], [4, 2, 8, 7, 5, 2, 1, 8, 5], [4, 2, 5, 1, 8, 2, 7, 5, 8], [2, 7, 4, 8, 1, 7, 5, 4, 1], [2, 7, 1, 5, 4, 7, 8, 1, 4], [2, 4, 7, 5, 1, 4, 8, 7, 1], [2, 4, 1, 8, 7, 4, 5, 1, 7], [2, 1, 7, 8, 4, 1, 5, 7, 4], [2, 1, 4, 5, 7, 1, 8, 4, 7], [1, 8, 5, 4, 2, 8, 7, 5, 2], [1, 8, 2, 7, 5, 8, 4, 2, 5], [1, 5, 8, 7, 2, 5, 4, 8, 2], [1, 5, 2, 4, 8, 5, 7, 2, 8], [1, 2, 8, 4, 5, 2, 7, 8, 5], [1, 2, 5, 7, 8, 2, 4, 5, 8]]

2, "partition", {{1}, {2, 3}, {4}, {7}, {5, 6}, {8, 9}}

1, "range", [1, 3, 4, 6, 7, 9], [[9, 7, 7, 6, 1, 1, 3, 4, 4], [9, 7, 7, 3, 4, 4, 6, 1, 1], [9, 4, 4, 6, 7, 7, 3, 1, 1], [9, 4, 4, 3, 1, 1, 6, 7, 7], [9, 1, 1, 6, 4, 4, 3, 7, 7], [9, 1, 1, 3, 7, 7, 6, 4, 4], [7, 9, 9, 4, 6, 6, 1, 3, 3], [7, 9, 9, 1, 3, 3, 4, 6, 6], [7, 6, 6, 4, 3, 3, 1, 9, 9], [7, 6, 6, 1, 9, 9, 4, 3, 3], [7, 3, 3, 4, 9, 9, 1, 6, 6], [7, 3, 3, 1, 6, 6, 4, 9, 9], [6, 7, 7, 9, 4, 4, 3, 1, 1], [6, 7, 7, 3, 1, 1, 9, 4, 4], [6, 4, 4, 9, 1, 1, 3, 7, 7], [6, 4, 4, 3, 7, 7, 9, 1, 1], [6, 1, 1, 9, 7, 7, 3, 4, 4], [6, 1, 1, 3, 4, 4, 9, 7, 7], [4, 9, 9, 7, 3, 3, 1, 6, 6], [4, 9, 9, 1, 6, 6, 7, 3, 3], [4, 6, 6, 7, 9, 9, 1, 3, 3], [4, 6, 6, 1, 3, 3, 7, 9, 9], [4, 3, 3, 7, 6, 6, 1, 9, 9], [4, 3, 3, 1, 9, 9, 7, 6, 6], [3, 7, 7, 9, 1, 1, 6, 4, 4], [3, 7, 7, 6, 4, 4, 9, 1, 1], [3, 4, 4, 9, 7, 7, 6, 1, 1], [3, 4, 4, 6, 1, 1, 9, 7, 7], [3, 1, 1, 9, 4, 4, 6, 7, 7], [3, 1, 1, 6, 7, 7, 9, 4, 4], [1, 9, 9, 7, 6, 6, 4, 3, 3], [1, 9, 9, 4, 3, 3, 7, 6, 6], [1, 6, 6, 7, 3, 3, 4, 9, 9], [1, 6, 6, 4, 9, 9, 7, 3, 3], [1, 3, 3, 7, 9, 9, 4, 6, 6], [1, 3, 3, 4, 6, 6, 7, 9, 9]]

2, "range", [1, 2, 4, 5, 7, 8], [[8, 7, 7, 5, 1, 1, 2, 4, 4], [8, 7, 7, 2, 4, 4, 5, 1, 1], [8, 4, 4, 5, 7, 7, 2, 1, 1], [8, 4, 4, 2, 1, 1, 5, 7, 7], [8, 1, 1, 5, 4, 4, 2, 7, 7], [8, 1, 1, 2, 7, 7, 5, 4, 4], [7, 8, 8, 4, 5, 5, 1, 2, 2], [7, 8, 8, 1, 2, 2, 4, 5, 5], [7, 5, 5, 4, 2, 2, 1, 8, 8], [7, 5, 5, 1, 8, 8, 4, 2, 2], [7, 2, 2, 4, 8, 8, 1, 5, 5], [7, 2, 2, 1, 5, 5, 4, 8, 8], [5, 7, 7, 8, 4, 4, 2, 1, 1], [5, 7, 7, 2, 1, 1, 8, 4, 4], [5, 4, 4, 8, 1, 1, 2, 7, 7], [5, 4, 4, 2, 7, 7, 8, 1, 1], [5, 1, 1, 8, 7, 7, 2, 4, 4], [5, 1, 1, 2, 4, 4, 8, 7, 7], [4, 8, 8, 7, 2, 2, 1, 5, 5], [4, 8, 8, 1, 5, 5, 7, 2, 2], [4, 5, 5, 7, 8, 8, 1, 2, 2], [4, 5, 5, 1, 2, 2, 7, 8, 8], [4, 2, 2, 7, 5, 5, 1, 8, 8], [4, 2, 2, 1, 8, 8, 7, 5, 5], [2, 7, 7, 8, 1, 1, 5, 4, 4], [2, 7, 7, 5, 4, 4, 8, 1, 1], [2, 4, 4, 8, 7, 7, 5, 1, 1], [2, 4, 4, 5, 1, 1, 8, 7, 7], [2, 1, 1, 8, 4, 4, 5, 7, 7], [2, 1, 1, 5, 7, 7, 8, 4, 4], [1, 8, 8, 7, 5, 5, 4, 2, 2], [1, 8, 8, 4, 2, 2, 7, 5, 5], [1, 5, 5, 7, 2, 2, 4, 8, 8], [1, 5, 5, 4, 8, 8, 7, 2, 2], [1, 2, 2, 7, 8, 8, 4, 5, 5], [1, 2, 2, 4, 5, 5, 7, 8, 8]]

"group has", 36, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 6], [2, 3, 4, 5]]

$g_2 =$ [[1, 6, 3, 2], [4, 5]]

$g_3 =$ [[1, 6], [2, 5, 4, 3]]

$g_4 =$ [[1, 6, 5, 2], [3, 4]]

$g_5 =$ [[1, 6, 3, 4], [2, 5]]

linear dimension, 18

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} -2h[2] & 0 & 0 & 2h[2] & 2h[2] & 0 & -6h[1] & 0 & 0 & 6h[1] & 3h[3] & 0 & 2h[2] & 0 & 3h[3] & 0 & 2h[2] & 6h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 6

Z[1] = $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 2

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 3

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 0 & 0 & 0 & 0 & 3. & -3. \\ 2. & 2. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 15t^5 + 26t^6 + 38t^7 + 59t^8 + 84t^9 + 121t^{10}$

n−choose−rank

{1, [1, 2, 3, 4, 5, 6]}, {2, [1, 2, 3, 4, 5, 7]}, {3, [1, 2, 3, 4, 5, 8]}, {4, [1, 2, 3, 4, 5, 9]}, {5, [1, 2, 3, 4, 6, 7]}, {6, [1, 2, 3, 4, 6, 8]}, {7, [1, 2, 3, 4, 6, 9]}, {8, [1, 2, 3, 4, 7, 8]}, {9, [1, 2, 3, 4, 7, 9]}, {10, [1, 2, 3, 4, 8, 9]}, {11, [1, 2, 3, 5, 6, 7]}, {12, [1, 2, 3, 5, 6, 8]}, {13, [1, 2, 3, 5, 6, 9]}, {14, [1, 2, 3, 5, 7, 8]}, {15, [1, 2, 3, 5, 7, 9]}, {16, [1, 2, 3, 5, 8, 9]}, {17, [1, 2, 3, 6, 7, 8]}, {18, [1, 2, 3, 6, 7, 9]}, {19, [1, 2, 3, 6, 8, 9]}, {20, [1, 2, 3, 7, 8, 9]}, {21, [1, 2, 4, 5, 6, 7]}, {22, [1, 2, 4, 5, 6, 8]}, {23, [1, 2, 4, 5, 6, 9]}, {24, [1, 2, 4, 5, 7, 8]}, {25, [1, 2, 4, 5, 7, 9]}, {26, [1, 2, 4, 5, 8, 9]}, {27, [1, 2, 4, 6, 7, 8]}, {28, [1, 2, 4, 6, 7, 9]}, {29, [1, 2, 4, 6, 8, 9]}, {30, [1, 2, 4, 7, 8, 9]}, {31, [1, 2, 5, 6, 7, 8]}, {32, [1, 2, 5, 6, 7, 9]}, {33, [1, 2, 5, 6, 8, 9]}, {34, [1, 2, 5, 7, 8, 9]}, {35, [1, 2, 6, 7, 8, 9]}, {36, [1, 3, 4, 5, 6, 7]}, {37, [1, 3, 4, 5, 6, 8]}, {38, [1, 3, 4, 5, 6, 9]}, {39, [1, 3, 4, 5, 7, 8]}, {40, [1, 3, 4, 5, 7, 9]}, {41, [1, 3, 4, 5, 8, 9]}, {42, [1, 3, 4, 6, 7, 8]}, {43, [1, 3, 4, 6, 7, 9]}, {44, [1, 3, 4, 6, 8, 9]}, {45, [1, 3, 4, 7, 8, 9]}, {46, [1, 3, 5, 6, 7, 8]}, {47, [1, 3, 5, 6, 7, 9]}, {48, [1, 3, 5, 6, 8, 9]}, {49, [1, 3, 5, 7, 8, 9]}, {50, [1, 3, 6, 7, 8, 9]}, {51, [1, 4, 5, 6, 7, 8]}, {52, [1, 4, 5, 6, 7, 9]}, {53, [1, 4, 5, 6, 8, 9]}, {54, [1, 4, 5, 7, 8, 9]}, {55, [1, 4, 6, 7, 8, 9]}, {56, [1, 5, 6, 7, 8, 9]}, {57, [2, 3, 4, 5, 6, 7]}, {58, [2, 3, 4, 5, 6, 8]}, {59, [2, 3, 4, 5, 6, 9]}, {60, [2, 3, 4, 5, 7, 8]}, {61, [2, 3, 4, 5, 7, 9]}, {62, [2, 3, 4, 5, 8, 9]}, {63, [2, 3, 4, 6, 7, 8]}, {64, [2, 3, 4, 6, 7, 9]}, {65, [2, 3, 4, 6, 8, 9]}, {66, [2, 3, 4, 7, 8, 9]}, {67, [2, 3, 5, 6, 7, 8]}, {68, [2, 3, 5, 6, 7, 9]}, {69, [2, 3, 5, 6, 8, 9]}, {70, [2, 3, 5, 7, 8, 9]}, {71, [2, 3, 6, 7, 8, 9]}, {72, [2, 4, 5, 6, 7, 8]}, {73, [2, 4, 5, 6, 7, 9]}, {74, [2, 4, 5, 6, 8, 9]}, {75, [2, 4, 5, 7, 8, 9]}, {76, [2, 4, 6, 7, 8, 9]}, {77, [2, 5, 6, 7, 8, 9]}, {78, [3, 4, 5, 6, 7, 8]}, {79, [3, 4, 5, 6, 7, 9]}, {80, [3, 4, 5, 6, 8, 9]}, {81, [3, 4, 5, 7, 8, 9]}, {82, [3, 4, 6, 7, 8, 9]}, {83, [3, 5, 6, 7, 8, 9]}, {84, [4, 5, 6, 7, 8, 9]}

KERNEL HIERARCHY

$\pi 6 =$

$\left(\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}\right.$

{24, 43}

$u6 =$
$\left(\begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 2 & 0 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{matrix}\right.$

{2, 9, 24, 25, 27, 28, 30, 36, 39, 40, 42, 43, 51, 55}

picheck $\begin{pmatrix} 3 & 2 & 1 & 3 & 2 & 1 & 3 & 2 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & \frac{1}{9} & \frac{1}{18} & \frac{1}{6} & \frac{1}{9} & \frac{1}{18} \end{pmatrix}$

$\pi 5 =$
$\left(\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}\right.$

$u5 =$
$\left(\begin{matrix} \frac{1}{6} & 0 & \frac{1}{6} & 0 & \frac{1}{6} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{2} & \frac{1}{2} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{2} \end{matrix}\right.$

picheck $\begin{pmatrix} 15 & 10 & 5 & 15 & 10 & 5 & 15 & 10 & 5 \end{pmatrix}$

$\pi 4 =$
$\left(\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 4 & 4 & 0 & 0 & 4 & 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 2 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 2 & 0 & 0 & 4 \end{matrix}\right.$

$u4 =$
$\left(\begin{matrix} \frac{1}{18} & \frac{1}{18} & 0 & \frac{1}{18} & 0 & \frac{1}{18} & \frac{1}{6} & \frac{1}{9} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{6} & \frac{1}{6} & \frac{1}{18} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{9} & \frac{1}{6} & \frac{1}{18} & \frac{1}{6} & \frac{1}{9} & \frac{1}{9} & \frac{1}{6} & \frac{1}{9} & \frac{1}{6} & \frac{1}{9} \end{matrix}\right.$

picheck $\begin{pmatrix} 60 & 40 & 20 & 60 & 40 & 20 & 60 & 40 & 20 \end{pmatrix}$

$\pi 3 =$
$\left(\begin{matrix} 0 & 12 & 12 & 0 & 12 & 12 & 0 & 6 & 0 & 6 & 6 & 0 & 6 & 12 & 6 & 18 & 12 & 6 & 0 & 12 & 12 & 0 & 6 & 0 & 6 & 12 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}\right.$

$u3 =$
$\left(\begin{matrix} \frac{1}{36} & \frac{1}{12} & \frac{1}{12} & \frac{1}{18} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{18} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{36} & \frac{1}{12} & \frac{1}{12} & \frac{1}{18} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & 1 \end{matrix}\right.$

picheck $\begin{pmatrix} 180 & 120 & 60 & 180 & 120 & 60 & 180 & 120 & 60 \end{pmatrix}$

$\pi 2 =$
$\left(\begin{matrix} 48 & 24 & 72 & 48 & 24 & 72 & 48 & 24 & 0 & 48 & 48 & 0 & 48 & 48 & 0 & 24 & 0 & 24 & 24 & 0 & 24 & 48 & 24 & 72 & 48 & 24 & 0 & 4 \end{matrix}\right.$

$u2 =$
$\left(\begin{matrix} \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{54} & \frac{1}{18} & \frac{1}{18} & \frac{1}{27} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{27} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & 5 \end{matrix}\right.$

picheck $\begin{pmatrix} 360 & 240 & 120 & 360 & 240 & 120 & 360 & 240 & 120 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 360 & 240 & 120 & 360 & 240 & 120 & 360 & 240 & 120 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{5}{108} & \frac{5}{108} & \frac{5}{108} & \frac{5}{108} & \frac{5}{108} & \frac{5}{108} & \frac{5}{108} & \frac{5}{108} & \frac{5}{108} \end{pmatrix}$

picheck $\begin{pmatrix} 360 & 240 & 120 & 360 & 240 & 120 & 360 & 240 & 120 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{2}{9} & 0 & 0 & \frac{1}{9} & 0 & 0 & 0 \\ 0 & \frac{4}{9} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{2}{9} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{2}{9} & 0 & 0 & \frac{1}{9} \\ 0 & \frac{2}{9} & 0 & 0 & \frac{4}{9} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{9} & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{2}{9} \\ 0 & 0 & 0 & 0 & \frac{2}{9} & 0 & 0 & \frac{4}{9} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 11 & 6 & 3 & \frac{17}{2} & 6 & 3 & \frac{17}{2} & 6 & 3 \\ 9 & \frac{22}{3} & \frac{61}{18} & 9 & \frac{17}{3} & \frac{28}{9} & 9 & \frac{17}{3} & \frac{17}{6} \\ 9 & \frac{61}{9} & \frac{11}{3} & 9 & \frac{17}{3} & \frac{17}{6} & 9 & \frac{56}{9} & \frac{17}{6} \\ \frac{17}{2} & 6 & 3 & 11 & 6 & 3 & \frac{17}{2} & 6 & 3 \\ 9 & \frac{17}{3} & \frac{17}{6} & 9 & \frac{22}{3} & \frac{61}{18} & 9 & \frac{17}{3} & \frac{28}{9} \\ 9 & \frac{56}{9} & \frac{17}{6} & 9 & \frac{61}{9} & \frac{11}{3} & 9 & \frac{17}{3} & \frac{17}{6} \\ \frac{17}{2} & 6 & 3 & \frac{17}{2} & 6 & 3 & 11 & 6 & 3 \\ 9 & \frac{17}{3} & \frac{28}{9} & 9 & \frac{17}{3} & \frac{17}{6} & 9 & \frac{22}{3} & \frac{61}{18} \\ 9 & \frac{17}{3} & \frac{17}{6} & 9 & \frac{56}{9} & \frac{17}{6} & 9 & \frac{61}{9} & \frac{11}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, -1, 1, 0, -1, 1, 0, -1, 1]$

$\ker N_C = \begin{pmatrix} 0 & 1 & -1 & 0 & 1 & -1 & 0 & 1 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & -s & s & 0 & -s & s & 0 & -s & s \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & 0 & t & 0 \\ t & 0 & 0 & -s & 0 & -s \\ t & s & 0 & 0 & 0 & 0 \\ -s & -t & -s & 0 & -t & 0 \\ -t & 0 & -t & 0 & 0 & s \\ -t & -s & -t & 0 & -s & 0 \\ 0 & t & s & 0 & 0 & 0 \\ 0 & 0 & t & s & 0 & 0 \\ 0 & 0 & t & 0 & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} -1 & 1 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & 0 & 0 & 0 & t \\ t & 0 & 0 & 0 & 0 & s & 0 \\ t & s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s & t & 0 & 0 \\ 0 & s & -s & t & s & -s & s \\ 0 & 0 & 0 & t & s & 0 & 0 \\ -s & s+t & 0 & -s & s & 0 & s \\ -t & t & s & -t & t & 0 & t \\ -t & t & 0 & -t & t & 0 & s+t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 3 & 0 & 0 & 3 & 0 & 3 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 1 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 9, 6, "vs", 6

$$\mathrm{CNM} = \begin{pmatrix} 0 & \frac{-2}{3} & \frac{-4}{3} & 0 & \frac{-2}{3} & \frac{-4}{3} & 0 & \frac{-2}{3} & \frac{-4}{3} \\ \frac{2}{3} & 0 & \frac{-5}{6} & \frac{2}{3} & 0 & \frac{-5}{6} & \frac{2}{3} & 0 & \frac{-5}{6} \\ \frac{4}{3} & \frac{5}{6} & 0 & \frac{4}{3} & \frac{5}{6} & 0 & \frac{4}{3} & \frac{5}{6} & 0 \\ 0 & \frac{-2}{3} & \frac{-4}{3} & 0 & \frac{-2}{3} & \frac{-4}{3} & 0 & \frac{-2}{3} & \frac{-4}{3} \\ \frac{2}{3} & 0 & \frac{-5}{6} & \frac{2}{3} & 0 & \frac{-5}{6} & \frac{2}{3} & 0 & \frac{-5}{6} \\ \frac{4}{3} & \frac{5}{6} & 0 & \frac{4}{3} & \frac{5}{6} & 0 & \frac{4}{3} & \frac{5}{6} & 0 \\ 0 & \frac{-2}{3} & \frac{-4}{3} & 0 & \frac{-2}{3} & \frac{-4}{3} & 0 & \frac{-2}{3} & \frac{-4}{3} \\ \frac{2}{3} & 0 & \frac{-5}{6} & \frac{2}{3} & 0 & \frac{-5}{6} & \frac{2}{3} & 0 & \frac{-5}{6} \\ \frac{4}{3} & \frac{5}{6} & 0 & \frac{4}{3} & \frac{5}{6} & 0 & \frac{4}{3} & \frac{5}{6} & 0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-2}{9} & 0 & 0 & \frac{-1}{9} & 0 & 0 & 0 \\ 0 & \frac{2}{9} & 0 & 0 & 0 & 0 & 0 & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-2}{9} & 0 & 0 & \frac{-1}{9} \\ 0 & \frac{1}{9} & 0 & 0 & \frac{2}{9} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{9} & 0 & 0 & 0 & 0 & 0 & \frac{-2}{9} \\ 0 & 0 & 0 & 0 & \frac{1}{9} & 0 & 0 & \frac{2}{9} & 0 \end{pmatrix} \quad \mathrm{Skew\ Omega} =$$

$$\begin{pmatrix} 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \\ 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} & \frac{1}{18} & 0 & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{5}{2} & \frac{4}{3} & \frac{2}{3} & \frac{5}{2} & \frac{4}{3} & \frac{2}{3} & \frac{5}{2} & \frac{4}{3} & \frac{2}{3} \\ \frac{4}{3} & \frac{5}{3} & 0 & \frac{4}{3} & \frac{5}{3} & 0 & \frac{4}{3} & \frac{5}{3} & 0 \\ \frac{2}{3} & 0 & \frac{5}{6} & \frac{2}{3} & 0 & \frac{5}{6} & \frac{2}{3} & 0 & \frac{5}{6} \\ \frac{5}{2} & \frac{4}{3} & \frac{2}{3} & \frac{5}{2} & \frac{4}{3} & \frac{2}{3} & \frac{5}{2} & \frac{4}{3} & \frac{2}{3} \\ \frac{4}{3} & \frac{5}{3} & 0 & \frac{4}{3} & \frac{5}{3} & 0 & \frac{4}{3} & \frac{5}{3} & 0 \\ \frac{2}{3} & 0 & \frac{5}{6} & \frac{2}{3} & 0 & \frac{5}{6} & \frac{2}{3} & 0 & \frac{5}{6} \\ \frac{5}{2} & \frac{4}{3} & \frac{2}{3} & \frac{5}{2} & \frac{4}{3} & \frac{2}{3} & \frac{5}{2} & \frac{4}{3} & \frac{2}{3} \\ \frac{4}{3} & \frac{5}{3} & 0 & \frac{4}{3} & \frac{5}{3} & 0 & \frac{4}{3} & \frac{5}{3} & 0 \\ \frac{2}{3} & 0 & \frac{5}{6} & \frac{2}{3} & 0 & \frac{5}{6} & \frac{2}{3} & 0 & \frac{5}{6} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 1 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", false

Ω

$\left(\frac{5}{36}\ \frac{1}{9}\ \frac{4}{9}\ \frac{1}{18}\ \frac{2}{9}\ \frac{1}{3}\ \frac{2}{9}\ 0\ \frac{7}{36}\ \frac{-1}{18}\ \frac{1}{9}\ 0\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{9}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{18}\right.$

T

$\left(\frac{2}{3}\ \frac{1}{3}\ 0\ 0\ \frac{2}{9}\ 0\ \frac{5}{6}\ \frac{2}{3}\ 0\ \frac{-1}{9}\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ \frac{1}{3}\ \frac{4}{9}\ 0\ 0\ \frac{1}{9}\ 0\ 0\ \frac{2}{9}\ \frac{2}{3}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right.$

"IS NM in Vec(K)?", true

NM

$\left(\frac{35}{4}\ \frac{13}{2}\ 24\ \frac{17}{6}\ \frac{107}{9}\ 18\ \frac{161}{12}\ \frac{5}{3}\ \frac{21}{2}\ \frac{-28}{9}\ \frac{17}{3}\ 0\ 3\ 6\ 11\ 3\ 6\ \frac{17}{2}\ \frac{17}{6}\ \frac{17}{3}\ 9\ \frac{11}{3}\ \frac{61}{9}\ 9\ \frac{17}{3}\ \frac{28}{9}\ \frac{17}{3}\ 9\right.$

"IS MN in Vec(K)?", false

MN

$\left(\frac{455}{36}\ \frac{163}{36}\ 18\ \frac{79}{18}\ \frac{163}{18}\ \frac{72}{5}\ \frac{455}{36}\ \frac{5}{3}\ \frac{36}{5}\ \frac{-89}{18}\ \frac{37}{9}\ 0\ \frac{18}{5}\ \frac{18}{5}\ 11\ \frac{18}{5}\ \frac{18}{5}\ \frac{17}{2}\ \frac{79}{18}\ \frac{79}{18}\ \frac{36}{5}\ \frac{47}{9}\ \frac{89}{18}\ \frac{36}{5}\ \frac{37}{9}\right.$

$\boldsymbol{\tau}$ = 15/1 , rank = 6 , ratio = 5/2 , n^2 / r = 27/2

$\boldsymbol{\tau}'$ = 66/1 , r' = 5/6 , $\boldsymbol{\tau} / n^2$ = 5/27

p^2 = 7/54 , min $\boldsymbol{\tau}$ = 21/2 , $\boldsymbol{\tau}$–check is positive? 9/2

max r = 54/7 , r–check is positive? 2/9

IS N0M0 a combination of T and Omega? , false

$$N_0 M_0 = \frac{1311}{3506} T + \frac{22822}{1753} \Omega$$

There are, 2, partitions and, 2, ranges, with a group size of, 36

KERNEL HAS LINEAR DIMENSION 40
out of total no. of elements equal to 144

dim span idems 4 vs no. of idems 4

"PT1" = {{1}, {5, 9}, {3, 8}, {4}, {7}, {2, 6}}

"PT2" = {{1}, {2, 3}, {4}, {7}, {5, 6}, {8, 9}}

"RG1" = {1, 3, 4, 6, 7, 9}

"RG2" = {1, 2, 4, 5, 7, 8}

$$M_C = \begin{pmatrix}
\frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} \\
\frac{-1}{6} & \frac{2}{3} & \frac{-1}{2} & \frac{-1}{6} & \frac{2}{3} & \frac{-1}{2} & \frac{-1}{6} & \frac{2}{3} & \frac{-1}{2} \\
\frac{-1}{12} & \frac{-1}{2} & \frac{7}{12} & \frac{-1}{12} & \frac{-1}{2} & \frac{7}{12} & \frac{-1}{12} & \frac{-1}{2} & \frac{7}{12} \\
\frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} \\
\frac{-1}{6} & \frac{2}{3} & \frac{-1}{2} & \frac{-1}{6} & \frac{2}{3} & \frac{-1}{2} & \frac{-1}{6} & \frac{2}{3} & \frac{-1}{2} \\
\frac{-1}{12} & \frac{-1}{2} & \frac{7}{12} & \frac{-1}{12} & \frac{-1}{2} & \frac{7}{12} & \frac{-1}{12} & \frac{-1}{2} & \frac{7}{12} \\
\frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} \\
\frac{-1}{6} & \frac{2}{3} & \frac{-1}{2} & \frac{-1}{6} & \frac{2}{3} & \frac{-1}{2} & \frac{-1}{6} & \frac{2}{3} & \frac{-1}{2} \\
\frac{-1}{12} & \frac{-1}{2} & \frac{7}{12} & \frac{-1}{12} & \frac{-1}{2} & \frac{7}{12} & \frac{-1}{12} & \frac{-1}{2} & \frac{7}{12}
\end{pmatrix}
\quad
N_C = \begin{pmatrix}
\frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{47}{54} & \frac{29}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{11}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{29}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{11}{54} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{29}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{11}{54} \\
\frac{-7}{54} & \frac{11}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{29}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{-7}{54} & \frac{-7}{54} \\
\frac{-7}{54} & \frac{-7}{54} & \frac{11}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{47}{54} & \frac{29}{54} \\
\frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{11}{54} & \frac{-7}{54} & \frac{-7}{54} & \frac{29}{54} & \frac{47}{54}
\end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix}
1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\
\frac{-1}{4} & 1 & \frac{-3}{4} & \frac{-1}{4} & 1 & \frac{-3}{4} & \frac{-1}{4} & 1 & \frac{-3}{4} \\
\frac{-1}{7} & \frac{-6}{7} & 1 & \frac{-1}{7} & \frac{-6}{7} & 1 & \frac{-1}{7} & \frac{-6}{7} & 1 \\
1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\
\frac{-1}{4} & 1 & \frac{-3}{4} & \frac{-1}{4} & 1 & \frac{-3}{4} & \frac{-1}{4} & 1 & \frac{-3}{4} \\
\frac{-1}{7} & \frac{-6}{7} & 1 & \frac{-1}{7} & \frac{-6}{7} & 1 & \frac{-1}{7} & \frac{-6}{7} & 1 \\
1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} & 1 & \frac{-2}{3} & \frac{-1}{3} \\
\frac{-1}{4} & 1 & \frac{-3}{4} & \frac{-1}{4} & 1 & \frac{-3}{4} & \frac{-1}{4} & 1 & \frac{-3}{4} \\
\frac{-1}{7} & \frac{-6}{7} & 1 & \frac{-1}{7} & \frac{-6}{7} & 1 & \frac{-1}{7} & \frac{-6}{7} & 1
\end{pmatrix}
\quad
N_C\text{–scaled} =$$

$$\begin{pmatrix}
1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} \\
\frac{-7}{47} & 1 & \frac{29}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{11}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} \\
\frac{-7}{47} & \frac{29}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{11}{47} & \frac{-7}{47} \\
\frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} \\
\frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & 1 & \frac{29}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{11}{47} \\
\frac{-7}{47} & \frac{11}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{29}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} \\
\frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & 1 & \frac{-7}{47} & \frac{-7}{47} \\
\frac{-7}{47} & \frac{-7}{47} & \frac{11}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & 1 & \frac{29}{47} \\
\frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{11}{47} & \frac{-7}{47} & \frac{-7}{47} & \frac{29}{47} & 1
\end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} \\ \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{-1}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} \\ \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{1}{12} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

$$\text{commutator} = \begin{pmatrix} 0 & \frac{-1}{12} & \frac{-1}{6} & 0 & \frac{-1}{12} & \frac{-1}{6} & 0 & \frac{-1}{12} & \frac{-1}{6} \\ \frac{1}{12} & 0 & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{12} \\ \frac{1}{6} & \frac{-1}{12} & 0 & \frac{1}{6} & \frac{-1}{12} & 0 & \frac{1}{6} & \frac{-1}{12} & 0 \\ 0 & \frac{-1}{12} & \frac{-1}{6} & 0 & \frac{-1}{12} & \frac{-1}{6} & 0 & \frac{-1}{12} & \frac{-1}{6} \\ \frac{1}{12} & 0 & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{12} \\ \frac{1}{6} & \frac{-1}{12} & 0 & \frac{1}{6} & \frac{-1}{12} & 0 & \frac{1}{6} & \frac{-1}{12} & 0 \\ 0 & \frac{-1}{12} & \frac{-1}{6} & 0 & \frac{-1}{12} & \frac{-1}{6} & 0 & \frac{-1}{12} & \frac{-1}{6} \\ \frac{1}{12} & 0 & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{12} \\ \frac{1}{6} & \frac{-1}{12} & 0 & \frac{1}{6} & \frac{-1}{12} & 0 & \frac{1}{6} & \frac{-1}{12} & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 3.395643924, 1.104356076]

Eigenvalues N_C

[0., 1.545819536, 0.2875137971, 1., 1.577350269, 0.4226497307, 1., 1.577350269, 0.4226497307]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 5.481980506, 3.518019494]

Eigenvalues N_C–scaled

[0., 1.776047977, 0.3303350006, 1.148936170, 1.812274777, 0.4855975627, 1.148936170, 1.812274777, 0.4855975627]

NullSpace M_C

{[0, 0, 0, 0, 0, 1, 0, 0, −1], [0, 1, 0, 0, 0, 0, 1, 0, 1], [0, −1, 0, 0, 0, 0, 0, 1, 0], [1, 1, 0, 0, 0, 0, 0, 0, 1], [0, 1, 0, 1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0, −1]}

NullSpace N_C

{[0, 1, −1, 0, 1, −1, 0, 1, −1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 10.76132355, 0.955137562, 3.283538886]

Eigenvalues N_0

[0., 2., 1.577350269, 0.4226497307, 1.577350269, 0.4226497307, 1., 1., 1.]

NullSpace M_0

{[0, −1, 0, 0, 0, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 0, 1], [0, 0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0, 0]}

NullSpace N_0

{[0, −1, 1, 0, −1, 1, 0, −1, 1]}

Eigenvalues M

[8.608980227, −0.684142302, 2.075162074, −2.500000000, −1.666666667, −0.8333333333, −2.500000000, −1.666666667, −0.8333333333]

Eigenvalues N

[0., 7.358898944, −1.358898944, −1., −0.4226497307, −1.577350269, −1., −0.4226497307, −1.577350269]

NullSpace M

{}

NullSpace N

{[0, −1, 1, 0, −1, 1, 0, −1, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 & 2 & 0 & 3 & 3 \\ 0 & 1 & 0 & 0 & 3 & 3 & 0 & 2 & 3 \\ 3 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 3 & 3 & 0 & 0 & 1 & 0 & 3 & 2 \\ 0 & 2 & 3 & 0 & 1 & 0 & 0 & 3 & 3 \\ 3 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 & 3 & 3 & 0 & 0 & 1 \\ 0 & 3 & 3 & 0 & 2 & 3 & 0 & 1 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2 : \quad \text{commutator} = \begin{pmatrix} 0 & 6 & 6 & 0 & 6 & 6 & 0 & 6 & 6 \\ -6 & 0 & 0 & -6 & 0 & 0 & -6 & 0 & 0 \\ -6 & 0 & 0 & -6 & 0 & 0 & -6 & 0 & 0 \\ 0 & 6 & 6 & 0 & 6 & 6 & 0 & 6 & 6 \\ -6 & 0 & 0 & -6 & 0 & 0 & -6 & 0 & 0 \\ -6 & 0 & 0 & -6 & 0 & 0 & -6 & 0 & 0 \\ 0 & 6 & 6 & 0 & 6 & 6 & 0 & 6 & 6 \\ -6 & 0 & 0 & -6 & 0 & 0 & -6 & 0 & 0 \\ -6 & 0 & 0 & -6 & 0 & 0 & -6 & 0 & 0 \end{pmatrix}$$

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240, [1, -1, 1, 1, -1, -1, -1, -1, -1]

