T - Run

[4, 3, 1, 2, 3, 4, 4, 3], [7, 8, 8, 7, 8, 7, 5, 6]

$\tilde{\pi}$ = [1, 1, 2, 2, 1, 1, 2, 2]
$\boldsymbol{\delta}$ = [1, 1, 3, 3, 1, 1, 3, 3]

POSSIBLE RANKS

1 x 12
2 x 6
3 x 4

BASE DETERMINANT 8325/65536, .1270294189

NullSpace of Δ

{2, 4, 5, 7}, {1, 3, 6, 8}

Nullspace of A

[{5, 7},{2, 4}] `,` [{6, 8},{1, 3}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[3] v[8] + v[4] v[7]

Degree 3
0

Degree 4
v[5] v[6] v[8] v[7] + v[1] v[2] v[3] v[4] + v[1] v[3] v[5] v[7] + v[2] v[4] v[6] v[8] + v[3] v[4] v[8] v[7]

Degree 5
v[3] v[5] v[6] v[8] v[7] + v[1] v[2] v[3] v[4] v[7] + v[4] v[5] v[6] v[8] v[7] + v[2] v[4] v[6]

v[8] v[7] + v[2] v[3] v[4] v[6] v[8] + v[1] v[3] v[5] v[8] v[7] + v[1] v[3] v[4] v[5] v[7] + v[1] v[2] v[3] v[4] v[8]

Degree 6
v[2] v[3] v[4] v[6] v[8] v[7] + v[1] v[3] v[4] v[5] v[8] v[7] + v[3] v[4] v[5] v[6] v[8] v[7] + v[1] v[2] v[3] v[4] v[8] v[7]

Degree 7
0

Degree 8
4 v[1] v[2] v[3] v[4] v[5] v[6] v[8] v[7]

==

{}

R: [4, 3, 1, 2, 3, 4, 4, 3]
B: [7, 8, 8, 7, 8, 7, 5, 6]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{225}{1048576}(1+s)^2(-1+s)(3+s)(4-s+s^2)(37+3s^2)(-20-3s+5s^2)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[1] v[2] v[3] v[4]
"B CYCLES", 1 + v[5] v[6] v[8] v[7]

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B^*

{[0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\,(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8])$

degree 2: $\frac{1}{6}\,(v[1]v[2] + 2v[3]v[4] + v[5]v[6] + 2v[7]v[8])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 4, 6, 8}, {2, 3, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT3" = {{1, 3, 6, 7}, {2, 4, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$\pi 2$ = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2]

supp $\pi 2$ = {1, 14, 23, 28}

$u2$ = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 1, 2, 2, 1, 2, 1, 3, 2, 1, 1, 2, 3]

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [3], [4], [3]]
Action of B on ranges, [[2], [1], [1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [3, 3, 2]

BPARTS [2, 1, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[2, 3, 4, 1, 2, 4]

B−BLOCKS,

[5, 5, 6, 6, 3, 1]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

b_1 = {1, 4, 6, 7}
b_2 = {1, 3, 6, 7}
b_3 = {2, 3, 5, 8}
b_4 = {2, 4, 5, 8}
b_5 = {1, 4, 6, 8}
b_6 = {2, 3, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 4, 4, 4

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 16, Shape: $8 \oplus 8/3$

$$\text{CLB} = \begin{pmatrix} 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4}}, true

Ω_B in Vec(K)? , {{5, 6, 7, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 4, 6, 8}, {2, 3, 5, 7}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 7, 8], [7, 8, 8, 7, 8, 7, 8, 7]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 5, 6], [5, 6, 6, 5, 6, 5, 6, 5]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3, 4, 3]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 1, 2], [1, 2, 2, 1, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

3, "partition", {{1, 3, 6, 7}, {2, 4, 5, 8}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 8, 7], [7, 8, 7, 8, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 6, 5], [5, 6, 5, 6, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 4, 3], [3, 4, 3, 4, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 2, 1], [1, 2, 1, 2, 2, 1, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

{1, 14, 23, 28}

$u2 =$

$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{2}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & \frac{2}{9} & \frac{1}{9} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{9} & \frac{32}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & \frac{32}{9} & \frac{16}{9} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{9} & \frac{32}{9} & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 2, 2, -1, -1, -2, -2]$

$$\ker N_c = \begin{pmatrix} -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & t & t & -t & -t \\ s & s & -s & -s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix}-2 & 2 & -1 & -1\end{pmatrix}$

$$\ker M_0 = \begin{pmatrix}-1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -1\\ 0 & 0 & 0 & 1\end{pmatrix} \begin{pmatrix}0 & 0 & s & -t\\ 0 & 0 & -s & t\\ -s & 0 & 0 & t\\ s & 0 & 0 & -t\\ 0 & 0 & -s & t\\ 0 & 0 & s & -t\\ 0 & -t & s & 0\\ 0 & t & -s & 0\end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix}0 & 0 & 0 & 1 & 0\\ 0 & 1 & 0 & -1 & 1\\ 0 & 1 & -1 & 0 & 1\\ 0 & 0 & 1 & 0 & 0\\ -1 & 1 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1\end{pmatrix} \begin{pmatrix}0 & t & s & 0 & 0\\ 0 & s & -s & 0 & s+t\\ 0 & 0 & 0 & s & t\\ 0 & s+t & 0 & -s & s\\ 0 & s & -s & 0 & s+t\\ 0 & t & s & 0 & 0\\ -t & t & s & 0 & t\\ t & s & -s & 0 & s\end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix}0 & 4 & 0 & 0 & 4\end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix}1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 0\\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 1\\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3}\\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 1\\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 0\\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 0\\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 1\end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix}1 & 0 & 0 & 0 & 0 & 1 & 1 & 0\\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0\\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0\\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1\end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 1 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & 1 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & 0 & 1 & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 1 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & 0 & 1 & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \; BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew } T =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix}$$

Skew Omega =

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(\frac{1}{3}\ \frac{1}{18}\ \frac{2}{9}\ \frac{1}{9}\ \frac{2}{9}\ \frac{1}{9}\ \frac{1}{3}\ \frac{1}{18}\ \frac{1}{9}\ \frac{2}{9}\ \frac{1}{6}\ 0\ \frac{2}{9}\ \frac{1}{9}\ 0\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM}\left(\frac{16}{3}\ \frac{8}{9}\ \frac{32}{9}\ \frac{16}{9}\ \frac{32}{9}\ \frac{16}{9}\ \frac{16}{3}\ \frac{8}{9}\ \frac{16}{9}\ \frac{32}{9}\ \frac{8}{3}\ 0\ \frac{32}{9}\ \frac{16}{9}\ 0\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN}\left(\frac{14}{3}\ \frac{10}{9}\ \frac{26}{9}\ \frac{26}{9}\ \frac{26}{9}\ \frac{26}{9}\ \frac{14}{3}\ \frac{10}{9}\ \frac{14}{9}\ \frac{22}{9}\ \frac{10}{3}\ \frac{2}{3}\ \frac{22}{9}\ \frac{14}{9}\ \frac{2}{3}\ \frac{10}{3}\right)$$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 5/36$, min $\boldsymbol{\tau} = 80/9$, $\boldsymbol{\tau}$-check is positive? 208/9

max r = 36/5 , r-check is positive? 13/18

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 3, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 16
out of total no. of elements equal to 24

dim span idems 12 vs no. of idems 12

"PT1" = {{1, 4, 6, 8}, {2, 3, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT3" = {{1, 3, 6, 7}, {2, 4, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{19}{36} & \frac{7}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{7}{36} & \frac{19}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_c\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_c\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{19}{31} & \frac{7}{31} & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & \frac{7}{31} & \frac{19}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[0., 0., 0., 0., 1.333333333, 2.888888889, 2.276142374, 0.3905242916]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 1.548387097, 3.354838710, 2.643262113, 0.453512081]

NullSpace M_C

{[0, 1, 1, 0, 1, 0, 1, 0], [0, 1, 1, 0, 1, 0, 0, 1], [1, −1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0]}

NullSpace N_C

{[−1, −1, 1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 0, 0, 0, 0, 1, 1], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 0., 4., 1.333333333, 2.276142374, 0.3905242916]

NullSpace M_0

{[0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0]}

NullSpace N_0

{[−1, −1, 0, 0, 0, 0, 1, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 4., −1.333333333, −0.3905242916, −2.276142374]

NullSpace M

{}

NullSpace N

{[-1, -1, 0, 0, 0, 0, 1, 1], [-1, -1, 1, 1, 0, 0, 0, 0], [0, -1, 0, 0, 1, 0, 0, 0], [-1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 2 & 1 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 1 & 2 & 2 & 1 & 0 & 3 \\ 2 & 1 & 2 & 1 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{3, 4}

R: [4, 3, 8, 7, 3, 4, 4, 3]
B: [7, 8, 1, 2, 8, 7, 5, 6]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{-1}{262144}$ $(30+6s+23s^2-12s^3+s^4)(-370+46s+99s^2-16s^3+s^4)(-3+s)(-1+s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 2, "vs", 4

"R CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])
"B CYCLES", 1 + v[5] v[6] v[7] v[8]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B^*

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8] \right)$
degree 2: $\frac{1}{6}\left(v[1]v[2] + 2\,v[3]v[4] + v[5]v[6] + 2\,v[7]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$\pi 2$ = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2]

supp $\pi 2$ = {1, 14, 23, 28}

$u2$ = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [3], [1], [3]]
Action of B on ranges, [[2], [1], [4], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 2]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[1, 2, 2, 1]

B–BLOCKS,

[3, 4, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N – check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 6, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: 3 ⊕ 10/8

$$\text{CLB} = \begin{pmatrix} 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 8}, {4, 7}}, true

Ω_B in Vec(K)? , {{5, 6, 7, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 7, 8], [7, 8, 7, 8, 8, 7, 8, 7]]

2, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 5, 6], [5, 6, 5, 6, 6, 5, 6, 5]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 3, 4], [3, 4, 3, 4, 4, 3, 4, 3]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 1, 2], [1, 2, 1, 2, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\,2\,] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 14, 23, 28}

$u2 =$
$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 2, 2, -1, -1, 0, 0]$

$$\ker N_C = \begin{pmatrix} 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -t & 0 & s & 0 & -t & 0 & t & -s+t \\ -t & 0 & s & 0 & -t & 0 & t & -s+t \\ 0 & t & 0 & -s & -t & 0 & s & 0 \\ -t & 0 & s & 0 & 0 & t & 0 & -s \end{pmatrix} \text{ RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 & -1 & 2 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & -t \\ -s & 0 & 0 & t \\ 0 & -t & 0 & s \\ 0 & t & 0 & -s \\ -s & 0 & 0 & t \\ s & 0 & 0 & -t \\ s & 0 & -t & 0 \\ -s & 0 & t & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & s & s+t & -s & 0 \\ 0 & t & 0 & s & 0 \\ 0 & s+t & t & 0 & -t \\ 0 & 0 & s & 0 & t \\ 0 & t & 0 & s & 0 \\ 0 & s & s+t & -s & 0 \\ -t & s+t & s+t & -s & 0 \\ t & 0 & 0 & s & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 4 & 0 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$$

$$T \left(0 \;\; \frac{1}{9} \;\; \frac{1}{3} \;\; \frac{1}{18} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{2}{9} \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$NM\left(0 \quad \frac{16}{9} \quad \frac{16}{3} \quad \frac{8}{9} \quad \frac{16}{9} \quad \frac{32}{9} \quad \frac{8}{3} \quad 0 \quad \frac{32}{9} \quad \frac{16}{9} \quad 0 \quad \frac{8}{3}\right)$

"IS MN in Vec(K)?", false

$MN\left(\frac{-2}{3} \quad \frac{26}{9} \quad \frac{14}{3} \quad \frac{10}{9} \quad \frac{14}{9} \quad \frac{22}{9} \quad \frac{10}{3} \quad \frac{2}{3} \quad \frac{22}{9} \quad \frac{14}{9} \quad \frac{2}{3} \quad \frac{10}{3}\right)$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[1, 0, 1, 0, 1, 0, 0, 1], [1, 0, 1, 0, 1, 0, 1, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0]}

NullSpace N_C

{[0, 0, −1, 0, 0, 0, 0, 1], [−1, 0, 1, 1, −1, 0, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 1, 0, −1, 0, 1, 0]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, −1, 0,

0]}

NullSpace N_0

{[−1, −1, 0, 1, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{}

NullSpace N

{[0, 0, −1, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [1, 0, −1, −1, 1, 0, 0, 0], [1, 1, −1, −1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

20, [1, 1, 1, −1, −1, 1, 1, 1]

==

{5, 6}

R: [4, 3, 1, 2, 8, 7, 4, 3]
B: [7, 8, 8, 7, 3, 4, 5, 6]

TRACE TWO = 1

det AT = $\frac{1}{16}(1+t)^2(-1+t)^2(t)^2$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{5}{268435456}(-2880-2496s-2340s^2-1052s^3-683s^4-153s^5-4s^6+4s^7+3s^8+s^9)(11840+1472s-1788s^2+980s^3+115s^4-117s^5-27s^6+5s^7)(-1+s)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", 1 + v[1] v[2] v[3] v[4]
"B CYCLES", 1 + v[3] v[4] v[5] v[6] v[7] v[8]

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [0., 0., −1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040

I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B^*

{[0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{5} & \frac{3}{5} & \frac{4}{15} & \frac{11}{15} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{3}{5} & \frac{2}{5} & \frac{11}{15} & \frac{4}{15} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{5} & \frac{3}{5} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{3}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{3}{5} & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{8}{15} & \frac{7}{15} \\ \frac{3}{5} & \frac{2}{5} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{7}{15} & \frac{8}{15} \\ \frac{4}{15} & \frac{11}{15} & \frac{2}{5} & \frac{3}{5} & \frac{8}{15} & \frac{7}{15} & 0 & 1 \\ \frac{11}{15} & \frac{4}{15} & \frac{3}{5} & \frac{2}{5} & \frac{7}{15} & \frac{8}{15} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8] \right)$

degree 2: $\frac{1}{6}\left(v[1]v[2] + 2v[3]v[4] + v[5]v[6] + 2v[7]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT2" = {{1, 4, 6, 8}, {2, 3, 5, 7}}

"PT3" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT4" = {{1, 4, 5, 7}, {2, 3, 6, 8}}

"PT5" = {{1, 3, 6, 7}, {2, 4, 5, 8}}

"PT6" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$\pi 2 = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 14, 23, 28}

$$u2 = [15, 10, 5, 6, 9, 4, 11, 5, 10, 9, 6, 11, 4, 15, 10, 5, 6, 9, 5, 10, 9, 6, 15, 8, 7, 7, 8, 15]$$

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [1], [4], [3]]
Action of B on ranges, [[2], [3], [1], [1]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$$

RPARTS [6, 6, 5, 5, 4, 4]

BPARTS [1, 4, 2, 3, 1, 4]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{5} & \frac{1}{15} & \frac{2}{15} & \frac{4}{15} & \frac{1}{5} & \frac{2}{15} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[4, 9, A, 8, 6, 7, 4, 6, 8, 7, 9, A]

B–BLOCKS,

[2, 8, 7, B, 3, C, 1, 5, 7, 8, C, B]

with invariant measure, [2, 1, 1, 3, 2, 3, 4, 4, 2, 2, 3, 3]

N by blocks, N – check: true

$b_1 = \{1, 4, 6, 7\}$
$b_2 = \{1, 4, 6, 8\}$
$b_3 = \{2, 3, 5, 7\}$
$b_4 = \{1, 3, 6, 7\}$
$b_5 = \{2, 3, 5, 8\}$
$b_6 = \{2, 4, 5, 8\}$
$b_7 = \{1, 4, 5, 7\}$
$b_8 = \{2, 3, 6, 8\}$
$b_9 = \{1, 3, 5, 7\}$
$b_{10} = \{2, 4, 6, 8\}$
$b_{11} = \{1, 4, 5, 8\}$
$b_{12} = \{2, 3, 6, 7\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 25, Shape: $18 \oplus 7/5$

$$CLB = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4}}, true

Ω_B in Vec(K)? , {{3, 4, 5, 6, 7, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [7, 8], [[8, 7, 7, 8, 8, 7, 7, 8], [7, 8, 8, 7, 7, 8, 8, 7]]

2, "range", [5, 6], [[6, 5, 5, 6, 6, 5, 5, 6], [5, 6, 6, 5, 5, 6, 6, 5]]

3, "range", [3, 4], [[4, 3, 3, 4, 4, 3, 3, 4], [3, 4, 4, 3, 3, 4, 4, 3]]

4, "range", [1, 2], [[2, 1, 1, 2, 2, 1, 1, 2], [1, 2, 2, 1, 1, 2, 2, 1]]

2, "partition", {{1, 4, 6, 8}, {2, 3, 5, 7}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 7, 8], [7, 8, 8, 7, 8, 7, 8, 7]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 5, 6], [5, 6, 6, 5, 6, 5, 6, 5]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3, 4, 3]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 1, 2], [1, 2, 2, 1, 2, 1, 2, 1]]

3, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

4, "partition", {{1, 4, 5, 7}, {2, 3, 6, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 8, 7, 8, 7], [7, 8, 8, 7, 7, 8, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 6, 5, 6, 5], [5, 6, 6, 5, 5, 6, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 4, 3, 4, 3], [3, 4, 4, 3, 3, 4, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 2, 1, 2, 1], [1, 2, 2, 1, 1, 2, 1, 2]]

5, "partition", {{1, 3, 6, 7}, {2, 4, 5, 8}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 8, 7], [7, 8, 7, 8, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 6, 5], [5, 6, 5, 6, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 4, 3], [3, 4, 3, 4, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 2, 1], [1, 2, 1, 2, 2, 1, 1, 2]]

6, "partition", {{1, 3, 5, 7}, {2, 4, 6, 8}}

1, "range", [7, 8], [[8, 7, 8, 7, 8, 7, 8, 7], [7, 8, 7, 8, 7, 8, 7, 8]]

2, "range", [5, 6], [[6, 5, 6, 5, 6, 5, 6, 5], [5, 6, 5, 6, 5, 6, 5, 6]]

3, "range", [3, 4], [[4, 3, 4, 3, 4, 3, 4, 3], [3, 4, 3, 4, 3, 4, 3, 4]]

4, "range", [1, 2], [[2, 1, 2, 1, 2, 1, 2, 1], [1, 2, 1, 2, 1, 2, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$

$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 14, 23, 28}

$u2 =$
$(15 \quad 10 \quad 5 \quad 6 \quad 9 \quad 4 \quad 11 \quad 5 \quad 10 \quad 9 \quad 6 \quad 11 \quad 4 \quad 15 \quad 10 \quad 5 \quad 6 \quad 9 \quad 5 \quad 10 \quad 9 \quad 6 \quad 15 \quad 8 \quad 7 \quad 7$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{15} & \frac{4}{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{4}{15} & \frac{11}{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{5} & \frac{2}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{15} & \frac{8}{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{8}{15} & \frac{7}{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{3}{5} & \frac{2}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{5} & \frac{3}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{7}{15} & \frac{8}{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{15} & \frac{7}{15} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{5} & \frac{2}{5} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{5} & \frac{3}{5} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{5} & \frac{2}{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{5} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{11}{15} & \frac{4}{15} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{4}{15} & \frac{11}{15} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

idem-checks

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & \frac{1}{10} & \frac{1}{15} & \frac{11}{45} & \frac{4}{45} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{15} & \frac{1}{10} & \frac{4}{45} & \frac{11}{45} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{18} & \frac{1}{9} & \frac{1}{5} & \frac{2}{15} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{9} & \frac{1}{18} & \frac{2}{15} & \frac{1}{5} \\ \frac{1}{10} & \frac{1}{15} & \frac{1}{9} & \frac{2}{9} & \frac{1}{6} & 0 & \frac{7}{45} & \frac{8}{45} \\ \frac{1}{15} & \frac{1}{10} & \frac{2}{9} & \frac{1}{9} & 0 & \frac{1}{6} & \frac{8}{45} & \frac{7}{45} \\ \frac{11}{90} & \frac{2}{45} & \frac{1}{5} & \frac{2}{15} & \frac{7}{90} & \frac{4}{45} & \frac{1}{3} & 0 \\ \frac{2}{45} & \frac{11}{90} & \frac{2}{15} & \frac{1}{5} & \frac{4}{45} & \frac{7}{90} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & \frac{8}{5} & \frac{16}{15} & \frac{176}{45} & \frac{64}{45} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{16}{15} & \frac{8}{5} & \frac{64}{45} & \frac{176}{45} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{8}{9} & \frac{16}{9} & \frac{16}{5} & \frac{32}{15} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{16}{9} & \frac{8}{9} & \frac{32}{15} & \frac{16}{5} \\ \frac{8}{5} & \frac{16}{15} & \frac{16}{9} & \frac{32}{9} & \frac{8}{3} & 0 & \frac{112}{45} & \frac{128}{45} \\ \frac{16}{15} & \frac{8}{5} & \frac{32}{9} & \frac{16}{9} & 0 & \frac{8}{3} & \frac{128}{45} & \frac{112}{45} \\ \frac{88}{45} & \frac{32}{45} & \frac{16}{5} & \frac{32}{15} & \frac{56}{45} & \frac{64}{45} & \frac{16}{3} & 0 \\ \frac{32}{45} & \frac{88}{45} & \frac{32}{15} & \frac{16}{5} & \frac{64}{45} & \frac{56}{45} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 1, 1, -1, -1, -1, -1]$

$$\ker N_c = \begin{pmatrix} -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & -s+t & -s+t & 0 & 0 & -t+s & -t+s \\ 0 & 0 & 0 & 0 & t & t & -t & -t \\ s & s & -s & -s & 0 & 0 & 0 & 0 \end{pmatrix}$$

RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -t & 0 & 0 & s \\ t & 0 & 0 & -s \\ t & -s & 0 & 0 \\ -t & s & 0 & 0 \\ s & 0 & 0 & -t \\ -s & 0 & 0 & t \\ 0 & 0 & -t & s \\ 0 & 0 & t & -s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & -t & t & t \\ -s & 0 & t & s & s \\ 0 & 0 & t & s & 0 \\ 0 & 0 & -t & t & s+t \\ -t & 0 & s & t & t \\ t & 0 & -s & s & s \\ s & -t & 0 & t & t \\ -s & t & 0 & s & s \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 0 & 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{3}{5} & \frac{2}{5} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{2}{5} & \frac{3}{5} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{4}{15} & \frac{11}{15} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{11}{15} & \frac{4}{15} & \frac{1}{3} & \frac{2}{3} \\ \frac{3}{5} & \frac{2}{5} & \frac{4}{15} & \frac{11}{15} & 1 & 0 & \frac{3}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{3}{5} & \frac{11}{15} & \frac{4}{15} & 0 & 1 & \frac{2}{5} & \frac{3}{5} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{3}{5} & \frac{2}{5} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{2}{5} & \frac{3}{5} & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & \frac{3}{5} & \frac{2}{5} & \frac{7}{15} & \frac{8}{15} \\ 0 & 1 & 1 & 0 & \frac{2}{5} & \frac{3}{5} & \frac{8}{15} & \frac{7}{15} \\ 0 & 1 & 1 & 0 & \frac{2}{5} & \frac{3}{5} & \frac{8}{15} & \frac{7}{15} \\ 1 & 0 & 0 & 1 & \frac{3}{5} & \frac{2}{5} & \frac{7}{15} & \frac{8}{15} \\ \frac{3}{5} & \frac{2}{5} & \frac{2}{5} & \frac{3}{5} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{5} & \frac{3}{5} & \frac{3}{5} & \frac{2}{5} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{7}{15} & \frac{8}{15} & \frac{8}{15} & \frac{7}{15} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{8}{15} & \frac{7}{15} & \frac{7}{15} & \frac{8}{15} & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{11}{90} & \frac{2}{45} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{2}{45} & \frac{11}{90} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{7}{90} & \frac{4}{45} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{4}{45} & \frac{7}{90} \\ \frac{-11}{90} & \frac{-2}{45} & 0 & 0 & \frac{-7}{90} & \frac{-4}{45} & 0 & 0 \\ \frac{-2}{45} & \frac{-11}{90} & 0 & 0 & \frac{-4}{45} & \frac{-7}{90} & 0 & 0 \end{pmatrix}$$

Skew Omega =

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{3}{5} & \frac{2}{5} & \frac{11}{15} & \frac{4}{15} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{5} & \frac{3}{5} & \frac{4}{15} & \frac{11}{15} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{3}{5} & \frac{2}{5} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{2}{5} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{7}{15} & \frac{8}{15} \\ \frac{2}{5} & \frac{3}{5} & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{8}{15} & \frac{7}{15} \\ \frac{11}{15} & \frac{4}{15} & \frac{3}{5} & \frac{2}{5} & \frac{7}{15} & \frac{8}{15} & 1 & 0 \\ \frac{4}{15} & \frac{11}{15} & \frac{2}{5} & \frac{3}{5} & \frac{8}{15} & \frac{7}{15} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(\frac{1}{3}\ \frac{7}{90}\ \frac{1}{5}\ \frac{11}{90}\ \frac{7}{45}\ \frac{1}{6}\ \frac{1}{9}\ \frac{1}{10}\ \frac{1}{5}\ \frac{1}{18}\ \frac{1}{3}\ \frac{1}{18}\ \frac{4}{45}\ \frac{11}{45}\ \frac{1}{15}\ \frac{1}{10}\ \frac{2}{9}\ \frac{1}{9}\ 0\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\text{NM}\left(\frac{16}{3}\ \frac{56}{45}\ \frac{16}{5}\ \frac{88}{45}\ \frac{112}{45}\ \frac{8}{3}\ \frac{16}{9}\ \frac{8}{5}\ \frac{16}{5}\ \frac{8}{9}\ \frac{16}{3}\ \frac{8}{9}\ \frac{64}{45}\ \frac{176}{45}\ \frac{16}{15}\ \frac{8}{5}\ \frac{32}{9}\ \frac{16}{9}\ 0\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{14}{3}\ \frac{82}{45}\ \frac{38}{15}\ \frac{146}{45}\ \frac{86}{45}\ \frac{10}{3}\ \frac{14}{9}\ \frac{34}{15}\ \frac{38}{15}\ \frac{10}{9}\ \frac{14}{3}\ \frac{10}{9}\ \frac{62}{45}\ \frac{118}{45}\ \frac{26}{15}\ \frac{34}{15}\ \frac{22}{9}\ \frac{14}{9}\ \frac{2}{3}\ \frac{10}{3}\right)$$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 5/36$, min $\boldsymbol{\tau} = 80/9$, $\boldsymbol{\tau}$–check is positive? 208/9

max r = 36/5 , r–check is positive? 13/18

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT2" = {{1, 4, 6, 8}, {2, 3, 5, 7}}

"PT3" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT4" = {{1, 4, 5, 7}, {2, 3, 6, 8}}

"PT5" = {{1, 3, 6, 7}, {2, 4, 5, 8}}

"PT6" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{83}{180} & \frac{47}{180} & \frac{107}{180} & \frac{23}{180} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{47}{180} & \frac{83}{180} & \frac{23}{180} & \frac{107}{180} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{83}{180} & \frac{47}{180} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{47}{180} & \frac{83}{180} \\ \frac{83}{180} & \frac{47}{180} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{59}{180} & \frac{71}{180} \\ \frac{47}{180} & \frac{83}{180} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{71}{180} & \frac{59}{180} \\ \frac{107}{180} & \frac{23}{180} & \frac{83}{180} & \frac{47}{180} & \frac{59}{180} & \frac{71}{180} & \frac{31}{36} & \frac{-5}{36} \\ \frac{23}{180} & \frac{107}{180} & \frac{47}{180} & \frac{83}{180} & \frac{71}{180} & \frac{59}{180} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_c\text{–scaled} = \begin{pmatrix} 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_c\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{83}{155} & \frac{47}{155} & \frac{107}{155} & \frac{23}{155} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & \frac{47}{155} & \frac{83}{155} & \frac{23}{155} & \frac{107}{155} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{83}{155} & \frac{47}{155} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & \frac{47}{155} & \frac{83}{155} \\ \frac{83}{155} & \frac{47}{155} & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{59}{155} & \frac{71}{155} \\ \frac{47}{155} & \frac{83}{155} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{71}{155} & \frac{59}{155} \\ \frac{107}{155} & \frac{23}{155} & \frac{83}{155} & \frac{47}{155} & \frac{59}{155} & \frac{71}{155} & 1 & \frac{-5}{31} \\ \frac{23}{155} & \frac{107}{155} & \frac{47}{155} & \frac{83}{155} & \frac{71}{155} & \frac{59}{155} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[0., 0., 0., 2.888888889, 0.3212986242, 0.7071405096, 1.362246242, 1.609314622]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C−scaled

[0., 0., 0., 3.354838710, 0.3731209831, 0.8211954308, 1.581963379, 1.868881498]

NullSpace M_C

{[1, −1, 0, 0, 0, 0, 0, 0], [0, 0, 1, −1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 1, 0, 1, 1, 0, 1, 0], [0, 1, 0, 1, 1, 0, 0, 1] }

NullSpace N_C

{[−1, −1, 0, 0, 1, 1, 0, 0], [−1, −1, 0, 0, 0, 0, 1, 1], [−1, −1, 1, 1, 0, 0, 0, 0]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 0.3212986242, 0.7071405096, 1.362246242, 1.609314622]

NullSpace M_0

{[0, 0, 0, 0, 0, 0, −1, 1], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0]}

NullSpace N_0

{[−1, −1, 0, 0, 1, 1, 0, 0], [−1, −1, 0, 0, 0, 0, 1, 1], [−1, −1, 1, 1, 0, 0, 0, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 4., −0.3212986242, −0.7071405096, −1.362246242, −1.609314622]

NullSpace M

{}

NullSpace N

{[−1, −1, 0, 0, 1, 1, 0, 0], [−1, −1, 0, 0, 0, 0, 1, 1], [−1, −1, 1, 1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 15 & 10 & 5 & 6 & 9 & 4 & 11 \\ 15 & 0 & 5 & 10 & 9 & 6 & 11 & 4 \\ 10 & 5 & 0 & 15 & 10 & 5 & 6 & 9 \\ 5 & 10 & 15 & 0 & 5 & 10 & 9 & 6 \\ 6 & 9 & 10 & 5 & 0 & 15 & 8 & 7 \\ 9 & 6 & 5 & 10 & 15 & 0 & 7 & 8 \\ 4 & 11 & 6 & 9 & 8 & 7 & 0 & 15 \\ 11 & 4 & 9 & 6 & 7 & 8 & 15 & 0 \end{pmatrix}$$

==

{7, 8}

R: [4, 3, 1, 2, 3, 4, 5, 6]
B: [7, 8, 8, 7, 8, 7, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{-1}{262144}\left(-370 + 46s + 99s^2 - 16s^3 + s^4\right)\left(-1 + s\right)\left(-3 + s\right)\left(30 + 6s\right.$

$+23s^2 - 12s^3 + s^4$)

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[1] v[2] v[3] v[4]
"B CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B^*

{[0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 1, 0, −1, 0, 0, 0], [1, 0, 0, 0, 0, −1, 0, 0], [0, 0, 0, 1, 0, −1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\,(v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8])$
degree 2: $\frac{1}{6}\,(v[1]v[2] + 2\,v[3]v[4] + v[5]v[6] + 2\,v[7]v[8])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$\pi 2$ = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2]

supp $\pi 2$ = {1, 14, 23, 28}

$u2$ = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[2], [3], [4], [3]]
Action of B on ranges, [[3], [1], [1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[3, 4, 2, 1]

B−BLOCKS,

[1, 2, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N − check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 6, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: $3 \oplus 10/8$

$$\text{CLB} = \begin{pmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4}}, true

Ω_B in Vec(K)? , {{3, 8}, {4, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 7, 8], [7, 8, 7, 8, 8, 7, 8, 7]]

2, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 5, 6], [5, 6, 5, 6, 6, 5, 6, 5]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 3, 4], [3, 4, 3, 4, 4, 3, 4, 3]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 1, 2], [1, 2, 1, 2, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 14, 23, 28}

$u2 =$
$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 0, 0, 1, 1, -2, -2]$

$$\ker N_C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s & 0 & -s & -s+t & s & 0 & -t & 0 \\ s & s & -s & -s & 0 & 0 & 0 & 0 \\ -s & 0 & t & 0 & 0 & s & 0 & -t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 & -2 & 0 & -2 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & s & 0 & -t \\ 0 & -s & 0 & t \\ -s & 0 & 0 & t \\ s & 0 & 0 & -t \\ 0 & -s & 0 & t \\ 0 & s & 0 & -t \\ 0 & t & -s & 0 \\ 0 & -t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} -1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & s & 0 & t & 0 \\ 0 & -s & 0 & s & s+t \\ -s & 0 & 0 & s & s+t \\ s & 0 & 0 & t & 0 \\ 0 & -s & 0 & s & s+t \\ 0 & s & 0 & t & 0 \\ 0 & t & -s & s & s \\ 0 & -t & s & t & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 0 & 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$$

$$T \left(0 \;\; \frac{1}{9} \;\; \frac{1}{3} \;\; \frac{1}{18} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{2}{9} \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM\begin{pmatrix} 0 & \frac{16}{9} & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{32}{9} & \frac{8}{3} & 0 & \frac{32}{9} & \frac{16}{9} & 0 & \frac{8}{3} \end{pmatrix}$$

"IS MN in Vec(K)?", false

$$MN\begin{pmatrix} \frac{-2}{3} & \frac{26}{9} & \frac{14}{3} & \frac{10}{9} & \frac{14}{9} & \frac{22}{9} & \frac{10}{3} & \frac{2}{3} & \frac{22}{9} & \frac{14}{9} & \frac{2}{3} & \frac{10}{3} \end{pmatrix}$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[0, 0, 0, 0, −1, 1, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [1, 0, 1, 0, 1, 0, 1, 0], [1, 0, 1, 0, 1, 0, 0, 1] }

NullSpace N_C

{[−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 1, 0, 0, 0, 1, 0]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[0, 0, 0, 0, 0, 0, −1, 1], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0,

0]}

NullSpace N_0

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 0, 0, 0, 1, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{}

NullSpace N

{[0, 0, 0, 0, 1, 1, −1, −1], [1, 0, 0, 0, 0, −1, 0, 0], [0, 0, 1, 0, 0, 0, 0, −1], [0, 0, 0, 1, 0, 0, −1, 0], [0, 1, 0, 0, 0, 1, −1, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 3, 4}

R: [4, 8, 8, 7, 3, 4, 4, 3]
B: [7, 3, 1, 2, 8, 7, 5, 6]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 7

Level 2 det = $\frac{1}{1048576}(-1+s)(5+s)(-26640+17472s-8712s^2+4352s^3-3390s^4+2614s^5-1353s^6+298s^7+14s^8-16s^9+s^{10})$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 7

B ranking is 5, "vs", 7

BBAR ranking 2, "vs", 4

"R CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])
"B CYCLES", 1 + v[5] v[6] v[7] v[8]

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, −1, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B^*

{[−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3   "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8] \right)$
degree 2: $\frac{1}{12}\left(v[1]v[2] + v[1]v[5] + v[2]v[6] + 2\,v[3]v[4] + 2\,v[3]v[7] + 2\,v[4]v[8] + v[5]v[6] + 2\,v[7]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$\pi 2 = [1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 4, 11, 14, 17, 22, 23, 28}

$$u2 = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[6], [5], [6], [2], [2], [1], [6], [2]]
Action of B on ranges, [[3], [4], [1], [5], [7], [8], [1], [5]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

RPARTS [2, 2]

BPARTS [2, 1]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2, 2, 1]

B-BLOCKS,

[3, 4, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N – check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 6, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 3 ⊕ 19/17

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 8}, {4, 7}}, true

Ω_B in Vec(K)? , {{5, 6, 7, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 7, 8], [7, 8, 7, 8, 8, 7, 8, 7]]

2, "range", [4, 8], [[8, 4, 8, 4, 4, 8, 4, 8], [4, 8, 4, 8, 8, 4, 8, 4]]

3, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 5, 6], [5, 6, 5, 6, 6, 5, 6, 5]]

4, "range", [2, 6], [[6, 2, 6, 2, 2, 6, 2, 6], [2, 6, 2, 6, 6, 2, 6, 2]]

5, "range", [3, 7], [[7, 3, 7, 3, 3, 7, 3, 7], [3, 7, 3, 7, 7, 3, 7, 3]]

6, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 3, 4], [3, 4, 3, 4, 4, 3, 4, 3]]

7, "range", [1, 5], [[5, 1, 5, 1, 1, 5, 1, 5], [1, 5, 1, 5, 5, 1, 5, 1]]

8, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 1, 2], [1, 2, 1, 2, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

3, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

4, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

5, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

6, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

7, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

8, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

{1, 4, 11, 14, 17, 22, 23, 28}

$u2 =$
$$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

picheck $\begin{pmatrix}2 & 2 & 4 & 4 & 2 & 2 & 4 & 4\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_5 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_6 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 1, 2, -1, -1, 0, 1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -t & 0 & s & t & 0 & -s & 0 \\ t & t & -s & -s & 0 & 0 & s-t & s-t \\ 0 & t & 0 & -s & 0 & t & s-t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & t-s & 0 & 0 & 0 & 0 & s-t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 1 & 1 & -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & s+t \\ 0 & -t-s \\ -t & -s \\ t & s \\ 0 & -t-s \\ 0 & s+t \\ t & s \\ -t & -s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s+t \\ 0 & s+t & 0 \\ t & s & 0 \\ -t & t & s+t \\ 0 & s+t & 0 \\ 0 & 0 & s+t \\ -t & t & s+t \\ t & s & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 7, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & 0 & 0 & 0 \\ \frac{4}{3} & \frac{8}{3} & 0 & 0 & 0 & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 \\ 0 & 0 & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & 0 & 0 & \frac{8}{3} & \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} \\ 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(0\ \frac{1}{9}\ 0\ \frac{1}{3}\ \frac{1}{9}\ \frac{1}{18}\ \frac{1}{9}\ \frac{2}{9}\ \frac{1}{6}\ 0\ \frac{1}{9}\ \frac{2}{9}\ \frac{1}{6}\ 0\ \frac{2}{9}\ \frac{1}{9}\ 0\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(0\ \ \frac{16}{9}\ \ 0\ \ \frac{16}{3}\ \ \frac{16}{9}\ \ \frac{8}{9}\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{32}{9}\ \ \frac{16}{9}\ \ 0\ \ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\frac{-2}{3}\ \ \frac{26}{9}\ \ \frac{-2}{3}\ \ \frac{14}{3}\ \ \frac{26}{9}\ \ \frac{10}{9}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{22}{9}\ \ \frac{14}{9}\ \ \frac{2}{3}\ \ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank } = 2 \text{ , ratio } = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 8, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 32

dim span idems 12 vs no. of idems 16

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} \\ \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[7.111111111, 5.333333333, 2.666666667, 5.333333333, 2.666666667, 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C–scaled

[2.600000000, 1.500000000, 1.200000000, 1.500000000, 1.200000000, 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[0, 1, 0, 1, 1, 0, 1, 0], [1, −1, 0, 0, −1, 1, 0, 0], [0, 1, 1, 0, 1, 0, 0, 1] }

NullSpace N_C

{[−1, −1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 0, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[10.66666667, 0., 0., 2.666666667, 2.666666667, 5.333333333, 5.333333333, 5.333333333]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[1, −1, 0, 0, −1, 1, 0, 0], [0, 0, 1, −1, 0, 0, −1, 1]}

NullSpace N_0

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 0, 1, 0, 0, 0, 1], [−1, −1, 1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0]}

Eigenvalues M

[0., 0., 0., 0., −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

NullSpace N

{[−1, −1, 0, 1, 0, 0, 0, 1], [0, 0, 0, −1, 0, 0, 1, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

40, [1, −1, 1, 1, −1, 1, −1, 1]

==

{2, 7, 8}

R: [4, 8, 1, 2, 3, 4, 5, 6]
B: [7, 3, 8, 7, 8, 7, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 7

Level 2 det = $\frac{1}{1048576}$ $(-26640 + 17472 s - 8712 s^2 + 4352 s^3 - 3390 s^4 + 2614 s^5$
$- 1353 s^6 + 298 s^7 + 14 s^8 - 16 s^9 + s^{10}) (5 + s) (-1 + s)$

RANK of R is 7

R ranking is 5, "vs", 7

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[2] v[4] v[6] v[8]
"B CYCLES", (1 + v[3] v[8]) (1 + v[4] v[7])

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[1, 0, 0, 0, 0, −1, 0, 0]}

NullSpace of B^*

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, −1, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8] \right)$

degree 2: $\frac{1}{12}\left(v[1]v[2] + v[1]v[5] + v[2]v[6] + 2\,v[3]v[4] + 2\,v[3]v[7] + 2\,v[4]v[8] + v[5]v[6] + 2\,v[7]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$\pi 2 = [1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 4, 11, 14, 17, 22, 23, 28}

$$u2 = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [4], [6], [2], [7], [8], [6], [2]]
Action of B on ranges, [[6], [5], [1], [5], [2], [1], [1], [5]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

RPARTS [2, 1]

BPARTS [2, 2]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 4, 2, 1]

B-BLOCKS,

[1, 2, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N – check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 6, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N):   3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 3 ⊕ 19/17

$$\mathrm{CLB} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4, 6, 8}}, true

Ω_B in Vec(K)? , {{3, 8}, {4, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 7, 8], [7, 8, 7, 8, 8, 7, 8, 7]]

2, "range", [4, 8], [[8, 4, 8, 4, 4, 8, 4, 8], [4, 8, 4, 8, 8, 4, 8, 4]]

3, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 5, 6], [5, 6, 5, 6, 6, 5, 6, 5]]

4, "range", [2, 6], [[6, 2, 6, 2, 2, 6, 2, 6], [2, 6, 2, 6, 6, 2, 6, 2]]

5, "range", [3, 7], [[7, 3, 7, 3, 3, 7, 3, 7], [3, 7, 3, 7, 7, 3, 7, 3]]

6, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 3, 4], [3, 4, 3, 4, 4, 3, 4, 3]]

7, "range", [1, 5], [[5, 1, 5, 1, 1, 5, 1, 5], [1, 5, 1, 5, 5, 1, 5, 1]]

8, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 1, 2], [1, 2, 1, 2, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

3, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

4, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

5, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

6, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

7, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

8, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

{1, 4, 11, 14, 17, 22, 23, 28}

$u2 =$
$$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_5 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_6 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, -1, 0, 1, 1, -2, -1]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -t+s & 0 & 0 & 0 & 0 & -s+t \\ -s & -s & t & s & 0 & 0 & 0 & -t+s \\ -s & 0 & t & 0 & 0 & s & 0 & -t \\ 0 & -s & 0 & t & s & 0 & -t & 0 \\ -s & -s & t & s & 0 & 0 & 0 & -t+s \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 & -1 & -2 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -s-t \\ 0 & t+s \\ s & t \\ -s & -t \\ 0 & t+s \\ 0 & -s-t \\ -s & -t \\ s & t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} t+s & t+s & -s-t \\ 0 & 0 & t+s \\ s & 0 & t \\ t & t+s & -t \\ 0 & 0 & t+s \\ t+s & t+s & -s-t \\ t & t+s & -t \\ s & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 & 0 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 7, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & 0 & 0 & 0 \\ \frac{4}{3} & \frac{8}{3} & 0 & 0 & 0 & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 \\ 0 & 0 & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & 0 & 0 & \frac{8}{3} & \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} \\ 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{12} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{12} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{12} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{12} \right)$$

$$T \left(0 \ \ \frac{1}{9} \ \ 0 \ \ \frac{1}{3} \ \ \frac{1}{9} \ \ \frac{1}{18} \ \ \frac{1}{9} \ \ \frac{2}{9} \ \ \frac{1}{6} \ \ 0 \ \ \frac{1}{9} \ \ \frac{2}{9} \ \ \frac{1}{6} \ \ 0 \ \ \frac{2}{9} \ \ \frac{1}{9} \ \ 0 \ \ \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM}\left(0\ \ \frac{16}{9}\ \ 0\ \ \frac{16}{3}\ \ \frac{16}{9}\ \ \frac{8}{9}\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{32}{9}\ \ \frac{16}{9}\ \ 0\ \ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN}\left(\frac{-2}{3}\ \ \frac{26}{9}\ \ \frac{-2}{3}\ \ \frac{14}{3}\ \ \frac{26}{9}\ \ \frac{10}{9}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{22}{9}\ \ \frac{14}{9}\ \ \frac{2}{3}\ \ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank } = 2 \text{ , ratio } = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 8, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 32

dim span idems 12 vs no. of idems 16

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} \\ \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[7.111111111, 5.333333333, 2.666666667, 5.333333333, 2.666666667, 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C−scaled

[2.600000000, 1.500000000, 1.200000000, 1.500000000, 1.200000000, 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[1, −1, 0, 0, −1, 1, 0, 0], [0, 1, 1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0, 1, 0] }

NullSpace N_C

{[−1, −1, 1, 0, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1], [−1, −1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues M_0

[10.66666667, 0., 0., 2.666666667, 2.666666667, 5.333333333, 5.333333333, 5.333333333]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[1, −1, 0, 0, −1, 1, 0, 0], [0, 0, −1, 1, 0, 0, 1, −1]}

NullSpace N_0

{[0, 0, −1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 1, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0]}

Eigenvalues M

[0., 0., 0., 0., −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

NullSpace N

{[0, 0, −1, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [−1, −1, 1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{3, 4, 5}

R: [4, 3, 8, 7, 8, 4, 4, 3]
B: [7, 8, 1, 2, 3, 7, 5, 6]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 7

Level 2 det = $\frac{1}{1048576}(-26640 + 17472s - 8712s^2 + 4352s^3 - 3390s^4 + 2614s^5 - 1353s^6 + 298s^7 + 14s^8 - 16s^9 + s^{10})(5+s)(-1+s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 7

B ranking is 5, "vs", 7

BBAR ranking 2, "vs", 4

"R CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])
"B CYCLES", 1 + v[1] v[3] v[5] v[7]

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[−1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0, 0, 0]}

NullSpace of B^*

{[−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\,(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8])$

degree 2: $\frac{1}{12}\,(v[1]v[2] + v[1]v[5] + v[2]v[6] + 2v[3]v[4] + 2v[3]v[7] + 2v[4]v[8] + v[5]v[6] + 2v[7]v[8])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$\pi 2 = [1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 4, 11, 14, 17, 22, 23, 28}

$$u2 = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[6], [5], [2], [6], [2], [1], [2], [6]]
Action of B on ranges, [[3], [4], [5], [1], [7], [8], [5], [1]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

RPARTS [2, 2]

BPARTS [2, 1]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2, 2, 1]

B-BLOCKS,

[3, 4, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N – check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 6, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 3 ⊕ 19/17

$$\mathrm{CLB} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 8}, {4, 7}}, true

Ω_B in Vec(K)? , {{1, 3, 5, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 7, 8], [7, 8, 7, 8, 8, 7, 8, 7]]

2, "range", [4, 8], [[8, 4, 8, 4, 4, 8, 4, 8], [4, 8, 4, 8, 8, 4, 8, 4]]

3, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 5, 6], [5, 6, 5, 6, 6, 5, 6, 5]]

4, "range", [2, 6], [[6, 2, 6, 2, 2, 6, 2, 6], [2, 6, 2, 6, 6, 2, 6, 2]]

5, "range", [3, 7], [[7, 3, 7, 3, 3, 7, 3, 7], [3, 7, 3, 7, 7, 3, 7, 3]]

6, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 3, 4], [3, 4, 3, 4, 4, 3, 4, 3]]

7, "range", [1, 5], [[5, 1, 5, 1, 1, 5, 1, 5], [1, 5, 1, 5, 5, 1, 5, 1]]

8, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 1, 2], [1, 2, 1, 2, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

3, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

4, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

5, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

6, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

7, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

8, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 4, 11, 14, 17, 22, 23, 28}

$u2 =$
$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix}2 & 2 & 4 & 4 & 2 & 2 & 4 & 4\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem−checks} \quad \text{N0−checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem−checks} \quad \text{N0−checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem−checks N0−checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix}$$

idem−checks N0−checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 1, 2, -1, -1, 0, 1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -t & 0 & s & 0 & 0 & t & 0 & -s \\ t & t & -s & -s & 0 & 0 & -t+s & -t+s \\ 0 & 0 & -s+t & 0 & 0 & 0 & 0 & -t+s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t & 0 & -s & 0 & t & 0 & -t & -t+s \end{pmatrix}$$

RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 2 & -1 & -1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 \\ 0 & -1 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} s+t & 0 \\ -s-t & 0 \\ -s & t \\ s & -t \\ -s-t & 0 \\ s+t & 0 \\ s & -t \\ -s & t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s+t \\ 0 & s+t & 0 \\ -t & s+t & t \\ t & 0 & s \\ 0 & s+t & 0 \\ 0 & 0 & s+t \\ t & 0 & s \\ -t & s+t & t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1&0&0&1&0&1&1&0 \\ 0&1&1&0&1&0&0&1 \\ 0&1&1&0&1&0&0&1 \\ 1&0&0&1&0&1&1&0 \\ 0&1&1&0&1&0&0&1 \\ 1&0&0&1&0&1&1&0 \\ 1&0&0&1&0&1&1&0 \\ 0&1&1&0&1&0&0&1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1&0&0&0&0&1&1&0 \\ 0&1&0&0&0&0&0&1 \\ 0&0&1&0&1&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&1&0&1&0&0&0 \\ 1&0&0&0&0&1&1&0 \\ 1&0&0&0&0&1&1&0 \\ 0&1&0&0&0&0&0&1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1&0&\frac{2}{3}&\frac{1}{3}&0&1&\frac{1}{3}&\frac{2}{3} \\ 0&1&\frac{1}{3}&\frac{2}{3}&1&0&\frac{2}{3}&\frac{1}{3} \\ \frac{2}{3}&\frac{1}{3}&1&0&\frac{1}{3}&\frac{2}{3}&0&1 \\ \frac{1}{3}&\frac{2}{3}&0&1&\frac{2}{3}&\frac{1}{3}&1&0 \\ 0&1&\frac{1}{3}&\frac{2}{3}&1&0&\frac{2}{3}&\frac{1}{3} \\ 1&0&\frac{2}{3}&\frac{1}{3}&0&1&\frac{1}{3}&\frac{2}{3} \\ \frac{1}{3}&\frac{2}{3}&0&1&\frac{2}{3}&\frac{1}{3}&1&0 \\ \frac{2}{3}&\frac{1}{3}&1&0&\frac{1}{3}&\frac{2}{3}&0&1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1&0&0&0&0&1&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 1&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 7, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & 0 & 0 & 0 \\ \frac{4}{3} & \frac{8}{3} & 0 & 0 & 0 & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 \\ 0 & 0 & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & 0 & 0 & \frac{8}{3} & \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} \\ 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12}\right)$$

$$T \left(0 \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{9} \;\; \frac{1}{18} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{2}{9} \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(0\ \ \frac{16}{9}\ \ 0\ \ \frac{16}{3}\ \ \frac{16}{9}\ \ \frac{8}{9}\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{32}{9}\ \ \frac{16}{9}\ \ 0\ \ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\frac{-2}{3}\ \ \frac{26}{9}\ \ \frac{-2}{3}\ \ \frac{14}{3}\ \ \frac{26}{9}\ \ \frac{10}{9}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{22}{9}\ \ \frac{14}{9}\ \ \frac{2}{3}\ \ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 8, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 32

dim span idems 12 vs no. of idems 16

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} \\ \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[7.111111111, 5.333333333, 2.666666667, 5.333333333, 2.666666667, 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C−scaled

[2.600000000, 1.500000000, 1.200000000, 1.500000000, 1.200000000, 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[0, 1, 0, 1, 1, 0, 1, 0], [1, −1, 0, 0, −1, 1, 0, 0], [0, 1, 1, 0, 1, 0, 0, 1] }

NullSpace N_C

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [−1, −1, 1, 0, 0, 0, 1, 0]}

Eigenvalues M_0

[10.66666667, 0., 0., 2.666666667, 2.666666667, 5.333333333, 5.333333333, 5.333333333]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[0, 0, 1, −1, 0, 0, −1, 1], [1, −1, 0, 0, −1, 1, 0, 0]}

NullSpace N_0

{[0, 0, 1, 0, 0, 0, 0, −1], [−1, −1, 0, 1, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 0, 0, 0, 0, 1, 1], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues M

[0., 0., 0., 0., −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{[0, 0, 0, −1, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace N

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 0, 1, 0, 0, 0, 1], [−1, −1, 1, 1, 0, 0, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{3, 4, 6}

R: [4, 3, 8, 7, 3, 7, 4, 3]
B: [7, 8, 1, 2, 8, 4, 5, 6]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 7

Level 2 det = $\frac{1}{1048576} (5+s)(-26640 + 17472s - 8712s^2 + 4352s^3 - 3390s^4 + 2614s^5 - 1353s^6 + 298s^7 + 14s^8 - 16s^9 + s^{10})(-1+s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 7

B ranking is 5, "vs", 7

BBAR ranking 2, "vs", 4

"R CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])
"B CYCLES", 1 + v[2] v[4] v[6] v[8]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, 0, −1, 0, 1, 0, 0]}

NullSpace of B^*

{[0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3   "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8] \right)$
degree 2: $\frac{1}{12}\left(v[1]v[2] + v[1]v[5] + v[2]v[6] + 2v[3]v[4] + 2v[3]v[7] + 2v[4]v[8] + v[5]v[6] + 2v[7]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$\pi 2 = [1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 4, 11, 14, 17, 22, 23, 28}

$$u2 = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[6], [5], [5], [5], [2], [1], [6], [6]]
Action of B on ranges, [[3], [4], [2], [2], [7], [8], [1], [1]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

RPARTS [2, 2]

BPARTS [2, 1]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2, 2, 1]

B-BLOCKS,

[3, 4, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N − check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 6, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 3 ⊕ 19/17

$$\mathrm{CLB} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 8}, {4, 7}}, true

Ω_B in Vec(K)? , {{2, 4, 6, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 7, 8], [7, 8, 7, 8, 8, 7, 8, 7]]

2, "range", [4, 8], [[8, 4, 8, 4, 4, 8, 4, 8], [4, 8, 4, 8, 8, 4, 8, 4]]

3, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 5, 6], [5, 6, 5, 6, 6, 5, 6, 5]]

4, "range", [2, 6], [[6, 2, 6, 2, 2, 6, 2, 6], [2, 6, 2, 6, 6, 2, 6, 2]]

5, "range", [3, 7], [[7, 3, 7, 3, 3, 7, 3, 7], [3, 7, 3, 7, 7, 3, 7, 3]]

6, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 3, 4], [3, 4, 3, 4, 4, 3, 4, 3]]

7, "range", [1, 5], [[5, 1, 5, 1, 1, 5, 1, 5], [1, 5, 1, 5, 5, 1, 5, 1]]

8, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 1, 2], [1, 2, 1, 2, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

3, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

4, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

5, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

6, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

7, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

8, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

{1, 4, 11, 14, 17, 22, 23, 28}

$u2 =$
$$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

picheck $\begin{pmatrix}2 & 2 & 4 & 4 & 2 & 2 & 4 & 4\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_5 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_6 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 2, 1, -1, -1, 1, 0]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s+t & 0 & 0 & -t+s & 0 \\ t & t & -s & -s & 0 & 0 & -t+s & -t+s \\ t & 0 & -s & 0 & t & 0 & -t & -t+s \\ -t & 0 & s & 0 & 0 & t & 0 & -s \end{pmatrix}$$

RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 & 1 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 \\ 0 & 1 \\ 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \\ -1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -s-t & 0 \\ s+t & 0 \\ s & -t \\ -s & t \\ s+t & 0 \\ -s-t & 0 \\ -s & t \\ s & -t \end{pmatrix}$$ RB checks

$$\ker M_c = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & s+t \\ 0 & s+t & 0 \\ -t & s+t & t \\ t & 0 & s \\ 0 & s+t & 0 \\ 0 & 0 & s+t \\ t & 0 & s \\ -t & s+t & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 7, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & 0 & 0 & 0 \\ \frac{4}{3} & \frac{8}{3} & 0 & 0 & 0 & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 \\ 0 & 0 & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & 0 & 0 & \frac{8}{3} & \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} \\ 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\right)$$

$$T\left(0\ \ \frac{1}{9}\ \ 0\ \ \frac{1}{3}\ \ \frac{1}{9}\ \ \frac{1}{18}\ \ \frac{1}{9}\ \ \frac{2}{9}\ \ \frac{1}{6}\ \ 0\ \ \frac{1}{9}\ \ \frac{2}{9}\ \ \frac{1}{6}\ \ 0\ \ \frac{2}{9}\ \ \frac{1}{9}\ \ 0\ \ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(0\ \ \frac{16}{9}\ \ 0\ \ \frac{16}{3}\ \ \frac{16}{9}\ \ \frac{8}{9}\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{32}{9}\ \ \frac{16}{9}\ \ 0\ \ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\frac{-2}{3}\ \ \frac{26}{9}\ \ \frac{-2}{3}\ \ \frac{14}{3}\ \ \frac{26}{9}\ \ \frac{10}{9}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{22}{9}\ \ \frac{14}{9}\ \ \frac{2}{3}\ \ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 8, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 32

dim span idems 12 vs no. of idems 16

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} \\ \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[7.111111111, 5.333333333, 2.666666667, 5.333333333, 2.666666667, 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C–scaled

[2.600000000, 1.500000000, 1.200000000, 1.500000000, 1.200000000, 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[1, 0, 0, 1, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1, 0, 1], [−1, 1, 0, 0, 1, −1, 0, 0] }

NullSpace N_C

{[0, 0, −1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [−1, −1, 1, 0, 0, 0, 1, 0]}

Eigenvalues M_0

[10.66666667, 0., 0., 2.666666667, 2.666666667, 5.333333333, 5.333333333, 5.333333333]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[1, −1, 0, 0, −1, 1, 0, 0], [0, 0, 1, −1, 0, 0, −1, 1]}

NullSpace N_0

{[1, 1, −1, −1, 0, 0, 0, 0], [0, 1, −1, −1, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues M

[0., 0., 0., 0., −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 1, 0, 0, −1, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

NullSpace N

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 1, 1, −1, −1, 0, 0], [0, 0, 1, 0, −1, −1, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

60, [1, 1, 1, −1, 1, 1, −1, −1]

==

{5, 7, 8}

R: [4, 3, 1, 2, 8, 4, 5, 6]
B: [7, 8, 8, 7, 3, 7, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 7

Level 2 det = $\frac{1}{1048576}(-26640 + 17472s - 8712s^2 + 4352s^3 - 3390s^4 + 2614s^5 - 1353s^6 + 298s^7 + 14s^8 - 16s^9 + s^{10})(5 + s)(-1 + s)$

RANK of R is 7

R ranking is 5, "vs", 7

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[1] v[2] v[3] v[4]
"B CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])

Eigenvalues

R: [0., 0., 0., 0., -1., 1., 1. I, -1. I]

B: [1., -1., 1., -1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[-1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, -1, 0, 0, 1], [1, 0, 0, -1, 0, 0, 0, 0], [0, -1, 1, 0, 0, 0, 0, 0], [0, 0, 0, -1, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$)
degree 2: $\frac{1}{12}$ ($v[1]v[2] + v[1]v[5] + v[2]v[6] + 2v[3]v[4] + 2v[3]v[7] + 2v[4]v[8] + v[5]v[6] + 2v[7]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$\pi 2 = [1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 4, 11, 14, 17, 22, 23, 28}

$$u2 = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [4], [2], [6], [7], [8], [2], [6]]
Action of B on ranges, [[6], [5], [5], [1], [2], [1], [5], [1]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

RPARTS [2, 1]

BPARTS [2, 2]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 4, 2, 1]

B-BLOCKS,

[1, 2, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N – check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 6, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 3 ⊕ 19/17

$$\mathrm{CLB} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4}}, true

Ω_B in Vec(K)? , {{3, 8}, {4, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 7, 8], [7, 8, 7, 8, 8, 7, 8, 7]]

2, "range", [4, 8], [[8, 4, 8, 4, 4, 8, 4, 8], [4, 8, 4, 8, 8, 4, 8, 4]]

3, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 5, 6], [5, 6, 5, 6, 6, 5, 6, 5]]

4, "range", [2, 6], [[6, 2, 6, 2, 2, 6, 2, 6], [2, 6, 2, 6, 6, 2, 6, 2]]

5, "range", [3, 7], [[7, 3, 7, 3, 3, 7, 3, 7], [3, 7, 3, 7, 7, 3, 7, 3]]

6, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 3, 4], [3, 4, 3, 4, 4, 3, 4, 3]]

7, "range", [1, 5], [[5, 1, 5, 1, 1, 5, 1, 5], [1, 5, 1, 5, 5, 1, 5, 1]]

8, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 1, 2], [1, 2, 1, 2, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

3, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

4, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

5, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

6, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

7, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

8, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2$ =
$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 4, 11, 14, 17, 22, 23, 28}

$u2$ =
$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, -1, 0, 1, 1, -2, -1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 1 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & -t & -s+t & s & 0 & -t & -s+t \\ 0 & s & 0 & -t & -s & 0 & t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s-t & 0 & 0 & 0 & 0 & -s+t \\ 0 & 0 & 0 & -s+t & s & s & -t & -s \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 0 & 1 & 1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & s+t \\ 0 & -s-t \\ s & -t \\ -s & t \\ 0 & -s-t \\ 0 & s+t \\ -s & t \\ s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & s+t & 0 \\ 0 & 0 & s+t \\ s & 0 & t \\ -s & s+t & s \\ 0 & 0 & s+t \\ 0 & s+t & 0 \\ -s & s+t & s \\ s & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 7, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & 0 & 0 & 0 \\ \frac{4}{3} & \frac{8}{3} & 0 & 0 & 0 & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 \\ 0 & 0 & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & 0 & 0 & \frac{8}{3} & \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} \\ 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$$

$$T \left(0 \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{9} \;\; \frac{1}{18} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{2}{9} \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM\left(0\ \frac{16}{9}\ 0\ \frac{16}{3}\ \frac{16}{9}\ \frac{8}{9}\ \frac{16}{9}\ \frac{32}{9}\ \frac{8}{3}\ 0\ \frac{16}{9}\ \frac{32}{9}\ \frac{8}{3}\ 0\ \frac{32}{9}\ \frac{16}{9}\ 0\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\frac{-2}{3}\ \frac{26}{9}\ \frac{-2}{3}\ \frac{14}{3}\ \frac{26}{9}\ \frac{10}{9}\ \frac{14}{9}\ \frac{22}{9}\ \frac{10}{3}\ \frac{2}{3}\ \frac{14}{9}\ \frac{22}{9}\ \frac{10}{3}\ \frac{2}{3}\ \frac{22}{9}\ \frac{14}{9}\ \frac{2}{3}\ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1\text{ , rank} = 2\text{ , ratio} = 16/1\text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1\text{ , } r' = 1/2\text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36\text{ , min } \boldsymbol{\tau} = 80/9\text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5\text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 8, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 32

dim span idems 12 vs no. of idems 16

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} \\ \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[7.111111111, 5.333333333, 2.666666667, 5.333333333, 2.666666667, 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C−scaled

[2.600000000, 1.500000000, 1.200000000, 1.500000000, 1.200000000, 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[1, 0, 0, 1, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1, 0, 1], [−1, 1, 0, 0, 1, −1, 0, 0] }

NullSpace N_C

{[0, 0, −1, 0, 0, 0, 0, 1], [−1, −1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 0, 0, 0, 1, 0]}

Eigenvalues M_0

[10.66666667, 0., 0., 2.666666667, 2.666666667, 5.333333333, 5.333333333, 5.333333333]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[1, −1, 0, 0, −1, 1, 0, 0], [0, 0, 1, −1, 0, 0, −1, 1]}

NullSpace N_0

{[−1, −1, 1, 0, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0]}

Eigenvalues M

[0., 0., 0., 0., −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{[0, 0, 0, 1, 0, 0, −1, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0]}

NullSpace N

{[0, 0, 1, 0, 0, 0, 0, −1], [−1, −1, 0, 1, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 0, 0, 0, 0, 1, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{6, 7, 8}

R: [4, 3, 1, 2, 3, 7, 5, 6]
B: [7, 8, 8, 7, 8, 4, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 7

Level 2 det = $\frac{1}{1048576}(-1+s)(-26640+17472s-8712s^2+4352s^3-3390s^4+2614s^5-1353s^6+298s^7+14s^8-16s^9+s^{10})(5+s)$

RANK of R is 7

R ranking is 5, "vs", 7

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[1] v[2] v[3] v[4]
"B CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])

Eigenvalues

R: [0., 0., 0., 0., -1., 1., 1. I, -1. I]

B: [1., -1., 1., -1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[0, 1, 0, 0, −1, 0, 0, 0]}

NullSpace of B^*

{[1, 0, 0, −1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, −1, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 1, 0, −1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8] \right)$

degree 2: $\frac{1}{12}\left(v[1]v[2] + v[1]v[5] + v[2]v[6] + 2v[3]v[4] + 2v[3]v[7] + 2v[4]v[8] + v[5]v[6] + 2v[7]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$\pi 2$ = [1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2]

supp $\pi 2$ = {1, 4, 11, 14, 17, 22, 23, 28}

$u2$ = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [4], [5], [5], [7], [8], [6], [6]]
Action of B on ranges, [[6], [5], [2], [2], [2], [1], [1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 4, 2, 1]

B-BLOCKS,

[1, 2, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N - check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 6, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 3 ⊕ 19/17

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4}}, true

Ω_B in Vec(K)? , {{3, 8}, {4, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 7, 8], [7, 8, 7, 8, 8, 7, 8, 7]]

2, "range", [4, 8], [[8, 4, 8, 4, 4, 8, 4, 8], [4, 8, 4, 8, 8, 4, 8, 4]]

3, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 5, 6], [5, 6, 5, 6, 6, 5, 6, 5]]

4, "range", [2, 6], [[6, 2, 6, 2, 2, 6, 2, 6], [2, 6, 2, 6, 6, 2, 6, 2]]

5, "range", [3, 7], [[7, 3, 7, 3, 3, 7, 3, 7], [3, 7, 3, 7, 7, 3, 7, 3]]

6, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 3, 4], [3, 4, 3, 4, 4, 3, 4, 3]]

7, "range", [1, 5], [[5, 1, 5, 1, 1, 5, 1, 5], [1, 5, 1, 5, 5, 1, 5, 1]]

8, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 1, 2], [1, 2, 1, 2, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

3, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

4, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

5, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

6, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

7, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

8, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 4, 11, 14, 17, 22, 23, 28}

$u2 =$
$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix}2 & 2 & 4 & 4 & 2 & 2 & 4 & 4\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 0, -1, 1, 1, -1, -2]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & 0 & -t & -s & 0 & t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s & 0 & -t & 0 & 0 & -s & 0 & t \\ 0 & 0 & s-t & s-t & -s & -s & t & t \\ 0 & 0 & s-t & 0 & -s & -s & s & t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & 0 & 1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & s+t \\ 0 & -s-t \\ -s & -t \\ s & t \\ 0 & -s-t \\ 0 & s+t \\ s & t \\ -s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s+t \\ 0 & s+t & 0 \\ s & t & 0 \\ -s & s & s+t \\ 0 & s+t & 0 \\ 0 & 0 & s+t \\ -s & s & s+t \\ s & t & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 7, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & 0 & 0 & 0 \\ \frac{4}{3} & \frac{8}{3} & 0 & 0 & 0 & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 \\ 0 & 0 & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & 0 & 0 & \frac{8}{3} & \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} \\ 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$$

$$T \left(0 \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{9} \;\; \frac{1}{18} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{2}{9} \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$NM\left(0\ \ \frac{16}{9}\ \ 0\ \ \frac{16}{3}\ \ \frac{16}{9}\ \ \frac{8}{9}\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{32}{9}\ \ \frac{16}{9}\ \ 0\ \ \frac{8}{3}\right)$

"IS MN in Vec(K)?", false

$MN\left(\frac{-2}{3}\ \ \frac{26}{9}\ \ \frac{-2}{3}\ \ \frac{14}{3}\ \ \frac{26}{9}\ \ \frac{10}{9}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{22}{9}\ \ \frac{14}{9}\ \ \frac{2}{3}\ \ \frac{10}{3}\right)$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 8, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 32

dim span idems 12 vs no. of idems 16

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} \\ \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[7.111111111, 5.333333333, 2.666666667, 5.333333333, 2.666666667, 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C−scaled

[2.600000000, 1.500000000, 1.200000000, 1.500000000, 1.200000000, 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[1, 0, 0, 1, 0, 1, 1, 0], [−1, 1, 0, 0, 1, −1, 0, 0], [1, 0, 1, 0, 0, 1, 0, 1] }

NullSpace N_C

{[0, −1, 0, 0, 1, 0, 0, 0], [0, 1, −1, −1, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1], [1, 1, −1, −1, 0, 0, 0, 0]}

Eigenvalues M_0

[10.66666667, 0., 0., 2.666666667, 2.666666667, 5.333333333, 5.333333333, 5.333333333]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[0, 0, 1, −1, 0, 0, −1, 1], [1, −1, 0, 0, −1, 1, 0, 0]}

NullSpace N_0

{[0, 0, −1, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [1, 1, −1, −1, 0, 0, 0, 0], [1, 0, −1, −1, 1, 0, 0, 0]}

Eigenvalues M

[0., 0., 0., 0., −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

NullSpace N

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [−1, −1, 0, 1, 0, 0, 0, 1], [0, 0, 0, −1, 0, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 3, 4, 5}

R: [4, 8, 8, 7, 8, 4, 4, 3]
B: [7, 3, 1, 2, 3, 7, 5, 6]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{-1}{262144} (30 + 6s + 23s^2 - 12s^3 + s^4) (-1 + s) (-370 + 46s + 99s^2 - 16s^3 + s^4) (-3 + s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 2, "vs", 4

"R CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])
"B CYCLES", 1 + v[1] v[3] v[5] v[7]

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

NullSpace of B^*

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\,(\,v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8]\,)$
degree 2: $\frac{1}{6}\,(\,v[1]v[5] + v[2]v[6] + 2\,v[3]v[7] + 2\,v[4]v[8]\,)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {4, 8}

"RG2" = {2, 6}

"RG3" = {3, 7}

"RG4" = {1, 5}

$\pi 2$ = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {4, 11, 17, 22}

$u2$ = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [1], [1], [1]]
Action of B on ranges, [[2], [3], [4], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 2]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2, 2, 1]

B-BLOCKS,

[3, 4, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N - check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 6, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \\ h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: $3 \oplus 10/8$

$$\text{CLB} = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 8}, {4, 7}}, true

Ω_B in Vec(K)? , {{1, 3, 5, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [4, 8], [[8, 4, 8, 4, 4, 8, 4, 8], [4, 8, 4, 8, 8, 4, 8, 4]]

2, "range", [2, 6], [[6, 2, 6, 2, 2, 6, 2, 6], [2, 6, 2, 6, 6, 2, 6, 2]]

3, "range", [3, 7], [[7, 3, 7, 3, 3, 7, 3, 7], [3, 7, 3, 7, 7, 3, 7, 3]]

4, "range", [1, 5], [[5, 1, 5, 1, 1, 5, 1, 5], [1, 5, 1, 5, 5, 1, 5, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

2, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

3, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\,2\,] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 11, 17, 22}

$u2 =$
$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 0, 2, -1, -1, 0, 2]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t & 0 & -t & 0 & t & 0 & -t & 0 \\ -t & 0 & s & 0 & 0 & t & 0 & -s \\ t & t & -t & -s & 0 & 0 & -t+s & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 & 0 & 2 & 2 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & t & -s \\ 0 & 0 & -t & s \\ -t & 0 & 0 & s \\ 0 & -t & s & 0 \\ 0 & 0 & -t & s \\ 0 & 0 & t & -s \\ t & 0 & 0 & -s \\ 0 & t & -s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t & s & 0 & 0 & 0 \\ -t & t & 0 & 0 & t+s \\ 0 & t & -t & 0 & t+s \\ s & t & 0 & -t & t \\ -t & t & 0 & 0 & t+s \\ t & s & 0 & 0 & 0 \\ 0 & s & t & 0 & 0 \\ -s & s & 0 & t & s \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \\ \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$$

$$T \left(0 \;\; \frac{1}{3} \;\; \frac{1}{9} \;\; \frac{1}{18} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{2}{9} \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM\left(0 \quad \frac{16}{3} \quad \frac{16}{9} \quad \frac{8}{9} \quad \frac{16}{9} \quad \frac{32}{9} \quad \frac{8}{3} \quad 0 \quad \frac{32}{9} \quad \frac{16}{9} \quad 0 \quad \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\frac{-2}{3} \quad \frac{14}{3} \quad \frac{26}{9} \quad \frac{10}{9} \quad \frac{14}{9} \quad \frac{22}{9} \quad \frac{10}{3} \quad \frac{2}{3} \quad \frac{22}{9} \quad \frac{14}{9} \quad \frac{2}{3} \quad \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {4, 8}

"RG2" = {2, 6}

"RG3" = {3, 7}

"RG4" = {1, 5}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \\ \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\ 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[1, 1, 1, 0, 0, 0, 0, 1], [1, 1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0]}

NullSpace N_C

{[0, 1, −1, −1, 0, 1, 0, 0], [1, 1, −1, −1, 0, 0, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[0, −1, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [1, 0, 0, 0, −1, 0, 0,

0]}

NullSpace N_0

{[0, 1, −1, −1, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1], [1, 1, −1, −1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{}

NullSpace N

{[−1, −1, 1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 3, 4, 6}

R: [4, 8, 8, 7, 3, 7, 4, 3]
B: [7, 3, 1, 2, 8, 4, 5, 6]

TRACE TWO = 2

det AT = $\frac{-1}{16}\,(-1+t)^4\,(t)^2$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 8

Level 2 det = $\frac{-5}{268435456}(444 - 24s + 235s^2 + 54s^3 + 68s^4 + 18s^5 + 5s^6)(-12 - 4s + s^2 + s^3)(-1+s)^2(5+s)^2(16+s^2)^2$

RANK of R is 4

R ranking is 1, "vs", 4

RBAR ranking 1, "vs", 4

RANK of B is 8

B ranking is 2, "vs", 8

BBAR ranking 2, "vs", 8

"R CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6] v[7] v[8]

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [-1. I, 1. I, -1., 1., -0.7071067810 - 0.7071067810 I, 0.7071067810 + 0.7071067810 I, -0.7071067810 + 0.7071067810 I, 0.7071067810 - 0.7071067810 I]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{}

NullSpace of R^*

{[-1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, -1, 0, 1, 0, 0], [0, -1, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, -1, 0, 0, 1]}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 8 \\ 0 & 0 & 8 & 0 & 0 & 0 & 8 & 8 \\ 4 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\ 4 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 8 & 0 & 0 & 0 & 8 \\ 0 & 0 & 8 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & 1 & 1 & 1 & \frac{1}{3} & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 1 & 1 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 1 & 1 & 1 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ \frac{1}{3} & 1 & 1 & 1 & \frac{2}{3} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 7, "RANK of M is ", 8

"RANK of the KERNEL is ", 4

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 4

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$)
degree 2: $\frac{1}{12}$ ($v[1]v[2] + v[1]v[5] + 2v[1]v[6] + 2v[2]v[5] + v[2]v[6] + 2v[3]v[4] + 2v[3]v[7] + 4v[3]v[8] + 4v[4]v[7] + 2v[4]v[8] + v[5]v[6] + 2v[7]v[8]$)
degree 3 : $\frac{1}{12}$ ($v[1]v[2]v[5] + v[1]v[2]v[6] + v[1]v[5]v[6] + v[2]v[5]v[6] + 2v[3]v[4]v[7] + 2v[3]v[4]v[8] + 2v[3]v[7]v[8] + 2v[4]v[7]v[8]$)
degree 4 : $\frac{1}{3}$ ($v[1]v[2]v[5]v[6] + 2v[3]v[4]v[7]v[8]$)

Group spectrum $1 + t + 2t^2 + t^3 + t^4$

KERNEL STRUCTURE

"PT1" = {{1, 7}, {4, 6}, {5, 8}, {2, 3}}

"PT2" = {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

"RG1" = {3, 4, 7, 8}

"RG2" = {1, 2, 5, 6}

$\pi4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi4$ = {10, 61}

$u4$ = [0, 0, 0, 0, 0, 2, 0, 0, 2, 3, 0, 1, 1, 2, 1, 2, 0, 0, 2, 2, 0, 0, 0, 2, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 0, 2, 0, 0, 0, 2, 2, 0, 0, 2, 1, 2, 1, 1, 0, 3, 2, 0, 0, 2, 0, 0, 0, 0, 0]

supp u4 = {6, 9, 10, 12, 13, 14, 15, 16, 19, 20, 24, 26, 28, 29, 31, 40, 42, 43, 45, 47, 51, 52, 55, 56, 57, 58, 59, 61, 62, 65}

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 1, 1, 6, 6, 4, 4, 3]

B-BLOCKS,

[7, 3, 5, 8, 4, 2, 6, 1]

with invariant measure, [2, 1, 2, 2, 1, 2, 1, 1]

N by blocks, N – check: true

b_1 = {1, 7}
b_2 = {2, 4}
b_3 = {4, 6}
b_4 = {5, 8}
b_5 = {6, 8}
b_6 = {2, 3}
b_7 = {1, 3}
b_8 = {5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 7, 7, 7

$$\text{Centralizer} = \begin{pmatrix} h[4] & h[3] & 0 & 0 & h[2] & h[1] & 0 & 0 \\ h[2] & h[4] & 0 & 0 & h[1] & h[3] & 0 & 0 \\ 0 & 0 & h[4] & h[3] & 0 & 0 & h[2] & h[1] \\ 0 & 0 & h[2] & h[4] & 0 & 0 & h[1] & h[3] \\ h[3] & h[1] & 0 & 0 & h[4] & h[2] & 0 & 0 \\ h[1] & h[2] & 0 & 0 & h[3] & h[4] & 0 & 0 \\ 0 & 0 & h[3] & h[1] & 0 & 0 & h[4] & h[2] \\ 0 & 0 & h[1] & h[2] & 0 & 0 & h[3] & h[4] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 12, Shape: 9 ⊕ 3/1

$$\text{CLB} = \begin{pmatrix} -1 & 1 & 1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 8}, {4, 7}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 4, 5, 6, 7, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 7}, {4, 6}, {5, 8}, {2, 3}}

1, "range", [3, 4, 7, 8], [[8, 7, 7, 3, 4, 3, 8, 4], [7, 3, 3, 4, 8, 4, 7, 8], [4, 8, 8, 7, 3, 7, 4, 3], [3, 4, 4, 8, 7, 8, 3, 7]]

2, "range", [1, 2, 5, 6], [[6, 5, 5, 1, 2, 1, 6, 2], [5, 1, 1, 2, 6, 2, 5, 6], [2, 6, 6, 5, 1, 5, 2, 1], [1, 2, 2, 6, 5, 6, 1, 5]]

2, "partition", {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

1, "range", [3, 4, 7, 8], [[8, 7, 8, 7, 4, 3, 4, 3], [7, 3, 7, 3, 8, 4, 8, 4], [4, 8, 4, 8, 3, 7, 3, 7], [3, 4, 3, 4, 7, 8, 7, 8]]

2, "range", [1, 2, 5, 6], [[6, 5, 6, 5, 2, 1, 2, 1], [5, 1, 5, 1, 6, 2, 6, 2], [2, 6, 2, 6, 1, 5, 1, 5], [1, 2, 1, 2, 5, 6, 5, 6]]

"group has", 4, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 3, 4, 2]]

$g_2 =$ []

$g_3 =$ [[1, 4], [2, 3]]

$g_4 =$ [[1, 2, 4, 3]]

linear dimension, 4

"Symmetric?", false

Is Z in Vec(K)? true
$\begin{pmatrix} h[3] & h[1] & h[4] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\,3\,] = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

4, "coeff", 1

$$Z[\,4\,] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
2, 3, true
2, 4, true
3, 4, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. \\ -1. & 1. & 1.I & -1.I \\ -1. & 1. & 1.I & -1.I \\ 1. & -1. & 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 4 & 0 & 0 \\ 0 & 6 & 2 & 0 \\ 0 & 4 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + t^3 + t^4$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 14t^5 + 22t^6 + 30t^7 + 43t^8 + 55t^9 + 73t^{10}$

n-choose-rank

{1, [1, 2, 3, 4]}, {2, [1, 2, 3, 5]}, {3, [1, 2, 3, 6]}, {4, [1, 2, 3, 7]}, {5, [1, 2, 3, 8]}, {6, [1, 2, 4, 5]}, {7, [1, 2, 4, 6]}, {8, [1, 2, 4, 7]}, {9, [1, 2, 4, 8]}, {10, [1, 2, 5, 6]}, {11, [1, 2, 5, 7]}, {12, [1, 2, 5, 8]}, {13, [1, 2, 6, 7]}, {14, [1, 2, 6, 8]}, {15, [1, 2, 7, 8]}, {16, [1, 3, 4, 5]}, {17, [1, 3, 4, 6]}, {18, [1, 3, 4, 7]}, {19, [1, 3, 4, 8]}, {20, [1, 3, 5, 6]}, {21, [1, 3, 5, 7]}, {22, [1, 3, 5, 8]}, {23, [1, 3, 6, 7]}, {24, [1, 3, 6, 8]}, {25, [1, 3, 7, 8]}, {26, [1, 4, 5, 6]}, {27, [1, 4, 5, 7]}, {28, [1, 4, 5, 8]}, {29, [1, 4, 6, 7]}, {30, [1, 4, 6, 8]}, {31, [1, 4, 7, 8]}, {32, [1, 5, 6, 7]}, {33, [1, 5, 6, 8]}, {34, [1, 5, 7, 8]}, {35, [1, 6, 7, 8]}, {36, [2, 3, 4, 5]}, {37, [2, 3, 4, 6]}, {38, [2, 3, 4, 7]}, {39, [2, 3, 4, 8]}, {40, [2, 3, 5, 6]}, {41, [2, 3, 5, 7]}, {42, [2, 3, 5, 8]}, {43, [2, 3, 6, 7]}, {44, [2, 3, 6, 8]}, {45, [2, 3, 7, 8]}, {46, [2, 4, 5, 6]}, {47, [2, 4, 5, 7]}, {48, [2, 4, 5, 8]}, {49, [2, 4, 6, 7]}, {50, [2, 4, 6, 8]}, {51, [2, 4, 7, 8]}, {52, [2, 5, 6, 7]}, {53, [2, 5, 6, 8]}, {54, [2, 5, 7, 8]}, {55, [2, 6, 7, 8]}, {56, [3, 4, 5, 6]}, {57, [3, 4, 5, 7]}, {58, [3, 4, 5, 8]}, {59, [3, 4, 6, 7]}, {60, [3, 4, 6, 8]}, {61, [3, 4, 7, 8]}, {62, [3, 5, 6, 7]}, {63, [3, 5, 6, 8]}, {64, [3, 5, 7, 8]}, {65, [3, 6, 7, 8]}, {66, [4, 5, 6, 7]}, {67, [4, 5, 6, 8]}, {68, [4, 5, 7, 8]}, {69, [4, 6, 7, 8]}, {70, [5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 4$ =
(0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

{10, 61}

$u4$ =
(0 0 0 0 0 2 0 0 2 3 0 1 1 2 1 2 0 0 2 2 0 0 0 2 0 1 0 1 1 0

{6, 9, 10, 12, 13, 14, 15, 16, 19, 20, 24, 26, 28, 29, 31, 40, 42, 43, 45, 47, 51, 52, 55, 56, 57, 58, 59, 61, 62, 65}

picheck $\begin{pmatrix}1 & 1 & 2 & 2 & 1 & 1 & 2 & 2\end{pmatrix}$

$\pi = \begin{pmatrix}\frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6}\end{pmatrix}$

$\pi 3 =$
$(0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0$

$u3 =$
$\Big(0\ \ \frac{1}{2}\ \ \frac{3}{4}\ \ \frac{3}{4}\ \ \frac{1}{4}\ \ \frac{3}{4}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ 0\ \ \frac{1}{2}\ \ \frac{3}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{3}{4}\ \ \frac{3}{4}\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{2}\ \ \frac{1}{4}\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{2}\ \ 0$

picheck $\begin{pmatrix}3 & 3 & 6 & 6 & 3 & 3 & 6 & 6\end{pmatrix}$

$\pi 2 =$
$\begin{pmatrix}2 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 4 & 0 & 0 & 4 & 4 & 0 & 0 & 4 & 4 & 2 & 0 & 0 & 0 & 0 & 4\end{pmatrix}$

$u2 =$
$\begin{pmatrix}\frac{3}{8} & \frac{1}{4} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} & \frac{3}{8} & \frac{1}{8} & \frac{1}{4} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{4} & \frac{1}{8} & \frac{3}{8} & \frac{1}{4} & \frac{3}{8}\end{pmatrix}$

picheck $\begin{pmatrix}6 & 6 & 12 & 12 & 6 & 6 & 12 & 12\end{pmatrix}$

$\pi 1 = \begin{pmatrix}6 & 6 & 12 & 12 & 6 & 6 & 12 & 12\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{9}{32} & \frac{9}{32} & \frac{9}{32} & \frac{9}{32} & \frac{9}{32} & \frac{9}{32} & \frac{9}{32} & \frac{9}{32}\end{pmatrix}$

picheck $\begin{pmatrix}6 & 6 & 12 & 12 & 6 & 6 & 12 & 12\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{9} & 0 & 0 & 0 & \frac{4}{9} & 0 \\ 0 & \frac{1}{3} & \frac{4}{9} & \frac{2}{9} & 0 & 0 & 0 & 0 \\ \frac{1}{9} & \frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{9} & 0 & \frac{2}{3} & 0 & \frac{2}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{9} & \frac{4}{9} \\ 0 & 0 & 0 & \frac{4}{9} & 0 & \frac{1}{3} & 0 & \frac{2}{9} \\ \frac{2}{9} & 0 & 0 & 0 & \frac{1}{9} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & \frac{8}{3} & \frac{56}{9} & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{64}{9} & \frac{16}{3} \\ \frac{8}{3} & 4 & \frac{64}{9} & \frac{56}{9} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{28}{9} & \frac{32}{9} & 8 & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{28}{9} & \frac{16}{3} & 8 & \frac{8}{3} & \frac{32}{9} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & 4 & \frac{8}{3} & \frac{56}{9} & \frac{64}{9} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{64}{9} & \frac{8}{3} & 4 & \frac{16}{3} & \frac{56}{9} \\ \frac{32}{9} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{28}{9} & \frac{8}{3} & 8 & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{32}{9} & \frac{28}{9} & \frac{16}{3} & 8 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 1, 1, -1, -1, 1, 1]$

$\ker N_c = \begin{pmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \end{pmatrix} \quad \begin{pmatrix} -t & -t & t & t & -t & -t & t & t \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & 0 & t & 0 \\ 0 & -t+s & -t & 0 & -t & 0 \\ -t & s & 0 & -t & 0 & -t \\ 0 & 0 & 0 & 0 & s & t \\ 0 & -s+t & -s & 0 & -s & 0 \\ 0 & 0 & t & 0 & s & 0 \\ 0 & 0 & s & t & 0 & 0 \\ t & -s & -s & 0 & -s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & 0 & 0 & t & 0 \\ 0 & t & 0 & 0 & 0 & 0 & s \\ -t & t & t & -t & -t & t & t+s \\ t & 0 & 0 & 0 & 0 & s & 0 \\ 0 & s & 0 & 0 & 0 & 0 & t \\ 0 & 0 & t & 0 & 0 & s & 0 \\ 0 & 0 & s & t & 0 & 0 & 0 \\ 0 & s & 0 & 0 & t & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 2 & 2 & 0 & 0 & 2 & 2 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 8, "vs", 4

$$CNM = \begin{pmatrix} 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{9} & 0 & 0 & 0 & \frac{2}{9} & 0 \\ 0 & 0 & \frac{2}{9} & \frac{1}{9} & 0 & 0 & 0 & 0 \\ \frac{-1}{9} & \frac{-2}{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{9} & 0 & 0 & 0 & \frac{-2}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{9} & \frac{2}{9} \\ 0 & 0 & 0 & \frac{2}{9} & 0 & 0 & 0 & \frac{1}{9} \\ \frac{-2}{9} & 0 & 0 & 0 & \frac{-1}{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-2}{9} & \frac{-1}{9} & 0 & 0 \end{pmatrix}$$

$$\text{Skew Omega} = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 32\Omega$$

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$$

$$T \left(0 \;\; 0 \;\; 0 \;\; \frac{2}{3} \;\; \frac{2}{9} \;\; \frac{1}{9} \;\; 0 \;\; \frac{4}{9} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{2}{9} \;\; 0 \;\; \frac{1}{3} \right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{16}{3}\ \ \frac{8}{3}\ \ \frac{16}{3}\ \ 8\ \ \frac{32}{9}\ \ \frac{28}{9}\ \ \frac{16}{3}\ \ \frac{64}{9}\ \ \frac{8}{3}\ \ \frac{8}{3}\ \ \frac{16}{3}\ \ \frac{56}{9}\ \ \frac{8}{3}\ \ 4\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\frac{23}{6}\ \ \frac{23}{6}\ \ \frac{23}{6}\ \ \frac{13}{2}\ \ \frac{101}{18}\ \ \frac{85}{18}\ \ \frac{25}{6}\ \ \frac{91}{18}\ \ \frac{25}{6}\ \ \frac{25}{6}\ \ \frac{25}{6}\ \ \frac{83}{18}\ \ \frac{25}{6}\ \ \frac{11}{2}\right)$$

$$\boldsymbol{\tau} = 16/1 \text{ , rank } = 4 \text{ , ratio } = 4/1 \text{ , } n^2 / r = 16/1$$

$$\boldsymbol{\tau}' = 48/1 \text{ , } r' = 3/4 \text{ , } \boldsymbol{\tau} / n^2 = 1/4$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 64/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 4/9$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 16\Omega$$

There are, 2, partitions and, 2, ranges, with a group size of, 4

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 16

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 7}, {4, 6}, {5, 8}, {2, 3}}

"PT2" = {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

"RG1" = {3, 4, 7, 8}

"RG2" = {1, 2, 5, 6}

$$M_C = \begin{pmatrix} \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 7.111111111]

Eigenvalues N_C

[0., 0.6666666667, 1.333333333, 0.8888888889, 1.745355992, 0.2546440077, 1.745355992, 0.2546440077]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_C–scaled

[0., 0.7741935484, 1.032258065, 1.548387097, 2.026865024, 0.2957156222, 2.026865024, 0.2957156222]

NullSpace M_C

{[−1, 1, 0, 0, 0, 0, 0, 0], [1, 0, 1, 0, 0, 0, 0, 0], [1, 0, 0, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0, 0, 1] }

NullSpace N_C

{[1, 1, −1, −1, 1, 1, −1, −1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 10.66666667, 5.333333333]

Eigenvalues N_0

[0., 2., 0.6666666667, 1.333333333, 1.745355992, 0.2546440077, 1.745355992, 0.2546440077]

NullSpace M_0

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

NullSpace N_0

{[−1, −1, 1, 1, −1, −1, 1, 1]}

Eigenvalues M

[8., 4., −1.333333333, −2.666666667, −1.333333333, −2.666666667, −1.333333333, −2.666666667]

Eigenvalues N

[0., −0.6666666667, −1.333333333, 6., −0.2546440077, −1.745355992, −0.2546440077, −1.745355992]

NullSpace M

{}

NullSpace N

{[1, 1, −1, −1, 1, 1, −1, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & -3 & 3 & -6 & 1 & 0 \\ 3 & 0 & 1 & 2 & -6 & 3 & 0 & -3 \\ 2 & 1 & 0 & 3 & -3 & 0 & 3 & -6 \\ -3 & 2 & 3 & 0 & 0 & 1 & -6 & 3 \\ 3 & -6 & -3 & 0 & 0 & 3 & 2 & 1 \\ -6 & 3 & 0 & 1 & 3 & 0 & -3 & 2 \\ 1 & 0 & 3 & -6 & 2 & -3 & 0 & 3 \\ 0 & -3 & -6 & 3 & 1 & 2 & 3 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 2 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 3 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 3 \\ 2 & 0 & 0 & 0 & 1 & 0 & 3 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 & 0 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

==

{2, 3, 5, 8}

R: [4, 8, 8, 2, 8, 4, 4, 6]
B: [7, 3, 1, 7, 3, 7, 5, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{225}{1048576} (37 + 3s^2)(4 - s + s^2)(3 + s)(-20 - 3s + 5s^2)(-1 + s)(1 + s)^2$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[2] v[4] v[6] v[8]
"B CYCLES", 1 + v[1] v[3] v[5] v[7]

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of R^*

{[0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B^*

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8])$

degree 2: $\frac{1}{6}(v[1]v[5] + v[2]v[6] + 2v[3]v[7] + 2v[4]v[8])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT3" = {{2, 5, 7, 8}, {1, 3, 4, 6}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$$\pi 2 = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0]$$

supp $\pi 2$ = {4, 11, 17, 22}

$$u2 = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 1, 1, 2, 3, 2, 2, 1, 2, 3, 3, 2, 1, 1, 2, 1]$$

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [1], [1], [1]]
Action of B on ranges, [[2], [4], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 1, 1]

BPARTS [3, 3, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[3, 4, 2, 1, 4, 3]

B–BLOCKS,

[6, 5, 6, 5, 1, 2]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 6, 7, 8}

b_4 = {2, 3, 4, 5}
b_5 = {2, 5, 7, 8}
b_6 = {1, 3, 4, 6}

dim(span of partition vectors), rank(N_0), rank(N): 4, 4, 4

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \\ h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 16, Shape: $8 \oplus 8/3$

$$\text{CLB} = \begin{pmatrix} 0 & 1 & -1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4, 6, 8}}, true

Ω_B in Vec(K)? , {{1, 3, 5, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 6, 7, 8}, {2, 3, 4, 5}}

1, "range", [4, 8], [[8, 4, 4, 4, 4, 8, 8, 8], [4, 8, 8, 8, 8, 4, 4, 4]]

2, "range", [3, 7], [[7, 3, 3, 3, 3, 7, 7, 7], [3, 7, 7, 7, 7, 3, 3, 3]]

3, "range", [2, 6], [[6, 2, 2, 2, 2, 6, 6, 6], [2, 6, 6, 6, 6, 2, 2, 2]]

4, "range", [1, 5], [[5, 1, 1, 1, 1, 5, 5, 5], [1, 5, 5, 5, 5, 1, 1, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

2, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

3, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

3, "partition", {{2, 5, 7, 8}, {1, 3, 4, 6}}

1, "range", [4, 8], [[8, 4, 8, 8, 4, 8, 4, 4], [4, 8, 4, 4, 8, 4, 8, 8]]

2, "range", [3, 7], [[7, 3, 7, 7, 3, 7, 3, 3], [3, 7, 3, 3, 7, 3, 7, 7]]

3, "range", [2, 6], [[6, 2, 6, 6, 2, 6, 2, 2], [2, 6, 2, 2, 6, 2, 6, 6]]

4, "range", [1, 5], [[5, 1, 5, 5, 1, 5, 1, 1], [1, 5, 1, 1, 5, 1, 5, 5]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 11, 17, 22}

$u2 =$

$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 1 & 1 & 2 & 3 & 2 & 2 & 1 & 2 & 3 & 3 & 2 & 1 & 1 & 2 & 1 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \end{pmatrix}$$

idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$

idem−checks N0−checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix}$$

idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & \frac{2}{9} & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & \frac{2}{9} & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{9} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{9} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{9} & 0 & \frac{1}{9} & \frac{1}{18} & \frac{2}{9} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & \frac{32}{9} & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{9} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{9} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{9} & 0 & \frac{16}{9} & \frac{8}{9} & \frac{32}{9} & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, -2, 2, -1, 1, -2, 2]$

$$\ker N_c = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t & 0 & -t & 0 & t & 0 & -t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s & 0 & -s & 0 & s & 0 & -s \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix}-1 & -2 & 1 & 2\end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -t & 0 & 0 & s \\ t & 0 & 0 & -s \\ 0 & 0 & -t & -s \\ -t & s & 0 & 0 \\ t & 0 & 0 & -s \\ -t & 0 & 0 & s \\ 0 & 0 & t & s \\ t & -s & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & t & 0 \\ t & 0 & 0 & -t & t+s \\ t & -t & 0 & 0 & t+s \\ s & 0 & -s & t & s \\ t & 0 & 0 & -t & t+s \\ s & 0 & 0 & t & 0 \\ s & t & 0 & 0 & 0 \\ t & 0 & s & -t & t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix}4 & 0 & 0 & 0 & 4\end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} & 0 & 1 & 1 & \frac{2}{3} \\ 0 & 1 & 1 & \frac{2}{3} & 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 1 & \frac{2}{3} & 1 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & 1 & \frac{2}{3} & 1 & 0 & 0 & \frac{1}{3} \\ 1 & 0 & 0 & \frac{1}{3} & 0 & 1 & 1 & \frac{2}{3} \\ 1 & 0 & 0 & \frac{1}{3} & 0 & 1 & 1 & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} & 1 \\ \frac{2}{3} & \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} & 1 \\ 1 & 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} & 1 \end{pmatrix}, B B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix}$$

Skew Omega =

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \\ \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(\frac{1}{3}\ \frac{2}{9}\ \frac{1}{18}\ \frac{1}{9}\ \frac{2}{9}\ \frac{1}{3}\ \frac{1}{9}\ \frac{1}{18}\ \frac{1}{9}\ \frac{2}{9}\ \frac{1}{6}\ 0\ \frac{2}{9}\ \frac{1}{9}\ 0\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM}\left(\frac{16}{3}\ \frac{32}{9}\ \frac{8}{9}\ \frac{16}{9}\ \frac{32}{9}\ \frac{16}{3}\ \frac{16}{9}\ \frac{8}{9}\ \frac{16}{9}\ \frac{32}{9}\ \frac{8}{3}\ 0\ \frac{32}{9}\ \frac{16}{9}\ 0\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN}\left(\frac{14}{3}\ \frac{26}{9}\ \frac{10}{9}\ \frac{26}{9}\ \frac{26}{9}\ \frac{14}{3}\ \frac{26}{9}\ \frac{10}{9}\ \frac{14}{9}\ \frac{22}{9}\ \frac{10}{3}\ \frac{2}{3}\ \frac{22}{9}\ \frac{14}{9}\ \frac{2}{3}\ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1\text{ , rank} = 2\text{ , ratio} = 16/1\text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1\text{ , } r' = 1/2\text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36\text{ , min } \boldsymbol{\tau} = 80/9\text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5\text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 3, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 16
out of total no. of elements equal to 24

dim span idems 12 vs no. of idems 12

"PT1" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT3" = {{2, 5, 7, 8}, {1, 3, 4, 6}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \\ \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{19}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_c\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\ 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_c\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{19}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{19}{31} & 1 & \frac{7}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & \frac{7}{31} & \frac{7}{31} & \frac{19}{31} & 1 & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{19}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[0., 0., 0., 0., 1.333333333, 2.888888889, 2.276142374, 0.3905242916]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 1.548387097, 3.354838710, 2.643262113, 0.453512081]

NullSpace M_C

{[0, 1, 0, 0, 0, −1, 0, 0], [1, 0, 1, 0, 0, 1, 0, 1], [0, 0, 0, 1, 0, 0, 0, −1], [1, 0, 0, 0, 0, 1, 1, 1], [−1, 0, 0, 0, 1, 0, 0, 0]}

NullSpace N_C

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 1, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 0, 1, 0, 0, 0, 1]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 0., 4., 1.333333333, 2.276142374, 0.3905242916]

NullSpace M_0

{[−1, 0, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1]}

NullSpace N_0

{[0, −1, 0, 1, 0, −1, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, −1, 0, 0], [0, −1, 1, 0, 0, −1, 1, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 4., −1.333333333, −0.3905242916, −2.276142374]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 1, 0, 0], [−1, −1, 1, 0, 0, 0, 1, 0], [−1, −1, 0, 1, 0, 0, 0, 1], [0, −1, 0, 0, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 1 & 1 & 2 & 3 & 2 \\ 1 & 2 & 1 & 0 & 2 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 2 & 2 & 1 & 0 & 1 \\ 2 & 1 & 2 & 3 & 1 & 2 & 1 & 0 \end{pmatrix}$$

==

{2, 3, 6, 7}

R: [4, 8, 8, 2, 3, 7, 5, 3]
B: [7, 3, 1, 7, 8, 4, 4, 6]

TRACE TWO = 1

det AT = $\frac{-1}{16}(1+t)^2(t)^2(-1+t)^2$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{-5}{268435456}$ $(80 - 24s - 13s^2 + s^3) (1 + s)^2 (-1 + s) (-148 + 8s + 7s^2 + 8s^3 + 5s^4) (-36 + 3s^2 + s^4) (80 - 8s + 3s^2 + s^3)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[3] v[8]
"B CYCLES", 1 + v[4] v[7]

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

NullSpace of B^*

{[0, 0, 0, 0, 0, −1, 1, 0], [−1, 0, 0, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} & 1 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{4} & 0 & \frac{3}{4} & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 1 \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{4} \\ \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 1 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{4} & 0 & \frac{3}{4} \\ \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8]$)
degree 2: $\frac{1}{6}$ ($v[1]v[6] + v[2]v[5] + 2\,v[3]v[8] + 2\,v[4]v[7]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 5, 6, 7}, {1, 2, 4, 8}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 2, 3, 7}, {4, 5, 6, 8}}

"PT4" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {3, 8}

"RG2" = {4, 7}

"RG3" = {2, 5}

"RG4" = {1, 6}

$$\pi 2 = [0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0]$$

supp $\pi 2$ = {5, 10, 18, 21}

$$u2 = [1, 2, 1, 3, 4, 3, 2, 1, 2, 4, 3, 2, 3, 3, 3, 2, 1, 4, 2, 3, 4, 1, 1, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[1], [3], [1], [2]]
Action of B on ranges, [[4], [2], [1], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

RPARTS [4, 3, 3, 2]

BPARTS [1, 1, 4, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 1, 6, 5, 1, 8, 4, 2]

B-BLOCKS,

[4, 6, 7, 8, 3, 5, 3, 7]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N - check: true

b_1 = {1, 2, 3, 7}
b_2 = {4, 5, 6, 8}
b_3 = {3, 5, 6, 7}
b_4 = {1, 2, 3, 4}
b_5 = {1, 4, 5, 8}
b_6 = {5, 6, 7, 8}
b_7 = {1, 2, 4, 8}
b_8 = {2, 3, 6, 7}

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & 0 & h[2] \\ 0 & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 \\ 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & 0 \\ h[2] & 0 & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: $23 \oplus 4/3$

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 8}}, true

Ω_B in Vec(K)? , {{4, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 5, 6, 7}, {1, 2, 4, 8}}

1, "range", [3, 8], [[8, 8, 3, 8, 3, 3, 3, 8], [3, 3, 8, 3, 8, 8, 8, 3]]

2, "range", [4, 7], [[7, 7, 4, 7, 4, 4, 4, 7], [4, 4, 7, 4, 7, 7, 7, 4]]

3, "range", [2, 5], [[5, 5, 2, 5, 2, 2, 2, 5], [2, 2, 5, 2, 5, 5, 5, 2]]

4, "range", [1, 6], [[6, 6, 1, 6, 1, 1, 1, 6], [1, 1, 6, 1, 6, 6, 6, 1]]

2, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [3, 8], [[8, 3, 3, 8, 8, 3, 3, 8], [3, 8, 8, 3, 3, 8, 8, 3]]

2, "range", [4, 7], [[7, 4, 4, 7, 7, 4, 4, 7], [4, 7, 7, 4, 4, 7, 7, 4]]

3, "range", [2, 5], [[5, 2, 2, 5, 5, 2, 2, 5], [2, 5, 5, 2, 2, 5, 5, 2]]

4, "range", [1, 6], [[6, 1, 1, 6, 6, 1, 1, 6], [1, 6, 6, 1, 1, 6, 6, 1]]

3, "partition", {{1, 2, 3, 7}, {4, 5, 6, 8}}

1, "range", [3, 8], [[8, 8, 8, 3, 3, 3, 8, 3], [3, 3, 3, 8, 8, 8, 3, 8]]

2, "range", [4, 7], [[7, 7, 7, 4, 4, 4, 7, 4], [4, 4, 4, 7, 7, 7, 4, 7]]

3, "range", [2, 5], [[5, 5, 5, 2, 2, 2, 5, 2], [2, 2, 2, 5, 5, 5, 2, 5]]

4, "range", [1, 6], [[6, 6, 6, 1, 1, 1, 6, 1], [1, 1, 1, 6, 6, 6, 1, 6]]

4, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [3, 8], [[8, 8, 8, 8, 3, 3, 3, 3], [3, 3, 3, 3, 8, 8, 8, 8]]

2, "range", [4, 7], [[7, 7, 7, 7, 4, 4, 4, 4], [4, 4, 4, 4, 7, 7, 7, 7]]

3, "range", [2, 5], [[5, 5, 5, 5, 2, 2, 2, 2], [2, 2, 2, 2, 5, 5, 5, 5]]

4, "range", [1, 6], [[6, 6, 6, 6, 1, 1, 1, 1], [1, 1, 1, 1, 6, 6, 6, 6]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{5, 10, 18, 21}

$u2 =$
$\begin{pmatrix} 1 & 2 & 1 & 3 & 4 & 3 & 2 & 1 & 2 & 4 & 3 & 2 & 3 & 3 & 3 & 2 & 1 & 4 & 2 & 3 & 4 & 1 & 1 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{8} & \frac{1}{6} & \frac{1}{4} & \frac{1}{24} & 0 & \frac{1}{12} & \frac{1}{6} \\ \frac{1}{8} & \frac{1}{6} & \frac{1}{4} & \frac{1}{6} & 0 & \frac{1}{24} & \frac{1}{6} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{8} & \frac{1}{3} & \frac{1}{12} & \frac{1}{24} & \frac{1}{12} & \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{12} & \frac{1}{12} & \frac{1}{3} & \frac{1}{12} & \frac{1}{24} & 0 & \frac{1}{4} \\ \frac{1}{24} & 0 & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{8} & \frac{1}{6} & \frac{1}{4} \\ 0 & \frac{1}{24} & \frac{1}{6} & \frac{1}{12} & \frac{1}{8} & \frac{1}{6} & \frac{1}{4} & \frac{1}{6} \\ \frac{1}{24} & \frac{1}{12} & \frac{1}{4} & 0 & \frac{1}{12} & \frac{1}{8} & \frac{1}{3} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{24} & 0 & \frac{1}{4} & \frac{1}{8} & \frac{1}{12} & \frac{1}{12} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 2 & \frac{8}{3} & 4 & \frac{2}{3} & 0 & \frac{4}{3} & \frac{8}{3} \\ 2 & \frac{8}{3} & 4 & \frac{8}{3} & 0 & \frac{2}{3} & \frac{8}{3} & \frac{4}{3} \\ \frac{4}{3} & 2 & \frac{16}{3} & \frac{4}{3} & \frac{2}{3} & \frac{4}{3} & 4 & 0 \\ 2 & \frac{4}{3} & \frac{4}{3} & \frac{16}{3} & \frac{4}{3} & \frac{2}{3} & 0 & 4 \\ \frac{2}{3} & 0 & \frac{4}{3} & \frac{8}{3} & \frac{8}{3} & 2 & \frac{8}{3} & 4 \\ 0 & \frac{2}{3} & \frac{8}{3} & \frac{4}{3} & 2 & \frac{8}{3} & 4 & \frac{8}{3} \\ \frac{2}{3} & \frac{4}{3} & 4 & 0 & \frac{4}{3} & 2 & \frac{16}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{2}{3} & 0 & 4 & 2 & \frac{4}{3} & \frac{4}{3} & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\boldsymbol{\pi\Delta} = [-1, 1, 1, -1, 1, -1, -1, 1]$

$$\ker N_c = \begin{pmatrix} 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & -1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & -t-s & s+t & 0 & 0 & s+t & -t-s \\ t & 0 & -t & 0 & 0 & t & 0 & -t \\ 0 & s & -t-s & t & s & 0 & t & -t-s \end{pmatrix} \quad \text{RB}$$

checks

$\boldsymbol{\pi\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s-t & 0 & 0 \\ s-t & 0 & 0 & 0 \\ s & 0 & 0 & -t \\ 0 & -t & -s & 0 \\ -s+t & 0 & 0 & 0 \\ 0 & -s+t & 0 & 0 \\ 0 & t & s & 0 \\ -s & 0 & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 1 & 0 & 0 & -1 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} t & s & 0 & 0 & 0 \\ s & s & 0 & -s+t & 0 \\ t+s & t+s & 0 & -s & -t \\ t+s & s & -s & 0 & 0 \\ t & t & 0 & s-t & 0 \\ s & t & 0 & 0 & 0 \\ 0 & t & s & 0 & 0 \\ 0 & 0 & 0 & s & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & \frac{3}{4} & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{3}{4} & 1 & 1 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{3}{4} & 1 & 1 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 1 & \frac{3}{4} & \frac{1}{2} & 0 & \frac{3}{4} \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{4} & 1 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{3}{4} & \frac{3}{4} & 0 & \frac{1}{4} & \frac{1}{2} & 1 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{4} & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & \frac{3}{4} & \frac{1}{4} & 1 & \frac{1}{4} & 0 & 0 & \frac{3}{4} \\ \frac{3}{4} & 1 & \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & \frac{3}{4} & 0 \\ 1 & \frac{3}{4} & \frac{1}{4} & 1 & \frac{1}{4} & 0 & 0 & \frac{3}{4} \\ \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{3}{4} & \frac{3}{4} & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{4} & 1 & 1 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{4} & 1 & 1 & \frac{1}{4} \\ \frac{3}{4} & \frac{1}{2} & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{8} & 0 & 0 & \frac{1}{24} & \frac{1}{12} \\ 0 & 0 & \frac{1}{8} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{24} \\ \frac{-1}{12} & \frac{-1}{8} & 0 & 0 & \frac{-1}{24} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{8} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{24} & 0 & 0 \\ 0 & 0 & \frac{1}{24} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{8} \\ 0 & 0 & \frac{1}{12} & \frac{1}{24} & 0 & 0 & \frac{1}{8} & \frac{1}{12} \\ \frac{-1}{24} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{8} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{24} & 0 & 0 & \frac{-1}{8} & \frac{-1}{12} & 0 & 0 \end{pmatrix} \text{ Skew Omega =}$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 0 \\ \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{4} & 0 & \frac{3}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} & 1 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & 0 & \frac{3}{4} & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(\frac{1}{3}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{8}\ \frac{1}{12}\ \frac{1}{3}\ \frac{1}{8}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{4}\ \frac{1}{6}\ \frac{1}{8}\ \frac{1}{6}\ \frac{1}{12}\ 0\ \frac{1}{24}\ \frac{1}{4}\ \frac{1}{6}\ \frac{1}{8}\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM}\left(\frac{16}{3}\ \frac{4}{3}\ \frac{4}{3}\ 2\ \frac{4}{3}\ \frac{16}{3}\ 2\ \frac{4}{3}\ \frac{8}{3}\ 4\ \frac{8}{3}\ 2\ \frac{8}{3}\ \frac{4}{3}\ 0\ \frac{2}{3}\ 4\ \frac{8}{3}\ 2\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN}\left(\frac{14}{3}\ \frac{2}{3}\ 2\ \frac{10}{3}\ \frac{2}{3}\ \frac{14}{3}\ \frac{10}{3}\ 2\ 2\ \frac{8}{3}\ \frac{10}{3}\ \frac{8}{3}\ 2\ \frac{4}{3}\ \frac{2}{3}\ \frac{4}{3}\ \frac{8}{3}\ 2\ \frac{8}{3}\ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 4, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 32

dim span idems 16 vs no. of idems 16

"PT1" = {{3, 5, 6, 7}, {1, 2, 4, 8}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 2, 3, 7}, {4, 5, 6, 8}}

"PT4" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {3, 8}

"RG2" = {4, 7}

"RG3" = {2, 5}

"RG4" = {1, 6}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix}
\frac{31}{36} & \frac{11}{18} & \frac{13}{36} & \frac{11}{18} & \frac{1}{9} & \frac{-5}{36} & \frac{1}{9} & \frac{13}{36} \\
\frac{11}{18} & \frac{31}{36} & \frac{11}{18} & \frac{13}{36} & \frac{-5}{36} & \frac{1}{9} & \frac{13}{36} & \frac{1}{9} \\
\frac{13}{36} & \frac{11}{18} & \frac{31}{36} & \frac{1}{9} & \frac{1}{9} & \frac{13}{36} & \frac{11}{18} & \frac{-5}{36} \\
\frac{11}{18} & \frac{13}{36} & \frac{1}{9} & \frac{31}{36} & \frac{13}{36} & \frac{1}{9} & \frac{-5}{36} & \frac{11}{18} \\
\frac{1}{9} & \frac{-5}{36} & \frac{1}{9} & \frac{13}{36} & \frac{31}{36} & \frac{11}{18} & \frac{13}{36} & \frac{11}{18} \\
\frac{-5}{36} & \frac{1}{9} & \frac{13}{36} & \frac{1}{9} & \frac{11}{18} & \frac{31}{36} & \frac{11}{18} & \frac{13}{36} \\
\frac{1}{9} & \frac{13}{36} & \frac{11}{18} & \frac{-5}{36} & \frac{13}{36} & \frac{11}{18} & \frac{31}{36} & \frac{1}{9} \\
\frac{13}{36} & \frac{1}{9} & \frac{-5}{36} & \frac{11}{18} & \frac{11}{18} & \frac{13}{36} & \frac{1}{9} & \frac{31}{36}
\end{pmatrix}$$

$$M_c\text{-scaled} = \begin{pmatrix}
1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\
\frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\
\frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\
\frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\
\frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\
1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\
\frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\
\frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1
\end{pmatrix} \quad N_c\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{22}{31} & \frac{13}{31} & \frac{22}{31} & \frac{4}{31} & \frac{-5}{31} & \frac{4}{31} & \frac{13}{31} \\ \frac{22}{31} & 1 & \frac{22}{31} & \frac{13}{31} & \frac{-5}{31} & \frac{4}{31} & \frac{13}{31} & \frac{4}{31} \\ \frac{13}{31} & \frac{22}{31} & 1 & \frac{4}{31} & \frac{4}{31} & \frac{13}{31} & \frac{22}{31} & \frac{-5}{31} \\ \frac{22}{31} & \frac{13}{31} & \frac{4}{31} & 1 & \frac{13}{31} & \frac{4}{31} & \frac{-5}{31} & \frac{22}{31} \\ \frac{4}{31} & \frac{-5}{31} & \frac{4}{31} & \frac{13}{31} & 1 & \frac{22}{31} & \frac{13}{31} & \frac{22}{31} \\ \frac{-5}{31} & \frac{4}{31} & \frac{13}{31} & \frac{4}{31} & \frac{22}{31} & 1 & \frac{22}{31} & \frac{13}{31} \\ \frac{4}{31} & \frac{13}{31} & \frac{22}{31} & \frac{-5}{31} & \frac{13}{31} & \frac{22}{31} & 1 & \frac{4}{31} \\ \frac{13}{31} & \frac{4}{31} & \frac{-5}{31} & \frac{22}{31} & \frac{22}{31} & \frac{13}{31} & \frac{4}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[2.888888889, 1.707106781, 0.2928932190, 1.707106781, 0.2928932190, 0., 0., 0.]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C−scaled

[3.354838710, 1.982446585, 0.3401340612, 1.982446585, 0.3401340612, 0., 0., 0.]

NullSpace M_C

{[0, 0, 0, −1, 0, 0, 1, 0], [1, 1, 0, 1, 0, 0, 0, 1], [1, 1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace N_C

{[1, −1, 0, 0, −1, 1, 0, 0], [0, −1, 1, 0, −1, 0, 0, 1], [0, −1, 0, 1, −1, 0, 1, 0]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[4., 1.707106781, 0.2928932190, 1.707106781, 0.2928932190, 0., 0., 0.]

NullSpace M_0

{[0, −1, 0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, −1, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, 0, 0, −1, 0, 0, 1, 0]}

NullSpace N_0

{[1, −1, 0, 0, −1, 1, 0, 0], [0, −1, 1, 0, −1, 0, 0, 1], [0, −1, 0, 1, −1, 0, 1, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[4., −0.2928932190, −1.707106781, −0.2928932190, −1.707106781, 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[−1, 1, 0, 0, 1, −1, 0, 0], [−1, 0, 0, 1, 0, −1, 1, 0], [−1, 0, 1, 0, 0, −1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 3 & 4 & 3 & 2 \\ 1 & 0 & 1 & 2 & 4 & 3 & 2 & 3 \\ 2 & 1 & 0 & 3 & 3 & 2 & 1 & 4 \\ 1 & 2 & 3 & 0 & 2 & 3 & 4 & 1 \\ 3 & 4 & 3 & 2 & 0 & 1 & 2 & 1 \\ 4 & 3 & 2 & 3 & 1 & 0 & 1 & 2 \\ 3 & 2 & 1 & 4 & 2 & 1 & 0 & 3 \\ 2 & 3 & 4 & 1 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 4, 5, 7}

R: [4, 8, 1, 7, 8, 4, 5, 3]
B: [7, 3, 8, 2, 3, 7, 4, 6]

TRACE TWO = 2

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{-3087}{1048576} (-20 - 5s + 7s^2) (37 + 4s + 7s^2) (-1 + s) ^2 (1 + s) ^2 (20$

$-3s+7s^2$)

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", 1 + v[1] v[3] v[4] v[5] v[7] v[8]
"B CYCLES", 1 + v[2] v[3] v[4] v[6] v[7] v[8]

Eigenvalues

R: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

B: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[-1, 0, 0, 0, 0, 1, 0, 0], [0, -1, 0, 0, 1, 0, 0, 0]}

NullSpace of B^*

{[-1, 0, 0, 0, 0, 1, 0, 0], [0, -1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 5 & 5 & 6 & 0 & 5 & 5 \\ 0 & 0 & 5 & 5 & 0 & 6 & 5 & 5 \\ 5 & 5 & 0 & 10 & 5 & 5 & 12 & 10 \\ 5 & 5 & 10 & 0 & 5 & 5 & 10 & 12 \\ 6 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 0 & 6 & 5 & 5 & 0 & 0 & 5 & 5 \\ 5 & 5 & 12 & 10 & 5 & 5 & 0 & 10 \\ 5 & 5 & 10 & 12 & 5 & 5 & 10 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 8

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$)

degree 2: $\frac{1}{24}$ ($v[1]v[3] + v[1]v[4] + 4v[1]v[5] + v[1]v[7] + v[1]v[8] + v[2]v[3] + v[2]v[4] + 4v[2]v[6] + v[2]v[7] + v[2]v[8] + 2v[3]v[4] + v[3]v[5] + v[3]v[6] + 8v[3]v[7] + 2v[3]v[8] + v[4]v[5] + v[4]v[6] + 2v[4]v[7] + 8v[4]v[8] + v[5]v[7] + v[5]v[8] + v[6]v[7] + v[6]v[8] + 2v[7]v[8]$)

degree 3 : $\frac{1}{16}$ ($3v[1]v[3]v[8] + 3v[1]v[4]v[7] + 3v[3]v[5]v[8] + 4v[4]v[6]v[8] + 4v[2]v[6]v[8] + 3v[3]v[4]v[6] + 4v[1]v[4]v[8] + 3v[3]v[6]v[8] + 8v[3]v[4]v[7] + 4v[2]v[4]v[8] + 4v[2]v[6]v[7] + 3v[2]v[3]v[4] + 4v[1]v[5]v[8] + 3v[4]v[6]v[7] + 3v[2]v[3]v[8] + 4v[2]v[3]v[7] + 4v[1]v[4]v[5] + 4v[1]v[5]v[7] + 3v[2]v[4]v[7] + 4v[1]v[3]v[7] + 4v[2]v[4]v[6] + 3v[2]v[7]v[8] + 4v[4]v[5]v[8] + 4v[3]v[6]v[7] + 4v[1]v[3]v[5] + 3v[5]v[7]v[8] + 3v[1]v[3]v[4] + 3v[4]v[5]v[7] + 3v[1]v[7]v[8] + 8v[3]v[4]v[8] + 3v[6]v[7]v[8] + 4v[3]v[5]v[7] + 8v[3]v[7]v[8] + 4v[2]v[3]v[6] + 3v[3]v[4]v[5] + 8v[4]v[7]v[8]$)

degree 4 : $\frac{1}{24}$ ($v[2]v[3]v[4]v[8] + v[1]v[4]v[5]v[7] + v[1]v[3]v[4]v[5] + v[3]v[4]v[5]v[8] + v[3]v[4]v[6]v[8] + v[4]v[6]v[7]v[8] + v[2]v[4]v[6]v[7] + 4v[1]v[3]v[5]v[7] + v[2]v[3]v[4]v[7] + v[1]v[3]v[4]v[7] + v[2]v[6]v[7]v[8] + v[2]v[3]v[7]v[8] + v[3]v[6]v[7]v[8] + v[1]v[3]v[7]v[8] + v[3]v[4]v[6]v[7] + v[3]v[5]v[7]v[8] + v[4]v[5]v[7]v[8] + v[1]v[5]v[7]v[8] + v[2]v[3]v[6]v[8] + v[1]v[3]v[4]v[8] + v[1]v[4]v[7]v[8] + 8v[3]v[4]v[7]v[8] + v[2]v[4]v[7]v[8] + v[3]v[4]v[5]v[7] + 4v[2]v[4]v[6]v[8] + v[2]v[3]v[4]v[6] + 4v[1]v[4]v[5]v[8] + 4v[2]v[3]v[6]v[7] + v[1]v[3]v[5]v[8]$)

degree 5 : $\frac{1}{12}$ ($v[1]v[3]v[4]v[5]v[7] + v[1]v[3]v[4]v[5]v[8] + v[1]v[3]v[4]v[7]v[8] + v[1]v[3]v[5]v[7]v[8] + v[1]v[4]v[5]v[7]v[8] + v[2]v[3]v[4]v[6]v[7] + v[2]v[3]v[4]v[6]v[8] + v[2]v[3]v[4]v[7]v[8] + v[2]v[3]v[6]v[7]v[8] + v[2]v[4]v[6]v[7]v[8] + v[3]v[4]v[5]v[7]v[8] + v[3]v[4]v[6]v[7]v[8]$)

degree 6 : $\frac{1}{2}$ ($v[4]$) ($v[3]$) ($v[1]v[5] + v[2]v[6]$) ($v[7]$) ($v[8]$)

Group spectrum $1 + t + 2t^2 + 3t^3 + 2t^4 + t^5 + t^6$

KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {4}, {3}, {7}, {8}}

"RG1" = {2, 3, 4, 6, 7, 8}

"RG2" = {1, 3, 4, 5, 7, 8}

$\pi 6$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0]

supp $\pi 6$ = {18, 25}

$u6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1]

supp u6 = {6, 18, 25, 28}

Action of R on ranges, [[2], [2]]
Action of B on ranges, [[1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[4, 5, 1, 6, 3, 2]

B–BLOCKS,

[6, 3, 5, 2, 1, 4]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

b_1 = {1, 6}
b_2 = {2, 5}
b_3 = {4}

$b_4 = \{3\}$
$b_5 = \{7\}$
$b_6 = \{8\}$

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \\ h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 16, Shape: $8 \oplus 8/3$

$$\text{CLB} = \begin{pmatrix} 0 & -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3, 4, 5, 7, 8}}, true

Ω_B in Vec(K)? , {{2, 3, 4, 6, 7, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {4}, {3}, {7}, {8}}

1, "range", [2, 3, 4, 6, 7, 8], [[8, 4, 6, 7, 4, 8, 2, 3], [8, 4, 6, 3, 4, 8, 2, 7], [8, 4, 2, 7, 4, 8, 6, 3], [8, 4, 2, 3, 4, 8, 6, 7], [7, 3, 8, 6, 3, 7, 4, 2], [7, 3, 8, 2, 3, 7, 4, 6], [7, 3, 4, 6, 3, 7, 8, 2], [7, 3, 4, 2, 3, 7, 8, 6], [6, 2, 7, 8, 2, 6, 3, 4], [6, 2, 7, 4, 2, 6, 3, 8], [6, 2, 3, 8, 2, 6, 7, 4], [6, 2, 3, 4, 2, 6, 7, 8], [4, 8, 6, 7, 8, 4, 2, 3], [4, 8, 6, 3, 8, 4, 2, 7], [4, 8, 2, 7, 8, 4, 6, 3], [4, 8, 2, 3, 8, 4, 6, 7], [3, 7, 8, 6, 7, 3, 4, 2], [3, 7, 8, 2, 7, 3, 4, 6], [3, 7, 4, 6, 7, 3, 8, 2], [3, 7, 4, 2, 7, 3, 8, 6], [2, 6, 7, 8, 6, 2, 3, 4], [2, 6, 7, 4, 6, 2, 3, 8], [2, 6, 3, 8, 6, 2, 7, 4], [2, 6, 3, 4, 6, 2, 7, 8]]

2, "range", [1, 3, 4, 5, 7, 8], [[8, 4, 5, 7, 4, 8, 1, 3], [8, 4, 5, 3, 4, 8, 1, 7], [8, 4, 1, 7, 4, 8, 5, 3], [8, 4, 1, 3, 4, 8, 5, 7], [7, 3, 8, 5, 3, 7, 4, 1], [7, 3, 8, 1, 3, 7, 4, 5], [7, 3, 4, 5, 3, 7, 8, 1], [7, 3, 4, 1, 3, 7, 8, 5], [5, 1, 7, 8, 1, 5, 3, 4], [5, 1, 7, 4, 1, 5, 3, 8], [5, 1, 3, 8, 1, 5, 7, 4], [5, 1, 3, 4, 1, 5, 7, 8], [4, 8, 5, 7, 8, 4, 1, 3], [4, 8, 5, 3, 8, 4, 1, 7], [4, 8, 1, 7, 8, 4, 5, 3], [4, 8, 1, 3, 8, 4, 5, 7], [3, 7, 8, 5, 7, 3, 4, 1], [3, 7, 8, 1, 7, 3, 4, 5], [3, 7, 4, 5, 7, 3, 8, 1], [3, 7, 4, 1, 7, 3, 8, 5], [1, 5, 7, 8, 5, 1, 3, 4], [1, 5, 7, 4, 5, 1, 3, 8], [1, 5, 3, 8, 5, 1, 7, 4], [1, 5, 3, 4, 5, 1, 7, 8]]

"group has", 24, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

g_1 = [[1, 3, 5], [2, 4, 6]]

g_2 = [[1, 3, 2, 4, 6, 5]]

g_3 = [[1, 3, 5, 4, 6, 2]]

g_4 = [[1, 3, 2], [4, 6, 5]]

g_5 = [[1, 2, 6], [3, 4, 5]]

linear dimension, 12

"Symmetric?", false

Is Z in Vec(K)? true
$\big(-2h[3] \;\; 2h[3] \;\; 2h[3] \;\; -2h[2] \;\; 2h[2] \;\; 2h[2] \;\; -4h[1] \;\; 4h[1] \;\; 4h[1] \;\; 2h[3] \;\; 2h[2] \;\; 4h[4]$

"Basis for Z(G)"

1, "coeff", 4

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 2

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

3, "coeff", 2

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

4, "coeff", 4

$$Z[\ 4\ ] = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true

1, 4, true
2, 3, true
2, 4, true
3, 4, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 0 & 0 & 0 & 2. & -1. + 1.732050808 I & -1. - 1.732050808 I \\ 0 & 0 & 0 & 2. & -1. + 1.732050808 I & -1. - 1.732050808 I \\ 1. & -1. & 1. & -1. & 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 4 & 4 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 7 & 7 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 3 & 7 \\ 2 & 0 & 0 & 2 & 2 & 0 & 0 & 2 & 4 & 8 & 8 & 20 & 0 & 2 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 4 & 4 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 7 & 7 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 3 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 4 & 4 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 3t^3 + 2t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 3t^2 + 7t^3 + 13t^4 + 23t^5 + 41t^6 + 63t^7 + 98t^8 + 146t^9 + 210t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}, {2, [1, 2, 3, 4, 5, 7]}, {3, [1, 2, 3, 4, 5, 8]}, {4, [1, 2, 3, 4, 6, 7]}, {5, [1, 2, 3, 4, 6, 8]}, {6, [1, 2, 3, 4, 7, 8]}, {7, [1, 2, 3, 5, 6, 7]}, {8, [1, 2, 3, 5, 6, 8]}, {9, [1, 2, 3, 5, 7, 8]}, {10, [1, 2, 3, 6, 7, 8]}, {11, [1, 2, 4, 5, 6, 7]}, {12, [1, 2, 4, 5, 6, 8]}, {13, [1, 2, 4, 5, 7, 8]}, {14, [1, 2, 4, 6, 7, 8]}, {15, [1, 2, 5, 6, 7, 8]}, {16, [1, 3, 4, 5, 6, 7]}, {17, [1, 3, 4, 5, 6, 8]}, {18, [1, 3, 4, 5, 7, 8]}, {19, [1, 3, 4, 6, 7, 8]}, {20, [1, 3, 5, 6, 7, 8]}, {21, [1, 4, 5, 6, 7, 8]}, {22, [2, 3, 4, 5, 6, 7]}, {23, [2, 3, 4, 5, 6, 8]}, {24, [2, 3, 4, 5, 7, 8]}, {25, [2, 3, 4, 6, 7, 8]}, {26, [2, 3, 5, 6, 7, 8]}, {27, [2, 4, 5, 6, 7, 8]}, {28, [3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 6 =$
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

{18, 25}

$u6$ =
(0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 1)

{6, 18, 25, 28}

picheck (1 1 2 2 1 1 2 2)

$\pi = \left(\frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$

$\pi 5$ =
(0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 0 0 1 0

$u5$ =
$\Big(0\ \ 0\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{6}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{6}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ 0\ \ 0\ \ \frac{1}{6}\ \ 0\ \ 0\ \ \frac{1}{6}$

picheck (5 5 10 10 5 5 10 10)

$\pi 4$ =
(0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 0 2 2 0 2 2 0 0 2 0 2 2 0 0

$u4$ =
$\Big(\frac{1}{18}\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ \frac{1}{18}\ \ \frac{1}{18}\ \ 0\ \ 0\ \ \frac{1}{18}$

picheck (20 20 40 40 20 20 40 40)

$\pi 3$ =
(0 0 0 0 0 0 6 6 0 6 6 6 0 6 6 0 6 6 0 0 6 6 0 6 6 6 0 6 6 6

$u3$ =
$\Big(\frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0\ \ 0\ \ \frac{1}{36}\ \ \frac{1}{36}\ \ 0$

picheck (60 60 120 120 60 60 120 120)

$\pi 2$ =
(0 24 24 24 0 24 24 24 24 0 24 24 24 48 24 24 48 48 24 24 48

$u2$ =
$\Big(\frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ 0\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ 0\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}\ \ \frac{1}{54}$

picheck $\begin{pmatrix}120 & 120 & 240 & 240 & 120 & 120 & 240 & 240\end{pmatrix}$

$\pi 1 = \begin{pmatrix}120 & 120 & 240 & 240 & 120 & 120 & 240 & 240\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324}\end{pmatrix}$

picheck $\begin{pmatrix}120 & 120 & 240 & 240 & 120 & 120 & 240 & 240\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} \frac{13}{3} & \frac{10}{3} & 7 & 7 & \frac{10}{3} & \frac{13}{3} & 7 & 7 \\ \frac{10}{3} & \frac{13}{3} & 7 & 7 & \frac{13}{3} & \frac{10}{3} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & \frac{26}{3} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{20}{3} & 7 \\ \frac{7}{2} & \frac{7}{2} & 7 & \frac{26}{3} & \frac{7}{2} & \frac{7}{2} & 7 & \frac{20}{3} \\ \frac{10}{3} & \frac{13}{3} & 7 & 7 & \frac{13}{3} & \frac{10}{3} & 7 & 7 \\ \frac{13}{3} & \frac{10}{3} & 7 & 7 & \frac{10}{3} & \frac{13}{3} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & \frac{20}{3} & 7 & \frac{7}{2} & \frac{7}{2} & \frac{26}{3} & 7 \\ \frac{7}{2} & \frac{7}{2} & 7 & \frac{20}{3} & \frac{7}{2} & \frac{7}{2} & 7 & \frac{26}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, 0, 0, 1, -1, 0, 0]$

$$\ker N_c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -s & 0 & 0 & -t \\ s & 0 & 0 & t \\ t & s & 0 & 0 \\ 0 & 0 & t & -s \\ s & 0 & 0 & t \\ -s & 0 & 0 & -t \\ -t & -s & 0 & 0 \\ 0 & 0 & -t & s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & t & 0 \\ t & 0 & 0 & -t & t+s \\ s & -s & 0 & 0 & t+s \\ t & 0 & -t & s & t \\ t & 0 & 0 & -t & t+s \\ s & 0 & 0 & t & 0 \\ t & s & 0 & 0 & 0 \\ s & 0 & t & -s & s \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 0 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$CNM = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \end{pmatrix} \quad \text{Skew } T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew}$$

$$Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 0 & \frac{5}{6} & \frac{5}{6} & 1 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 1 & \frac{5}{6} & \frac{5}{6} & 0 & 1 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & 2 & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 2 & \frac{5}{3} \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & 2 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & 2 \\ 1 & 0 & \frac{5}{6} & \frac{5}{6} & 1 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 1 & \frac{5}{6} & \frac{5}{6} & 0 & 1 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & 2 & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 2 & \frac{5}{3} \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & 2 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", false

$$\Omega \left(\frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$$

$$T \left(0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{2} \right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{7}{2}\ \ \frac{26}{3}\ \ 7\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ 7\ \ \frac{26}{3}\ \ \frac{7}{2}\ \ \frac{7}{2}\ \ 7\ \ 7\ \ \frac{13}{3}\ \ \frac{10}{3}\ \ 7\ \ 7\ \ \frac{10}{3}\ \ \frac{13}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(3\ \ \frac{26}{3}\ \ 6\ \ 3\ \ 3\ \ 3\ \ 6\ \ \frac{26}{3}\ \ 3\ \ 3\ \ 6\ \ 6\ \ \frac{13}{3}\ \ \frac{10}{3}\ \ 6\ \ 6\ \ \frac{10}{3}\ \ \frac{13}{3}\right)$$

$\boldsymbol{\tau}$ = 12/1 , rank = 6 , ratio = 2/1 , n^2 / r = 32/3

$\boldsymbol{\tau}'$ = 52/1 , r' = 5/6 , $\boldsymbol{\tau} / n^2$ = 3/16

p^2 = 5/36 , min $\boldsymbol{\tau}$ = 80/9 , $\boldsymbol{\tau}$–check is positive? 28/9

max r = 36/5 , r–check is positive? 1/6

IS N0M0 a combination of T and Omega? , false

$$N_0 M_0 = \frac{14}{51} T + \frac{176}{17} \Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 24

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 48

dim span idems 2 vs no. of idems 2

"PT1" = {{1, 6}, {2, 5}, {4}, {3}, {7}, {8}}

"RG1" = {2, 3, 4, 6, 7, 8}

"RG2" = {1, 3, 4, 5, 7, 8}

$$M_c = \begin{pmatrix} \frac{5}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{5}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{2}{9} \\ \frac{5}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{5}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{2}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} & 1 & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-4}{5} & 1 & \frac{-1}{10} & \frac{-1}{10} & \frac{-4}{5} & 1 & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\ 1 & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} & 1 & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-4}{5} & 1 & \frac{-1}{10} & \frac{-1}{10} & \frac{-4}{5} & 1 & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{2}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{2}{9} \end{pmatrix} \quad M_c N_c =$$

$$\begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 2., 0.6666666667, 0.4444444444]

Eigenvalues N_C

[2., 1.691868003, 0.1970208860, 0., 0., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 1.400000000, 3.600000000]

Eigenvalues N_C–scaled

[2.322580645, 1.964749939, 0.2287984489, 0., 0., 1.161290323, 1.161290323, 1.161290323]

NullSpace M_C

{[0, 0, 0, 1, 0, 0, 0, −1], [0, 0, 0, 0, 1, 1, 1, 1], [0, 0, 1, 0, 0, 0, −1, 0], [0, 1, 0, 0, 0, −1, 0, 0], [1, 0, 0, 0, 0, 1, 1, 1]}

NullSpace N_C

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

Eigenvalues M_0

[0., 0., 0., 0., 2., 0.6666666667, 8.935416159, 0.397917175]

Eigenvalues N_0

[0., 0., 2., 2., 1., 1., 1., 1.]

NullSpace M_0

{[0, 0, −1, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [−1, 0, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 1, 0, 0]}

NullSpace N_0

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues M

[1., −1.333333333, 7.142286814, −0.808953480, −1., −2., −1., −2.]

Eigenvalues N

[−2., 6.531128874, −1.531128874, 0., 0., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0&1&0&0&1&0&0&0\\ 1&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 1&0&0&0&0&1&0&0\\ 0&1&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0 \end{pmatrix} \quad \begin{pmatrix} 0&0&1&1&0&0&1&1\\ 0&0&1&1&0&0&1&1\\ 1&1&0&1&1&1&0&1\\ 1&1&1&0&1&1&1&0\\ 0&0&1&1&0&0&1&1\\ 0&0&1&1&0&0&1&1\\ 1&1&0&1&1&1&0&1\\ 1&1&1&0&1&1&1&0 \end{pmatrix}$$

Commutator(s)

1, 2 :   commutator = $\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 & -1 & -1 & 0 & 0 \\ -1 & -1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 & -1 & -1 & 0 & 0 \\ -1 & -1 & 0 & 0 & -1 & -1 & 0 & 0 \end{pmatrix}$

==

{2, 4, 6, 8}

R: [4, 8, 1, 7, 3, 7, 4, 6]
B: [7, 3, 8, 2, 8, 4, 5, 3]

TRACE TWO = 1

det AT = $\frac{-1}{16}(t)^2(-1+t)^2(1+t)^2$

AT = $\begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{-5}{268435456}(-148+8s+7s^2+8s^3+5s^4)(80-8s+3s^2+s^3)(-1+s)(1+s)^2(80-24s-13s^2+s^3)(-36+3s^2+s^4)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[4] v[7]
"B CYCLES", 1 + v[3] v[8]

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of R^*

{[0, 0, 0, −1, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B^*

{[0, −1, 0, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} & 1 & \frac{1}{4} & \frac{1}{2} \\ \frac{3}{4} & 0 & \frac{3}{4} & \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 \\ \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{3}{4} \\ \frac{1}{4} & 1 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} \\ 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & 1 & \frac{3}{4} & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8]$)

degree 2: $\frac{1}{6}$ ($v[1]v[6] + v[2]v[5] + 2\,v[3]v[8] + 2\,v[4]v[7]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT2" = {{2, 3, 4, 6}, {1, 5, 7, 8}}

"PT3" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"PT4" = {{1, 3, 4, 5}, {2, 6, 7, 8}}

"RG1" = {3, 8}

"RG2" = {4, 7}

"RG3" = {2, 5}

"RG4" = {1, 6}

$\pi 2$ = [0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {5, 10, 18, 21}

$u2$ = [3, 2, 3, 1, 4, 1, 2, 3, 2, 4, 1, 2, 1, 1, 1, 2, 3, 4, 2, 1, 4, 3, 3, 2, 3, 3, 2, 1]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[4], [2], [1], [2]]
Action of B on ranges, [[1], [3], [1], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 2, 1, 3]

BPARTS [4, 3, 1, 4]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[8, 6, 7, 1, 8, 5, 4, 1]

B-BLOCKS,

[7, 3, 2, 2, 3, 4, 5, 6]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

b_1 = {2, 3, 4, 6}
b_2 = {1, 3, 4, 5}
b_3 = {2, 6, 7, 8}
b_4 = {1, 2, 7, 8}
b_5 = {3, 4, 5, 6}
b_6 = {1, 3, 5, 7}
b_7 = {2, 4, 6, 8}
b_8 = {1, 5, 7, 8}

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & 0 & h[1] \\ 0 & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 \\ 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & 0 \\ h[1] & 0 & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: $23 \oplus 4/3$

$$\text{CLB} = \begin{pmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{4, 7}}, true

Ω_B in Vec(K)? , {{3, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 7, 8}, {3, 4, 5, 6}}

1, "range", [3, 8], [[8, 8, 3, 3, 3, 3, 8, 8], [3, 3, 8, 8, 8, 8, 3, 3]]

2, "range", [4, 7], [[7, 7, 4, 4, 4, 4, 7, 7], [4, 4, 7, 7, 7, 7, 4, 4]]

3, "range", [2, 5], [[5, 5, 2, 2, 2, 2, 5, 5], [2, 2, 5, 5, 5, 5, 2, 2]]

4, "range", [1, 6], [[6, 6, 1, 1, 1, 1, 6, 6], [1, 1, 6, 6, 6, 6, 1, 1]]

2, "partition", {{2, 3, 4, 6}, {1, 5, 7, 8}}

1, "range", [3, 8], [[8, 3, 3, 3, 8, 3, 8, 8], [3, 8, 8, 8, 3, 8, 3, 3]]

2, "range", [4, 7], [[7, 4, 4, 4, 7, 4, 7, 7], [4, 7, 7, 7, 4, 7, 4, 4]]

3, "range", [2, 5], [[5, 2, 2, 2, 5, 2, 5, 5], [2, 5, 5, 5, 2, 5, 2, 2]]

4, "range", [1, 6], [[6, 1, 1, 1, 6, 1, 6, 6], [1, 6, 6, 6, 1, 6, 1, 1]]

3, "partition", {{1, 3, 5, 7}, {2, 4, 6, 8}}

1, "range", [3, 8], [[8, 3, 8, 3, 8, 3, 8, 3], [3, 8, 3, 8, 3, 8, 3, 8]]

2, "range", [4, 7], [[7, 4, 7, 4, 7, 4, 7, 4], [4, 7, 4, 7, 4, 7, 4, 7]]

3, "range", [2, 5], [[5, 2, 5, 2, 5, 2, 5, 2], [2, 5, 2, 5, 2, 5, 2, 5]]

4, "range", [1, 6], [[6, 1, 6, 1, 6, 1, 6, 1], [1, 6, 1, 6, 1, 6, 1, 6]]

4, "partition", {{1, 3, 4, 5}, {2, 6, 7, 8}}

1, "range", [3, 8], [[8, 3, 8, 8, 8, 3, 3, 3], [3, 8, 3, 3, 3, 8, 8, 8]]

2, "range", [4, 7], [[7, 4, 7, 7, 7, 4, 4, 4], [4, 7, 4, 4, 4, 7, 7, 7]]

3, "range", [2, 5], [[5, 2, 5, 5, 5, 2, 2, 2], [2, 5, 2, 2, 2, 5, 5, 5]]

4, "range", [1, 6], [[6, 1, 6, 6, 6, 1, 1, 1], [1, 6, 1, 1, 1, 6, 6, 6]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{5, 10, 18, 21}

$u2 =$
$\begin{pmatrix} 3 & 2 & 3 & 1 & 4 & 1 & 2 & 3 & 2 & 4 & 1 & 2 & 1 & 1 & 1 & 2 & 3 & 4 & 2 & 1 & 4 & 3 & 3 & 2 & 3 & 3 & 2 & 1 \end{pmatrix}$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\ \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$ idem-checks

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{24} & \frac{1}{6} & \frac{1}{12} & \frac{1}{8} & 0 & \frac{1}{4} & \frac{1}{6} \\ \frac{1}{24} & \frac{1}{6} & \frac{1}{12} & \frac{1}{6} & 0 & \frac{1}{8} & \frac{1}{6} & \frac{1}{4} \\ \frac{1}{12} & \frac{1}{24} & \frac{1}{3} & \frac{1}{4} & \frac{1}{8} & \frac{1}{12} & \frac{1}{12} & 0 \\ \frac{1}{24} & \frac{1}{12} & \frac{1}{4} & \frac{1}{3} & \frac{1}{12} & \frac{1}{8} & 0 & \frac{1}{12} \\ \frac{1}{8} & 0 & \frac{1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{1}{24} & \frac{1}{6} & \frac{1}{12} \\ 0 & \frac{1}{8} & \frac{1}{6} & \frac{1}{4} & \frac{1}{24} & \frac{1}{6} & \frac{1}{12} & \frac{1}{6} \\ \frac{1}{8} & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{12} & \frac{1}{8} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{1}{12} & \frac{1}{4} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & \frac{2}{3} & \frac{8}{3} & \frac{4}{3} & 2 & 0 & 4 & \frac{8}{3} \\ \frac{2}{3} & \frac{8}{3} & \frac{4}{3} & \frac{8}{3} & 0 & 2 & \frac{8}{3} & 4 \\ \frac{4}{3} & \frac{2}{3} & \frac{16}{3} & 4 & 2 & \frac{4}{3} & \frac{4}{3} & 0 \\ \frac{2}{3} & \frac{4}{3} & 4 & \frac{16}{3} & \frac{4}{3} & 2 & 0 & \frac{4}{3} \\ 2 & 0 & 4 & \frac{8}{3} & \frac{8}{3} & \frac{2}{3} & \frac{8}{3} & \frac{4}{3} \\ 0 & 2 & \frac{8}{3} & 4 & \frac{2}{3} & \frac{8}{3} & \frac{4}{3} & \frac{8}{3} \\ 2 & \frac{4}{3} & \frac{4}{3} & 0 & \frac{4}{3} & \frac{2}{3} & \frac{16}{3} & 4 \\ \frac{4}{3} & 2 & 0 & \frac{4}{3} & \frac{2}{3} & \frac{4}{3} & 4 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, -1, 1, -1, 1, 1, -1]$

$$\ker N_c = \begin{pmatrix} 0 & -1 & 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & -t-s & s & t & 0 & s & -t-s \\ s & 0 & -s & 0 & 0 & s & 0 & -s \\ 0 & 0 & -t-s & s+t & 0 & 0 & s+t & -t-s \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & s-t & 0 & 0 \\ t-s & 0 & 0 & 0 \\ -t & 0 & 0 & -s \\ 0 & -s & -t & 0 \\ s-t & 0 & 0 & 0 \\ 0 & t-s & 0 & 0 \\ 0 & s & t & 0 \\ t & 0 & 0 & s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & t & 0 \\ t & 0 & 0 & t & s-t \\ s & 0 & -s & s & t \\ t & -t & 0 & s+t & 0 \\ s & 0 & 0 & s & t-s \\ t & 0 & 0 & s & 0 \\ s & t & 0 & 0 & 0 \\ t & 0 & s & t & -t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = \begin{pmatrix} 4 & 0 & 0 & 4 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{3}{4} & 0 & 1 & \frac{3}{4} \\ \frac{1}{4} & 1 & \frac{1}{2} & \frac{3}{4} & 0 & \frac{3}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & 1 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & \frac{3}{4} & \frac{3}{4} & 1 & \frac{1}{4} & 1 & 0 & \frac{1}{4} \\ \frac{3}{4} & 0 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{4} & \frac{3}{4} & \frac{1}{2} \\ 0 & \frac{3}{4} & \frac{3}{4} & 1 & \frac{1}{4} & 1 & 0 & \frac{1}{4} \\ 1 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{3}{4} & 0 & 1 & \frac{3}{4} \\ \frac{3}{4} & \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & \frac{1}{4} & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & 1 & 0 & \frac{1}{4} & 0 & \frac{3}{4} & \frac{3}{4} & 1 \\ \frac{3}{4} & 0 & 1 & \frac{3}{4} & 1 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{3}{4} & 0 & 1 & \frac{3}{4} & 1 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{3}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{3}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} & 1 & \frac{3}{4} \\ \frac{1}{4} & 1 & 0 & \frac{1}{4} & 0 & \frac{3}{4} & \frac{3}{4} & 1 \end{pmatrix}, \; B B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{24} & 0 & 0 & \frac{1}{8} & \frac{1}{12} \\ 0 & 0 & \frac{1}{24} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{8} \\ \frac{-1}{12} & \frac{-1}{24} & 0 & 0 & \frac{-1}{8} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{24} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{8} & 0 & 0 \\ 0 & 0 & \frac{1}{8} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{24} \\ 0 & 0 & \frac{1}{12} & \frac{1}{8} & 0 & 0 & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{8} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{24} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{8} & 0 & 0 & \frac{-1}{24} & \frac{-1}{12} & 0 & 0 \end{pmatrix} \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & 1 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{4} & 1 & \frac{3}{4} & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{4} \\ \frac{3}{4} & 0 & \frac{3}{4} & \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} & 1 & \frac{1}{4} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} & 1 & \frac{3}{4} \\ \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(\frac{1}{4}\ \frac{1}{6}\ \frac{1}{8}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{4}\ \frac{1}{12}\ \frac{1}{8}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{24}\ \frac{1}{6}\ \frac{1}{4}\ 0\ \frac{1}{8}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{24}\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM}\left(4\ \frac{8}{3}\ 2\ \frac{4}{3}\ \frac{8}{3}\ 4\ \frac{4}{3}\ 2\ \frac{8}{3}\ \frac{4}{3}\ \frac{8}{3}\ \frac{2}{3}\ \frac{8}{3}\ 4\ 0\ 2\ \frac{4}{3}\ \frac{8}{3}\ \frac{2}{3}\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN}\left(4\ \frac{8}{3}\ \frac{8}{3}\ \frac{4}{3}\ \frac{8}{3}\ 4\ \frac{4}{3}\ \frac{8}{3}\ 2\ \frac{4}{3}\ \frac{10}{3}\ \frac{4}{3}\ 2\ \frac{8}{3}\ \frac{2}{3}\ \frac{8}{3}\ \frac{4}{3}\ 2\ \frac{4}{3}\ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1\ ,\ \mathrm{rank} = 2\ ,\ \mathrm{ratio} = 16/1\ ,\ n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1\ ,\ r' = 1/2\ ,\ \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36\ ,\ \min\ \boldsymbol{\tau} = 80/9\ ,\ \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\max\ \mathrm{r} = 36/5\ ,\ r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 4, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 32

dim span idems 16 vs no. of idems 16

"PT1" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT2" = {{2, 3, 4, 6}, {1, 5, 7, 8}}

"PT3" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"PT4" = {{1, 3, 4, 5}, {2, 6, 7, 8}}

"RG1" = {3, 8}

"RG2" = {4, 7}

"RG3" = {2, 5}

"RG4" = {1, 6}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{1}{9} & \frac{13}{36} & \frac{1}{9} & \frac{11}{18} & \frac{-5}{36} & \frac{11}{18} & \frac{13}{36} \\ \frac{1}{9} & \frac{31}{36} & \frac{1}{9} & \frac{13}{36} & \frac{-5}{36} & \frac{11}{18} & \frac{13}{36} & \frac{11}{18} \\ \frac{13}{36} & \frac{1}{9} & \frac{31}{36} & \frac{11}{18} & \frac{11}{18} & \frac{13}{36} & \frac{1}{9} & \frac{-5}{36} \\ \frac{1}{9} & \frac{13}{36} & \frac{11}{18} & \frac{31}{36} & \frac{13}{36} & \frac{11}{18} & \frac{-5}{36} & \frac{1}{9} \\ \frac{11}{18} & \frac{-5}{36} & \frac{11}{18} & \frac{13}{36} & \frac{31}{36} & \frac{1}{9} & \frac{13}{36} & \frac{1}{9} \\ \frac{-5}{36} & \frac{11}{18} & \frac{13}{36} & \frac{11}{18} & \frac{1}{9} & \frac{31}{36} & \frac{1}{9} & \frac{13}{36} \\ \frac{11}{18} & \frac{13}{36} & \frac{1}{9} & \frac{-5}{36} & \frac{13}{36} & \frac{1}{9} & \frac{31}{36} & \frac{11}{18} \\ \frac{13}{36} & \frac{11}{18} & \frac{-5}{36} & \frac{1}{9} & \frac{1}{9} & \frac{13}{36} & \frac{11}{18} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{4}{31} & \frac{13}{31} & \frac{4}{31} & \frac{22}{31} & \frac{-5}{31} & \frac{22}{31} & \frac{13}{31} \\ \frac{4}{31} & 1 & \frac{4}{31} & \frac{13}{31} & \frac{-5}{31} & \frac{22}{31} & \frac{13}{31} & \frac{22}{31} \\ \frac{13}{31} & \frac{4}{31} & 1 & \frac{22}{31} & \frac{22}{31} & \frac{13}{31} & \frac{4}{31} & \frac{-5}{31} \\ \frac{4}{31} & \frac{13}{31} & \frac{22}{31} & 1 & \frac{13}{31} & \frac{22}{31} & \frac{-5}{31} & \frac{4}{31} \\ \frac{22}{31} & \frac{-5}{31} & \frac{22}{31} & \frac{13}{31} & 1 & \frac{4}{31} & \frac{13}{31} & \frac{4}{31} \\ \frac{-5}{31} & \frac{22}{31} & \frac{13}{31} & \frac{22}{31} & \frac{4}{31} & 1 & \frac{4}{31} & \frac{13}{31} \\ \frac{22}{31} & \frac{13}{31} & \frac{4}{31} & \frac{-5}{31} & \frac{13}{31} & \frac{4}{31} & 1 & \frac{22}{31} \\ \frac{13}{31} & \frac{22}{31} & \frac{-5}{31} & \frac{4}{31} & \frac{4}{31} & \frac{13}{31} & \frac{22}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[2.888888889, 1.707106781, 0.2928932190, 1.707106781, 0.2928932190, 0., 0., 0.]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C−scaled

[3.354838710, 1.982446585, 0.3401340612, 1.982446585, 0.3401340612, 0., 0., 0.]

NullSpace M_C

{[1, 1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [1, 1, 1, 0, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

NullSpace N_C

{[1, 0, 0, −1, 0, 1, −1, 0], [0, 1, 0, −1, 1, 0, −1, 0], [0, 0, 1, −1, 0, 0, −1, 1]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[4., 1.707106781, 0.2928932190, 1.707106781, 0.2928932190, 0., 0., 0.]

NullSpace M_0

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0, −1, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

NullSpace N_0

{[1, −1, 0, 0, −1, 1, 0, 0], [0, −1, 1, 0, −1, 0, 0, 1], [0, −1, 0, 1, −1, 0, 1, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[4., −0.2928932190, −1.707106781, −0.2928932190, −1.707106781, 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[0, −1, 1, 0, −1, 0, 0, 1], [0, −1, 0, 1, −1, 0, 1, 0], [1, −1, 0, 0, −1, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 3 & 1 & 4 & 1 & 2 \\ 3 & 0 & 3 & 2 & 4 & 1 & 2 & 1 \\ 2 & 3 & 0 & 1 & 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 0 & 2 & 1 & 4 & 3 \\ 1 & 4 & 1 & 2 & 0 & 3 & 2 & 3 \\ 4 & 1 & 2 & 1 & 3 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 & 2 & 3 & 0 & 1 \\ 2 & 1 & 4 & 3 & 3 & 2 & 1 & 0 \end{pmatrix}$$

==

80, [1, −1, 1, −1, 1, 1, −1, −1]

==

{2, 5, 7, 8}

R: [4, 8, 1, 2, 8, 4, 5, 6]
B: [7, 3, 8, 7, 3, 7, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{-1}{262144}$ $(-370 + 46s + 99s^2 - 16s^3 + s^4) (30 + 6s + 23s^2 - 12s^3 + s^4) (-1 + s) (-3 + s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[2] v[4] v[6] v[8]
"B CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$)
degree 2: $\frac{1}{6}$ ($v[1]v[5] + v[2]v[6] + 2v[3]v[7] + 2v[4]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {4, 8}

"RG2" = {2, 6}

"RG3" = {3, 7}

"RG4" = {1, 5}

$\pi 2$ = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {4, 11, 17, 22}

$u2$ = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[2], [1], [4], [1]]
Action of B on ranges, [[3], [3], [1], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[4, 3, 1, 2]

B-BLOCKS,

[4, 3, 3, 4]

with invariant measure, [1, 1, 2, 2]

N by blocks, N − check: true

b_1 = {1, 3, 6, 8}
b_2 = {2, 4, 5, 7}
b_3 = {1, 4, 6, 7}
b_4 = {2, 3, 5, 8}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \\ h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: $3 \oplus 10/8$

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 1 & 1 & -1 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4, 6, 8}}, true

Ω_B in Vec(K)? , {{3, 8}, {4, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [4, 8], [[8, 4, 8, 4, 4, 8, 4, 8], [4, 8, 4, 8, 8, 4, 8, 4]]

2, "range", [2, 6], [[6, 2, 6, 2, 2, 6, 2, 6], [2, 6, 2, 6, 6, 2, 6, 2]]

3, "range", [3, 7], [[7, 3, 7, 3, 3, 7, 3, 7], [3, 7, 3, 7, 7, 3, 7, 3]]

4, "range", [1, 5], [[5, 1, 5, 1, 1, 5, 1, 5], [1, 5, 1, 5, 5, 1, 5, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

2, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

3, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\,2\,] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 11, 17, 22}

$u2 =$
$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem–checks   N0–checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix}$$

idem–checks   N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, -2, 0, 1, 1, -2, 0]$

$$\ker N_C = \begin{pmatrix} -1 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} s & 0 & -t & -s+t & s & 0 & -t & -s+t \\ -s & 0 & t & 0 & 0 & s & 0 & -t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s & s & -t & -s & 0 & 0 & 0 & -s+t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -2 & 0 & 1 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & t & -s \\ 0 & 0 & -t & s \\ -s & 0 & 0 & t \\ 0 & -s & t & 0 \\ 0 & 0 & -t & s \\ 0 & 0 & t & -s \\ s & 0 & 0 & -t \\ 0 & s & -t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -t & s+t & 0 & 0 & t \\ t & 0 & 0 & 0 & s \\ 0 & s & -s & 0 & s+t \\ -t & s+t & 0 & -s & s+t \\ t & 0 & 0 & 0 & s \\ -t & s+t & 0 & 0 & t \\ 0 & t & s & 0 & 0 \\ t & 0 & 0 & s & 0 \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \text{ Skew T =}$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \text{ Skew Omega =}$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \\ \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12}\right)$$

$$T \left(0 \;\; \frac{1}{3} \;\; \frac{1}{9} \;\; \frac{1}{18} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{2}{9} \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(0\ \ \frac{16}{3}\ \ \frac{16}{9}\ \ \frac{8}{9}\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{32}{9}\ \ \frac{16}{9}\ \ 0\ \ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\frac{-2}{3}\ \ \frac{14}{3}\ \ \frac{26}{9}\ \ \frac{10}{9}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{22}{9}\ \ \frac{14}{9}\ \ \frac{2}{3}\ \ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {4, 8}

"RG2" = {2, 6}

"RG3" = {3, 7}

"RG4" = {1, 5}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \\ \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\ 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C−scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[−1, 0, 0, 0, 1, 0, 0, 0], [1, 0, 1, 1, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [1, 1, 1, 1, 0, 0, 0, 0]}

NullSpace N_C

{[0, 0, −1, 0, 0, 0, 0, 1], [−1, −1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 1, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[0, −1, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [−1, 0, 0, 0, 1, 0, 0,

0]}

NullSpace N_0

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 0, 1, 0, 0, 0, 1], [−1, −1, 1, 1, 0, 0, 0, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{}

NullSpace N

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 0, 0, −1, −1, 1, 1], [0, 0, 1, 0, −1, −1, 1, 0], [0, 0, 0, 1, 0, 0, −1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 6, 7, 8}

R: [4, 8, 1, 2, 3, 7, 5, 6]
B: [7, 3, 8, 7, 8, 4, 4, 3]

TRACE TWO = 2

det AT = $\frac{-1}{16} (t)^2 (1 + t)^4$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 8

Level 2 det = $\frac{-5}{268435456} (444 - 24s + 235s^2 + 54s^3 + 68s^4 + 18s^5 + 5s^6)(16 + s^2)^2 (5 + s)^2 (-1 + s)^2 (-12 - 4s + s^2 + s^3)$

RANK of R is 8

R ranking is 2, "vs", 8

RBAR ranking 2, "vs", 8

RANK of B is 4

B ranking is 1, "vs", 4

BBAR ranking 1, "vs", 4

"R CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6] v[7] v[8]
"B CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])

Eigenvalues

R: [−1. I, 1. I, −1., 1., −0.7071067810 − 0.7071067810 I, 0.7071067810 + 0.7071067810 I, −0.7071067810 + 0.7071067810 I, 0.7071067810 − 0.7071067810 I]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{}

NullSpace of B^*

{[0, 0, −1, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 8 \\ 0 & 0 & 8 & 0 & 0 & 0 & 8 & 8 \\ 4 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\ 4 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 8 & 0 & 0 & 0 & 8 \\ 0 & 0 & 8 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & \frac{2}{3} & 1 & 1 & 1 & \frac{1}{3} \\ \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} & 1 & 1 & 1 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 1 & 1 & 1 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & \frac{1}{3} & 1 & 1 & 1 & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 7, "RANK of M is ", 8

"RANK of the KERNEL is ", 4

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 4

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8] \right)$

degree 2: $\frac{1}{12}\left(v[1]v[2] + v[1]v[5] + 2v[1]v[6] + 2v[2]v[5] + v[2]v[6] + 2v[3]v[4] + 2v[3]v[7] + 4v[3]v[8] + 4v[4]v[7] + 2v[4]v[8] + v[5]v[6] + 2v[7]v[8] \right)$

degree 3 : $\frac{1}{12}\left(v[1]v[2]v[5] + v[1]v[2]v[6] + v[1]v[5]v[6] + v[2]v[5]v[6] + 2v[3]v[4]v[7] + 2v[3]v[4]v[8] + 2v[3]v[7]v[8] + 2v[4]v[7]v[8] \right)$

degree 4 : $\frac{1}{3}\left(v[1]v[2]v[5]v[6] + 2v[3]v[4]v[7]v[8] \right)$

Group spectrum $1 + t + 2t^2 + t^3 + t^4$

KERNEL STRUCTURE

"PT1" = {{2, 8}, {6, 7}, {1, 4}, {3, 5}}

"PT2" = {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

"RG1" = {3, 4, 7, 8}

"RG2" = {1, 2, 5, 6}

$\pi 4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 4$ = {10, 61}

$u4$ = [0, 0, 2, 2, 0, 0, 0, 0, 0, 3, 2, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 1, 0, 1, 1, 0, 1, 0, 2, 2, 0, 0, 2, 2, 0, 1, 0, 1, 1, 0, 1, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 2, 3, 0, 0, 0, 0, 0, 2, 2, 0, 0]

supp u4 = {3, 4, 10, 11, 12, 13, 15, 24, 25, 26, 28, 29, 31, 33, 34, 37, 38, 40, 42, 43, 45, 46, 47, 56, 58, 59, 60, 61, 67, 68}

Action of R on ranges, [[2], [1]]
Action of B on ranges, [[1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [1, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 6, 1, 3, 8, 5, 4, 7]

B–BLOCKS,

[8, 4, 8, 6, 1, 4, 6, 1]

with invariant measure, [2, 1, 1, 2, 1, 2, 1, 2]

N by blocks, N – check: true

$b_1 = \{2, 8\}$
$b_2 = \{2, 4\}$
$b_3 = \{6, 8\}$
$b_4 = \{6, 7\}$
$b_5 = \{1, 3\}$
$b_6 = \{1, 4\}$
$b_7 = \{5, 7\}$
$b_8 = \{3, 5\}$

dim(span of partition vectors), rank(N_0), rank(N):   7, 7, 7

$$\text{Centralizer} = \begin{pmatrix} h[3] & h[2] & 0 & 0 & h[4] & h[1] & 0 & 0 \\ h[4] & h[3] & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & h[3] & h[2] & 0 & 0 & h[4] & h[1] \\ 0 & 0 & h[4] & h[3] & 0 & 0 & h[1] & h[2] \\ h[2] & h[1] & 0 & 0 & h[3] & h[4] & 0 & 0 \\ h[1] & h[4] & 0 & 0 & h[2] & h[3] & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & h[3] & h[4] \\ 0 & 0 & h[1] & h[4] & 0 & 0 & h[2] & h[3] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 12, Shape: $9 \oplus\ 3/1$

$$\text{CLB} = \begin{pmatrix} 1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4, 5, 6, 7, 8}}, true

Ω_B in Vec(K)? , {{3, 8}, {4, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{2, 8}, {6, 7}, {1, 4}, {3, 5}}

1, "range", [3, 4, 7, 8], [[8, 7, 4, 8, 4, 3, 3, 7], [7, 3, 8, 7, 8, 4, 4, 3], [4, 8, 3, 4, 3, 7, 7, 8], [3, 4, 7, 3, 7, 8, 8, 4]]

2, "range", [1, 2, 5, 6], [[6, 5, 2, 6, 2, 1, 1, 5], [5, 1, 6, 5, 6, 2, 2, 1], [2, 6, 1, 2, 1, 5, 5, 6], [1, 2, 5, 1, 5, 6, 6, 2]]

2, "partition", {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

1, "range", [3, 4, 7, 8], [[8, 7, 8, 7, 4, 3, 4, 3], [7, 3, 7, 3, 8, 4, 8, 4], [4, 8, 4, 8, 3, 7, 3, 7], [3, 4, 3, 4, 7, 8, 7, 8]]

2, "range", [1, 2, 5, 6], [[6, 5, 6, 5, 2, 1, 2, 1], [5, 1, 5, 1, 6, 2, 6, 2], [2, 6, 2, 6, 1, 5, 1, 5], [1, 2, 1, 2, 5, 6, 5, 6]]

"group has", 4, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2, 4, 3]]

$g_2 =$ [[1, 4], [2, 3]]

$g_3 =$ []

$g_4 =$ [[1, 3, 4, 2]]

linear dimension, 4

"Symmetric?", false

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[4] & h[1] & h[3] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

4, "coeff", 1

$$Z[\ 4\ ] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
2, 3, true
2, 4, true
3, 4, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. \\ -1. & 1. & 1.\,I & -1.\,I \\ -1. & 1. & 1.\,I & -1.\,I \\ 1. & -1. & 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 4 & 0 \\ 0 & 2 & 6 & 0 \\ 0 & 0 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + t^3 + t^4$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 14t^5 + 22t^6 + 30t^7 + 43t^8 + 55t^9 + 73t^{10}$

n-choose-rank

{1, [1, 2, 3, 4]}, {2, [1, 2, 3, 5]}, {3, [1, 2, 3, 6]}, {4, [1, 2, 3, 7]}, {5, [1, 2, 3, 8]}, {6, [1, 2, 4, 5]}, {7, [1, 2, 4, 6]}, {8, [1, 2, 4, 7]}, {9, [1, 2, 4, 8]}, {10, [1, 2, 5, 6]}, {11, [1, 2, 5, 7]}, {12, [1, 2, 5, 8]}, {13, [1, 2, 6, 7]}, {14, [1, 2, 6, 8]}, {15, [1, 2, 7, 8]}, {16, [1, 3, 4, 5]}, {17, [1, 3, 4, 6]}, {18, [1, 3, 4, 7]}, {19, [1, 3, 4, 8]}, {20, [1, 3, 5, 6]}, {21, [1, 3, 5, 7]}, {22, [1, 3, 5, 8]}, {23, [1, 3, 6, 7]}, {24, [1, 3, 6, 8]}, {25, [1, 3, 7, 8]}, {26, [1, 4, 5, 6]}, {27, [1, 4, 5, 7]}, {28, [1, 4, 5, 8]}, {29, [1, 4, 6, 7]}, {30, [1, 4, 6, 8]}, {31, [1, 4, 7, 8]}, {32, [1, 5, 6, 7]}, {33, [1, 5, 6, 8]}, {34, [1, 5, 7, 8]}, {35, [1, 6, 7, 8]}, {36, [2, 3, 4, 5]}, {37, [2, 3, 4, 6]}, {38, [2, 3, 4, 7]}, {39, [2, 3, 4, 8]}, {40, [2, 3, 5, 6]}, {41, [2, 3, 5, 7]}, {42, [2, 3, 5, 8]}, {43, [2, 3, 6, 7]}, {44, [2, 3, 6, 8]}, {45, [2, 3, 7, 8]}, {46, [2, 4, 5, 6]}, {47, [2, 4, 5, 7]}, {48, [2, 4, 5, 8]}, {49, [2, 4, 6, 7]}, {50, [2, 4, 6, 8]}, {51, [2, 4, 7, 8]}, {52, [2, 5, 6, 7]}, {53, [2, 5, 6, 8]}, {54, [2, 5, 7, 8]}, {55, [2, 6, 7, 8]}, {56, [3, 4, 5, 6]}, {57, [3, 4, 5, 7]}, {58, [3, 4, 5, 8]}, {59, [3, 4, 6, 7]}, {60, [3, 4, 6, 8]}, {61, [3, 4, 7, 8]}, {62, [3, 5, 6, 7]}, {63, [3, 5, 6, 8]}, {64, [3, 5, 7, 8]}, {65, [3, 6, 7, 8]}, {66, [4, 5, 6, 7]}, {67, [4, 5, 6, 8]}, {68, [4, 5, 7, 8]}, {69, [4, 6, 7, 8]}, {70, [5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 4$ =
(0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

{10, 61}

$u4$ =
(0 0 2 2 0 0 0 0 0 3 2 1 1 0 1 0 0 0 0 0 0 0 0 2 2 1 0 1 1 0

{3, 4, 10, 11, 12, 13, 15, 24, 25, 26, 28, 29, 31, 33, 34, 37, 38, 40, 42, 43, 45, 46, 47, 56, 58, 59, 60, 61, 67, 68} picheck $\left(1 \quad 1 \quad 2 \quad 2 \quad 1 \quad 1 \quad 2 \quad 2\right)$

$\pi = \left(\frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6}\right)$

$\pi 3 =$
$(0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$u3 =$
$\left(\frac{1}{2} \quad 0 \quad \frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4} \quad \frac{1}{4} \quad 0 \quad 0 \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{3}{4} \quad \frac{1}{2} \quad \frac{3}{4} \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{4} \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{3}{4} \quad \frac{3}{4} \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{2}\right.$

picheck $\left(3 \quad 3 \quad 6 \quad 6 \quad 3 \quad 3 \quad 6 \quad 6\right)$

$\pi 2 =$
$\left(2 \quad 0 \quad 0 \quad 2 \quad 2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 2 \quad 0 \quad 0 \quad 4 \quad 0 \quad 0 \quad 4 \quad 4 \quad 0 \quad 0 \quad 4 \quad 4 \quad 2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 4\right)$

$u2 =$
$\left(\frac{3}{8} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{4} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{8} \quad \frac{3}{8} \quad \frac{1}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{4} \quad \frac{3}{8} \quad \frac{1}{8} \quad \frac{1}{4} \quad \frac{3}{8}\right)$

picheck $\left(6 \quad 6 \quad 12 \quad 12 \quad 6 \quad 6 \quad 12 \quad 12\right)$

$\pi 1 = \left(6 \quad 6 \quad 12 \quad 12 \quad 6 \quad 6 \quad 12 \quad 12\right)$

$u1 = \left(\frac{9}{32} \quad \frac{9}{32} \quad \frac{9}{32} \quad \frac{9}{32} \quad \frac{9}{32} \quad \frac{9}{32} \quad \frac{9}{32} \quad \frac{9}{32}\right)$

picheck $\left(6 \quad 6 \quad 12 \quad 12 \quad 6 \quad 6 \quad 12 \quad 12\right)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \end{pmatrix}$$ idem-checks

$$PP_2 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{9} & \frac{4}{9} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{9} & 0 & 0 & 0 & \frac{4}{9} \\ \frac{1}{9} & 0 & \frac{2}{3} & 0 & \frac{2}{9} & 0 & 0 & 0 \\ \frac{2}{9} & \frac{1}{9} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{9} & 0 & \frac{1}{3} & 0 & \frac{2}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{4}{9} & \frac{2}{9} \\ 0 & 0 & 0 & 0 & \frac{1}{9} & \frac{2}{9} & \frac{2}{3} & 0 \\ 0 & \frac{2}{9} & 0 & 0 & 0 & \frac{1}{9} & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & \frac{8}{3} & \frac{56}{9} & \frac{64}{9} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & 4 & \frac{16}{3} & \frac{56}{9} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{64}{9} \\ \frac{28}{9} & \frac{8}{3} & 8 & \frac{16}{3} & \frac{32}{9} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{32}{9} & \frac{28}{9} & \frac{16}{3} & 8 & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{64}{9} & \frac{16}{3} & 4 & \frac{8}{3} & \frac{56}{9} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & 4 & \frac{64}{9} & \frac{56}{9} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{28}{9} & \frac{32}{9} & 8 & \frac{16}{3} \\ \frac{8}{3} & \frac{32}{9} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & \frac{28}{9} & \frac{16}{3} & 8 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, -1, -1, 1, 1, -1, -1]$

$\ker N_c = \begin{pmatrix} -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} s & s & -s & -s & s & s & -s & -s \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} 0 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -t & 0 & 0 & -t & s-t & 0 \\ s & 0 & 0 & t & 0 & 0 \\ t & -s & -s & 0 & 0 & -s \\ -t & 0 & s & -t & -t & 0 \\ t & 0 & 0 & s & 0 & 0 \\ -s & 0 & 0 & -s & t-s & 0 \\ 0 & s & 0 & 0 & t & 0 \\ 0 & 0 & 0 & t & 0 & s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} -1 & -1 & 1 & -1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & t & 0 & 0 & 0 & s \\ 0 & 0 & 0 & 0 & t & s & 0 \\ -s & -s & s & -s & s & s+t & s \\ s & 0 & t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s & t & 0 \\ 0 & 0 & s & 0 & 0 & 0 & t \\ 0 & 0 & 0 & s & 0 & 0 & t \\ 0 & s & 0 & 0 & t & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 2 & 0 & 2 & 2 & 2 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&1\\ 0&0&1&0&1&0&0&0\\ 1&0&0&1&0&0&0&0\\ 0&0&1&0&1&0&0&0\\ 0&0&0&0&0&1&1&0\\ 0&0&0&0&0&1&1&0\\ 0&1&0&0&0&0&0&1 \end{pmatrix}, BB^* = \begin{pmatrix} 1&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&1\\ 0&0&1&0&1&0&0&0\\ 1&0&0&1&0&0&0&0\\ 0&0&1&0&1&0&0&0\\ 0&0&0&0&0&1&1&0\\ 0&0&0&0&0&1&1&0\\ 0&1&0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 4, "vs", 4

$$CNM = \begin{pmatrix} 0&0&\frac{4}{3}&\frac{4}{3}&0&0&\frac{4}{3}&\frac{4}{3}\\ 0&0&\frac{4}{3}&\frac{4}{3}&0&0&\frac{4}{3}&\frac{4}{3}\\ \frac{-4}{3}&\frac{-4}{3}&0&0&\frac{-4}{3}&\frac{-4}{3}&0&0\\ \frac{-4}{3}&\frac{-4}{3}&0&0&\frac{-4}{3}&\frac{-4}{3}&0&0\\ 0&0&\frac{4}{3}&\frac{4}{3}&0&0&\frac{4}{3}&\frac{4}{3}\\ 0&0&\frac{4}{3}&\frac{4}{3}&0&0&\frac{4}{3}&\frac{4}{3}\\ \frac{-4}{3}&\frac{-4}{3}&0&0&\frac{-4}{3}&\frac{-4}{3}&0&0\\ \frac{-4}{3}&\frac{-4}{3}&0&0&\frac{-4}{3}&\frac{-4}{3}&0&0 \end{pmatrix} \quad Skew\ T = \begin{pmatrix} 0&0&\frac{1}{9}&\frac{2}{9}&0&0&0&0\\ 0&0&0&\frac{1}{9}&0&0&0&\frac{2}{9}\\ \frac{-1}{9}&0&0&0&\frac{-2}{9}&0&0&0\\ \frac{-2}{9}&\frac{-1}{9}&0&0&0&0&0&0\\ 0&0&\frac{2}{9}&0&0&0&\frac{1}{9}&0\\ 0&0&0&0&0&0&\frac{2}{9}&\frac{1}{9}\\ 0&0&0&0&\frac{-1}{9}&\frac{-2}{9}&0&0\\ 0&\frac{-2}{9}&0&0&0&\frac{-1}{9}&0&0 \end{pmatrix}$$

$$\text{Skew Omega} = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 32\Omega$$

$$\Omega \left(\frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{12} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{12} \ \ \frac{1}{6} \ \ \frac{1}{6} \ \ \frac{1}{12} \ \ \frac{1}{12} \right)$$

$$T \left(0 \ \ \frac{2}{9} \ \ 0 \ \ \frac{2}{3} \ \ 0 \ \ \frac{1}{9} \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ \frac{4}{9} \ \ \frac{2}{9} \ \ 0 \ \ \frac{1}{3} \right)$$

"IS NM in Vec(K)?", true

$NM\left(\frac{16}{3} \quad \frac{32}{9} \quad \frac{16}{3} \quad 8 \quad \frac{8}{3} \quad \frac{28}{9} \quad \frac{16}{3} \quad \frac{16}{3} \quad \frac{8}{3} \quad \frac{8}{3} \quad \frac{64}{9} \quad \frac{56}{9} \quad \frac{8}{3} \quad 4\right)$

"IS MN in Vec(K)?", false

$MN\left(\frac{23}{6} \quad \frac{101}{18} \quad \frac{23}{6} \quad \frac{13}{2} \quad \frac{23}{6} \quad \frac{85}{18} \quad \frac{25}{6} \quad \frac{25}{6} \quad \frac{25}{6} \quad \frac{25}{6} \quad \frac{91}{18} \quad \frac{83}{18} \quad \frac{25}{6} \quad \frac{11}{2}\right)$

$$\boldsymbol{\tau} = 16/1 \text{ , rank } = 4 \text{ , ratio } = 4/1 \text{ , } n^2 / r = 16/1$$

$$\boldsymbol{\tau}' = 48/1 \text{ , } r' = 3/4 \text{ , } \boldsymbol{\tau} / n^2 = 1/4$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 64/9$$

$$\text{max r } = 36/5 \text{ , } r\text{-check is positive? } 4/9$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 16\Omega$$

There are, 2, partitions and, 2, ranges, with a group size of, 4

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 16

dim span idems 4 vs no. of idems 4

"PT1" = {{2, 8}, {6, 7}, {1, 4}, {3, 5}}

"PT2" = {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

"RG1" = {3, 4, 7, 8}

"RG2" = {1, 2, 5, 6}

$$M_C = \begin{pmatrix} \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 7.111111111]

Eigenvalues N_C

[0., 0.6666666667, 1.333333333, 0.8888888889, 1.745355992, 0.2546440077, 1.745355992, 0.2546440077]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_C−scaled

[0., 0.7741935484, 1.032258065, 1.548387097, 2.026865024, 0.2957156222, 2.026865024, 0.2957156222]

NullSpace M_C

{[−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [1, 0, 0, 1, 0, 0, 0, 0], [1, 0, 1, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0, 0, 1] }

NullSpace N_C

{[−1, −1, 1, 1, −1, −1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 10.66666667, 5.333333333]

Eigenvalues N_0

[0., 2., 0.6666666667, 1.333333333, 1.745355992, 0.2546440077, 1.745355992, 0.2546440077]

NullSpace M_0

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

NullSpace N_0

{[−1, −1, 1, 1, −1, −1, 1, 1]}

Eigenvalues M

[8., 4., −1.333333333, −2.666666667, −1.333333333, −2.666666667, −1.333333333, −2.666666667]

Eigenvalues N

[0., −0.6666666667, −1.333333333, 6., −0.2546440077, −1.745355992, −0.2546440077, −1.745355992]

NullSpace M

{}

NullSpace N

{[−1, −1, 1, 1, −1, −1, 1, 1]}

Harmonic Basis

$$\begin{pmatrix} 0&0&0&0&0&3&2&1\\ 0&0&2&0&3&0&1&0\\ 0&2&0&0&0&1&0&3\\ 0&0&0&0&1&2&3&0\\ 0&3&0&1&0&0&0&2\\ 3&0&1&2&0&0&0&0\\ 2&1&0&3&0&0&0&0\\ 1&0&3&0&2&0&0&0 \end{pmatrix} \quad \begin{pmatrix} 0&6&4&2&6&-3&0&3\\ 6&0&0&4&-3&6&3&2\\ 4&0&0&6&2&3&6&-3\\ 2&4&6&0&3&0&-3&6\\ 6&-3&2&3&0&6&4&0\\ -3&6&3&0&6&0&2&4\\ 0&3&6&-3&4&2&0&6\\ 3&2&-3&6&0&4&6&0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

==

{3, 4, 5, 6}

R: [4, 3, 8, 7, 8, 7, 4, 3]
B: [7, 8, 1, 2, 3, 4, 5, 6]

TRACE TWO = 3

det AT = $\frac{1}{16}(-1+t)^4(t)^2$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 8

Level 2 det = $\frac{5}{268435456}(2+s)(-1+s)^3(4+s)^2(-4+s)^2(6+3s+s^2)(5+s)^2(-3+s)(148-8s+47s^2+8s^3+5s^4)$

RANK of R is 4

R ranking is 1, "vs", 4

RBAR ranking 1, "vs", 4

RANK of B is 8

B ranking is 2, "vs", 8

BBAR ranking 2, "vs", 8

"R CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])
"B CYCLES", (1 + v[1] v[3] v[5] v[7]) (1 + v[2] v[4] v[6] v[8])

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [-1., 1., 1. I, -1. I, -1., 1., 1. I, -1. I]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{}

NullSpace of R^*

{[0, 0, −1, 0, 1, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 3

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 8 \\ 0 & 0 & 8 & 0 & 0 & 0 & 8 & 8 \\ 4 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\ 4 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 8 & 0 & 0 & 0 & 8 \\ 0 & 0 & 8 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & 1 & 1 & 1 & \frac{1}{3} & 1 \\ 1 & 0 & 1 & \frac{2}{3} & 1 & 1 & 1 & \frac{1}{3} \\ \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} & 1 & 1 & 1 \\ 1 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} & 1 & 1 \\ 1 & 1 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} & 1 \\ 1 & 1 & 1 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ \frac{1}{3} & 1 & 1 & 1 & \frac{2}{3} & 1 & 0 & 1 \\ 1 & \frac{1}{3} & 1 & 1 & 1 & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 8

"RANK of the KERNEL is ", 4

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 4

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$)
degree 2: $\frac{1}{6}$ ($v[1]v[2] + v[1]v[5] + v[1]v[6] + v[2]v[5] + v[2]v[6] + 2v[3]v[4] + 2v[3]v[7] + 2v[3]v[8] + 2v[4]v[7] + 2v[4]v[8] + v[5]v[6] + 2v[7]v[8]$)
degree 3 : $\frac{1}{12}$ ($v[1]v[2]v[5] + v[1]v[2]v[6] + v[1]v[5]v[6] + v[2]v[5]v[6] + 2v[3]v[4]v[7] + 2v[3]v[4]v[8] + 2v[3]v[7]v[8] + 2v[4]v[7]v[8]$)
degree 4 : $\frac{1}{3}$ ($v[1]v[2]v[5]v[6] + 2v[3]v[4]v[7]v[8]$)

Group spectrum $1 + t + 3t^2 + t^3 + t^4$

KERNEL STRUCTURE

"PT1" = {{2, 8}, {1, 7}, {4, 6}, {3, 5}}

"PT2" = {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

"RG1" = {3, 4, 7, 8}

"RG2" = {1, 2, 5, 6}

$\pi 4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 4$ = {10, 61}

$u4$ = [2, 0, 2, 0, 0, 2, 0, 0, 0, 3, 0, 1, 1, 0, 1, 0, 0, 0, 2, 0, 0, 0, 0, 2, 0, 1, 0, 3, 1, 0, 1, 0, 2, 0, 0, 0, 0, 2, 0, 1, 0, 1, 3, 0, 1, 0, 2, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 1, 1, 0, 3, 0, 0, 0, 2, 0, 0, 2, 0, 2]

supp u4 = {1, 3, 6, 10, 12, 13, 15, 19, 24, 26, 28, 29, 31, 33, 38, 40, 42, 43, 45, 47, 52, 56, 58, 59, 61, 65, 68, 70}

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[8, 4, 2, 2, 8, 1, 4, 1]

B–BLOCKS,

[3, 6, 4, 5, 1, 8, 2, 7]

with invariant measure, [2, 2, 1, 2, 1, 1, 1, 2]

N by blocks, N – check: true

$b_1 = \{2, 8\}$
$b_2 = \{1, 7\}$
$b_3 = \{2, 4\}$
$b_4 = \{4, 6\}$
$b_5 = \{6, 8\}$
$b_6 = \{1, 3\}$
$b_7 = \{5, 7\}$
$b_8 = \{3, 5\}$

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[3] & h[2] & 0 & 0 & h[4] & h[1] & 0 & 0 \\ h[2] & h[3] & 0 & 0 & h[1] & h[4] & 0 & 0 \\ 0 & 0 & h[3] & h[2] & 0 & 0 & h[4] & h[1] \\ 0 & 0 & h[2] & h[3] & 0 & 0 & h[1] & h[4] \\ h[4] & h[1] & 0 & 0 & h[3] & h[2] & 0 & 0 \\ h[1] & h[4] & 0 & 0 & h[2] & h[3] & 0 & 0 \\ 0 & 0 & h[4] & h[1] & 0 & 0 & h[3] & h[2] \\ 0 & 0 & h[1] & h[4] & 0 & 0 & h[2] & h[3] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: $6 \oplus 4/2$

$$\text{CLB} = \begin{pmatrix} -1 & -1 & 1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 8}, {4, 7}}, true

Ω_B in Vec(K)? , {{1, 3, 5, 7}, {2, 4, 6, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I−V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{2, 8}, {1, 7}, {4, 6}, {3, 5}}

1, "range", [3, 4, 7, 8], [[8, 7, 4, 3, 4, 3, 8, 7], [7, 8, 3, 4, 3, 4, 7, 8], [4, 3, 8, 7, 8, 7, 4, 3], [3, 4, 7, 8, 7, 8, 3, 4]]

2, "range", [1, 2, 5, 6], [[6, 5, 2, 1, 2, 1, 6, 5], [5, 6, 1, 2, 1, 2, 5, 6], [2, 1, 6, 5, 6, 5, 2, 1], [1, 2, 5, 6, 5, 6, 1, 2]]

2, "partition", {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

1, "range", [3, 4, 7, 8], [[8, 7, 8, 7, 4, 3, 4, 3], [7, 8, 7, 8, 3, 4, 3, 4], [4, 3, 4, 3, 8, 7, 8, 7], [3, 4, 3, 4, 7, 8, 7, 8]]

2, "range", [1, 2, 5, 6], [[6, 5, 6, 5, 2, 1, 2, 1], [5, 6, 5, 6, 1, 2, 1, 2], [2, 1, 2, 1, 6, 5, 6, 5], [1, 2, 1, 2, 5, 6, 5, 6]]

"group has", 4, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2], [3, 4]]

$g_2 =$ []

$g_3 =$ [[1, 4], [2, 3]]

$g_4 =$ [[1, 3], [2, 4]]

linear dimension, 4

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] & h[4] & h[3] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\,3\,] = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

4, "coeff", 1

$$Z[\,4\,] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
2, 3, true
2, 4, true
3, 4, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. \\ 1. & -1. & 1. & -1. \\ 1. & -1. & 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 4 & 0 & 0 \\ 2 & 6 & 2 & 2 \\ 0 & 4 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 3t^2 + t^3 + t^4$

Molien Series to order 10: $1 + t + 4t^2 + 5t^3 + 11t^4 + 14t^5 + 24t^6 + 30t^7 + 45t^8 + 55t^9 + 76t^{10}$

n-choose-rank

{1, [1, 2, 3, 4]}, {2, [1, 2, 3, 5]}, {3, [1, 2, 3, 6]}, {4, [1, 2, 3, 7]}, {5, [1, 2, 3, 8]}, {6, [1, 2, 4, 5]}, {7, [1, 2, 4, 6]}, {8, [1, 2, 4, 7]}, {9, [1, 2, 4, 8]}, {10, [1, 2, 5, 6]}, {11, [1, 2, 5, 7]}, {12, [1, 2, 5, 8]}, {13, [1, 2, 6, 7]}, {14, [1, 2, 6, 8]}, {15, [1, 2, 7, 8]}, {16, [1, 3, 4, 5]}, {17, [1, 3, 4, 6]}, {18, [1, 3, 4, 7]}, {19, [1, 3, 4, 8]}, {20, [1, 3, 5, 6]}, {21, [1, 3, 5, 7]}, {22, [1, 3, 5, 8]}, {23, [1, 3, 6, 7]}, {24, [1, 3, 6, 8]}, {25, [1, 3, 7, 8]}, {26, [1, 4, 5, 6]}, {27, [1, 4, 5, 7]}, {28, [1, 4, 5, 8]}, {29, [1, 4, 6, 7]}, {30, [1, 4, 6, 8]}, {31, [1, 4, 7, 8]}, {32, [1, 5, 6, 7]}, {33, [1, 5, 6, 8]}, {34, [1, 5, 7, 8]}, {35, [1, 6, 7, 8]}, {36, [2, 3, 4, 5]}, {37, [2, 3, 4, 6]}, {38, [2, 3, 4, 7]}, {39, [2, 3, 4, 8]}, {40, [2, 3, 5, 6]}, {41, [2, 3, 5, 7]}, {42, [2, 3, 5, 8]}, {43, [2, 3, 6, 7]}, {44, [2, 3, 6, 8]}, {45, [2, 3, 7, 8]}, {46, [2, 4, 5, 6]}, {47, [2, 4, 5, 7]}, {48, [2, 4, 5, 8]}, {49, [2, 4, 6, 7]}, {50, [2, 4, 6, 8]}, {51, [2, 4, 7, 8]}, {52, [2, 5, 6, 7]}, {53, [2, 5, 6, 8]}, {54, [2, 5, 7, 8]}, {55, [2, 6, 7, 8]}, {56, [3, 4, 5, 6]}, {57, [3, 4, 5, 7]}, {58, [3, 4, 5, 8]}, {59, [3, 4, 6, 7]}, {60, [3, 4, 6, 8]}, {61, [3, 4, 7, 8]}, {62, [3, 5, 6, 7]}, {63, [3, 5, 6, 8]}, {64, [3, 5, 7, 8]}, {65, [3, 6, 7, 8]}, {66, [4, 5, 6, 7]}, {67, [4, 5, 6, 8]}, {68, [4, 5, 7, 8]}, {69, [4, 6, 7, 8]}, {70, [5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 4$ =
$\left(0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0\right.$

{10, 61}

$u4$ =
$\left(2 \;\; 0 \;\; 2 \;\; 0 \;\; 0 \;\; 2 \;\; 0 \;\; 0 \;\; 0 \;\; 3 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 2 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 2 \;\; 0 \;\; 1 \;\; 0 \;\; 3 \;\; 1 \;\; 0\right.$

{1, 3, 6, 10, 12, 13, 15, 19, 24, 26, 28, 29, 31, 33, 38, 40, 42, 43, 45, 47, 52, 56, 58, 59, 61, 65, 68, 70}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \left(\frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6}\right)$

$\pi3 =$
$(0 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0$

$u3 =$
$\left(\frac{1}{2} \;\; \frac{1}{2} \;\; \frac{3}{4} \;\; \frac{3}{4} \;\; \frac{1}{4} \;\; \frac{1}{4} \;\; \frac{1}{2} \;\; 0 \;\; \frac{1}{2} \;\; 0 \;\; \frac{1}{2} \;\; \frac{3}{4} \;\; \frac{1}{4} \;\; \frac{1}{4} \;\; \frac{3}{4} \;\; \frac{3}{4} \;\; 0 \;\; \frac{3}{4} \;\; \frac{1}{4} \;\; \frac{1}{2} \;\; \frac{1}{4} \;\; \frac{1}{2} \;\; \frac{1}{4} \;\; \frac{3}{4} \;\; \frac{3}{4} \;\; \frac{1}{4} \;\; \frac{1}{2} \;\; 0\right.$

picheck $(3 \;\; 3 \;\; 6 \;\; 6 \;\; 3 \;\; 3 \;\; 6 \;\; 6)$

$\pi2 =$
$(2 \;\; 0 \;\; 0 \;\; 2 \;\; 2 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 2 \;\; 2 \;\; 0 \;\; 0 \;\; 4 \;\; 0 \;\; 0 \;\; 4 \;\; 4 \;\; 0 \;\; 0 \;\; 4 \;\; 4 \;\; 2 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 4)$

$u2 =$
$\left(\frac{3}{8} \;\; \frac{1}{4} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{1}{8} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{1}{4} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{1}{8} \;\; \frac{3}{8} \;\; \frac{1}{8} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{1}{8} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{1}{4} \;\; \frac{3}{8} \;\; \frac{3}{8} \;\; \frac{1}{4} \;\; \frac{3}{8}\right)$

picheck $(6 \;\; 6 \;\; 12 \;\; 12 \;\; 6 \;\; 6 \;\; 12 \;\; 12)$

$\pi1 = (6 \;\; 6 \;\; 12 \;\; 12 \;\; 6 \;\; 6 \;\; 12 \;\; 12)$

$u1 = \left(\frac{9}{32} \;\; \frac{9}{32} \;\; \frac{9}{32} \;\; \frac{9}{32} \;\; \frac{9}{32} \;\; \frac{9}{32} \;\; \frac{9}{32} \;\; \frac{9}{32}\right)$

picheck $(6 \;\; 6 \;\; 12 \;\; 12 \;\; 6 \;\; 6 \;\; 12 \;\; 12)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \end{pmatrix}$$ idem-checks

$$PP_2 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{9} & 0 & 0 & 0 & \frac{4}{9} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{9} & 0 & 0 & 0 & \frac{4}{9} \\ \frac{1}{9} & 0 & \frac{2}{3} & 0 & \frac{2}{9} & 0 & 0 & 0 \\ 0 & \frac{1}{9} & 0 & \frac{2}{3} & 0 & \frac{2}{9} & 0 & 0 \\ 0 & 0 & \frac{4}{9} & 0 & \frac{1}{3} & 0 & \frac{2}{9} & 0 \\ 0 & 0 & 0 & \frac{4}{9} & 0 & \frac{1}{3} & 0 & \frac{2}{9} \\ \frac{2}{9} & 0 & 0 & 0 & \frac{1}{9} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{2}{9} & 0 & 0 & 0 & \frac{1}{9} & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & \frac{8}{3} & \frac{56}{9} & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{64}{9} & \frac{16}{3} \\ \frac{8}{3} & 4 & \frac{16}{3} & \frac{56}{9} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{64}{9} \\ \frac{28}{9} & \frac{8}{3} & 8 & \frac{16}{3} & \frac{32}{9} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{28}{9} & \frac{16}{3} & 8 & \frac{8}{3} & \frac{32}{9} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{64}{9} & \frac{16}{3} & 4 & \frac{8}{3} & \frac{56}{9} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{64}{9} & \frac{8}{3} & 4 & \frac{16}{3} & \frac{56}{9} \\ \frac{32}{9} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{28}{9} & \frac{8}{3} & 8 & \frac{16}{3} \\ \frac{8}{3} & \frac{32}{9} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & \frac{28}{9} & \frac{16}{3} & 8 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 1, 1, -1, -1, 1, 1]$

$$\ker N_c = \begin{pmatrix} 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & 0 & -t & 0 & t & 0 & -t \\ -t & 0 & t & 0 & -t & 0 & t & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t & 0 & s & 0 & 0 & 0 \\ -s & -s+t & -s & 0 & 0 & 0 \\ 0 & s & 0 & -t & -t & -t \\ s & 0 & 0 & 0 & t & 0 \\ -t & -t+s & -t & 0 & 0 & 0 \\ s & 0 & t & 0 & 0 & 0 \\ 0 & 0 & s & t & 0 & 0 \\ -s & -s & -s & 0 & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s & t & 0 \\ t & -t & t & -t & t & t+s & -t \\ 0 & 0 & s & t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & t & s & 0 \\ t & 0 & s & 0 & 0 & 0 & 0 \\ s & t & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 2 & 0 & 2 & 0 & 2 & 2 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 1 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 1 \end{pmatrix}, \; BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 8, "vs", 4

$$CNM = \begin{pmatrix} 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew } T = \begin{pmatrix} 0 & 0 & \frac{1}{9} & 0 & 0 & 0 & \frac{2}{9} & 0 \\ 0 & 0 & 0 & \frac{1}{9} & 0 & 0 & 0 & \frac{2}{9} \\ \frac{-1}{9} & 0 & 0 & 0 & \frac{-2}{9} & 0 & 0 & 0 \\ 0 & \frac{-1}{9} & 0 & 0 & 0 & \frac{-2}{9} & 0 & 0 \\ 0 & 0 & \frac{2}{9} & 0 & 0 & 0 & \frac{1}{9} & 0 \\ 0 & 0 & 0 & \frac{2}{9} & 0 & 0 & 0 & \frac{1}{9} \\ \frac{-2}{9} & 0 & 0 & 0 & \frac{-1}{9} & 0 & 0 & 0 \\ 0 & \frac{-2}{9} & 0 & 0 & 0 & \frac{-1}{9} & 0 & 0 \end{pmatrix}$$

$$\text{Skew Omega} = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 1 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 1 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 4T + 32\Omega$$

$$\Omega\left(\frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12}\right)$$

$$T\left(0 \;\; \frac{2}{3} \;\; 0 \;\; \frac{1}{9} \;\; 0 \;\; \frac{4}{9} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{2}{9} \;\; 0 \;\; \frac{1}{3}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{16}{3}\ \ 8\ \ \frac{8}{3}\ \ \frac{28}{9}\ \ \frac{16}{3}\ \ \frac{64}{9}\ \ \frac{8}{3}\ \ \frac{8}{3}\ \ \frac{16}{3}\ \ \frac{56}{9}\ \ \frac{8}{3}\ \ 4\right)$$

"IS MN in Vec(K)?", false

$$MN\left(4\ \ \frac{19}{3}\ \ 4\ \ \frac{41}{9}\ \ 4\ \ \frac{47}{9}\ \ 4\ \ \frac{13}{3}\ \ 4\ \ \frac{43}{9}\ \ 4\ \ \frac{17}{3}\right)$$

$$\boldsymbol{\tau} = 16/1 \text{ , rank } = 4 \text{ , ratio } = 4/1 \text{ , } n^2 / r = 16/1$$

$$\boldsymbol{\tau}' = 48/1 \text{ , } r' = 3/4 \text{ , } \boldsymbol{\tau} / n^2 = 1/4$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 64/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 4/9$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 16\Omega$$

There are, 2, partitions and, 2, ranges, with a group size of, 4

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 4 vs no. of idems 4

"PT1" = {{2, 8}, {1, 7}, {4, 6}, {3, 5}}

"PT2" = {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

"RG1" = {3, 4, 7, 8}

"RG2" = {1, 2, 5, 6}

$$M_C = \begin{pmatrix} \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 7.111111111]

Eigenvalues N_C

[2., 0.8888888889, 0., 0., 1.333333333, 0.6666666667, 1.333333333, 0.6666666667
]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_C−scaled

[2.322580645, 1.032258065, 0., 0., 1.548387097, 0.7741935484, 1.548387097,
0.7741935484]

NullSpace M_C

{[1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 1], [1, 0, 1, 0, 0, 0, 0, 0],
[−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [1, 0, 0, 1, 0, 0, 0, 0]}

NullSpace N_C

{[−1, 0, 1, 0, −1, 0, 1, 0], [0, −1, 0, 1, 0, −1, 0, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 10.66666667, 5.333333333]

Eigenvalues N_0

[2., 0.6666666667, 1.333333333, 2., 0.6666666667, 1.333333333, 0., 0.]

NullSpace M_0

{[1, 0, 0, 0, −1, 0, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 0, 1, 0, 0, −1, 0], [0, 0, 0, 0, −1, 1, 0,
0], [0, 0, 0, 0, 0, 0, −1, 1], [0, 0, 1, 0, 0, 0, −1, 0]}

NullSpace N_0

{[0, −1, 0, 1, 0, −1, 0, 1], [−1, 0, 1, 0, −1, 0, 1, 0]}

Eigenvalues M

[8., 4., −1.333333333, −2.666666667, −1.333333333, −2.666666667,
−1.333333333, −2.666666667]

Eigenvalues N

[6., −2., 0., 0., −0.6666666667, −1.333333333, −0.6666666667, −1.333333333]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 1, 0, −1, 0, 1], [1, 0, −1, 0, 1, 0, −1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 2 & 0 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 3 & 0 & 1 \\ 2 & 0 & 0 & 0 & 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 3 \\ 3 & 0 & 1 & 0 & 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 1 & 0 & 0 & 0 & 2 \\ 1 & 0 & 3 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 2 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -2 & 0 & 0 & 0 & 1 & 0 & -1 \\ -2 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2 & 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -2 & 0 & 0 \\ 1 & 0 & -1 & 0 & -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & -2 \\ -1 & 0 & 1 & 0 & 0 & 0 & -2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 3 & 0 & 1 & 0 & 0 & 0 & 2 \\ 3 & 0 & 1 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 & 0 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 3 & 0 & 1 \\ 0 & 0 & 2 & 0 & 3 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 3 \\ 2 & 0 & 0 & 0 & 1 & 0 & 3 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$1, 3: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$2, 3: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

==

{3, 4, 7, 8}

R: [4, 3, 8, 7, 3, 4, 5, 6]
B: [7, 8, 1, 2, 8, 7, 4, 3]

TRACE TWO = 2

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det $= \frac{-3087}{1048576} (-20 - 5s + 7s^2) (-1 + s)^2 (1 + s)^2 (37 + 4s + 7s^2) (20 - 3s + 7s^2)$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", 1 + v[3] v[4] v[5] v[6] v[7] v[8]
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[7] v[8]

Eigenvalues

R: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

B: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of R^*

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B^*

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 6 & 5 & 5 & 0 & 0 & 5 & 5 \\ 6 & 0 & 5 & 5 & 0 & 0 & 5 & 5 \\ 5 & 5 & 0 & 12 & 5 & 5 & 10 & 10 \\ 5 & 5 & 12 & 0 & 5 & 5 & 10 & 10 \\ 0 & 0 & 5 & 5 & 0 & 6 & 5 & 5 \\ 0 & 0 & 5 & 5 & 6 & 0 & 5 & 5 \\ 5 & 5 & 10 & 10 & 5 & 5 & 0 & 12 \\ 5 & 5 & 10 & 10 & 5 & 5 & 12 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 8

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{12}\,(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8])$

degree 2: $\frac{1}{6}\,(4v[1]v[2] + v[1]v[3] + v[1]v[4] + v[1]v[7] + v[1]v[8] + v[2]v[3] + v[2]v[4] + v[2]v[7] + v[2]v[8] + 8v[3]v[4] + v[3]v[5] + v[3]v[6] + 2v[3]v[7] + 2v[3]v[8] + v[4]v[5] + v[4]v[6] + 2v[4]v[7] + 2v[4]v[8] + 4v[5]v[6] + v[5]v[7] + v[5]v[8] + v[6]v[7] + v[6]v[8] + 8v[7]v[8])$

degree 3 : $\frac{1}{12}\,(4v[1]v[2]v[3] + 3v[1]v[3]v[8] + 3v[1]v[4]v[7] + 3v[3]v[5]v[8] + 3v[4]v[6]v[8] + 4v[3]v[4]v[6] + 3v[1]v[4]v[8] + 4v[5]v[6]v[7] + 3v[3]v[6]v[8] + 8v[3]v[4]v[7] + 4v[1]v[2]v[4] + 3v[2]v[4]v[8] + 4v[4]v[5]v[6] + 4v[2]v[3]v[4] + 3v[4]v[6]v[7] + 3v[2]v[3]v[8] + 3v[2]v[3]v[7] + 3v[2]v[4]v[7] + 4v[1]v[2]v[8] + 3v[1]v[3]v[7] + 4v[2]v[7]v[8] + 3v[4]v[5]v[8] + 3v[3]v[6]v[7] + 4v[5]v[6]v[8] + 4v[5]v[7]v[8] + 4v[3]v[5]v[6] + 4v[1]v[3]v[4] + 3v[4]v[5]v[7] + 4v[1]v[7]v[8] + 8v[3]v[4]v[8] + 4v[1]v[2]v[7] + 4v[6]v[7]v[8] + 3v[3]v[5]v[7] + 8v[3]v[7]v[8] + 4v[3]v[4]v[5] + 8v[4]v[7]v[8])$

degree 4 : $\frac{1}{6}$ ($v[2]v[3]v[4]v[8] + v[3]v[4]v[5]v[8] + v[1]v[2]v[4]v[7] + v[3]v[4]v[6]v[8] + v[1]v[2]v[3]v[8] + 4v[1]v[2]v[3]v[4] + v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[8] + v[2]v[3]v[4]v[7] + v[1]v[3]v[4]v[7] + v[2]v[3]v[7]v[8] + v[3]v[6]v[7]v[8] + v[1]v[3]v[7]v[8] + v[3]v[4]v[6]v[7] + v[3]v[5]v[7]v[8] + v[4]v[5]v[7]v[8] + v[4]v[5]v[6]v[7] + v[3]v[5]v[6]v[7] + v[1]v[2]v[3]v[7] + v[1]v[3]v[4]v[8] + v[1]v[4]v[7]v[8] + 8v[3]v[4]v[7]v[8] + v[2]v[4]v[7]v[8] + 4v[3]v[4]v[5]v[6] + v[4]v[5]v[6]v[8] + v[3]v[4]v[5]v[7] + v[3]v[5]v[6]v[8] + 4v[1]v[2]v[7]v[8] + 4v[5]v[6]v[7]v[8]$)

degree 5 : $\frac{1}{12}$ ($v[1]v[2]v[3]v[4]v[7] + v[1]v[2]v[3]v[4]v[8] + v[1]v[2]v[3]v[7]v[8] + v[1]v[2]v[4]v[7]v[8] + v[1]v[3]v[4]v[7]v[8] + v[2]v[3]v[4]v[7]v[8] + v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[8] + v[3]v[4]v[5]v[7]v[8] + v[3]v[4]v[6]v[7]v[8] + v[3]v[5]v[6]v[7]v[8] + v[4]v[5]v[6]v[7]v[8]$)

degree 6 : $\frac{1}{2}$ ($v[1]v[2] + v[5]v[6]$) ($v[4]$) ($v[3]$) ($v[7]$) ($v[8]$)

Group spectrum $1 + t + 2t^2 + 3t^3 + 2t^4 + t^5 + t^6$

KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {8}, {4}, {3}, {7}}

"RG1" = {3, 4, 5, 6, 7, 8}

"RG2" = {1, 2, 3, 4, 7, 8}

$\pi 6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]

supp $\pi 6$ = {6, 28}

$u6$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1]

supp u6 = {6, 18, 25, 28}

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[3, 6, 5, 1, 2, 4]

B−BLOCKS,

[5, 4, 2, 6, 3, 1]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{1, 6\}$
$b_2 = \{2, 5\}$
$b_3 = \{8\}$
$b_4 = \{4\}$
$b_5 = \{3\}$
$b_6 = \{7\}$

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 16, Shape: $8 \oplus 8/3$

$$CLB = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 4, 5, 6, 7, 8}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 4, 7, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {8}, {4}, {3}, {7}}

1, "range", [3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 7, 8, 4, 3], [8, 7, 6, 5, 7, 8, 3, 4], [8, 7, 5, 6, 7, 8, 4, 3], [8, 7, 5, 6, 7, 8, 3, 4], [7, 8, 6, 5, 8, 7, 4, 3], [7, 8, 6, 5, 8, 7, 3, 4], [7, 8, 5, 6, 8, 7, 4, 3], [7, 8, 5, 6, 8, 7, 3, 4], [6, 5, 4, 3, 5, 6, 8, 7], [6, 5, 4, 3, 5, 6, 7, 8], [6, 5, 3, 4, 5, 6, 8, 7], [6, 5, 3, 4, 5, 6, 7, 8], [5, 6, 4, 3, 6, 5, 8, 7], [5, 6, 4, 3, 6, 5, 7, 8], [5, 6, 3, 4, 6, 5, 8, 7], [5, 6, 3, 4, 6, 5, 7, 8], [4, 3, 8, 7, 3, 4, 6, 5], [4, 3, 8, 7, 3, 4, 5, 6], [4, 3, 7, 8, 3, 4, 6, 5], [4, 3, 7, 8, 3, 4, 5, 6], [3, 4, 8, 7, 4, 3, 6, 5], [3, 4, 8, 7, 4, 3, 5, 6], [3, 4, 7, 8, 4, 3, 6, 5], [3, 4, 7, 8, 4, 3, 5, 6]]

2, "range", [1, 2, 3, 4, 7, 8], [[8, 7, 2, 1, 7, 8, 4, 3], [8, 7, 2, 1, 7, 8, 3, 4], [8, 7, 1, 2, 7, 8, 4, 3], [8, 7, 1, 2, 7, 8, 3, 4], [7, 8, 2, 1, 8, 7, 4, 3], [7, 8, 2, 1, 8, 7, 3, 4], [7, 8, 1, 2, 8, 7, 4, 3], [7, 8, 1, 2, 8, 7, 3, 4], [4, 3, 8, 7, 3, 4, 2, 1], [4, 3, 8, 7, 3, 4, 1, 2], [4, 3, 7, 8, 3, 4, 2, 1], [4, 3, 7, 8, 3, 4, 1, 2], [3, 4, 8, 7, 4, 3, 2, 1], [3, 4, 8, 7, 4, 3, 1, 2], [3, 4, 7, 8, 4, 3, 2, 1], [3, 4, 7, 8, 4, 3, 1, 2], [2, 1, 4, 3, 1, 2, 8, 7], [2, 1, 4, 3, 1, 2, 7, 8], [2, 1, 3, 4, 1, 2, 8, 7], [2, 1, 3, 4, 1, 2, 7, 8], [1, 2, 4, 3, 2, 1, 8, 7], [1, 2, 4, 3, 2, 1, 7, 8], [1, 2, 3, 4, 2, 1, 8, 7], [1, 2, 3, 4, 2, 1, 7, 8]] "group has", 24, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

g_1 = [[1, 4, 6], [2, 3, 5]]

g_2 = [[1, 4, 6, 2, 3, 5]]

g_3 = [[1, 3, 5, 2, 4, 6]]

g_4 = [[1, 3, 5], [2, 4, 6]]

g_5 = [[1, 4, 5, 2, 3, 6]]

linear dimension, 12

"Symmetric?", false

Is Z in Vec(K)? true

$\begin{pmatrix} -2h[3] & 2h[3] & 2h[3] & 2h[3] & -4h[1] & 4h[1] & 4h[1] & 4h[2] & -2h[4] & 2h[4] & 2h[4] & 2h[4] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 4

Z[1] = $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 4

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 2

$$Z[\,3\,] = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

4, "coeff", 2

$$Z[\,4\,] = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
2, 3, true
2, 4, true
3, 4, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. & 1. & -1. \\ 0 & 0 & 0 & 2. & -1. + 1.732050808I & -1. - 1.732050808I \\ 0 & 0 & 0 & 2. & -1. + 1.732050808I & -1. - 1.732050808I \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&0&0&0&0&0&2&4&4&6&0&2&2&4&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&3&7&7&15&3&3&3&7&0&0&0&0&0&0&0&0 \\ 2&0&0&2&0&2&2&0&4&8&8&20&0&4&4&8&2&0&0&2&0&2&2&0 \\ 0&0&0&0&0&0&0&0&3&7&7&15&3&3&3&7&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&2&4&4&6&0&2&2&4&0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 3t^3 + 2t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 3t^2 + 7t^3 + 13t^4 + 23t^5 + 41t^6 + 63t^7 + 98t^8 + 146t^9 + 210t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}, {2, [1, 2, 3, 4, 5, 7]}, {3, [1, 2, 3, 4, 5, 8]}, {4, [1, 2, 3, 4, 6, 7]}, {5, [1, 2, 3, 4, 6, 8]}, {6, [1, 2, 3, 4, 7, 8]}, {7, [1, 2, 3, 5, 6, 7]}, {8, [1, 2, 3, 5, 6, 8]}, {9, [1, 2, 3, 5, 7, 8]}, {10, [1, 2, 3, 6, 7, 8]}, {11, [1, 2, 4, 5, 6, 7]}, {12, [1, 2, 4, 5, 6, 8]}, {13, [1, 2, 4, 5, 7, 8]}, {14, [1, 2, 4, 6, 7, 8]}, {15, [1, 2, 5, 6, 7, 8]}, {16, [1, 3, 4, 5, 6, 7]}, {17, [1, 3, 4, 5, 6, 8]}, {18, [1, 3, 4, 5, 7, 8]}, {19, [1, 3, 4, 6, 7, 8]}, {20, [1, 3, 5, 6, 7, 8]}, {21, [1, 4, 5, 6, 7, 8]}, {22, [2, 3, 4, 5, 6, 7]}, {23, [2, 3, 4, 5, 6, 8]}, {24, [2, 3, 4, 5, 7, 8]}, {25, [2, 3, 4, 6, 7, 8]}, {26, [2, 3, 5, 6, 7, 8]}, {27, [2, 4, 5, 6, 7, 8]}, {28, [3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 6$ =

$$\begin{pmatrix} 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

{6, 28}

$u6$ =

$$\begin{pmatrix} 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&1&0&0&1 \end{pmatrix}$$

{6, 18, 25, 28}

picheck $\begin{pmatrix} 1&1&2&2&1&1&2&2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12}&\frac{1}{12}&\frac{1}{6}&\frac{1}{6}&\frac{1}{12}&\frac{1}{12}&\frac{1}{6}&\frac{1}{6} \end{pmatrix}$

$\pi 5$ =

(0 0 1 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0

$u5$ =

$\left(0\ 0\ \frac{1}{6}\ \frac{1}{6}\ 0\ 0\ 0\ 0\ 0\ \frac{1}{6}\ 0\ 0\ 0\ 0\ 0\ \frac{1}{6}\ 0\ 0\ 0\ 0\ 0\ \frac{1}{6}\ \frac{1}{6}\ 0\ 0\ \frac{1}{6}\ 0\ 0\ \frac{1}{6}\right.$

picheck (5 5 10 10 5 5 10 10)

$\pi 4$ =

(2 0 0 2 2 0 0 2 2 0 0 0 0 0 2 0 0 2 2 0 0 0 0 0 2 0 0 0 0 0

$u4$ =

$\left(\frac{1}{18}\ 0\ 0\ \frac{1}{18}\ \frac{1}{18}\ 0\ 0\ \frac{1}{18}\ \frac{1}{18}\ 0\ 0\ 0\ 0\ 0\ \frac{1}{18}\ \frac{1}{18}\ 0\ \frac{1}{18}\ \frac{1}{18}\ 0\ \frac{1}{18}\ \frac{1}{18}\ 0\ 0\ \frac{1}{18}\right.$

picheck (20 20 40 40 20 20 40 40)

$\pi 3$ =

(6 6 0 0 6 6 6 0 0 6 6 0 0 6 6 0 0 0 0 0 6 6 0 0 6 6 0 0 6 6

$u3$ =

$\left(\frac{1}{36}\ \frac{1}{36}\ 0\ 0\ \frac{1}{36}\ \frac{1}{36}\ \frac{1}{36}\ \frac{1}{36}\ 0\ \frac{1}{36}\ \frac{1}{36}\ \frac{1}{36}\ 0\ \frac{1}{36}\ \frac{1}{36}\ 0\ \frac{1}{36}\ \frac{1}{36}\ 0\ 0\ \frac{1}{36}\ \frac{1}{36}\ 0\right.$

picheck (60 60 120 120 60 60 120 120)

$\pi 2$ =

(24 24 24 0 0 24 24 24 24 0 0 24 24 48 24 24 48 48 24 24 48

$u2$ =

$\left(\frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ 0\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ 0\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\ \frac{1}{54}\right.$

picheck (120 120 240 240 120 120 240 240)

$\pi 1 = (120\ \ 120\ \ 240\ \ 240\ \ 120\ \ 120\ \ 240\ \ 240)$

$u1 = \left(\frac{5}{324}\ \frac{5}{324}\ \frac{5}{324}\ \frac{5}{324}\ \frac{5}{324}\ \frac{5}{324}\ \frac{5}{324}\ \frac{5}{324}\right)$

picheck (120 120 240 240 120 120 240 240)

Column Projections

$$P_1 = \begin{pmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{2}&0&0&0&0&\frac{1}{2}&0&0\\ 0&\frac{1}{2}&0&0&\frac{1}{2}&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&\frac{1}{2}&0&0&\frac{1}{2}&0&0&0\\ \frac{1}{2}&0&0&0&0&\frac{1}{2}&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} \frac{13}{3} & \frac{10}{3} & 7 & 7 & \frac{10}{3} & \frac{13}{3} & 7 & 7 \\ \frac{10}{3} & \frac{13}{3} & 7 & 7 & \frac{13}{3} & \frac{10}{3} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & \frac{26}{3} & \frac{20}{3} & \frac{7}{2} & \frac{7}{2} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & \frac{20}{3} & \frac{26}{3} & \frac{7}{2} & \frac{7}{2} & 7 & 7 \\ \frac{10}{3} & \frac{13}{3} & 7 & 7 & \frac{13}{3} & \frac{10}{3} & 7 & 7 \\ \frac{13}{3} & \frac{10}{3} & 7 & 7 & \frac{10}{3} & \frac{13}{3} & 7 & 7 \\ \frac{7}{2} & \frac{7}{2} & 7 & 7 & \frac{7}{2} & \frac{7}{2} & \frac{26}{3} & \frac{20}{3} \\ \frac{7}{2} & \frac{7}{2} & 7 & 7 & \frac{7}{2} & \frac{7}{2} & \frac{20}{3} & \frac{26}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 0, 0, 1, 1, 0, 0]$

$$\ker N_c = \begin{pmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & 0 & -t \\ 0 & -s & 0 & t \\ -t & 0 & 0 & s \\ t & 0 & 0 & -s \\ 0 & -s & 0 & t \\ 0 & s & 0 & -t \\ 0 & t & -s & 0 \\ 0 & -t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ -1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -s & 0 & s+t & s \\ 0 & s & 0 & 0 & t \\ 0 & 0 & t & 0 & s \\ 0 & 0 & -t & s+t & t \\ 0 & s & 0 & 0 & t \\ 0 & -s & 0 & s+t & s \\ -s & -t & 0 & s+t & s+t \\ s & t & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 0 & 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$CNM = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \\ \frac{-5}{6} & \frac{-5}{6} & 0 & 0 & \frac{-5}{6} & \frac{-5}{6} & 0 & 0 \end{pmatrix} \quad Skew\,T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad Skew$$

$$Omega = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 1 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ 1 & 1 & \frac{5}{6} & \frac{5}{6} & 0 & 0 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & 2 & 2 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} \\ \frac{5}{6} & \frac{5}{6} & 2 & 2 & \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 1 & 1 & \frac{5}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{6} & 1 & 1 & \frac{5}{6} & \frac{5}{6} \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 2 & 2 \\ \frac{5}{6} & \frac{5}{6} & \frac{5}{3} & \frac{5}{3} & \frac{5}{6} & \frac{5}{6} & 2 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", false

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \right)$$

$$T \left(1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{2} \right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \left(\frac{26}{3} \;\; \frac{7}{2} \;\; 7 \;\; \frac{7}{2} \;\; \frac{7}{2} \;\; 7 \;\; \frac{7}{2} \;\; \frac{26}{3} \;\; \frac{7}{2} \;\; \frac{7}{2} \;\; 7 \;\; 7 \;\; \frac{13}{3} \;\; \frac{10}{3} \;\; 7 \;\; 7 \;\; \frac{10}{3} \;\; \frac{13}{3} \right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN} \left(\frac{26}{3} \;\; 3 \;\; 6 \;\; 3 \;\; 3 \;\; 6 \;\; 3 \;\; \frac{26}{3} \;\; 3 \;\; 3 \;\; 6 \;\; 6 \;\; \frac{13}{3} \;\; \frac{10}{3} \;\; 6 \;\; 6 \;\; \frac{10}{3} \;\; \frac{13}{3} \right)$$

$$\boldsymbol{\tau} = 12/1 \text{ , rank} = 6 \text{ , ratio} = 2/1 \text{ , } n^2 / r = 32/3$$

$$\boldsymbol{\tau}' = 52/1 \text{ , } r' = 5/6 \text{ , } \boldsymbol{\tau} / n^2 = 3/16$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 28/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 1/6$$

IS N0M0 a combination of T and Omega? , false

$$N_0 M_0 = \frac{14}{51} T + \frac{176}{17} \Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 24

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 48

dim span idems 2 vs no. of idems 2

"PT1" = {{1, 6}, {2, 5}, {8}, {4}, {3}, {7}}

"RG1" = {3, 4, 5, 6, 7, 8}

"RG2" = {1, 2, 3, 4, 7, 8}

$$M_C = \begin{pmatrix} \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{5}{9} & \frac{5}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-1}{10} & \frac{-1}{10} & \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} \\ 1 & 1 & \frac{-1}{10} & \frac{-1}{10} & \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} & 1 & 1 & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-4}{5} & \frac{-4}{5} & \frac{-1}{10} & \frac{-1}{10} & 1 & 1 & \frac{-1}{10} & \frac{-1}{10} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{2}{9} & \frac{2}{9} \end{pmatrix} \quad M_C N_C =$$

$$\begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ 0 & 0 & \frac{1}{18} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 2., 0.6666666667, 0.4444444444]

Eigenvalues N_C

[2., 1.691868003, 0.1970208860, 0., 0., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 1.400000000, 3.600000000]

Eigenvalues N_C–scaled

[2.322580645, 1.964749939, 0.2287984489, 0., 0., 1.161290323, 1.161290323, 1.161290323]

NullSpace M_C

{[0, 0, 0, 0, 1, −1, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [1, 0, 1, 0, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1, 0, 1], [0, 0, −1, 1, 0, 0, 0, 0] }

NullSpace N_C

{[1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

Eigenvalues M_0

[0., 0., 0., 0., 2., 0.6666666667, 8.935416159, 0.397917175]

Eigenvalues N_0

[0., 0., 2., 2., 1., 1., 1., 1.]

NullSpace M_0

{[0, 0, 0, 0, 0, 0, 1, −1], [0, 0, 0, 0, 1, −1, 0, 0], [0, 0, 1, −1, 0, 0, 0, 0], [1, −1, 0, 0, 0, 0, 0, 0]}

NullSpace N_0

{[0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Eigenvalues M

[1., −1.333333333, 7.142286814, −0.808953480, −1., −2., −1., −2.]

Eigenvalues N

[−2., 6.531128874, −1.531128874, 0., 0., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

==

{3, 5, 6, 7}

R: [4, 3, 8, 2, 8, 7, 5, 3]
B: [7, 8, 1, 7, 3, 4, 4, 6]

TRACE TWO = 1

det AT = $\frac{1}{16}(1+t)^2(-1+t)^2(t)^2$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{5}{268435456}(-1+s)(2+s)(1+s)(-444+24s-181s^2-38s^3-4s^4-2s^5+5s^6)(6400-2560s+1120s^2-192s^3+121s^4-10s^5+s^6)(6-s+s^2)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[3] v[8]
"B CYCLES", 1 + v[4] v[7]

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 0, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0, 0, 0]}

NullSpace of B^*

{[−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&0&0&0&8&0&0\\ 0&0&0&0&8&0&0&0\\ 0&0&0&0&0&0&0&16\\ 0&0&0&0&0&0&16&0\\ 0&8&0&0&0&0&0&0\\ 8&0&0&0&0&0&0&0\\ 0&0&0&16&0&0&0&0\\ 0&0&16&0&0&0&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&\frac{1}{2}&\frac{3}{5}&\frac{3}{10}&\frac{1}{2}&1&\frac{7}{10}&\frac{2}{5}\\ \frac{1}{2}&0&\frac{7}{10}&\frac{3}{5}&1&\frac{1}{2}&\frac{2}{5}&\frac{3}{10}\\ \frac{3}{5}&\frac{7}{10}&0&\frac{1}{2}&\frac{3}{10}&\frac{2}{5}&\frac{1}{2}&1\\ \frac{3}{10}&\frac{3}{5}&\frac{1}{2}&0&\frac{2}{5}&\frac{7}{10}&1&\frac{1}{2}\\ \frac{1}{2}&1&\frac{3}{10}&\frac{2}{5}&0&\frac{1}{2}&\frac{3}{5}&\frac{7}{10}\\ 1&\frac{1}{2}&\frac{2}{5}&\frac{7}{10}&\frac{1}{2}&0&\frac{3}{10}&\frac{3}{5}\\ \frac{7}{10}&\frac{2}{5}&\frac{1}{2}&1&\frac{3}{5}&\frac{3}{10}&0&\frac{1}{2}\\ \frac{2}{5}&\frac{3}{10}&1&\frac{1}{2}&\frac{7}{10}&\frac{3}{5}&\frac{1}{2}&0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 1 "Trace mark", 0, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$)
degree 2: $\frac{1}{6}$ ($v[1]v[6] + v[2]v[5] + 2v[3]v[8] + 2v[4]v[7]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 5, 6, 7}, {1, 2, 4, 8}}

"PT2" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT3" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT4" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"PT5" = {{1, 3, 4, 5}, {2, 6, 7, 8}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {3, 8}

"RG2" = {4, 7}

"RG3" = {2, 5}

"RG4" = {1, 6}

$$\pi 2 = [0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0]$$

supp $\pi 2$ = {5, 10, 18, 21}

$$u2 = [5, 6, 3, 5, 10, 7, 4, 7, 6, 10, 5, 4, 3, 5, 3, 4, 5, 10, 4, 7, 10, 5, 5, 6, 7, 3, 6, 5]$$

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[1], [3], [1], [2]]
Action of B on ranges, [[4], [2], [1], [2]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

RPARTS [5, 2, 4, 5, 2, 1]

BPARTS [3, 6, 3, 5, 1, 1]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{10} & \frac{1}{5} & \frac{1}{10} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[7, 8, 6, 5, 2, 1, 8, 7, A, 2, C, 1]

B-BLOCKS,

[5, 6, 5, 6, 9, B, 3, 4, B, 1, 9, 2]

with invariant measure, [2, 2, 1, 1, 2, 2, 2, 2, 2, 1, 2, 1]

N by blocks, N - check: true

b_1 = {1, 3, 4, 5}
b_2 = {2, 6, 7, 8}
b_3 = {1, 2, 3, 4}
b_4 = {5, 6, 7, 8}
b_5 = {3, 5, 6, 7}
b_6 = {1, 2, 4, 8}
b_7 = {1, 2, 7, 8}
b_8 = {3, 4, 5, 6}

b_9 = {1, 4, 5, 8}
b_{10} = {1, 3, 5, 7}
b_{11} = {2, 3, 6, 7}
b_{12} = {2, 4, 6, 8}

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & 0 & h[1] \\ 0 & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 \\ 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & 0 \\ h[1] & 0 & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: 23 ⊕ 4/3

$$\text{CLB} = \begin{pmatrix} 0 & 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 8}}, true

Ω_B in Vec(K)? , {{4, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 5, 6, 7}, {1, 2, 4, 8}}

1, "range", [3, 8], [[8, 8, 3, 8, 3, 3, 3, 8], [3, 3, 8, 3, 8, 8, 8, 3]]

2, "range", [4, 7], [[7, 7, 4, 7, 4, 4, 4, 7], [4, 4, 7, 4, 7, 7, 7, 4]]

3, "range", [2, 5], [[5, 5, 2, 5, 2, 2, 2, 5], [2, 2, 5, 2, 5, 5, 5, 2]]

4, "range", [1, 6], [[6, 6, 1, 6, 1, 1, 1, 6], [1, 1, 6, 1, 6, 6, 6, 1]]

2, "partition", {{1, 2, 7, 8}, {3, 4, 5, 6}}

1, "range", [3, 8], [[8, 8, 3, 3, 3, 3, 8, 8], [3, 3, 8, 8, 8, 8, 3, 3]]

2, "range", [4, 7], [[7, 7, 4, 4, 4, 4, 7, 7], [4, 4, 7, 7, 7, 7, 4, 4]]

3, "range", [2, 5], [[5, 5, 2, 2, 2, 2, 5, 5], [2, 2, 5, 5, 5, 5, 2, 2]]

4, "range", [1, 6], [[6, 6, 1, 1, 1, 1, 6, 6], [1, 1, 6, 6, 6, 6, 1, 1]]

3, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [3, 8], [[8, 3, 3, 8, 8, 3, 3, 8], [3, 8, 8, 3, 3, 8, 8, 3]]

2, "range", [4, 7], [[7, 4, 4, 7, 7, 4, 4, 7], [4, 7, 7, 4, 4, 7, 7, 4]]

3, "range", [2, 5], [[5, 2, 2, 5, 5, 2, 2, 5], [2, 5, 5, 2, 2, 5, 5, 2]]

4, "range", [1, 6], [[6, 1, 1, 6, 6, 1, 1, 6], [1, 6, 6, 1, 1, 6, 6, 1]]

4, "partition", {{1, 3, 5, 7}, {2, 4, 6, 8}}

1, "range", [3, 8], [[8, 3, 8, 3, 8, 3, 8, 3], [3, 8, 3, 8, 3, 8, 3, 8]]

2, "range", [4, 7], [[7, 4, 7, 4, 7, 4, 7, 4], [4, 7, 4, 7, 4, 7, 4, 7]]

3, "range", [2, 5], [[5, 2, 5, 2, 5, 2, 5, 2], [2, 5, 2, 5, 2, 5, 2, 5]]

4, "range", [1, 6], [[6, 1, 6, 1, 6, 1, 6, 1], [1, 6, 1, 6, 1, 6, 1, 6]]

5, "partition", {{1, 3, 4, 5}, {2, 6, 7, 8}}

1, "range", [3, 8], [[8, 3, 8, 8, 8, 3, 3, 3], [3, 8, 3, 3, 3, 8, 8, 8]]

2, "range", [4, 7], [[7, 4, 7, 7, 7, 4, 4, 4], [4, 7, 4, 4, 4, 7, 7, 7]]

3, "range", [2, 5], [[5, 2, 5, 5, 5, 2, 2, 2], [2, 5, 2, 2, 2, 5, 5, 5]]

4, "range", [1, 6], [[6, 1, 6, 6, 6, 1, 1, 1], [1, 6, 1, 1, 1, 6, 6, 6]]

6, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [3, 8], [[8, 8, 8, 8, 3, 3, 3, 3], [3, 3, 3, 3, 8, 8, 8, 8]]

2, "range", [4, 7], [[7, 7, 7, 7, 4, 4, 4, 4], [4, 4, 4, 4, 7, 7, 7, 7]]

3, "range", [2, 5], [[5, 5, 5, 5, 2, 2, 2, 2], [2, 2, 2, 2, 5, 5, 5, 5]]

4, "range", [1, 6], [[6, 6, 6, 6, 1, 1, 1, 1], [1, 1, 1, 1, 6, 6, 6, 6]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

{5, 10, 18, 21}

$u2 =$
$$\begin{pmatrix} 5 & 6 & 3 & 5 & 10 & 7 & 4 & 7 & 6 & 10 & 5 & 4 & 3 & 5 & 3 & 4 & 5 & 10 & 4 & 7 & 10 & 5 & 5 & 6 & 7 & 3 & 6 & 5 \end{pmatrix}$$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & 0 & \frac{3}{5} \\ 0 & 0 & \frac{3}{10} & 0 & 0 & 0 & 0 & \frac{7}{10} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{7}{10} & 0 & 0 & 0 & 0 & \frac{3}{10} \\ 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & 0 & \frac{2}{5} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{7}{10} & 0 & 0 & \frac{3}{10} & 0 \\ 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & \frac{2}{5} & 0 \\ 0 & 0 & 0 & \frac{3}{10} & 0 & 0 & \frac{7}{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{10} & 0 & 0 & \frac{7}{10} & 0 & 0 & 0 \\ 0 & \frac{2}{5} & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{3}{5} & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 \\ 0 & \frac{7}{10} & 0 & 0 & \frac{3}{10} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{2}{5} & 0 & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 \\ \frac{7}{10} & 0 & 0 & 0 & 0 & \frac{3}{10} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{3}{10} & 0 & 0 & 0 & 0 & \frac{7}{10} & 0 & 0 \\ \frac{3}{5} & 0 & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{12} & \frac{2}{15} & \frac{7}{30} & \frac{1}{12} & 0 & \frac{1}{10} & \frac{1}{5} \\ \frac{1}{12} & \frac{1}{6} & \frac{1}{10} & \frac{2}{15} & 0 & \frac{1}{12} & \frac{1}{5} & \frac{7}{30} \\ \frac{1}{15} & \frac{1}{20} & \frac{1}{3} & \frac{1}{6} & \frac{7}{60} & \frac{1}{10} & \frac{1}{6} & 0 \\ \frac{7}{60} & \frac{1}{15} & \frac{1}{6} & \frac{1}{3} & \frac{1}{10} & \frac{1}{20} & 0 & \frac{1}{6} \\ \frac{1}{12} & 0 & \frac{7}{30} & \frac{1}{5} & \frac{1}{6} & \frac{1}{12} & \frac{2}{15} & \frac{1}{10} \\ 0 & \frac{1}{12} & \frac{1}{5} & \frac{1}{10} & \frac{1}{12} & \frac{1}{6} & \frac{7}{30} & \frac{2}{15} \\ \frac{1}{20} & \frac{1}{10} & \frac{1}{6} & 0 & \frac{1}{15} & \frac{7}{60} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{10} & \frac{7}{60} & 0 & \frac{1}{6} & \frac{1}{20} & \frac{1}{15} & \frac{1}{6} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & \frac{32}{15} & \frac{56}{15} & \frac{4}{3} & 0 & \frac{8}{5} & \frac{16}{5} \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{5} & \frac{32}{15} & 0 & \frac{4}{3} & \frac{16}{5} & \frac{56}{15} \\ \frac{16}{15} & \frac{4}{5} & \frac{16}{3} & \frac{8}{3} & \frac{28}{15} & \frac{8}{5} & \frac{8}{3} & 0 \\ \frac{28}{15} & \frac{16}{15} & \frac{8}{3} & \frac{16}{3} & \frac{8}{5} & \frac{4}{5} & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & \frac{56}{15} & \frac{16}{5} & \frac{8}{3} & \frac{4}{3} & \frac{32}{15} & \frac{8}{5} \\ 0 & \frac{4}{3} & \frac{16}{5} & \frac{8}{5} & \frac{4}{3} & \frac{8}{3} & \frac{56}{15} & \frac{32}{15} \\ \frac{4}{5} & \frac{8}{5} & \frac{8}{3} & 0 & \frac{16}{15} & \frac{28}{15} & \frac{16}{3} & \frac{8}{3} \\ \frac{8}{5} & \frac{28}{15} & 0 & \frac{8}{3} & \frac{4}{5} & \frac{16}{15} & \frac{8}{3} & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, 1, -1, 1, -1, -1, 1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & -1 & 1 & 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 1 & 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} -t & s & -s & t & s & -t & t & -s \\ -t & 0 & -s & s+t & 0 & -t & s+t & -s \\ -t & 0 & t & 0 & 0 & -t & 0 & t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & -s+t & 0 \\ -s+t & 0 & 0 & 0 \\ s & -t & 0 & 0 \\ 0 & 0 & t & -s \\ -t+s & 0 & 0 & 0 \\ 0 & 0 & -t+s & 0 \\ 0 & 0 & -t & s \\ -s & t & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & -s+t & s & 0 & 0 \\ s & 0 & t & 0 & 0 \\ t & 0 & s+t & 0 & -t \\ s & t & s & -s & 0 \\ t & 0 & s & 0 & 0 \\ t & -t+s & t & 0 & 0 \\ t & -t & t & s & 0 \\ s & 0 & 0 & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 4 & 0 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{2}{5} & \frac{1}{2} & 0 & \frac{3}{5} & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 & \frac{3}{10} & 0 & \frac{1}{2} & \frac{7}{10} & 1 \\ \frac{1}{2} & 0 & 1 & \frac{7}{10} & 1 & \frac{1}{2} & \frac{3}{10} & 0 \\ \frac{2}{5} & \frac{3}{10} & \frac{7}{10} & 1 & \frac{7}{10} & \frac{3}{5} & 0 & \frac{3}{10} \\ \frac{1}{2} & 0 & 1 & \frac{7}{10} & 1 & \frac{1}{2} & \frac{3}{10} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{5} & \frac{1}{2} & 1 & \frac{2}{5} & \frac{1}{2} \\ \frac{3}{5} & \frac{7}{10} & \frac{3}{10} & 0 & \frac{3}{10} & \frac{2}{5} & 1 & \frac{7}{10} \\ \frac{1}{2} & 1 & 0 & \frac{3}{10} & 0 & \frac{1}{2} & \frac{7}{10} & 1 \end{pmatrix}, \quad \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{10} & 1 & \frac{1}{2} & 0 & 0 & \frac{7}{10} \\ \frac{1}{2} & 1 & \frac{3}{5} & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{2}{5} \\ \frac{3}{10} & \frac{3}{5} & 1 & \frac{3}{10} & \frac{2}{5} & \frac{7}{10} & \frac{7}{10} & 0 \\ 1 & \frac{1}{2} & \frac{3}{10} & 1 & \frac{1}{2} & 0 & 0 & \frac{7}{10} \\ \frac{1}{2} & 0 & \frac{2}{5} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{5} \\ 0 & \frac{1}{2} & \frac{7}{10} & 0 & \frac{1}{2} & 1 & 1 & \frac{3}{10} \\ 0 & \frac{1}{2} & \frac{7}{10} & 0 & \frac{1}{2} & 1 & 1 & \frac{3}{10} \\ \frac{7}{10} & \frac{2}{5} & 0 & \frac{7}{10} & \frac{3}{5} & \frac{3}{10} & \frac{3}{10} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{15} & \frac{7}{60} & 0 & 0 & \frac{1}{20} & \frac{1}{10} \\ 0 & 0 & \frac{1}{20} & \frac{1}{15} & 0 & 0 & \frac{1}{10} & \frac{7}{60} \\ \frac{-1}{15} & \frac{-1}{20} & 0 & 0 & \frac{-7}{60} & \frac{-1}{10} & 0 & 0 \\ \frac{-7}{60} & \frac{-1}{15} & 0 & 0 & \frac{-1}{10} & \frac{-1}{20} & 0 & 0 \\ 0 & 0 & \frac{7}{60} & \frac{1}{10} & 0 & 0 & \frac{1}{15} & \frac{1}{20} \\ 0 & 0 & \frac{1}{10} & \frac{1}{20} & 0 & 0 & \frac{7}{60} & \frac{1}{15} \\ \frac{-1}{20} & \frac{-1}{10} & 0 & 0 & \frac{-1}{15} & \frac{-7}{60} & 0 & 0 \\ \frac{-1}{10} & \frac{-7}{60} & 0 & 0 & \frac{-1}{20} & \frac{-1}{15} & 0 & 0 \end{pmatrix}$$

Skew Omega =

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{2}{5} & \frac{7}{10} & \frac{1}{2} & 0 & \frac{3}{10} & \frac{3}{5} \\ \frac{1}{2} & 1 & \frac{3}{10} & \frac{2}{5} & 0 & \frac{1}{2} & \frac{3}{5} & \frac{7}{10} \\ \frac{2}{5} & \frac{3}{10} & 1 & \frac{1}{2} & \frac{7}{10} & \frac{3}{5} & \frac{1}{2} & 0 \\ \frac{7}{10} & \frac{2}{5} & \frac{1}{2} & 1 & \frac{3}{5} & \frac{3}{10} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{7}{10} & \frac{3}{5} & 1 & \frac{1}{2} & \frac{2}{5} & \frac{3}{10} \\ 0 & \frac{1}{2} & \frac{3}{5} & \frac{3}{10} & \frac{1}{2} & 1 & \frac{7}{10} & \frac{2}{5} \\ \frac{3}{10} & \frac{3}{5} & \frac{1}{2} & 0 & \frac{2}{5} & \frac{7}{10} & 1 & \frac{1}{2} \\ \frac{3}{5} & \frac{7}{10} & 0 & \frac{1}{2} & \frac{3}{10} & \frac{2}{5} & \frac{1}{2} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(\frac{1}{3}\ \frac{1}{6}\ \frac{1}{15}\ \frac{7}{60}\ \frac{1}{6}\ \frac{1}{3}\ \frac{1}{20}\ \frac{1}{15}\ \frac{2}{15}\ \frac{1}{10}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{5}\ \frac{1}{10}\ 0\ \frac{1}{12}\ \frac{7}{30}\ \frac{2}{15}\ \frac{1}{12}\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM}\left(\frac{16}{3}\ \frac{8}{3}\ \frac{16}{15}\ \frac{28}{15}\ \frac{8}{3}\ \frac{16}{3}\ \frac{4}{5}\ \frac{16}{15}\ \frac{32}{15}\ \frac{8}{5}\ \frac{8}{3}\ \frac{4}{3}\ \frac{16}{5}\ \frac{8}{5}\ 0\ \frac{4}{3}\ \frac{56}{15}\ \frac{32}{15}\ \frac{4}{3}\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN}\left(\frac{14}{3}\ 2\ \frac{22}{15}\ \frac{46}{15}\ 2\ \frac{14}{3}\ \frac{14}{15}\ \frac{22}{15}\ \frac{26}{15}\ \frac{22}{15}\ \frac{10}{3}\ 2\ \frac{34}{15}\ \frac{22}{15}\ \frac{2}{3}\ 2\ \frac{38}{15}\ \frac{26}{15}\ 2\ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{3, 5, 6, 7}, {1, 2, 4, 8}}

"PT2" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT3" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT4" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"PT5" = {{1, 3, 4, 5}, {2, 6, 7, 8}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {3, 8}

"RG2" = {4, 7}

"RG3" = {2, 5}

"RG4" = {1, 6}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{13}{36} & \frac{47}{180} & \frac{101}{180} & \frac{13}{36} & \frac{-5}{36} & \frac{29}{180} & \frac{83}{180} \\ \frac{13}{36} & \frac{31}{36} & \frac{29}{180} & \frac{47}{180} & \frac{-5}{36} & \frac{13}{36} & \frac{83}{180} & \frac{101}{180} \\ \frac{47}{180} & \frac{29}{180} & \frac{31}{36} & \frac{13}{36} & \frac{101}{180} & \frac{83}{180} & \frac{13}{36} & \frac{-5}{36} \\ \frac{101}{180} & \frac{47}{180} & \frac{13}{36} & \frac{31}{36} & \frac{83}{180} & \frac{29}{180} & \frac{-5}{36} & \frac{13}{36} \\ \frac{13}{36} & \frac{-5}{36} & \frac{101}{180} & \frac{83}{180} & \frac{31}{36} & \frac{13}{36} & \frac{47}{180} & \frac{29}{180} \\ \frac{-5}{36} & \frac{13}{36} & \frac{83}{180} & \frac{29}{180} & \frac{13}{36} & \frac{31}{36} & \frac{101}{180} & \frac{47}{180} \\ \frac{29}{180} & \frac{83}{180} & \frac{13}{36} & \frac{-5}{36} & \frac{47}{180} & \frac{101}{180} & \frac{31}{36} & \frac{13}{36} \\ \frac{83}{180} & \frac{101}{180} & \frac{-5}{36} & \frac{13}{36} & \frac{29}{180} & \frac{47}{180} & \frac{13}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_c\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_c\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{13}{31} & \frac{47}{155} & \frac{101}{155} & \frac{13}{31} & \frac{-5}{31} & \frac{29}{155} & \frac{83}{155} \\ \frac{13}{31} & 1 & \frac{29}{155} & \frac{47}{155} & \frac{-5}{31} & \frac{13}{31} & \frac{83}{155} & \frac{101}{155} \\ \frac{47}{155} & \frac{29}{155} & 1 & \frac{13}{31} & \frac{101}{155} & \frac{83}{155} & \frac{13}{31} & \frac{-5}{31} \\ \frac{101}{155} & \frac{47}{155} & \frac{13}{31} & 1 & \frac{83}{155} & \frac{29}{155} & \frac{-5}{31} & \frac{13}{31} \\ \frac{13}{31} & \frac{-5}{31} & \frac{101}{155} & \frac{83}{155} & 1 & \frac{13}{31} & \frac{47}{155} & \frac{29}{155} \\ \frac{-5}{31} & \frac{13}{31} & \frac{83}{155} & \frac{29}{155} & \frac{13}{31} & 1 & \frac{101}{155} & \frac{47}{155} \\ \frac{29}{155} & \frac{83}{155} & \frac{13}{31} & \frac{-5}{31} & \frac{47}{155} & \frac{101}{155} & 1 & \frac{13}{31} \\ \frac{83}{155} & \frac{101}{155} & \frac{-5}{31} & \frac{13}{31} & \frac{29}{155} & \frac{47}{155} & \frac{13}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[2.888888889, 1.447213595, 0.5527864046, 1.447213595, 0.5527864046, 0., 0., 0.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C–scaled

[3.354838710, 1.680635143, 0.6419455026, 1.680635143, 0.6419455026, 0., 0., 0.]

NullSpace M_C

{[−1, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 1, 1, 0, 0, 1], [0, 0, 0, −1, 0, 0, 1, 0], [0, 1, 0, 0, −1, 0, 0, 0], [1, 0, 1, 1, 1, 0, 0, 0]}

NullSpace N_C

{[0, 0, −1, 1, 0, 0, 1, −1], [0, 1, −1, 0, 1, 0, 0, −1], [1, 0, −1, 0, 0, 1, 0, −1]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[4., 1.447213595, 0.5527864046, 1.447213595, 0.5527864046, 0., 0., 0.]

NullSpace M_0

{[0, 0, 0, −1, 0, 0, 1, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, 1, 0, 0, −1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

NullSpace N_0

{[0, 0, 1, −1, 0, 0, −1, 1], [0, 1, 0, −1, 1, 0, −1, 0], [1, 0, 0, −1, 0, 1, −1, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[4., −0.5527864046, −1.447213595, −0.5527864046, −1.447213595, 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[0, 1, −1, 0, 1, 0, 0, −1], [0, 0, −1, 1, 0, 0, 1, −1], [1, 0, −1, 0, 0, 1, 0, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 5 & 6 & 3 & 5 & 10 & 7 & 4 \\ 5 & 0 & 7 & 6 & 10 & 5 & 4 & 3 \\ 6 & 7 & 0 & 5 & 3 & 4 & 5 & 10 \\ 3 & 6 & 5 & 0 & 4 & 7 & 10 & 5 \\ 5 & 10 & 3 & 4 & 0 & 5 & 6 & 7 \\ 10 & 5 & 4 & 7 & 5 & 0 & 3 & 6 \\ 7 & 4 & 5 & 10 & 6 & 3 & 0 & 5 \\ 4 & 3 & 10 & 5 & 7 & 6 & 5 & 0 \end{pmatrix}$$

==

{3, 5, 6, 8}

R: [4, 3, 8, 2, 8, 7, 4, 6]
B: [7, 8, 1, 7, 3, 4, 5, 3]

TRACE TWO = 1

det AT = $\frac{1}{16}(1+t)^2(-1+t)^2(t)^2$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{5}{268435456}$ $(11840 + 1472s - 1788s^2 + 980s^3 + 115s^4 - 117s^5 - 27s^6 + 5s^7) (-2880 - 2496s - 2340s^2 - 1052s^3 - 683s^4 - 153s^5 - 4s^6 + 4s^7 + 3s^8 + s^9) (-1 + s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 3, "vs", 6

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 1, "vs", 4

"R CYCLES", 1 + v[2] v[3] v[4] v[6] v[7] v[8]
"B CYCLES", 1 + v[1] v[3] v[5] v[7]

Eigenvalues

R: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

B: [0., 0., 0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of R^*

{[-1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 1, 0, -1, 0, 0, 0]}

NullSpace of B^*

{[-1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, -1, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{2}{5} & \frac{2}{3} & \frac{4}{15} & 1 & \frac{3}{5} & \frac{1}{3} & \frac{11}{15} \\ \frac{2}{5} & 0 & \frac{2}{3} & \frac{8}{15} & \frac{3}{5} & 1 & \frac{1}{3} & \frac{7}{15} \\ \frac{2}{3} & \frac{2}{3} & 0 & \frac{2}{5} & \frac{1}{3} & \frac{1}{3} & 1 & \frac{3}{5} \\ \frac{4}{15} & \frac{8}{15} & \frac{2}{5} & 0 & \frac{11}{15} & \frac{7}{15} & \frac{3}{5} & 1 \\ 1 & \frac{3}{5} & \frac{1}{3} & \frac{11}{15} & 0 & \frac{2}{5} & \frac{2}{3} & \frac{4}{15} \\ \frac{3}{5} & 1 & \frac{1}{3} & \frac{7}{15} & \frac{2}{5} & 0 & \frac{2}{3} & \frac{8}{15} \\ \frac{1}{3} & \frac{1}{3} & 1 & \frac{3}{5} & \frac{2}{3} & \frac{2}{3} & 0 & \frac{2}{5} \\ \frac{11}{15} & \frac{7}{15} & \frac{3}{5} & 1 & \frac{4}{15} & \frac{8}{15} & \frac{2}{5} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3   "Trace mark", 2, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{12}\,(\,v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8]\,)$
degree 2: $\frac{1}{6}\,(\,v[1]v[5] + v[2]v[6] + 2\,v[3]v[7] + 2\,v[4]v[8]\,)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT2" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT3" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT4" = {{1, 3, 4, 6}, {2, 5, 7, 8}}

"PT5" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$\pi 2$ = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {4, 11, 17, 22}

$u2$ = [6, 10, 4, 15, 9, 5, 11, 10, 8, 9, 15, 5, 7, 6, 5, 5, 15, 9, 11, 7, 9, 15, 6, 10, 4, 10, 8, 6]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [1], [2], [1]]
Action of B on ranges, [[2], [4], [1], [2]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

RPARTS [1, 5, 2, 1, 3, 5]

BPARTS [6, 6, 4, 5, 4, 5]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{5} & \frac{1}{15} & \frac{2}{15} & \frac{1}{5} & \frac{4}{15} & \frac{2}{15} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[9, A, B, 6, 7, 1, 2, C, 7, 6, C, B]

B-BLOCKS,

[3, 8, 7, 7, 6, 3, 8, 6, 4, 5, 4, 5]

with invariant measure, [2, 2, 3, 2, 2, 4, 4, 3, 1, 1, 3, 3]

N by blocks, N - check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 4, 6}
b_4 = {1, 2, 3, 4}
b_5 = {5, 6, 7, 8}
b_6 = {1, 2, 4, 7}
b_7 = {3, 5, 6, 8}
b_8 = {2, 5, 7, 8}

b_9 = {1, 6, 7, 8}
b_{10} = {2, 3, 4, 5}
b_{11} = {1, 2, 7, 8}
b_{12} = {3, 4, 5, 6}

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \\ h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 25, Shape: 18 ⊕ 7/5

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 1 & 1 & -1 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3, 4, 6, 7, 8}}, true

Ω_B in Vec(K)? , {{1, 3, 5, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 7, 8}, {3, 4, 5, 6}}

1, "range", [4, 8], [[8, 8, 4, 4, 4, 4, 8, 8], [4, 4, 8, 8, 8, 8, 4, 4]]

2, "range", [3, 7], [[7, 7, 3, 3, 3, 3, 7, 7], [3, 3, 7, 7, 7, 7, 3, 3]]

3, "range", [2, 6], [[6, 6, 2, 2, 2, 2, 6, 6], [2, 2, 6, 6, 6, 6, 2, 2]]

4, "range", [1, 5], [[5, 5, 1, 1, 1, 1, 5, 5], [1, 1, 5, 5, 5, 5, 1, 1]]

2, "partition", {{1, 6, 7, 8}, {2, 3, 4, 5}}

1, "range", [4, 8], [[8, 4, 4, 4, 4, 8, 8, 8], [4, 8, 8, 8, 8, 4, 4, 4]]

2, "range", [3, 7], [[7, 3, 3, 3, 3, 7, 7, 7], [3, 7, 7, 7, 7, 3, 3, 3]]

3, "range", [2, 6], [[6, 2, 2, 2, 2, 6, 6, 6], [2, 6, 6, 6, 6, 2, 2, 2]]

4, "range", [1, 5], [[5, 1, 1, 1, 1, 5, 5, 5], [1, 5, 5, 5, 5, 1, 1, 1]]

3, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

2, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

3, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

4, "partition", {{1, 3, 4, 6}, {2, 5, 7, 8}}

1, "range", [4, 8], [[8, 4, 8, 8, 4, 8, 4, 4], [4, 8, 4, 4, 8, 4, 8, 8]]

2, "range", [3, 7], [[7, 3, 7, 7, 3, 7, 3, 3], [3, 7, 3, 3, 7, 3, 7, 7]]

3, "range", [2, 6], [[6, 2, 6, 6, 2, 6, 2, 2], [2, 6, 2, 2, 6, 2, 6, 6]]

4, "range", [1, 5], [[5, 1, 5, 5, 1, 5, 1, 1], [1, 5, 1, 1, 5, 1, 5, 5]]

5, "partition", {{1, 2, 4, 7}, {3, 5, 6, 8}}

1, "range", [4, 8], [[8, 8, 4, 8, 4, 4, 8, 4], [4, 4, 8, 4, 8, 8, 4, 8]]

2, "range", [3, 7], [[7, 7, 3, 7, 3, 3, 7, 3], [3, 3, 7, 3, 7, 7, 3, 7]]

3, "range", [2, 6], [[6, 6, 2, 6, 2, 2, 6, 2], [2, 2, 6, 2, 6, 6, 2, 6]]

4, "range", [1, 5], [[5, 5, 1, 5, 1, 1, 5, 1], [1, 1, 5, 1, 5, 5, 1, 5]]

6, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [4, 8], [[8, 8, 8, 8, 4, 4, 4, 4], [4, 4, 4, 4, 8, 8, 8, 8]]

2, "range", [3, 7], [[7, 7, 7, 7, 3, 3, 3, 3], [3, 3, 3, 3, 7, 7, 7, 7]]

3, "range", [2, 6], [[6, 6, 6, 6, 2, 2, 2, 2], [2, 2, 2, 2, 6, 6, 6, 6]]

4, "range", [1, 5], [[5, 5, 5, 5, 1, 1, 1, 1], [1, 1, 1, 1, 5, 5, 5, 5]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

{4, 11, 17, 22}

$u2 =$
$$\begin{pmatrix} 6 & 10 & 4 & 15 & 9 & 5 & 11 & 10 & 8 & 9 & 15 & 5 & 7 & 6 & 5 & 5 & 15 & 9 & 11 & 7 & 9 & 15 & 6 & 10 & 4 & 1 \end{pmatrix}$$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{11}{15} & 0 & 0 & 0 & \frac{4}{15} \\ 0 & 0 & 0 & \frac{7}{15} & 0 & 0 & 0 & \frac{8}{15} \\ 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{15} & 0 & 0 & 0 & \frac{11}{15} \\ 0 & 0 & 0 & \frac{8}{15} & 0 & 0 & 0 & \frac{7}{15} \\ 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{7}{15} & 0 & 0 & 0 & \frac{8}{15} & 0 & 0 \\ 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{8}{15} & 0 & 0 & 0 & \frac{7}{15} & 0 & 0 \end{pmatrix}$$

idem-checks   N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{11}{15} & 0 & 0 & 0 & \frac{4}{15} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{4}{15} & 0 & 0 & 0 & \frac{11}{15} & 0 & 0 & 0 \end{pmatrix}$$

idem-checks   N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{10} & \frac{1}{9} & \frac{11}{45} & 0 & \frac{1}{15} & \frac{2}{9} & \frac{4}{45} \\ \frac{1}{10} & \frac{1}{6} & \frac{1}{9} & \frac{7}{45} & \frac{1}{15} & 0 & \frac{2}{9} & \frac{8}{45} \\ \frac{1}{18} & \frac{1}{18} & \frac{1}{3} & \frac{1}{5} & \frac{1}{9} & \frac{1}{9} & 0 & \frac{2}{15} \\ \frac{11}{90} & \frac{7}{90} & \frac{1}{5} & \frac{1}{3} & \frac{2}{45} & \frac{4}{45} & \frac{2}{15} & 0 \\ 0 & \frac{1}{15} & \frac{2}{9} & \frac{4}{45} & \frac{1}{6} & \frac{1}{10} & \frac{1}{9} & \frac{11}{45} \\ \frac{1}{15} & 0 & \frac{2}{9} & \frac{8}{45} & \frac{1}{10} & \frac{1}{6} & \frac{1}{9} & \frac{7}{45} \\ \frac{1}{9} & \frac{1}{9} & 0 & \frac{2}{15} & \frac{1}{18} & \frac{1}{18} & \frac{1}{3} & \frac{1}{5} \\ \frac{2}{45} & \frac{4}{45} & \frac{2}{15} & 0 & \frac{11}{90} & \frac{7}{90} & \frac{1}{5} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & \frac{8}{5} & \frac{16}{9} & \frac{176}{45} & 0 & \frac{16}{15} & \frac{32}{9} & \frac{64}{45} \\ \frac{8}{5} & \frac{8}{3} & \frac{16}{9} & \frac{112}{45} & \frac{16}{15} & 0 & \frac{32}{9} & \frac{128}{45} \\ \frac{8}{9} & \frac{8}{9} & \frac{16}{3} & \frac{16}{5} & \frac{16}{9} & \frac{16}{9} & 0 & \frac{32}{15} \\ \frac{88}{45} & \frac{56}{45} & \frac{16}{5} & \frac{16}{3} & \frac{32}{45} & \frac{64}{45} & \frac{32}{15} & 0 \\ 0 & \frac{16}{15} & \frac{32}{9} & \frac{64}{45} & \frac{8}{3} & \frac{8}{5} & \frac{16}{9} & \frac{176}{45} \\ \frac{16}{15} & 0 & \frac{32}{9} & \frac{128}{45} & \frac{8}{5} & \frac{8}{3} & \frac{16}{9} & \frac{112}{45} \\ \frac{16}{9} & \frac{16}{9} & 0 & \frac{32}{15} & \frac{8}{9} & \frac{8}{9} & \frac{16}{3} & \frac{16}{5} \\ \frac{32}{45} & \frac{64}{45} & \frac{32}{15} & 0 & \frac{88}{45} & \frac{56}{45} & \frac{16}{5} & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, -1, 1, -1, 1, -1, 1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & -1 & 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} -t & s & t & -s & -t & s & t & -s \\ -t & 0 & t & 0 & -t & 0 & t & 0 \\ -t & 0 & s & t-s & -t & 0 & s & t-s \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -t & s & 0 \\ 0 & s & -t & 0 \\ t & 0 & -s & 0 \\ 0 & -t & 0 & s \\ 0 & t & -s & 0 \\ 0 & -s & t & 0 \\ -t & 0 & s & 0 \\ 0 & t & 0 & -s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t & s & 0 & 0 & 0 \\ -s & s & 0 & 0 & s+t \\ 0 & t & -t & 0 & s+t \\ t & s & 0 & -s & s \\ -t & t & 0 & 0 & s+t \\ s & t & 0 & 0 & 0 \\ 0 & s & t & 0 & 0 \\ -t & t & 0 & s & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & \frac{3}{5} & 0 & \frac{7}{15} & 0 & \frac{2}{5} & 1 & \frac{8}{15} \\ \frac{3}{5} & 1 & \frac{2}{5} & \frac{1}{3} & \frac{2}{5} & 0 & \frac{3}{5} & \frac{2}{3} \\ 0 & \frac{2}{5} & 1 & \frac{8}{15} & 1 & \frac{3}{5} & 0 & \frac{7}{15} \\ \frac{7}{15} & \frac{1}{3} & \frac{8}{15} & 1 & \frac{8}{15} & \frac{2}{3} & \frac{7}{15} & 0 \\ 0 & \frac{2}{5} & 1 & \frac{8}{15} & 1 & \frac{3}{5} & 0 & \frac{7}{15} \\ \frac{2}{5} & 0 & \frac{3}{5} & \frac{2}{3} & \frac{3}{5} & 1 & \frac{2}{5} & \frac{1}{3} \\ 1 & \frac{3}{5} & 0 & \frac{7}{15} & 0 & \frac{2}{5} & 1 & \frac{8}{15} \\ \frac{8}{15} & \frac{2}{3} & \frac{7}{15} & 0 & \frac{7}{15} & \frac{1}{3} & \frac{8}{15} & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{3}{5} & \frac{2}{3} & 1 & 0 & \frac{2}{5} & \frac{1}{3} & 0 \\ \frac{3}{5} & 1 & \frac{4}{15} & \frac{3}{5} & \frac{2}{5} & 0 & \frac{11}{15} & \frac{2}{5} \\ \frac{2}{3} & \frac{4}{15} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{11}{15} & 0 & \frac{1}{3} \\ 1 & \frac{3}{5} & \frac{2}{3} & 1 & 0 & \frac{2}{5} & \frac{1}{3} & 0 \\ 0 & \frac{2}{5} & \frac{1}{3} & 0 & 1 & \frac{3}{5} & \frac{2}{3} & 1 \\ \frac{2}{5} & 0 & \frac{11}{15} & \frac{2}{5} & \frac{3}{5} & 1 & \frac{4}{15} & \frac{3}{5} \\ \frac{1}{3} & \frac{11}{15} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{4}{15} & 1 & \frac{2}{3} \\ 0 & \frac{2}{5} & \frac{1}{3} & 0 & 1 & \frac{3}{5} & \frac{2}{3} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{11}{90} & 0 & 0 & \frac{1}{9} & \frac{2}{45} \\ 0 & 0 & \frac{1}{18} & \frac{7}{90} & 0 & 0 & \frac{1}{9} & \frac{4}{45} \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & 0 & 0 \\ \frac{-11}{90} & \frac{-7}{90} & 0 & 0 & \frac{-2}{45} & \frac{-4}{45} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{2}{45} & 0 & 0 & \frac{1}{18} & \frac{11}{90} \\ 0 & 0 & \frac{1}{9} & \frac{4}{45} & 0 & 0 & \frac{1}{18} & \frac{7}{90} \\ \frac{-1}{9} & \frac{-1}{9} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \\ \frac{-2}{45} & \frac{-4}{45} & 0 & 0 & \frac{-11}{90} & \frac{-7}{90} & 0 & 0 \end{pmatrix}$$

Skew Omega =

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \\ \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{3}{5} & \frac{1}{3} & \frac{11}{15} & 0 & \frac{2}{5} & \frac{2}{3} & \frac{4}{15} \\ \frac{3}{5} & 1 & \frac{1}{3} & \frac{7}{15} & \frac{2}{5} & 0 & \frac{2}{3} & \frac{8}{15} \\ \frac{1}{3} & \frac{1}{3} & 1 & \frac{3}{5} & \frac{2}{3} & \frac{2}{3} & 0 & \frac{2}{5} \\ \frac{11}{15} & \frac{7}{15} & \frac{3}{5} & 1 & \frac{4}{15} & \frac{8}{15} & \frac{2}{5} & 0 \\ 0 & \frac{2}{5} & \frac{2}{3} & \frac{4}{15} & 1 & \frac{3}{5} & \frac{1}{3} & \frac{11}{15} \\ \frac{2}{5} & 0 & \frac{2}{3} & \frac{8}{15} & \frac{3}{5} & 1 & \frac{1}{3} & \frac{7}{15} \\ \frac{2}{3} & \frac{2}{3} & 0 & \frac{2}{5} & \frac{1}{3} & \frac{1}{3} & 1 & \frac{3}{5} \\ \frac{4}{15} & \frac{8}{15} & \frac{2}{5} & 0 & \frac{11}{15} & \frac{7}{15} & \frac{3}{5} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(\frac{1}{3}\ \frac{1}{5}\ \frac{7}{90}\ \frac{11}{90}\ \frac{1}{5}\ \frac{1}{3}\ \frac{1}{18}\ \frac{1}{18}\ \frac{7}{45}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{10}\ \frac{4}{45}\ \frac{2}{9}\ \frac{1}{15}\ 0\ \frac{11}{45}\ \frac{1}{9}\ \frac{1}{10}\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\text{NM}\left(\frac{16}{3}\ \frac{16}{5}\ \frac{56}{45}\ \frac{88}{45}\ \frac{16}{5}\ \frac{16}{3}\ \frac{8}{9}\ \frac{8}{9}\ \frac{112}{45}\ \frac{16}{9}\ \frac{8}{3}\ \frac{8}{5}\ \frac{64}{45}\ \frac{32}{9}\ \frac{16}{15}\ 0\ \frac{176}{45}\ \frac{16}{9}\ \frac{8}{5}\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{14}{3}\ \frac{38}{15}\ \frac{82}{45}\ \frac{146}{45}\ \frac{38}{15}\ \frac{14}{3}\ \frac{10}{9}\ \frac{10}{9}\ \frac{86}{45}\ \frac{14}{9}\ \frac{10}{3}\ \frac{34}{15}\ \frac{62}{45}\ \frac{22}{9}\ \frac{26}{15}\ \frac{2}{3}\ \frac{118}{45}\ \frac{14}{9}\ \frac{34}{15}\ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT2" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT3" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT4" = {{1, 3, 4, 6}, {2, 5, 7, 8}}

"PT5" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \\ \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{83}{180} & \frac{7}{36} & \frac{107}{180} & \frac{-5}{36} & \frac{47}{180} & \frac{19}{36} & \frac{23}{180} \\ \frac{83}{180} & \frac{31}{36} & \frac{7}{36} & \frac{59}{180} & \frac{47}{180} & \frac{-5}{36} & \frac{19}{36} & \frac{71}{180} \\ \frac{7}{36} & \frac{7}{36} & \frac{31}{36} & \frac{83}{180} & \frac{19}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{47}{180} \\ \frac{107}{180} & \frac{59}{180} & \frac{83}{180} & \frac{31}{36} & \frac{23}{180} & \frac{71}{180} & \frac{47}{180} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{47}{180} & \frac{19}{36} & \frac{23}{180} & \frac{31}{36} & \frac{83}{180} & \frac{7}{36} & \frac{107}{180} \\ \frac{47}{180} & \frac{-5}{36} & \frac{19}{36} & \frac{71}{180} & \frac{83}{180} & \frac{31}{36} & \frac{7}{36} & \frac{59}{180} \\ \frac{19}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{47}{180} & \frac{7}{36} & \frac{7}{36} & \frac{31}{36} & \frac{83}{180} \\ \frac{23}{180} & \frac{71}{180} & \frac{47}{180} & \frac{-5}{36} & \frac{107}{180} & \frac{59}{180} & \frac{83}{180} & \frac{31}{36} \end{pmatrix}$$

$$M_c\text{–scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\ 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_c\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{83}{155} & \frac{7}{31} & \frac{107}{155} & \frac{-5}{31} & \frac{47}{155} & \frac{19}{31} & \frac{23}{155} \\ \frac{83}{155} & 1 & \frac{7}{31} & \frac{59}{155} & \frac{47}{155} & \frac{-5}{31} & \frac{19}{31} & \frac{71}{155} \\ \frac{7}{31} & \frac{7}{31} & 1 & \frac{83}{155} & \frac{19}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{47}{155} \\ \frac{107}{155} & \frac{59}{155} & \frac{83}{155} & 1 & \frac{23}{155} & \frac{71}{155} & \frac{47}{155} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{47}{155} & \frac{19}{31} & \frac{23}{155} & 1 & \frac{83}{155} & \frac{7}{31} & \frac{107}{155} \\ \frac{47}{155} & \frac{-5}{31} & \frac{19}{31} & \frac{71}{155} & \frac{83}{155} & 1 & \frac{7}{31} & \frac{59}{155} \\ \frac{19}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{47}{155} & \frac{7}{31} & \frac{7}{31} & 1 & \frac{83}{155} \\ \frac{23}{155} & \frac{71}{155} & \frac{47}{155} & \frac{-5}{31} & \frac{107}{155} & \frac{59}{155} & \frac{83}{155} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[0., 0., 0., 2.888888889, 0.3212986242, 0.7071405096, 1.362246242, 1.609314622]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C−scaled

[0., 0., 0., 3.354838710, 0.3731209831, 0.8211954308, 1.581963379, 1.868881498]

NullSpace M_C

{[0, 1, 0, 0, 0, −1, 0, 0], [0, 0, 1, 0, 0, 0, −1, 0], [0, 0, 0, 1, 1, 1, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [1, 0, 0, 1, 0, 1, 1, 0] }

NullSpace N_C

{[−1, 0, 1, 0, −1, 0, 1, 0], [−1, 1, 0, 0, −1, 1, 0, 0], [−1, 0, 0, 1, −1, 0, 0, 1]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 0.3212986242, 0.7071405096, 1.362246242, 1.609314622]

NullSpace M_0

{[0, −1, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, −1]}

NullSpace N_0

{[−1, 1, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, −1, 0, 1, 0], [−1, 0, 0, 1, −1, 0, 0, 1]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 4., −0.3212986242, −0.7071405096, −1.362246242, −1.609314622]

NullSpace M

{}

NullSpace N

{[1, −1, 0, 0, 1, −1, 0, 0], [0, −1, 1, 0, 0, −1, 1, 0], [0, −1, 0, 1, 0, −1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 6 & 10 & 4 & 15 & 9 & 5 & 11 \\ 6 & 0 & 10 & 8 & 9 & 15 & 5 & 7 \\ 10 & 10 & 0 & 6 & 5 & 5 & 15 & 9 \\ 4 & 8 & 6 & 0 & 11 & 7 & 9 & 15 \\ 15 & 9 & 5 & 11 & 0 & 6 & 10 & 4 \\ 9 & 15 & 5 & 7 & 6 & 0 & 10 & 8 \\ 5 & 5 & 15 & 9 & 10 & 10 & 0 & 6 \\ 11 & 7 & 9 & 15 & 4 & 8 & 6 & 0 \end{pmatrix}$$

==

{4, 5, 6, 7}

R: [4, 3, 1, 7, 8, 7, 5, 3]
B: [7, 8, 8, 2, 3, 4, 4, 6]

TRACE TWO = 1

det AT = $\frac{1}{16}(-1+t)^2(1+t)^2(t)^2$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{5}{268435456}(-1+s)(-2880-2496s-2340s^2-1052s^3-683s^4-153s^5-4s^6+4s^7+3s^8+s^9)(11840+1472s-1788s^2+980s^3+115s^4-117s^5-27s^6+5s^7)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 3, "vs", 6

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 1, "vs", 4

"R CYCLES", 1 + v[1] v[3] v[4] v[5] v[7] v[8]
"B CYCLES", 1 + v[2] v[4] v[6] v[8]

Eigenvalues

R: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

B: [0., 0., 0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 1, 0, -1, 0, 0], [0, -1, 0, 0, 0, 0, 0, 1]}

NullSpace of B^*

{[0, -1, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, -1, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{2}{5} & \frac{8}{15} & \frac{2}{3} & 1 & \frac{3}{5} & \frac{7}{15} & \frac{1}{3} \\ \frac{2}{5} & 0 & \frac{4}{15} & \frac{2}{3} & \frac{3}{5} & 1 & \frac{11}{15} & \frac{1}{3} \\ \frac{8}{15} & \frac{4}{15} & 0 & \frac{2}{5} & \frac{7}{15} & \frac{11}{15} & 1 & \frac{3}{5} \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{5} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{3}{5} & 1 \\ 1 & \frac{3}{5} & \frac{7}{15} & \frac{1}{3} & 0 & \frac{2}{5} & \frac{8}{15} & \frac{2}{3} \\ \frac{3}{5} & 1 & \frac{11}{15} & \frac{1}{3} & \frac{2}{5} & 0 & \frac{4}{15} & \frac{2}{3} \\ \frac{7}{15} & \frac{11}{15} & 1 & \frac{3}{5} & \frac{8}{15} & \frac{4}{15} & 0 & \frac{2}{5} \\ \frac{1}{3} & \frac{1}{3} & \frac{3}{5} & 1 & \frac{2}{3} & \frac{2}{3} & \frac{2}{5} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3   "Trace mark", 2, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8] \right)$

degree 2: $\frac{1}{6}\left(v[1]v[5] + v[2]v[6] + 2v[3]v[7] + 2v[4]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 3, 8}, {4, 5, 6, 7}}

"PT2" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT3" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT4" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT5" = {{1, 3, 4, 6}, {2, 5, 7, 8}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$\pi 2$ = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {4, 11, 17, 22}

$u2$ = [6, 8, 10, 15, 9, 7, 5, 4, 10, 9, 15, 11, 5, 6, 7, 11, 15, 9, 5, 5, 9, 15, 6, 8, 10, 4, 10, 6]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[2], [4], [2], [1]]
Action of B on ranges, [[3], [1], [1], [2]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

RPARTS [4, 2, 2, 5, 1, 1]

BPARTS [3, 6, 1, 3, 6, 1]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{4}{15} & \frac{1}{5} & \frac{1}{5} & \frac{2}{15} & \frac{1}{15} & \frac{2}{15} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 6, 9, 9, A, A, C, B, 2, 1, C, B]

B-BLOCKS,

[7, 8, 5, A, 9, 4, 9, A, 8, 7, 4, 5]

with invariant measure, [2, 2, 1, 2, 2, 1, 3, 3, 4, 4, 3, 3]

N by blocks, N - check: true

b_1 = {1, 4, 6, 7}
b_2 = {2, 3, 5, 8}
b_3 = {1, 3, 4, 6}
b_4 = {1, 2, 3, 4}
b_5 = {5, 6, 7, 8}
b_6 = {2, 5, 7, 8}
b_7 = {1, 6, 7, 8}
b_8 = {2, 3, 4, 5}

b_9 = {1, 2, 3, 8}
b_{10} = {4, 5, 6, 7}
b_{11} = {1, 2, 7, 8}
b_{12} = {3, 4, 5, 6}

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \\ h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 25, Shape: 18 ⊕ 7/5

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3, 4, 5, 7, 8}}, true

Ω_B in Vec(K)? , {{2, 4, 6, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 3, 8}, {4, 5, 6, 7}}

1, "range", [4, 8], [[8, 8, 8, 4, 4, 4, 4, 8], [4, 4, 4, 8, 8, 8, 8, 4]]

2, "range", [3, 7], [[7, 7, 7, 3, 3, 3, 3, 7], [3, 3, 3, 7, 7, 7, 7, 3]]

3, "range", [2, 6], [[6, 6, 6, 2, 2, 2, 2, 6], [2, 2, 2, 6, 6, 6, 6, 2]]

4, "range", [1, 5], [[5, 5, 5, 1, 1, 1, 1, 5], [1, 1, 1, 5, 5, 5, 5, 1]]

2, "partition", {{1, 2, 7, 8}, {3, 4, 5, 6}}

1, "range", [4, 8], [[8, 8, 4, 4, 4, 4, 8, 8], [4, 4, 8, 8, 8, 8, 4, 4]]

2, "range", [3, 7], [[7, 7, 3, 3, 3, 3, 7, 7], [3, 3, 7, 7, 7, 7, 3, 3]]

3, "range", [2, 6], [[6, 6, 2, 2, 2, 2, 6, 6], [2, 2, 6, 6, 6, 6, 2, 2]]

4, "range", [1, 5], [[5, 5, 1, 1, 1, 1, 5, 5], [1, 1, 5, 5, 5, 5, 1, 1]]

3, "partition", {{1, 6, 7, 8}, {2, 3, 4, 5}}

1, "range", [4, 8], [[8, 4, 4, 4, 4, 8, 8, 8], [4, 8, 8, 8, 8, 4, 4, 4]]

2, "range", [3, 7], [[7, 3, 3, 3, 3, 7, 7, 7], [3, 7, 7, 7, 7, 3, 3, 3]]

3, "range", [2, 6], [[6, 2, 2, 2, 2, 6, 6, 6], [2, 6, 6, 6, 6, 2, 2, 2]]

4, "range", [1, 5], [[5, 1, 1, 1, 1, 5, 5, 5], [1, 5, 5, 5, 5, 1, 1, 1]]

4, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

2, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

3, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

5, "partition", {{1, 3, 4, 6}, {2, 5, 7, 8}}

1, "range", [4, 8], [[8, 4, 8, 8, 4, 8, 4, 4], [4, 8, 4, 4, 8, 4, 8, 8]]

2, "range", [3, 7], [[7, 3, 7, 7, 3, 7, 3, 3], [3, 7, 3, 3, 7, 3, 7, 7]]

3, "range", [2, 6], [[6, 2, 6, 6, 2, 6, 2, 2], [2, 6, 2, 2, 6, 2, 6, 6]]

4, "range", [1, 5], [[5, 1, 5, 5, 1, 5, 1, 1], [1, 5, 1, 1, 5, 1, 5, 5]]

6, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [4, 8], [[8, 8, 8, 8, 4, 4, 4, 4], [4, 4, 4, 4, 8, 8, 8, 8]]

2, "range", [3, 7], [[7, 7, 7, 7, 3, 3, 3, 3], [3, 3, 3, 3, 7, 7, 7, 7]]

3, "range", [2, 6], [[6, 6, 6, 6, 2, 2, 2, 2], [2, 2, 2, 2, 6, 6, 6, 6]]

4, "range", [1, 5], [[5, 5, 5, 5, 1, 1, 1, 1], [1, 1, 1, 1, 5, 5, 5, 5]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

{4, 11, 17, 22}

$u2 =$
$$\begin{pmatrix} 6 & 8 & 10 & 15 & 9 & 7 & 5 & 4 & 10 & 9 & 15 & 11 & 5 & 6 & 7 & 11 & 15 & 9 & 5 & 5 & 9 & 15 & 6 & 8 & 10 & 4 \end{pmatrix}$$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem−checks} \quad \text{N0−checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{7}{15} & 0 & 0 & 0 & \frac{8}{15} & 0 \\ 0 & 0 & \frac{11}{15} & 0 & 0 & 0 & \frac{4}{15} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 \\ 0 & 0 & \frac{8}{15} & 0 & 0 & 0 & \frac{7}{15} & 0 \\ 0 & 0 & \frac{4}{15} & 0 & 0 & 0 & \frac{11}{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 \end{pmatrix} \quad \text{idem−checks} \quad \text{N0−checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{11}{15} & 0 & 0 & 0 & \frac{4}{15} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{4}{15} & 0 & 0 & 0 & \frac{11}{15} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 \\ \frac{7}{15} & 0 & 0 & 0 & \frac{8}{15} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 \\ \frac{8}{15} & 0 & 0 & 0 & \frac{7}{15} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{10} & \frac{7}{45} & \frac{1}{9} & 0 & \frac{1}{15} & \frac{8}{45} & \frac{2}{9} \\ \frac{1}{10} & \frac{1}{6} & \frac{11}{45} & \frac{1}{9} & \frac{1}{15} & 0 & \frac{4}{45} & \frac{2}{9} \\ \frac{7}{90} & \frac{11}{90} & \frac{1}{3} & \frac{1}{5} & \frac{4}{45} & \frac{2}{45} & 0 & \frac{2}{15} \\ \frac{1}{18} & \frac{1}{18} & \frac{1}{5} & \frac{1}{3} & \frac{1}{9} & \frac{1}{9} & \frac{2}{15} & 0 \\ 0 & \frac{1}{15} & \frac{8}{45} & \frac{2}{9} & \frac{1}{6} & \frac{1}{10} & \frac{7}{45} & \frac{1}{9} \\ \frac{1}{15} & 0 & \frac{4}{45} & \frac{2}{9} & \frac{1}{10} & \frac{1}{6} & \frac{11}{45} & \frac{1}{9} \\ \frac{4}{45} & \frac{2}{45} & 0 & \frac{2}{15} & \frac{7}{90} & \frac{11}{90} & \frac{1}{3} & \frac{1}{5} \\ \frac{1}{9} & \frac{1}{9} & \frac{2}{15} & 0 & \frac{1}{18} & \frac{1}{18} & \frac{1}{5} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & \frac{8}{5} & \frac{112}{45} & \frac{16}{9} & 0 & \frac{16}{15} & \frac{128}{45} & \frac{32}{9} \\ \frac{8}{5} & \frac{8}{3} & \frac{176}{45} & \frac{16}{9} & \frac{16}{15} & 0 & \frac{64}{45} & \frac{32}{9} \\ \frac{56}{45} & \frac{88}{45} & \frac{16}{3} & \frac{16}{5} & \frac{64}{45} & \frac{32}{45} & 0 & \frac{32}{15} \\ \frac{8}{9} & \frac{8}{9} & \frac{16}{5} & \frac{16}{3} & \frac{16}{9} & \frac{16}{9} & \frac{32}{15} & 0 \\ 0 & \frac{16}{15} & \frac{128}{45} & \frac{32}{9} & \frac{8}{3} & \frac{8}{5} & \frac{112}{45} & \frac{16}{9} \\ \frac{16}{15} & 0 & \frac{64}{45} & \frac{32}{9} & \frac{8}{5} & \frac{8}{3} & \frac{176}{45} & \frac{16}{9} \\ \frac{64}{45} & \frac{32}{45} & 0 & \frac{32}{15} & \frac{56}{45} & \frac{88}{45} & \frac{16}{3} & \frac{16}{5} \\ \frac{16}{9} & \frac{16}{9} & \frac{32}{15} & 0 & \frac{8}{9} & \frac{8}{9} & \frac{16}{5} & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, 1, -1, 1, -1, 1, -1]$

$$\ker N_C = \begin{pmatrix} -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & s-t & -s+t & 0 & 0 & s-t & -s+t \\ s & 0 & -t & -s+t & s & 0 & -t & -s+t \\ 0 & t & s-t & -s & 0 & t & s-t & -s \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & t & -s \\ 0 & 0 & -s & t \\ -s & 0 & 0 & t \\ 0 & -t & s & 0 \\ 0 & 0 & -t & s \\ 0 & 0 & s & -t \\ s & 0 & 0 & -t \\ 0 & t & -s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ -1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & t & t & 0 & -t \\ -t & t & t & 0 & s \\ -t & t & s+t & 0 & 0 \\ 0 & s & s & t & -s \\ -s & s & s & 0 & t \\ t & s & s & 0 & -s \\ t & s & 0 & 0 & 0 \\ 0 & t & t & -t & s \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 4 & 0 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & \frac{3}{5} & \frac{1}{3} & \frac{2}{5} & 0 & \frac{2}{5} & \frac{2}{3} & \frac{3}{5} \\ \frac{3}{5} & 1 & \frac{7}{15} & 0 & \frac{2}{5} & 0 & \frac{8}{15} & 1 \\ \frac{1}{3} & \frac{7}{15} & 1 & \frac{8}{15} & \frac{2}{3} & \frac{8}{15} & 0 & \frac{7}{15} \\ \frac{2}{5} & 0 & \frac{8}{15} & 1 & \frac{3}{5} & 1 & \frac{7}{15} & 0 \\ 0 & \frac{2}{5} & \frac{2}{3} & \frac{3}{5} & 1 & \frac{3}{5} & \frac{1}{3} & \frac{2}{5} \\ \frac{2}{5} & 0 & \frac{8}{15} & 1 & \frac{3}{5} & 1 & \frac{7}{15} & 0 \\ \frac{2}{3} & \frac{8}{15} & 0 & \frac{7}{15} & \frac{1}{3} & \frac{7}{15} & 1 & \frac{8}{15} \\ \frac{3}{5} & 1 & \frac{7}{15} & 0 & \frac{2}{5} & 0 & \frac{8}{15} & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{3}{5} & \frac{3}{5} & \frac{4}{15} & 0 & \frac{2}{5} & \frac{2}{5} & \frac{11}{15} \\ \frac{3}{5} & 1 & 1 & \frac{2}{3} & \frac{2}{5} & 0 & 0 & \frac{1}{3} \\ \frac{3}{5} & 1 & 1 & \frac{2}{3} & \frac{2}{5} & 0 & 0 & \frac{1}{3} \\ \frac{4}{15} & \frac{2}{3} & \frac{2}{3} & 1 & \frac{11}{15} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{2}{5} & \frac{2}{5} & \frac{11}{15} & 1 & \frac{3}{5} & \frac{3}{5} & \frac{4}{15} \\ \frac{2}{5} & 0 & 0 & \frac{1}{3} & \frac{3}{5} & 1 & 1 & \frac{2}{3} \\ \frac{2}{5} & 0 & 0 & \frac{1}{3} & \frac{3}{5} & 1 & 1 & \frac{2}{3} \\ \frac{11}{15} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{4}{15} & \frac{2}{3} & \frac{2}{3} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{7}{90} & \frac{1}{18} & 0 & 0 & \frac{4}{45} & \frac{1}{9} \\ 0 & 0 & \frac{11}{90} & \frac{1}{18} & 0 & 0 & \frac{2}{45} & \frac{1}{9} \\ \frac{-7}{90} & \frac{-11}{90} & 0 & 0 & \frac{-4}{45} & \frac{-2}{45} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{18} & 0 & 0 & \frac{-1}{9} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{4}{45} & \frac{1}{9} & 0 & 0 & \frac{7}{90} & \frac{1}{18} \\ 0 & 0 & \frac{2}{45} & \frac{1}{9} & 0 & 0 & \frac{11}{90} & \frac{1}{18} \\ \frac{-4}{45} & \frac{-2}{45} & 0 & 0 & \frac{-7}{90} & \frac{-11}{90} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{9} & 0 & 0 & \frac{-1}{18} & \frac{-1}{18} & 0 & 0 \end{pmatrix}$$

Skew Omega =

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \\ \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{3}{5} & \frac{7}{15} & \frac{1}{3} & 0 & \frac{2}{5} & \frac{8}{15} & \frac{2}{3} \\ \frac{3}{5} & 1 & \frac{11}{15} & \frac{1}{3} & \frac{2}{5} & 0 & \frac{4}{15} & \frac{2}{3} \\ \frac{7}{15} & \frac{11}{15} & 1 & \frac{3}{5} & \frac{8}{15} & \frac{4}{15} & 0 & \frac{2}{5} \\ \frac{1}{3} & \frac{1}{3} & \frac{3}{5} & 1 & \frac{2}{3} & \frac{2}{3} & \frac{2}{5} & 0 \\ 0 & \frac{2}{5} & \frac{8}{15} & \frac{2}{3} & 1 & \frac{3}{5} & \frac{7}{15} & \frac{1}{3} \\ \frac{2}{5} & 0 & \frac{4}{15} & \frac{2}{3} & \frac{3}{5} & 1 & \frac{11}{15} & \frac{1}{3} \\ \frac{8}{15} & \frac{4}{15} & 0 & \frac{2}{5} & \frac{7}{15} & \frac{11}{15} & 1 & \frac{3}{5} \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{5} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{3}{5} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(\frac{1}{3}\ \frac{1}{5}\ \frac{1}{18}\ \frac{1}{18}\ \frac{1}{5}\ \frac{1}{3}\ \frac{11}{90}\ \frac{7}{90}\ \frac{1}{9}\ \frac{11}{45}\ \frac{1}{6}\ \frac{1}{10}\ \frac{2}{9}\ \frac{8}{45}\ \frac{1}{15}\ 0\ \frac{1}{9}\ \frac{7}{45}\ \frac{1}{10}\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM}\left(\frac{16}{3}\ \frac{16}{5}\ \frac{8}{9}\ \frac{8}{9}\ \frac{16}{5}\ \frac{16}{3}\ \frac{88}{45}\ \frac{56}{45}\ \frac{16}{9}\ \frac{176}{45}\ \frac{8}{3}\ \frac{8}{5}\ \frac{32}{9}\ \frac{128}{45}\ \frac{16}{15}\ 0\ \frac{16}{9}\ \frac{112}{45}\ \frac{8}{5}\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{14}{3}\ \frac{38}{15}\ \frac{10}{9}\ \frac{10}{9}\ \frac{38}{15}\ \frac{14}{3}\ \frac{146}{45}\ \frac{82}{45}\ \frac{14}{9}\ \frac{118}{45}\ \frac{10}{3}\ \frac{34}{15}\ \frac{22}{9}\ \frac{94}{45}\ \frac{26}{15}\ \frac{2}{3}\ \frac{14}{9}\ \frac{86}{45}\ \frac{34}{15}\ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1,\ \text{rank} = 2,\ \text{ratio} = 16/1,\ n^2/r = 32/1$$

$$\boldsymbol{\tau}' = 32/1,\ r' = 1/2,\ \boldsymbol{\tau}/n^2 = 1/2$$

$$p^2 = 5/36,\ \min \boldsymbol{\tau} = 80/9,\ \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\max r = 36/5,\ r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 2, 3, 8}, {4, 5, 6, 7}}

"PT2" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT3" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT4" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT5" = {{1, 3, 4, 6}, {2, 5, 7, 8}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \\ \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{83}{180} & \frac{59}{180} & \frac{7}{36} & \frac{-5}{36} & \frac{47}{180} & \frac{71}{180} & \frac{19}{36} \\ \frac{83}{180} & \frac{31}{36} & \frac{107}{180} & \frac{7}{36} & \frac{47}{180} & \frac{-5}{36} & \frac{23}{180} & \frac{19}{36} \\ \frac{59}{180} & \frac{107}{180} & \frac{31}{36} & \frac{83}{180} & \frac{71}{180} & \frac{23}{180} & \frac{-5}{36} & \frac{47}{180} \\ \frac{7}{36} & \frac{7}{36} & \frac{83}{180} & \frac{31}{36} & \frac{19}{36} & \frac{19}{36} & \frac{47}{180} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{47}{180} & \frac{71}{180} & \frac{19}{36} & \frac{31}{36} & \frac{83}{180} & \frac{59}{180} & \frac{7}{36} \\ \frac{47}{180} & \frac{-5}{36} & \frac{23}{180} & \frac{19}{36} & \frac{83}{180} & \frac{31}{36} & \frac{107}{180} & \frac{7}{36} \\ \frac{71}{180} & \frac{23}{180} & \frac{-5}{36} & \frac{47}{180} & \frac{59}{180} & \frac{107}{180} & \frac{31}{36} & \frac{83}{180} \\ \frac{19}{36} & \frac{19}{36} & \frac{47}{180} & \frac{-5}{36} & \frac{7}{36} & \frac{7}{36} & \frac{83}{180} & \frac{31}{36} \end{pmatrix}$$

$$M_c\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\ 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_c\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{83}{155} & \frac{59}{155} & \frac{7}{31} & \frac{-5}{31} & \frac{47}{155} & \frac{71}{155} & \frac{19}{31} \\ \frac{83}{155} & 1 & \frac{107}{155} & \frac{7}{31} & \frac{47}{155} & \frac{-5}{31} & \frac{23}{155} & \frac{19}{31} \\ \frac{59}{155} & \frac{107}{155} & 1 & \frac{83}{155} & \frac{71}{155} & \frac{23}{155} & \frac{-5}{31} & \frac{47}{155} \\ \frac{7}{31} & \frac{7}{31} & \frac{83}{155} & 1 & \frac{19}{31} & \frac{19}{31} & \frac{47}{155} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{47}{155} & \frac{71}{155} & \frac{19}{31} & 1 & \frac{83}{155} & \frac{59}{155} & \frac{7}{31} \\ \frac{47}{155} & \frac{-5}{31} & \frac{23}{155} & \frac{19}{31} & \frac{83}{155} & 1 & \frac{107}{155} & \frac{7}{31} \\ \frac{71}{155} & \frac{23}{155} & \frac{-5}{31} & \frac{47}{155} & \frac{59}{155} & \frac{107}{155} & 1 & \frac{83}{155} \\ \frac{19}{31} & \frac{19}{31} & \frac{47}{155} & \frac{-5}{31} & \frac{7}{31} & \frac{7}{31} & \frac{83}{155} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[0., 0., 0., 2.888888889, 0.3212986242, 0.7071405096, 1.362246242, 1.609314622]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C–scaled

[0., 0., 0., 3.354838710, 0.3731209831, 0.8211954308, 1.581963379, 1.868881498]

NullSpace M_C

{[0, 0, −1, 0, 0, 0, 1, 0], [1, 1, 1, 0, 0, 0, 0, 1], [1, 1, 1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 1, 0, 0]}

NullSpace N_C

{[−1, 0, 0, 1, −1, 0, 0, 1], [−1, 0, 1, 0, −1, 0, 1, 0], [−1, 1, 0, 0, −1, 1, 0, 0]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 0.3212986242, 0.7071405096, 1.362246242, 1.609314622]

NullSpace M_0

{[0, −1, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [−1, 0, 0, 0, 1, 0, 0, 0]}

NullSpace N_0

{[−1, 0, 0, 1, −1, 0, 0, 1], [−1, 1, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, −1, 0, 1, 0]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 4., −0.3212986242, −0.7071405096, −1.362246242, −1.609314622]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 1, 0, −1, 0, 1], [0, −1, 1, 0, 0, −1, 1, 0], [1, −1, 0, 0, 1, −1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 6 & 8 & 10 & 15 & 9 & 7 & 5 \\ 6 & 0 & 4 & 10 & 9 & 15 & 11 & 5 \\ 8 & 4 & 0 & 6 & 7 & 11 & 15 & 9 \\ 10 & 10 & 6 & 0 & 5 & 5 & 9 & 15 \\ 15 & 9 & 7 & 5 & 0 & 6 & 8 & 10 \\ 9 & 15 & 11 & 5 & 6 & 0 & 4 & 10 \\ 7 & 11 & 15 & 9 & 8 & 4 & 0 & 6 \\ 5 & 5 & 9 & 15 & 10 & 10 & 6 & 0 \end{pmatrix}$$

==

{4, 5, 6, 8}

R: [4, 3, 1, 7, 8, 7, 4, 6]
B: [7, 8, 8, 2, 3, 4, 5, 3]

TRACE TWO = 1

det AT = $\frac{1}{16}(-1+t)^2(1+t)^2(t)^2$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{5}{268435456}(6400-2560s+1120s^2-192s^3+121s^4-10s^5+s^6)(6-s+s^2)(-444+24s-181s^2-38s^3-4s^4-2s^5+5s^6)(-1+s)(2+s)(1+s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[4] v[7]
"B CYCLES", 1 + v[3] v[8]

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of R^*

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 1, 0, 0]}

NullSpace of B^*

{[0, −1, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{2} & \frac{3}{5} & \frac{7}{10} & \frac{1}{2} & 1 & \frac{3}{10} & \frac{2}{5} \\ \frac{1}{2} & 0 & \frac{3}{10} & \frac{3}{5} & 1 & \frac{1}{2} & \frac{2}{5} & \frac{7}{10} \\ \frac{3}{5} & \frac{3}{10} & 0 & \frac{1}{2} & \frac{7}{10} & \frac{2}{5} & \frac{1}{2} & 1 \\ \frac{7}{10} & \frac{3}{5} & \frac{1}{2} & 0 & \frac{2}{5} & \frac{3}{10} & 1 & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{7}{10} & \frac{2}{5} & 0 & \frac{1}{2} & \frac{3}{5} & \frac{3}{10} \\ 1 & \frac{1}{2} & \frac{2}{5} & \frac{3}{10} & \frac{1}{2} & 0 & \frac{7}{10} & \frac{3}{5} \\ \frac{3}{10} & \frac{2}{5} & \frac{1}{2} & 1 & \frac{3}{5} & \frac{7}{10} & 0 & \frac{1}{2} \\ \frac{2}{5} & \frac{7}{10} & 1 & \frac{1}{2} & \frac{3}{10} & \frac{3}{5} & \frac{1}{2} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 1 "Trace mark", 0, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\left(v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8] \right)$
degree 2: $\frac{1}{6}\left(v[1]v[6] + v[2]v[5] + 2\,v[3]v[8] + 2\,v[4]v[7] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 5, 7, 8}, {2, 3, 4, 6}}

"PT4" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"PT5" = {{4, 5, 6, 8}, {1, 2, 3, 7}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {3, 8}

"RG2" = {4, 7}

"RG3" = {2, 5}

"RG4" = {1, 6}

$$\pi 2 = [0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0]$$

supp $\pi 2$ = {5, 10, 18, 21}

$$u2 = [5, 6, 7, 5, 10, 3, 4, 3, 6, 10, 5, 4, 7, 5, 7, 4, 5, 10, 4, 3, 10, 5, 5, 6, 3, 7, 6, 5]$$

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[4], [2], [1], [2]]
Action of B on ranges, [[1], [3], [1], [2]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

RPARTS [1, 4, 1, 3, 3, 5]

BPARTS [6, 2, 5, 3, 2, 5]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{10} & \frac{1}{5} & \frac{1}{10} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[B, A, C, 8, 1, 2, 9, A, 7, 9, 7, B]

B−BLOCKS,

[A, B, 8, C, 6, 5, 3, 6, 4, C, 8, 5]

with invariant measure, [1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2]

N by blocks, N − check: true

b_1 = {1, 3, 5, 7}
b_2 = {2, 4, 6, 8}
b_3 = {1, 2, 3, 4}
b_4 = {5, 6, 7, 8}
b_5 = {1, 4, 5, 8}
b_6 = {2, 3, 6, 7}
b_7 = {1, 2, 7, 8}
b_8 = {4, 5, 6, 8}

b_9 = {3, 4, 5, 6}
b_{10} = {1, 5, 7, 8}
b_{11} = {2, 3, 4, 6}
b_{12} = {1, 2, 3, 7}

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & h[1] & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & 0 & h[2] \\ 0 & 0 & 0 & h[1] & 0 & 0 & h[2] & 0 \\ 0 & h[2] & 0 & 0 & h[1] & 0 & 0 & 0 \\ h[2] & 0 & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & h[1] & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: 23 ⊕ 4/3

$$\text{CLB} = \begin{pmatrix} 0 & -1 & 0 & 1 & 1 \\ -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{4, 7}}, true

Ω_B in Vec(K)? , {{3, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 7, 8}, {3, 4, 5, 6}}

1, "range", [3, 8], [[8, 8, 3, 3, 3, 3, 8, 8], [3, 3, 8, 8, 8, 8, 3, 3]]

2, "range", [4, 7], [[7, 7, 4, 4, 4, 4, 7, 7], [4, 4, 7, 7, 7, 7, 4, 4]]

3, "range", [2, 5], [[5, 5, 2, 2, 2, 2, 5, 5], [2, 2, 5, 5, 5, 5, 2, 2]]

4, "range", [1, 6], [[6, 6, 1, 1, 1, 1, 6, 6], [1, 1, 6, 6, 6, 6, 1, 1]]

2, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [3, 8], [[8, 3, 3, 8, 8, 3, 3, 8], [3, 8, 8, 3, 3, 8, 8, 3]]

2, "range", [4, 7], [[7, 4, 4, 7, 7, 4, 4, 7], [4, 7, 7, 4, 4, 7, 7, 4]]

3, "range", [2, 5], [[5, 2, 2, 5, 5, 2, 2, 5], [2, 5, 5, 2, 2, 5, 5, 2]]

4, "range", [1, 6], [[6, 1, 1, 6, 6, 1, 1, 6], [1, 6, 6, 1, 1, 6, 6, 1]]

3, "partition", {{1, 5, 7, 8}, {2, 3, 4, 6}}

1, "range", [3, 8], [[8, 3, 3, 3, 8, 3, 8, 8], [3, 8, 8, 8, 3, 8, 3, 3]]

2, "range", [4, 7], [[7, 4, 4, 4, 7, 4, 7, 7], [4, 7, 7, 7, 4, 7, 4, 4]]

3, "range", [2, 5], [[5, 2, 2, 2, 5, 2, 5, 5], [2, 5, 5, 5, 2, 5, 2, 2]]

4, "range", [1, 6], [[6, 1, 1, 1, 6, 1, 6, 6], [1, 6, 6, 6, 1, 6, 1, 1]]

4, "partition", {{1, 3, 5, 7}, {2, 4, 6, 8}}

1, "range", [3, 8], [[8, 3, 8, 3, 8, 3, 8, 3], [3, 8, 3, 8, 3, 8, 3, 8]]

2, "range", [4, 7], [[7, 4, 7, 4, 7, 4, 7, 4], [4, 7, 4, 7, 4, 7, 4, 7]]

3, "range", [2, 5], [[5, 2, 5, 2, 5, 2, 5, 2], [2, 5, 2, 5, 2, 5, 2, 5]]

4, "range", [1, 6], [[6, 1, 6, 1, 6, 1, 6, 1], [1, 6, 1, 6, 1, 6, 1, 6]]

5, "partition", {{4, 5, 6, 8}, {1, 2, 3, 7}}

1, "range", [3, 8], [[8, 8, 8, 3, 3, 3, 8, 3], [3, 3, 3, 8, 8, 8, 3, 8]]

2, "range", [4, 7], [[7, 7, 7, 4, 4, 4, 7, 4], [4, 4, 4, 7, 7, 7, 4, 7]]

3, "range", [2, 5], [[5, 5, 5, 2, 2, 2, 5, 2], [2, 2, 2, 5, 5, 5, 2, 5]]

4, "range", [1, 6], [[6, 6, 6, 1, 1, 1, 6, 1], [1, 1, 1, 6, 6, 6, 1, 6]]

6, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [3, 8], [[8, 8, 8, 8, 3, 3, 3, 3], [3, 3, 3, 3, 8, 8, 8, 8]]

2, "range", [4, 7], [[7, 7, 7, 7, 4, 4, 4, 4], [4, 4, 4, 4, 7, 7, 7, 7]]

3, "range", [2, 5], [[5, 5, 5, 5, 2, 2, 2, 2], [2, 2, 2, 2, 5, 5, 5, 5]]

4, "range", [1, 6], [[6, 6, 6, 6, 1, 1, 1, 1], [1, 1, 1, 1, 6, 6, 6, 6]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

{5, 10, 18, 21}

$u2 =$
$$\begin{pmatrix} 5 & 6 & 7 & 5 & 10 & 3 & 4 & 3 & 6 & 10 & 5 & 4 & 7 & 5 & 7 & 4 & 5 & 10 & 4 & 3 & 10 & 5 & 5 & 6 & 3 & 7 & 6 & 5 \end{pmatrix}$$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & 0 & \frac{3}{5} \\ 0 & 0 & \frac{7}{10} & 0 & 0 & 0 & 0 & \frac{3}{10} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{3}{10} & 0 & 0 & 0 & 0 & \frac{7}{10} \\ 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & 0 & \frac{2}{5} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{3}{10} & 0 & 0 & \frac{7}{10} & 0 \\ 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & \frac{2}{5} & 0 \\ 0 & 0 & 0 & \frac{7}{10} & 0 & 0 & \frac{3}{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{7}{10} & 0 & 0 & \frac{3}{10} & 0 & 0 & 0 \\ 0 & \frac{2}{5} & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{3}{5} & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 \\ 0 & \frac{3}{10} & 0 & 0 & \frac{7}{10} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{2}{5} & 0 & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 \\ \frac{3}{10} & 0 & 0 & 0 & 0 & \frac{7}{10} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{7}{10} & 0 & 0 & 0 & 0 & \frac{3}{10} & 0 & 0 \\ \frac{3}{5} & 0 & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{1}{12} & \frac{2}{15} & \frac{1}{10} & \frac{1}{12} & 0 & \frac{7}{30} & \frac{1}{5} \\ \frac{1}{12} & \frac{1}{6} & \frac{7}{30} & \frac{2}{15} & 0 & \frac{1}{12} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{15} & \frac{7}{60} & \frac{1}{3} & \frac{1}{6} & \frac{1}{20} & \frac{1}{10} & \frac{1}{6} & 0 \\ \frac{1}{20} & \frac{1}{15} & \frac{1}{6} & \frac{1}{3} & \frac{1}{10} & \frac{7}{60} & 0 & \frac{1}{6} \\ \frac{1}{12} & 0 & \frac{1}{10} & \frac{1}{5} & \frac{1}{6} & \frac{1}{12} & \frac{2}{15} & \frac{7}{30} \\ 0 & \frac{1}{12} & \frac{1}{5} & \frac{7}{30} & \frac{1}{12} & \frac{1}{6} & \frac{1}{10} & \frac{2}{15} \\ \frac{7}{60} & \frac{1}{10} & \frac{1}{6} & 0 & \frac{1}{15} & \frac{1}{20} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{10} & \frac{1}{20} & 0 & \frac{1}{6} & \frac{7}{60} & \frac{1}{15} & \frac{1}{6} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & \frac{32}{15} & \frac{8}{5} & \frac{4}{3} & 0 & \frac{56}{15} & \frac{16}{5} \\ \frac{4}{3} & \frac{8}{3} & \frac{56}{15} & \frac{32}{15} & 0 & \frac{4}{3} & \frac{16}{5} & \frac{8}{5} \\ \frac{16}{15} & \frac{28}{15} & \frac{16}{3} & \frac{8}{3} & \frac{4}{5} & \frac{8}{5} & \frac{8}{3} & 0 \\ \frac{4}{5} & \frac{16}{15} & \frac{8}{3} & \frac{16}{3} & \frac{8}{5} & \frac{28}{15} & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & \frac{8}{5} & \frac{16}{5} & \frac{8}{3} & \frac{4}{3} & \frac{32}{15} & \frac{56}{15} \\ 0 & \frac{4}{3} & \frac{16}{5} & \frac{56}{15} & \frac{4}{3} & \frac{8}{3} & \frac{8}{5} & \frac{32}{15} \\ \frac{28}{15} & \frac{8}{5} & \frac{8}{3} & 0 & \frac{16}{15} & \frac{4}{5} & \frac{16}{3} & \frac{8}{3} \\ \frac{8}{5} & \frac{4}{5} & 0 & \frac{8}{3} & \frac{28}{15} & \frac{16}{15} & \frac{8}{3} & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, -1, 1, -1, 1, 1, -1]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 & -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} s & 0 & t & -s-t & 0 & s & -s-t & t \\ 0 & t & 0 & -t & t & 0 & -t & 0 \\ 0 & 0 & s+t & -s-t & 0 & 0 & -s-t & s+t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & -s+t & 0 \\ 0 & 0 & 0 & -s+t \\ 0 & -s & 0 & t \\ -t & 0 & s & 0 \\ 0 & 0 & 0 & -t+s \\ 0 & 0 & -t+s & 0 \\ t & 0 & -s & 0 \\ 0 & s & 0 & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & 0 & t & 0 \\ 0 & s & 0 & s & -s+t \\ -s & s & 0 & s & t \\ 0 & t & -t & t+s & 0 \\ 0 & t & 0 & t & -t+s \\ 0 & t & 0 & s & 0 \\ 0 & s & t & 0 & 0 \\ s & t & 0 & t & -t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^\dagger = \begin{pmatrix} 0 & 4 & 0 & 4 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{10} & 0 & \frac{1}{2} & 0 & 1 & \frac{7}{10} \\ \frac{1}{2} & 1 & \frac{2}{5} & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{5} \\ \frac{3}{10} & \frac{2}{5} & 1 & \frac{7}{10} & \frac{3}{5} & \frac{7}{10} & \frac{3}{10} & 0 \\ 0 & \frac{1}{2} & \frac{7}{10} & 1 & \frac{1}{2} & 1 & 0 & \frac{3}{10} \\ \frac{1}{2} & 0 & \frac{3}{5} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{2}{5} \\ 0 & \frac{1}{2} & \frac{7}{10} & 1 & \frac{1}{2} & 1 & 0 & \frac{3}{10} \\ 1 & \frac{1}{2} & \frac{3}{10} & 0 & \frac{1}{2} & 0 & 1 & \frac{7}{10} \\ \frac{7}{10} & \frac{3}{5} & 0 & \frac{3}{10} & \frac{2}{5} & \frac{3}{10} & \frac{7}{10} & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{5} & \frac{1}{2} & 0 & \frac{2}{5} & \frac{1}{2} \\ \frac{1}{2} & 1 & 1 & \frac{3}{10} & 0 & \frac{1}{2} & \frac{7}{10} & 0 \\ \frac{1}{2} & 1 & 1 & \frac{3}{10} & 0 & \frac{1}{2} & \frac{7}{10} & 0 \\ \frac{3}{5} & \frac{3}{10} & \frac{3}{10} & 1 & \frac{7}{10} & \frac{2}{5} & 0 & \frac{7}{10} \\ \frac{1}{2} & 0 & 0 & \frac{7}{10} & 1 & \frac{1}{2} & \frac{3}{10} & 1 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{2}{5} & \frac{1}{2} & 1 & \frac{3}{5} & \frac{1}{2} \\ \frac{2}{5} & \frac{7}{10} & \frac{7}{10} & 0 & \frac{3}{10} & \frac{3}{5} & 1 & \frac{3}{10} \\ \frac{1}{2} & 0 & 0 & \frac{7}{10} & 1 & \frac{1}{2} & \frac{3}{10} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{15} & \frac{1}{20} & 0 & 0 & \frac{7}{60} & \frac{1}{10} \\ 0 & 0 & \frac{7}{60} & \frac{1}{15} & 0 & 0 & \frac{1}{10} & \frac{1}{20} \\ \frac{-1}{15} & \frac{-7}{60} & 0 & 0 & \frac{-1}{20} & \frac{-1}{10} & 0 & 0 \\ \frac{-1}{20} & \frac{-1}{15} & 0 & 0 & \frac{-1}{10} & \frac{-7}{60} & 0 & 0 \\ 0 & 0 & \frac{1}{20} & \frac{1}{10} & 0 & 0 & \frac{1}{15} & \frac{7}{60} \\ 0 & 0 & \frac{1}{10} & \frac{7}{60} & 0 & 0 & \frac{1}{20} & \frac{1}{15} \\ \frac{-7}{60} & \frac{-1}{10} & 0 & 0 & \frac{-1}{15} & \frac{-1}{20} & 0 & 0 \\ \frac{-1}{10} & \frac{-1}{20} & 0 & 0 & \frac{-7}{60} & \frac{-1}{15} & 0 & 0 \end{pmatrix}$$

Skew Omega =

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 & 0 & 0 \\ \frac{8}{3} & 0 & 0 & 0 & 0 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{16}{3} & 0 & 0 & \frac{16}{3} & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 & 0 & 0 & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{2}{5} & \frac{3}{10} & \frac{1}{2} & 0 & \frac{7}{10} & \frac{3}{5} \\ \frac{1}{2} & 1 & \frac{7}{10} & \frac{2}{5} & 0 & \frac{1}{2} & \frac{3}{5} & \frac{3}{10} \\ \frac{2}{5} & \frac{7}{10} & 1 & \frac{1}{2} & \frac{3}{10} & \frac{3}{5} & \frac{1}{2} & 0 \\ \frac{3}{10} & \frac{2}{5} & \frac{1}{2} & 1 & \frac{3}{5} & \frac{7}{10} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{3}{10} & \frac{3}{5} & 1 & \frac{1}{2} & \frac{2}{5} & \frac{7}{10} \\ 0 & \frac{1}{2} & \frac{3}{5} & \frac{7}{10} & \frac{1}{2} & 1 & \frac{3}{10} & \frac{2}{5} \\ \frac{7}{10} & \frac{3}{5} & \frac{1}{2} & 0 & \frac{2}{5} & \frac{3}{10} & 1 & \frac{1}{2} \\ \frac{3}{5} & \frac{3}{10} & 0 & \frac{1}{2} & \frac{7}{10} & \frac{2}{5} & \frac{1}{2} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(\frac{1}{3}\ \frac{1}{6}\ \frac{1}{15}\ \frac{1}{20}\ \frac{1}{6}\ \frac{1}{3}\ \frac{7}{60}\ \frac{1}{15}\ \frac{2}{15}\ \frac{7}{30}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{5}\ \frac{7}{30}\ 0\ \frac{1}{12}\ \frac{1}{10}\ \frac{2}{15}\ \frac{1}{12}\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM}\left(\frac{16}{3}\ \frac{8}{3}\ \frac{16}{15}\ \frac{4}{5}\ \frac{8}{3}\ \frac{16}{3}\ \frac{28}{15}\ \frac{16}{15}\ \frac{32}{15}\ \frac{56}{15}\ \frac{8}{3}\ \frac{4}{3}\ \frac{16}{5}\ \frac{56}{15}\ 0\ \frac{4}{3}\ \frac{8}{5}\ \frac{32}{15}\ \frac{4}{3}\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN}\left(\frac{14}{3}\ 2\ \frac{22}{15}\ \frac{14}{15}\ 2\ \frac{14}{3}\ \frac{46}{15}\ \frac{22}{15}\ \frac{26}{15}\ \frac{38}{15}\ \frac{10}{3}\ 2\ \frac{34}{15}\ \frac{38}{15}\ \frac{2}{3}\ 2\ \frac{22}{15}\ \frac{26}{15}\ 2\ \frac{10}{3}\right)$$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 5/36$, min $\boldsymbol{\tau} = 80/9$, $\boldsymbol{\tau}$-check is positive? 208/9

max r = 36/5 , r-check is positive? 13/18

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 5, 7, 8}, {2, 3, 4, 6}}

"PT4" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"PT5" = {{4, 5, 6, 8}, {1, 2, 3, 7}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {3, 8}

"RG2" = {4, 7}

"RG3" = {2, 5}

"RG4" = {1, 6}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{13}{36} & \frac{47}{180} & \frac{29}{180} & \frac{13}{36} & \frac{-5}{36} & \frac{101}{180} & \frac{83}{180} \\ \frac{13}{36} & \frac{31}{36} & \frac{101}{180} & \frac{47}{180} & \frac{-5}{36} & \frac{13}{36} & \frac{83}{180} & \frac{29}{180} \\ \frac{47}{180} & \frac{101}{180} & \frac{31}{36} & \frac{13}{36} & \frac{29}{180} & \frac{83}{180} & \frac{13}{36} & \frac{-5}{36} \\ \frac{29}{180} & \frac{47}{180} & \frac{13}{36} & \frac{31}{36} & \frac{83}{180} & \frac{101}{180} & \frac{-5}{36} & \frac{13}{36} \\ \frac{13}{36} & \frac{-5}{36} & \frac{29}{180} & \frac{83}{180} & \frac{31}{36} & \frac{13}{36} & \frac{47}{180} & \frac{101}{180} \\ \frac{-5}{36} & \frac{13}{36} & \frac{83}{180} & \frac{101}{180} & \frac{13}{36} & \frac{31}{36} & \frac{29}{180} & \frac{47}{180} \\ \frac{101}{180} & \frac{83}{180} & \frac{13}{36} & \frac{-5}{36} & \frac{47}{180} & \frac{29}{180} & \frac{31}{36} & \frac{13}{36} \\ \frac{83}{180} & \frac{29}{180} & \frac{-5}{36} & \frac{13}{36} & \frac{101}{180} & \frac{47}{180} & \frac{13}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{13}{31} & \frac{47}{155} & \frac{29}{155} & \frac{13}{31} & \frac{-5}{31} & \frac{101}{155} & \frac{83}{155} \\ \frac{13}{31} & 1 & \frac{101}{155} & \frac{47}{155} & \frac{-5}{31} & \frac{13}{31} & \frac{83}{155} & \frac{29}{155} \\ \frac{47}{155} & \frac{101}{155} & 1 & \frac{13}{31} & \frac{29}{155} & \frac{83}{155} & \frac{13}{31} & \frac{-5}{31} \\ \frac{29}{155} & \frac{47}{155} & \frac{13}{31} & 1 & \frac{83}{155} & \frac{101}{155} & \frac{-5}{31} & \frac{13}{31} \\ \frac{13}{31} & \frac{-5}{31} & \frac{29}{155} & \frac{83}{155} & 1 & \frac{13}{31} & \frac{47}{155} & \frac{101}{155} \\ \frac{-5}{31} & \frac{13}{31} & \frac{83}{155} & \frac{101}{155} & \frac{13}{31} & 1 & \frac{29}{155} & \frac{47}{155} \\ \frac{101}{155} & \frac{83}{155} & \frac{13}{31} & \frac{-5}{31} & \frac{47}{155} & \frac{29}{155} & 1 & \frac{13}{31} \\ \frac{83}{155} & \frac{29}{155} & \frac{-5}{31} & \frac{13}{31} & \frac{101}{155} & \frac{47}{155} & \frac{13}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[2.888888889, 1.447213595, 0.5527864046, 1.447213595, 0.5527864046, 0., 0., 0.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C–scaled

[3.354838710, 1.680635143, 0.6419455026, 1.680635143, 0.6419455026, 0., 0., 0.]

NullSpace M_C

{[0, 0, −1, 0, 0, 0, 0, 1], [1, 1, 1, 1, 0, 0, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, 1, 1, 1, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

NullSpace N_C

{[−1, 0, 1, 0, 0, −1, 0, 1], [−1, 0, 0, 1, 0, −1, 1, 0], [−1, 1, 0, 0, 1, −1, 0, 0]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[4., 1.447213595, 0.5527864046, 1.447213595, 0.5527864046, 0., 0., 0.]

NullSpace M_0

{[0, 0, 0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

NullSpace N_0

{[0, 0, −1, 1, 0, 0, 1, −1], [0, 1, −1, 0, 1, 0, 0, −1], [1, 0, −1, 0, 0, 1, 0, −1]}

Eigenvalues M

[−2.666666667, 2.666666667, −5.333333333, 5.333333333, −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[4., −0.5527864046, −1.447213595, −0.5527864046, −1.447213595, 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[1, −1, 0, 0, −1, 1, 0, 0], [0, −1, 1, 0, −1, 0, 0, 1], [0, −1, 0, 1, −1, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 5 & 6 & 7 & 5 & 10 & 3 & 4 \\ 5 & 0 & 3 & 6 & 10 & 5 & 4 & 7 \\ 6 & 3 & 0 & 5 & 7 & 4 & 5 & 10 \\ 7 & 6 & 5 & 0 & 4 & 3 & 10 & 5 \\ 5 & 10 & 7 & 4 & 0 & 5 & 6 & 3 \\ 10 & 5 & 4 & 3 & 5 & 0 & 7 & 6 \\ 3 & 4 & 5 & 10 & 6 & 7 & 0 & 5 \\ 4 & 7 & 10 & 5 & 3 & 6 & 5 & 0 \end{pmatrix}$$

==

{5, 6, 7, 8}

R: [4, 3, 1, 2, 8, 7, 5, 6]
B: [7, 8, 8, 7, 3, 4, 4, 3]

TRACE TWO = 3

det AT = $\frac{1}{16}(1+t)^4(t)^2$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 8

Level 2 det = $\frac{5}{268435456}(148-8s+47s^2+8s^3+5s^4)(-4+s)^2(5+s)^2(-3+s)(2+s)(6+3s+s^2)(-1+s)^3(4+s)^2$

RANK of R is 8

R ranking is 2, "vs", 8

RBAR ranking 2, "vs", 8

RANK of B is 4

B ranking is 1, "vs", 4

BBAR ranking 1, "vs", 4

"R CYCLES", (1 + v[5] v[6] v[7] v[8]) (1 + v[1] v[2] v[3] v[4])
"B CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])

Eigenvalues

R: [−1., 1., 1. I, −1. I, −1., 1., 1. I, −1. I]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{}

NullSpace of B^*

{[−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, −1], [0, 0, 0, 0, 0, 1, −1, 0], [0, −1, 1, 0, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 3

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 8 \\ 0 & 0 & 8 & 0 & 0 & 0 & 8 & 8 \\ 4 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\ 4 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 8 & 0 & 0 & 0 & 8 \\ 0 & 0 & 8 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 1 & 1 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 1 & 1 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & 1 & 1 & 1 & 1 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 1 & 1 & 1 & 1 & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 1 & 1 & 1 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 1 & 1 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 8

"RANK of the KERNEL is ", 4

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 4

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$)
degree 2: $\frac{1}{6}$ ($v[1]v[2] + v[1]v[5] + v[1]v[6] + v[2]v[5] + v[2]v[6] + 2v[3]v[4] + 2v[3]v[7]$
$+ 2v[3]v[8] + 2v[4]v[7] + 2v[4]v[8] + v[5]v[6] + 2v[7]v[8]$)
degree 3 : $\frac{1}{12}$ ($v[1]v[2]v[5] + v[1]v[2]v[6] + v[1]v[5]v[6] + v[2]v[5]v[6] + 2v[3]v[4]v[7]$
$+ 2v[3]v[4]v[8] + 2v[3]v[7]v[8] + 2v[4]v[7]v[8]$)
degree 4 : $\frac{1}{3}$ ($v[1]v[2]v[5]v[6] + 2v[3]v[4]v[7]v[8]$)

Group spectrum $1 + t + 3t^2 + t^3 + t^4$

KERNEL STRUCTURE

"PT1" = {{5, 8}, {2, 3}, {6, 7}, {1, 4}}

"PT2" = {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

"RG1" = {3, 4, 7, 8}

"RG2" = {1, 2, 5, 6}

$\pi 4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0,

0, 0, 0]

supp $\pi 4$ = {10, 61}

$u4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 2, 1, 1, 2, 3, 0, 0, 0, 0, 2, 2, 0, 0, 2, 2, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 2, 2, 0, 0, 2, 2, 0, 0, 0, 0, 3, 2, 1, 1, 2, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp u4 = {10, 11, 12, 13, 14, 15, 20, 21, 24, 25, 26, 28, 29, 31, 40, 42, 43, 45, 46, 47, 50, 51, 56, 57, 58, 59, 60, 61}

Action of R on ranges, [[2], [1]]
Action of B on ranges, [[1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [1, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[7, 8, 2, 1, 3, 4, 6, 5]

B-BLOCKS,

[5, 4, 4, 2, 7, 2, 5, 7]

with invariant measure, [1, 2, 1, 2, 2, 1, 2, 1]

N by blocks, N - check: true

b_1 = {2, 4}
b_2 = {5, 8}
b_3 = {6, 8}
b_4 = {2, 3}
b_5 = {6, 7}
b_6 = {1, 3}
b_7 = {1, 4}
b_8 = {5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[4] & h[3] & 0 & 0 & h[2] & h[1] & 0 & 0 \\ h[3] & h[4] & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & h[4] & h[3] & 0 & 0 & h[2] & h[1] \\ 0 & 0 & h[3] & h[4] & 0 & 0 & h[1] & h[2] \\ h[2] & h[1] & 0 & 0 & h[4] & h[3] & 0 & 0 \\ h[1] & h[2] & 0 & 0 & h[3] & h[4] & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & h[4] & h[3] \\ 0 & 0 & h[1] & h[2] & 0 & 0 & h[3] & h[4] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: $6 \oplus 4/2$

$$\text{CLB} = \begin{pmatrix} -1 & 1 & 1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4}, {5, 6, 7, 8}}, true

Ω_B in Vec(K)? , {{3, 8}, {4, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I−V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix}\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8}\end{pmatrix}$ vs $\begin{pmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix}0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4}\end{pmatrix}$ vs $\begin{pmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{5, 8}, {2, 3}, {6, 7}, {1, 4}}

1, "range", [3, 4, 7, 8], [[8, 7, 7, 8, 4, 3, 3, 4], [7, 8, 8, 7, 3, 4, 4, 3], [4, 3, 3, 4, 8, 7, 7, 8], [3, 4, 4, 3, 7, 8, 8, 7]]

2, "range", [1, 2, 5, 6], [[6, 5, 5, 6, 2, 1, 1, 2], [5, 6, 6, 5, 1, 2, 2, 1], [2, 1, 1, 2, 6, 5, 5, 6], [1, 2, 2, 1, 5, 6, 6, 5]]

2, "partition", {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

1, "range", [3, 4, 7, 8], [[8, 7, 8, 7, 4, 3, 4, 3], [7, 8, 7, 8, 3, 4, 3, 4], [4, 3, 4, 3, 8, 7, 8, 7], [3, 4, 3, 4, 7, 8, 7, 8]]

2, "range", [1, 2, 5, 6], [[6, 5, 6, 5, 2, 1, 2, 1], [5, 6, 5, 6, 1, 2, 1, 2], [2, 1, 2, 1, 6, 5, 6, 5], [1, 2, 1, 2, 5, 6, 5, 6]]

"group has", 4, "elements" Group element 1,1 = $\begin{pmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{pmatrix}$

$g_1 =$ [[1, 3], [2, 4]]

$g_2 =$ [[1, 4], [2, 3]]

$g_3 =$ []

$g_4 =$ [[1, 2], [3, 4]]

linear dimension, 4

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix}h[3] & h[4] & h[1] & h[2]\end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

4, "coeff", 1

$$Z[\ 4\ ] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
2, 3, true
2, 4, true
3, 4, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. \\ 1. & -1. & 1. & -1. \\ 1. & -1. & 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 4 & 0 \\ 2 & 2 & 6 & 2 \\ 0 & 0 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 3t^2 + t^3 + t^4$

Molien Series to order 10: $1 + t + 4t^2 + 5t^3 + 11t^4 + 14t^5 + 24t^6 + 30t^7 + 45t^8 + 55t^9 + 76t^{10}$

n-choose-rank

{1, [1, 2, 3, 4]}, {2, [1, 2, 3, 5]}, {3, [1, 2, 3, 6]}, {4, [1, 2, 3, 7]}, {5, [1, 2, 3, 8]}, {6, [1, 2, 4, 5]}, {7, [1, 2, 4, 6]}, {8, [1, 2, 4, 7]}, {9, [1, 2, 4, 8]}, {10, [1, 2, 5, 6]}, {11, [1, 2, 5, 7]}, {12, [1, 2, 5, 8]}, {13, [1, 2, 6, 7]}, {14, [1, 2, 6, 8]}, {15, [1, 2, 7, 8]}, {16, [1, 3, 4, 5]}, {17, [1, 3, 4, 6]}, {18, [1, 3, 4, 7]}, {19, [1, 3, 4, 8]}, {20, [1, 3, 5, 6]}, {21, [1, 3, 5, 7]}, {22, [1, 3, 5, 8]}, {23, [1, 3, 6, 7]}, {24, [1, 3, 6, 8]}, {25, [1, 3, 7, 8]}, {26, [1, 4, 5, 6]}, {27, [1, 4, 5, 7]}, {28, [1, 4, 5, 8]}, {29, [1, 4, 6, 7]}, {30, [1, 4, 6, 8]}, {31, [1, 4, 7, 8]}, {32, [1, 5, 6, 7]}, {33, [1, 5, 6, 8]}, {34, [1, 5, 7, 8]}, {35, [1, 6, 7, 8]}, {36, [2, 3, 4, 5]}, {37, [2, 3, 4, 6]}, {38, [2, 3, 4, 7]}, {39, [2, 3, 4, 8]}, {40, [2, 3, 5, 6]}, {41, [2, 3, 5, 7]}, {42, [2, 3, 5, 8]}, {43, [2, 3, 6, 7]}, {44, [2, 3, 6, 8]}, {45, [2, 3, 7, 8]}, {46, [2, 4, 5, 6]}, {47, [2, 4, 5, 7]}, {48, [2, 4, 5, 8]}, {49, [2, 4, 6, 7]}, {50, [2, 4, 6, 8]}, {51, [2, 4, 7, 8]}, {52, [2, 5, 6, 7]}, {53, [2, 5, 6, 8]}, {54, [2, 5, 7, 8]}, {55, [2, 6, 7, 8]}, {56, [3, 4, 5, 6]}, {57, [3, 4, 5, 7]}, {58, [3, 4, 5, 8]}, {59, [3, 4, 6, 7]}, {60, [3, 4, 6, 8]}, {61, [3, 4, 7, 8]}, {62, [3, 5, 6, 7]}, {63, [3, 5, 6, 8]}, {64, [3, 5, 7, 8]}, {65, [3, 6, 7, 8]}, {66, [4, 5, 6, 7]}, {67, [4, 5, 6, 8]}, {68, [4, 5, 7, 8]}, {69, [4, 6, 7, 8]}, {70, [5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 4$ =
(0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

{10, 61}

$u4$ =
(0 0 0 0 0 0 0 0 0 3 2 1 1 2 3 0 0 0 0 2 2 0 0 2 2 1 0 1 1 0

{10, 11, 12, 13, 14, 15, 20, 21, 24, 25, 26, 28, 29, 31, 40, 42, 43, 45, 46, 47, 50, 51, 56, 57, 58, 59, 60, 61}

picheck (1 1 2 2 1 1 2 2)

$\pi = \left(\frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\ \ \frac{1}{12}\ \ \frac{1}{12}\ \ \frac{1}{6}\ \ \frac{1}{6}\right)$

$\pi 3 =$
$(0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0$

$u3 =$
$\left(0\ \ 0\ \ \frac{3}{4}\ \ \frac{3}{4}\ \ \frac{3}{4}\ \ \frac{3}{4}\ \ 0\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{3}{4}\ \ \frac{1}{2}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{2}\ \ \frac{3}{4}\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{2}\ \ \frac{1}{2}\right.$

picheck $(3\ \ 3\ \ 6\ \ 6\ \ 3\ \ 3\ \ 6\ \ 6)$

$\pi 2 =$
$(2\ \ 0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 2\ \ 2\ \ 0\ \ 0\ \ 4\ \ 0\ \ 0\ \ 4\ \ 4\ \ 0\ \ 0\ \ 4\ \ 4\ \ 2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 4)$

$u2 =$
$\left(\frac{3}{8}\ \ \frac{1}{4}\ \ \frac{1}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{1}{8}\ \ \frac{1}{4}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{3}{8}\ \ \frac{1}{4}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{4}\ \ \frac{3}{8}\right)$

picheck $(6\ \ 6\ \ 12\ \ 12\ \ 6\ \ 6\ \ 12\ \ 12)$

$\pi 1 = (6\ \ 6\ \ 12\ \ 12\ \ 6\ \ 6\ \ 12\ \ 12)$

$u1 = \left(\frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\ \ \frac{9}{32}\right)$

picheck $(6\ \ 6\ \ 12\ \ 12\ \ 6\ \ 6\ \ 12\ \ 12)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{9} & \frac{4}{9} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{4}{9} & \frac{2}{9} & 0 & 0 & 0 & 0 \\ \frac{1}{9} & \frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{9} & \frac{1}{9} & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{9} & \frac{4}{9} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{4}{9} & \frac{2}{9} \\ 0 & 0 & 0 & 0 & \frac{1}{9} & \frac{2}{9} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & \frac{8}{3} & \frac{56}{9} & \frac{64}{9} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & 4 & \frac{64}{9} & \frac{56}{9} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{28}{9} & \frac{32}{9} & 8 & \frac{16}{3} & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{32}{9} & \frac{28}{9} & \frac{16}{3} & 8 & \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & 4 & \frac{8}{3} & \frac{56}{9} & \frac{64}{9} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{8}{3} & 4 & \frac{64}{9} & \frac{56}{9} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{28}{9} & \frac{32}{9} & 8 & \frac{16}{3} \\ \frac{8}{3} & \frac{8}{3} & \frac{16}{3} & \frac{16}{3} & \frac{32}{9} & \frac{28}{9} & \frac{16}{3} & 8 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, -1, -1, 1, 1, -1, -1]$

$$\ker N_c = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & s & s & -s & -s \\ s & s & -s & -s & 0 & 0 & 0 & 0 \end{pmatrix}$$ RB

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & -1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t & 0 & 0 & s & 0 & 0 \\ -s & -s+t & 0 & -s & 0 & 0 \\ 0 & t & -s & 0 & -s & -s \\ t & 0 & s & 0 & 0 & 0 \\ -t & -t+s & 0 & -t & 0 & 0 \\ s & 0 & 0 & t & 0 & 0 \\ 0 & 0 & 0 & t & s & 0 \\ -t & -t & 0 & -t & 0 & s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & s & 0 & 0 & 0 & t & 0 \\ s & 0 & 0 & 0 & t & 0 & 0 \\ s & s & -s & -s & s+t & s & -s \\ 0 & 0 & 0 & s & 0 & t & 0 \\ t & 0 & 0 & 0 & s & 0 & 0 \\ 0 & t & 0 & 0 & 0 & s & 0 \\ 0 & t & s & 0 & 0 & 0 & 0 \\ t & 0 & 0 & 0 & 0 & 0 & s \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 2 & 2 & 0 & 0 & 2 & 2 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 4, "vs", 4

$$CNM = \begin{pmatrix} 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \\ \frac{-4}{3} & \frac{-4}{3} & 0 & 0 & \frac{-4}{3} & \frac{-4}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew } T = \begin{pmatrix} 0 & 0 & \frac{1}{9} & \frac{2}{9} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{9} & \frac{1}{9} & 0 & 0 & 0 & 0 \\ \frac{-1}{9} & \frac{-2}{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-2}{9} & \frac{-1}{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{9} & \frac{2}{9} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{9} & \frac{1}{9} \\ 0 & 0 & 0 & 0 & \frac{-1}{9} & \frac{-2}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-2}{9} & \frac{-1}{9} & 0 & 0 \end{pmatrix}$$

$$\text{Skew Omega} = \begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ \frac{4}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 4T + 32\Omega$$

$$\Omega\left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12}\right)$$

$$T\left(0 \;\; 0 \;\; \frac{2}{3} \;\; \frac{1}{9} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{4}{9} \;\; \frac{2}{9} \;\; 0 \;\; \frac{1}{3}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(\frac{16}{3}\ \ \frac{8}{3}\ \ 8\ \ \frac{28}{9}\ \ \frac{16}{3}\ \ \frac{16}{3}\ \ \frac{8}{3}\ \ \frac{8}{3}\ \ \frac{64}{9}\ \ \frac{56}{9}\ \ \frac{8}{3}\ \ 4\right)$$

"IS MN in Vec(K)?", false

$$MN\left(4\ \ 4\ \ \frac{19}{3}\ \ \frac{41}{9}\ \ 4\ \ 4\ \ 4\ \ 4\ \ \frac{47}{9}\ \ \frac{43}{9}\ \ \frac{13}{3}\ \ \frac{17}{3}\right)$$

$$\boldsymbol{\tau} = 16/1 \text{ , rank } = 4 \text{ , ratio } = 4/1 \text{ , } n^2 / r = 16/1$$

$$\boldsymbol{\tau}' = 48/1 \text{ , } r' = 3/4 \text{ , } \boldsymbol{\tau} / n^2 = 1/4$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 64/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 4/9$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 16\Omega$$

There are, 2, partitions and, 2, ranges, with a group size of, 4

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 4 vs no. of idems 4

"PT1" = {{5, 8}, {2, 3}, {6, 7}, {1, 4}}

"PT2" = {{2, 4}, {6, 8}, {1, 3}, {5, 7}}

"RG1" = {3, 4, 7, 8}

"RG2" = {1, 2, 5, 6}

$$M_C = \begin{pmatrix} \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{8}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 7.111111111]

Eigenvalues N_C

[2., 0.8888888889, 0., 0., 1.333333333, 0.6666666667, 1.333333333, 0.6666666667
]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_C−scaled

[2.322580645, 1.032258065, 0., 0., 1.548387097, 0.7741935484, 1.548387097,
0.7741935484]

NullSpace M_C

{[0, 0, 0, 0, 0, 1, 0, 1], [0, 0, 0, 0, 0, 0, 1, −1], [0, 0, 0, 0, 1, 0, 0, 1], [1, 0, 0, 0, 0, 0, 0, 1],
[0, 0, 1, 0, 0, 0, 0, −1], [0, 0, 0, 1, 0, 0, 0, −1], [0, 1, 0, 0, 0, 0, 0, 1]}

NullSpace N_C

{[0, 0, 0, 0, −1, −1, 1, 1], [−1, −1, 1, 1, 0, 0, 0, 0]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 10.66666667, 5.333333333]

Eigenvalues N_0

[2., 0.6666666667, 1.333333333, 2., 0.6666666667, 1.333333333, 0., 0.]

NullSpace M_0

{[−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1,
0], [0, 0, −1, 0, 0, 0, 0, 1], [0, 0, −1, 1, 0, 0, 0, 0]}

NullSpace N_0

{[0, 0, 0, 0, −1, −1, 1, 1], [1, 1, −1, −1, 0, 0, 0, 0]}

Eigenvalues M

[8., 4., −1.333333333, −2.666666667, −1.333333333, −2.666666667,
−1.333333333, −2.666666667]

Eigenvalues N

[6., −2., 0., 0., −0.6666666667, −1.333333333, −0.6666666667, −1.333333333]

NullSpace M

{}

NullSpace N

{[−1, −1, 1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, −1, 1, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 3 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 3 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & -2 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & -2 \\ -2 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & -2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & -2 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 3 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 & 3 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$1, 3 : \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$2, 3 : \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

==

100, [1, −1, −1, −1, −1, −1, 1, 1]

==

{2, 3, 4, 5, 6}

R: [4, 8, 8, 7, 8, 7, 4, 3]
B: [7, 3, 1, 2, 3, 4, 5, 6]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 7

Level 2 det = $\frac{1}{1048576}(-1+s)(5+s)(-26640+17472s-8712s^2+4352s^3-3390s^4+2614s^5-1353s^6+298s^7+14s^8-16s^9+s^{10})$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 7

B ranking is 5, "vs", 7

BBAR ranking 2, "vs", 4

"R CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])
"B CYCLES", 1 + v[1] v[3] v[5] v[7]

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of R^*

{[0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 0, 1, 0, −1, 0, 0], [0, 0, 1, 0, −1, 0, 0, 0], [1, 0, 0, 0, 0, 0, −1, 0]}

NullSpace of B^*

{[0, −1, 0, 0, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 8 & 0 & 0 & 8 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}\,(\,v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8]\,)$

degree 2: $\frac{1}{12}\,(\,v[1]v[2] + v[1]v[5] + v[2]v[6] + 2\,v[3]v[4] + 2\,v[3]v[7] + 2\,v[4]v[8] + v[5]v[6] + 2\,v[7]v[8]\,)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$\pi 2 = [1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 4, 11, 14, 17, 22, 23, 28}

$$u2 = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[6], [5], [1], [1], [2], [1], [2], [2]]
Action of B on ranges, [[3], [4], [6], [6], [7], [8], [5], [5]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

RPARTS [2, 2]

BPARTS [2, 1]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 2, 3, 2]

B-BLOCKS,

[3, 1, 4, 2]

with invariant measure, [1, 2, 2, 1]

N by blocks, N - check: true

b_1 = {1, 3, 6, 8}
b_2 = {1, 4, 6, 7}
b_3 = {2, 3, 5, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 3 ⊕ 19/17

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 8}, {4, 7}}, true

Ω_B in Vec(K)? , {{1, 3, 5, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} \\ \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 7, 8], [7, 8, 7, 8, 8, 7, 8, 7]]

2, "range", [4, 8], [[8, 4, 8, 4, 4, 8, 4, 8], [4, 8, 4, 8, 8, 4, 8, 4]]

3, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 5, 6], [5, 6, 5, 6, 6, 5, 6, 5]]

4, "range", [2, 6], [[6, 2, 6, 2, 2, 6, 2, 6], [2, 6, 2, 6, 6, 2, 6, 2]]

5, "range", [3, 7], [[7, 3, 7, 3, 3, 7, 3, 7], [3, 7, 3, 7, 7, 3, 7, 3]]

6, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 3, 4], [3, 4, 3, 4, 4, 3, 4, 3]]

7, "range", [1, 5], [[5, 1, 5, 1, 1, 5, 1, 5], [1, 5, 1, 5, 5, 1, 5, 1]]

8, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 1, 2], [1, 2, 1, 2, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

3, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

4, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

5, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

6, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

7, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

8, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

{1, 4, 11, 14, 17, 22, 23, 28}

$u2 =$
$$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem–checks} \quad \text{N0–checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem–checks} \quad \text{N0–checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem–checks} \quad \text{N0–checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, 0, 1, -1, -1, 1, 2]$

$$\ker N_c = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -t & 0 & s & t & 0 & -s & 0 \\ -t & -t & t & t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -t & 0 & s & 0 & 0 & t & 0 & -s \\ -t & -t & t & s & 0 & 0 & t-s & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & -1 & 2 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -s-t & 0 \\ t+s & 0 \\ s & t \\ -s & -t \\ t+s & 0 \\ -s-t & 0 \\ -s & -t \\ s & t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & t+s \\ 0 & t+s & 0 \\ t & s & 0 \\ -t & t & t+s \\ 0 & t+s & 0 \\ 0 & 0 & t+s \\ -t & t & t+s \\ t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 7, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & 0 & 0 & 0 \\ \frac{4}{3} & \frac{8}{3} & 0 & 0 & 0 & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 \\ 0 & 0 & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & 0 & 0 & \frac{8}{3} & \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} \\ 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(0\ \frac{1}{9}\ 0\ \frac{1}{3}\ \frac{1}{9}\ \frac{1}{18}\ \frac{1}{9}\ \frac{2}{9}\ \frac{1}{6}\ 0\ \frac{1}{9}\ \frac{2}{9}\ \frac{1}{6}\ 0\ \frac{2}{9}\ \frac{1}{9}\ 0\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM}\left(0\ \ \frac{16}{9}\ \ 0\ \ \frac{16}{3}\ \ \frac{16}{9}\ \ \frac{8}{9}\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{32}{9}\ \ \frac{16}{9}\ \ 0\ \ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$\mathrm{MN}\left(\frac{-2}{3}\ \ \frac{26}{9}\ \ \frac{-2}{3}\ \ \frac{14}{3}\ \ \frac{26}{9}\ \ \frac{10}{9}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{22}{9}\ \ \frac{14}{9}\ \ \frac{2}{3}\ \ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 8, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 32

dim span idems 12 vs no. of idems 16

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} \\ \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[7.111111111, 5.333333333, 2.666666667, 5.333333333, 2.666666667, 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C−scaled

[2.600000000, 1.500000000, 1.200000000, 1.500000000, 1.200000000, 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[1, −1, 0, 0, −1, 1, 0, 0], [0, 1, 1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0, 1, 0] }

NullSpace N_C

{[0, 0, 1, 0, 0, 0, 0, −1], [0, 0, 0, 1, −1, −1, 0, 1], [0, 0, 0, 0, −1, −1, 1, 1], [0, 1, 0, 0, −1, 0, 0, 0], [1, 0, 0, 0, 0, −1, 0, 0]}

Eigenvalues M_0

[10.66666667, 0., 0., 2.666666667, 2.666666667, 5.333333333, 5.333333333, 5.333333333]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[0, 0, 1, −1, 0, 0, −1, 1], [1, −1, 0, 0, −1, 1, 0, 0]}

NullSpace N_0

{[−1, −1, 0, 1, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues M

[0., 0., 0., 0., −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, 0, 0, −1, 0, 0, 1, 0]}

NullSpace N

{[−1, −1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, −1, 0, 1, 0, 0, 0, 1], [0, 0, 0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 5, 6, 7, 8}

R: [4, 8, 1, 2, 8, 7, 5, 6]
B: [7, 3, 8, 7, 3, 4, 4, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 7

$$\text{Level 2 det} = \frac{1}{1048576}\left(-26640 + 17472s - 8712s^2 + 4352s^3 - 3390s^4 + 2614s^5 - 1353s^6 + 298s^7 + 14s^8 - 16s^9 + s^{10}\right)(-1+s)(5+s)$$

RANK of R is 7

R ranking is 5, "vs", 7

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[5] v[6] v[7] v[8]
"B CYCLES", (1 + v[4] v[7]) (1 + v[3] v[8])

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 1, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 1, 0, 0, −1, 0, 0, 0]}

NullSpace of B^*

{[−1, 0, 0, 1, 0, 0, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1], [0, 0, 0, 0, 0, 1, −1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&4&0&0&4&0&0&0\\ 4&0&0&0&0&4&0&0\\ 0&0&0&8&0&0&8&0\\ 0&0&8&0&0&0&0&8\\ 4&0&0&0&0&4&0&0\\ 0&4&0&0&4&0&0&0\\ 0&0&8&0&0&0&0&8\\ 0&0&0&8&0&0&8&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&1&\frac{2}{3}&\frac{1}{3}&1&0&\frac{1}{3}&\frac{2}{3}\\ 1&0&\frac{1}{3}&\frac{2}{3}&0&1&\frac{2}{3}&\frac{1}{3}\\ \frac{2}{3}&\frac{1}{3}&0&1&\frac{1}{3}&\frac{2}{3}&1&0\\ \frac{1}{3}&\frac{2}{3}&1&0&\frac{2}{3}&\frac{1}{3}&0&1\\ 1&0&\frac{1}{3}&\frac{2}{3}&0&1&\frac{2}{3}&\frac{1}{3}\\ 0&1&\frac{2}{3}&\frac{1}{3}&1&0&\frac{1}{3}&\frac{2}{3}\\ \frac{1}{3}&\frac{2}{3}&1&0&\frac{2}{3}&\frac{1}{3}&0&1\\ \frac{2}{3}&\frac{1}{3}&0&1&\frac{1}{3}&\frac{2}{3}&1&0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2v[3] + 2v[4] + v[5] + v[6] + 2v[7] + 2v[8]$)
degree 2: $\frac{1}{12}$ ($v[1]v[2] + v[1]v[5] + v[2]v[6] + 2v[3]v[4] + 2v[3]v[7] + 2v[4]v[8] + v[5]v[6] + 2v[7]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$\pi 2 = [1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 4, 11, 14, 17, 22, 23, 28}

$$u2 = [3, 2, 1, 3, 0, 1, 2, 1, 2, 0, 3, 2, 1, 3, 1, 2, 3, 0, 2, 1, 0, 3, 3, 2, 1, 1, 2, 3]$$

supp u2 = {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [4], [1], [1], [7], [8], [2], [2]]
Action of B on ranges, [[6], [5], [6], [6], [2], [1], [5], [5]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

RPARTS [2, 1]

BPARTS [2, 2]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 1, 4, 2]

B-BLOCKS,

[3, 2, 3, 2]

with invariant measure, [1, 2, 2, 1]

N by blocks, N – check: true

b_1 = {1, 3, 6, 8}
b_2 = {1, 4, 6, 7}
b_3 = {2, 3, 5, 8}
b_4 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N):   3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 3 ⊕ 19/17

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 6, 7, 8}}, true

Ω_B in Vec(K)? , {{3, 8}, {4, 7}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 7, 8], [7, 8, 7, 8, 8, 7, 8, 7]]

2, "range", [4, 8], [[8, 4, 8, 4, 4, 8, 4, 8], [4, 8, 4, 8, 8, 4, 8, 4]]

3, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 5, 6], [5, 6, 5, 6, 6, 5, 6, 5]]

4, "range", [2, 6], [[6, 2, 6, 2, 2, 6, 2, 6], [2, 6, 2, 6, 6, 2, 6, 2]]

5, "range", [3, 7], [[7, 3, 7, 3, 3, 7, 3, 7], [3, 7, 3, 7, 7, 3, 7, 3]]

6, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 3, 4], [3, 4, 3, 4, 4, 3, 4, 3]]

7, "range", [1, 5], [[5, 1, 5, 1, 1, 5, 1, 5], [1, 5, 1, 5, 5, 1, 5, 1]]

8, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 1, 2], [1, 2, 1, 2, 2, 1, 2, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

3, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

4, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

5, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

6, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

7, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

8, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{1, 4, 11, 14, 17, 22, 23, 28}

$u2 =$
$\begin{pmatrix} 3 & 2 & 1 & 3 & 0 & 1 & 2 & 1 & 2 & 0 & 3 & 2 & 1 & 3 & 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 2 & 4 & 4 & 2 & 2 & 4 & 4 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\left(2 \quad 2 \quad 4 \quad 4 \quad 2 \quad 2 \quad 4 \quad 4\right)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, -2, -1, 1, 1, -1, 0]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -s & 0 & t & 0 & 0 & s & 0 & -t \\ s & 0 & -t & 0 & s & 0 & -s & t-s \\ s & s & -t & -t & 0 & 0 & t-s & t-s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s-t & 0 & 0 & t-s & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & -1 & -1 & 1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 1 \\ 0 & -1 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} s+t & 0 \\ -t-s & 0 \\ -t & s \\ t & -s \\ -t-s & 0 \\ s+t & 0 \\ t & -s \\ -t & s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s+t & 0 \\ s+t & 0 & 0 \\ s+t & s & -s \\ 0 & t & s \\ s+t & 0 & 0 \\ 0 & s+t & 0 \\ 0 & t & s \\ s+t & s & -s \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 7, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{18} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{1}{18} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{4}{3} & 0 & 0 & \frac{4}{3} & 0 & 0 & 0 \\ \frac{4}{3} & \frac{8}{3} & 0 & 0 & 0 & \frac{4}{3} & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & 0 \\ 0 & 0 & \frac{8}{3} & \frac{16}{3} & 0 & 0 & 0 & \frac{8}{3} \\ \frac{4}{3} & 0 & 0 & 0 & \frac{8}{3} & \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 & 0 & \frac{4}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & 0 & 0 & 0 & \frac{16}{3} & \frac{8}{3} \\ 0 & 0 & 0 & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{12} \;\; \frac{1}{12}\right)$$

$$T\left(0 \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{3} \;\; \frac{1}{9} \;\; \frac{1}{18} \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{1}{9} \;\; \frac{2}{9} \;\; \frac{1}{6} \;\; 0 \;\; \frac{2}{9} \;\; \frac{1}{9} \;\; 0 \;\; \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$NM\left(0\ \ \frac{16}{9}\ \ 0\ \ \frac{16}{3}\ \ \frac{16}{9}\ \ \frac{8}{9}\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{16}{9}\ \ \frac{32}{9}\ \ \frac{8}{3}\ \ 0\ \ \frac{32}{9}\ \ \frac{16}{9}\ \ 0\ \ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

$$MN\left(\frac{-2}{3}\ \ \frac{26}{9}\ \ \frac{-2}{3}\ \ \frac{14}{3}\ \ \frac{26}{9}\ \ \frac{10}{9}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{14}{9}\ \ \frac{22}{9}\ \ \frac{10}{3}\ \ \frac{2}{3}\ \ \frac{22}{9}\ \ \frac{14}{9}\ \ \frac{2}{3}\ \ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36 \text{ , min } \boldsymbol{\tau} = 80/9 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5 \text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 8, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 32

dim span idems 12 vs no. of idems 16

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"RG1" = {7, 8}

"RG2" = {4, 8}

"RG3" = {5, 6}

"RG4" = {2, 6}

"RG5" = {3, 7}

"RG6" = {3, 4}

"RG7" = {1, 5}

"RG8" = {1, 2}

$$M_C = \begin{pmatrix} \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{8}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{8}{9} & \frac{32}{9} \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} \\ \frac{2}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{2}{5} & \frac{-2}{5} & \frac{-2}{5} & \frac{2}{5} & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{1}{4} & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[7.111111111, 5.333333333, 2.666666667, 5.333333333, 2.666666667, 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 0., 1.333333333, 2.666666667, 2.888888889]

Eigenvalues M_C–scaled

[2.600000000, 1.500000000, 1.200000000, 1.500000000, 1.200000000, 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 0., 1.548387097, 3.096774194, 3.354838710]

NullSpace M_C

{[0, 1, 1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0, 1, 0], [1, −1, 0, 0, −1, 1, 0, 0] }

NullSpace N_C

{[1, 1, −1, −1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, 1, −1, −1, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, 0, 0, −1, 0, 0, 1, 0]}

Eigenvalues M_0

[10.66666667, 0., 0., 2.666666667, 2.666666667, 5.333333333, 5.333333333, 5.333333333]

Eigenvalues N_0

[0., 0., 0., 0., 0., 4., 1.333333333, 2.666666667]

NullSpace M_0

{[1, −1, 0, 0, −1, 1, 0, 0], [0, 0, 1, −1, 0, 0, −1, 1]}

NullSpace N_0

{[1, 1, 0, 0, 0, 0, −1, −1], [1, 0, 0, 0, 1, 0, −1, −1], [0, 0, 1, 0, 0, 0, 0, −1], [−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 1, 0, 0, −1, 0]}

Eigenvalues M

[0., 0., 0., 0., −2.666666667, 2.666666667, −5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 0., 0., −1.333333333, 4., −2.666666667]

NullSpace M

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, 0, 0, −1, 0, 0, 1, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

NullSpace N

{[0, 0, −1, 0, 0, 0, 0, 1], [−1, −1, 1, 0, 0, 0, 1, 0], [−1, −1, 1, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 & 0 & 3 & 2 & 1 \\ 0 & 3 & 2 & 1 & 3 & 0 & 1 & 2 \\ 1 & 2 & 3 & 0 & 2 & 1 & 0 & 3 \\ 2 & 1 & 0 & 3 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

120, [1, 1, 1, −1, −1, −1, −1, −1]

==

{3, 4, 5, 6, 7, 8}

R: [4, 3, 8, 7, 8, 7, 5, 6]
B: [7, 8, 1, 2, 3, 4, 4, 3]

TRACE TWO = 1

det AT = $\frac{1}{16}(-1+t)^2(1+t)^2(t)^2$

$$AT = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{5}{268435456}(-1+s)(-2880-2496s-2340s^2-1052s^3-683s^4-153s^5-4s^6+4s^7+3s^8+s^9)(11840+1472s-1788s^2+980s^3+115s^4-117s^5-27s^6+5s^7)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", 1 + v[5] v[6] v[7] v[8]
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[7] v[8]

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [0., 0., −1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of R^*

{[0, 0, 0, −1, 0, 1, 0, 0], [0, 0, −1, 0, 1, 0, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 0, 0, 1], [0, 0, 0, 0, 0, −1, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & \frac{8}{15} & \frac{7}{15} & \frac{2}{5} & \frac{3}{5} & \frac{2}{3} & \frac{1}{3} \\ 1 & 0 & \frac{7}{15} & \frac{8}{15} & \frac{3}{5} & \frac{2}{5} & \frac{1}{3} & \frac{2}{3} \\ \frac{8}{15} & \frac{7}{15} & 0 & 1 & \frac{4}{15} & \frac{11}{15} & \frac{2}{5} & \frac{3}{5} \\ \frac{7}{15} & \frac{8}{15} & 1 & 0 & \frac{11}{15} & \frac{4}{15} & \frac{3}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{15} & \frac{11}{15} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{3}{5} & \frac{2}{5} & \frac{11}{15} & \frac{4}{15} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{5} & \frac{3}{5} & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{1}{3} & \frac{2}{3} & \frac{3}{5} & \frac{2}{5} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{12}$ ($v[1] + v[2] + 2\,v[3] + 2\,v[4] + v[5] + v[6] + 2\,v[7] + 2\,v[8]$)

degree 2: $\frac{1}{6}$ ($v[1]v[2] + 2\,v[3]v[4] + v[5]v[6] + 2\,v[7]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 5, 8}, {2, 4, 6, 7}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 4, 6, 8}, {2, 3, 5, 7}}

"PT4" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT5" = {{1, 3, 6, 7}, {2, 4, 5, 8}}

"PT6" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$\pi 2 = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {1, 14, 23, 28}

$$u2 = [15, 8, 7, 6, 9, 10, 5, 7, 8, 9, 6, 5, 10, 15, 4, 11, 6, 9, 11, 4, 9, 6, 15, 10, 5, 5, 10, 15]$$

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[2], [1], [1], [3]]
Action of B on ranges, [[3], [3], [4], [1]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$$

RPARTS [3, 6, 1, 3, 6, 1]

BPARTS [4, 2, 2, 5, 1, 1]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{4}{15} & \frac{1}{5} & \frac{1}{5} & \frac{2}{15} & \frac{1}{15} & \frac{2}{15} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[4, B, A, 9, C, 3, 3, 4, C, B, 9, A]

B-BLOCKS,

[9, 1, 9, A, 6, A, 8, 7, 5, 2, 8, 7]

with invariant measure, [1, 2, 2, 2, 2, 1, 3, 3, 4, 4, 3, 3]

N by blocks, N - check: true

$b_1 = \{1, 3, 6, 7\}$
$b_2 = \{1, 4, 6, 7\}$
$b_3 = \{1, 3, 5, 7\}$
$b_4 = \{2, 4, 6, 8\}$
$b_5 = \{2, 3, 5, 8\}$
$b_6 = \{2, 4, 5, 8\}$
$b_7 = \{1, 4, 5, 8\}$
$b_8 = \{2, 3, 6, 7\}$
$b_9 = \{1, 3, 5, 8\}$
$b_{10} = \{2, 4, 6, 7\}$
$b_{11} = \{1, 4, 6, 8\}$
$b_{12} = \{2, 3, 5, 7\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & 0 & 0 & 0 & 0 & 0 & 0 \\ h[1] & h[2] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h[2] & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & h[1] & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & h[1] & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & h[2] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & 0 & 0 & 0 & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 25, Shape: $18 \oplus 7/5$

$$CLB = \begin{pmatrix} 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 6, 7, 8}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 4, 7, 8}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{-1}{6} & 0 & \frac{-1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{-1}{6} & 0 & \frac{-1}{3} \\ \frac{-5}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{12} & \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{12} & \frac{-5}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{6} & 0 & \frac{-1}{3} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{-1}{6} & 0 & \frac{-1}{3} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{12} & \frac{-1}{4} & \frac{1}{6} & \frac{-1}{2} & \frac{5}{12} & \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{12} & \frac{5}{12} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega–checks true, true

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 5, 8}, {2, 4, 6, 7}}

1, "range", [7, 8], [[8, 7, 8, 7, 8, 7, 7, 8], [7, 8, 7, 8, 7, 8, 8, 7]]

2, "range", [5, 6], [[6, 5, 6, 5, 6, 5, 5, 6], [5, 6, 5, 6, 5, 6, 6, 5]]

3, "range", [3, 4], [[4, 3, 4, 3, 4, 3, 3, 4], [3, 4, 3, 4, 3, 4, 4, 3]]

4, "range", [1, 2], [[2, 1, 2, 1, 2, 1, 1, 2], [1, 2, 1, 2, 1, 2, 2, 1]]

2, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [7, 8], [[8, 7, 7, 8, 8, 7, 7, 8], [7, 8, 8, 7, 7, 8, 8, 7]]

2, "range", [5, 6], [[6, 5, 5, 6, 6, 5, 5, 6], [5, 6, 6, 5, 5, 6, 6, 5]]

3, "range", [3, 4], [[4, 3, 3, 4, 4, 3, 3, 4], [3, 4, 4, 3, 3, 4, 4, 3]]

4, "range", [1, 2], [[2, 1, 1, 2, 2, 1, 1, 2], [1, 2, 2, 1, 1, 2, 2, 1]]

3, "partition", {{1, 4, 6, 8}, {2, 3, 5, 7}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 7, 8], [7, 8, 8, 7, 8, 7, 8, 7]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 5, 6], [5, 6, 6, 5, 6, 5, 6, 5]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3, 4, 3]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 1, 2], [1, 2, 2, 1, 2, 1, 2, 1]]

4, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [7, 8], [[8, 7, 7, 8, 7, 8, 8, 7], [7, 8, 8, 7, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 5, 6, 5, 6, 6, 5], [5, 6, 6, 5, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4, 4, 3], [3, 4, 4, 3, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 1, 2, 1, 2, 2, 1], [1, 2, 2, 1, 2, 1, 1, 2]]

5, "partition", {{1, 3, 6, 7}, {2, 4, 5, 8}}

1, "range", [7, 8], [[8, 7, 8, 7, 7, 8, 8, 7], [7, 8, 7, 8, 8, 7, 7, 8]]

2, "range", [5, 6], [[6, 5, 6, 5, 5, 6, 6, 5], [5, 6, 5, 6, 6, 5, 5, 6]]

3, "range", [3, 4], [[4, 3, 4, 3, 3, 4, 4, 3], [3, 4, 3, 4, 4, 3, 3, 4]]

4, "range", [1, 2], [[2, 1, 2, 1, 1, 2, 2, 1], [1, 2, 1, 2, 2, 1, 1, 2]]

6, "partition", {{1, 3, 5, 7}, {2, 4, 6, 8}}

1, "range", [7, 8], [[8, 7, 8, 7, 8, 7, 8, 7], [7, 8, 7, 8, 7, 8, 7, 8]]

2, "range", [5, 6], [[6, 5, 6, 5, 6, 5, 6, 5], [5, 6, 5, 6, 5, 6, 5, 6]]

3, "range", [3, 4], [[4, 3, 4, 3, 4, 3, 4, 3], [3, 4, 3, 4, 3, 4, 3, 4]]

4, "range", [1, 2], [[2, 1, 2, 1, 2, 1, 2, 1], [1, 2, 1, 2, 1, 2, 1, 2]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$

$\begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2\end{pmatrix}$

{1, 14, 23, 28}

$u2 =$
$(15 \quad 8 \quad 7 \quad 6 \quad 9 \quad 10 \quad 5 \quad 7 \quad 8 \quad 9 \quad 6 \quad 5 \quad 10 \quad 15 \quad 4 \quad 11 \quad 6 \quad 9 \quad 11 \quad 4 \quad 9 \quad 6 \quad 15 \quad 10 \quad 5 \quad 5$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix}1 & 1 & 2 & 2 & 1 & 1 & 2 & 2\end{pmatrix}$

$\pi = \begin{pmatrix}\frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6}\end{pmatrix}$

$\pi 1 = \begin{pmatrix}1 & 1 & 2 & 2 & 1 & 1 & 2 & 2\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2} & \frac{15}{2}\end{pmatrix}$

picheck $\begin{pmatrix}1 & 1 & 2 & 2 & 1 & 1 & 2 & 2\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{5} & \frac{2}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{3}{5} & \frac{2}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{5} & \frac{3}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{11}{15} & \frac{4}{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4}{15} & \frac{11}{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{7}{15} & \frac{8}{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{8}{15} & \frac{7}{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{11}{15} & \frac{4}{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{15} & \frac{11}{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{5} & \frac{2}{5} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{5} & \frac{3}{5} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{7}{15} & \frac{8}{15} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{15} & \frac{7}{15} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{5} & \frac{2}{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{5} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix}$$ idem-checks

$$PP_2 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & \frac{1}{3} & 0 & \frac{1}{6} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{7}{45} & \frac{8}{45} & \frac{1}{10} & \frac{1}{15} & \frac{1}{9} & \frac{2}{9} \\ 0 & \frac{1}{6} & \frac{8}{45} & \frac{7}{45} & \frac{1}{15} & \frac{1}{10} & \frac{2}{9} & \frac{1}{9} \\ \frac{7}{90} & \frac{4}{45} & \frac{1}{3} & 0 & \frac{11}{90} & \frac{2}{45} & \frac{1}{5} & \frac{2}{15} \\ \frac{4}{45} & \frac{7}{90} & 0 & \frac{1}{3} & \frac{2}{45} & \frac{11}{90} & \frac{2}{15} & \frac{1}{5} \\ \frac{1}{10} & \frac{1}{15} & \frac{11}{45} & \frac{4}{45} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{15} & \frac{1}{10} & \frac{4}{45} & \frac{11}{45} & 0 & \frac{1}{6} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{1}{5} & \frac{2}{15} & \frac{1}{18} & \frac{1}{9} & \frac{1}{3} & 0 \\ \frac{1}{9} & \frac{1}{18} & \frac{2}{15} & \frac{1}{5} & \frac{1}{9} & \frac{1}{18} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{8}{3} & 0 & \frac{112}{45} & \frac{128}{45} & \frac{8}{5} & \frac{16}{15} & \frac{16}{9} & \frac{32}{9} \\ 0 & \frac{8}{3} & \frac{128}{45} & \frac{112}{45} & \frac{16}{15} & \frac{8}{5} & \frac{32}{9} & \frac{16}{9} \\ \frac{56}{45} & \frac{64}{45} & \frac{16}{3} & 0 & \frac{88}{45} & \frac{32}{45} & \frac{16}{5} & \frac{32}{15} \\ \frac{64}{45} & \frac{56}{45} & 0 & \frac{16}{3} & \frac{32}{45} & \frac{88}{45} & \frac{32}{15} & \frac{16}{5} \\ \frac{8}{5} & \frac{16}{15} & \frac{176}{45} & \frac{64}{45} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{32}{9} \\ \frac{16}{15} & \frac{8}{5} & \frac{64}{45} & \frac{176}{45} & 0 & \frac{8}{3} & \frac{32}{9} & \frac{16}{9} \\ \frac{8}{9} & \frac{16}{9} & \frac{16}{5} & \frac{32}{15} & \frac{8}{9} & \frac{16}{9} & \frac{16}{3} & 0 \\ \frac{16}{9} & \frac{8}{9} & \frac{32}{15} & \frac{16}{5} & \frac{16}{9} & \frac{8}{9} & 0 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, -1, -1, -1, 1, 1, 1, 1]$

$$\ker N_c = \begin{pmatrix} -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} t & t & -s & -s & 0 & 0 & -t+s & -t+s \\ 0 & 0 & -s+t & -s+t & 0 & 0 & -t+s & -t+s \\ 0 & 0 & -s+t & -s+t & s & s & -t & -t \end{pmatrix}$$

RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & -t \\ -s & 0 & 0 & t \\ 0 & 0 & -t & s \\ 0 & 0 & t & -s \\ -t & 0 & 0 & s \\ t & 0 & 0 & -s \\ t & -s & 0 & 0 \\ -t & s & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & t & 0 & s \\ 0 & s+t & -t & s+t & -s \\ 0 & s+t & -s & s & 0 \\ 0 & 0 & s & t & 0 \\ 0 & s+t & -s & s+t & -t \\ 0 & 0 & s & 0 & t \\ -s & s & 0 & s & t \\ s & t & 0 & t & -t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 0 & 4 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & \frac{3}{5} & \frac{2}{5} & \frac{3}{5} & \frac{2}{5} & \frac{4}{15} & \frac{11}{15} \\ 0 & 1 & \frac{2}{5} & \frac{3}{5} & \frac{2}{5} & \frac{3}{5} & \frac{11}{15} & \frac{4}{15} \\ \frac{3}{5} & \frac{2}{5} & 1 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{5} & \frac{3}{5} & 0 & 1 & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{3}{5} & \frac{2}{5} & 1 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{5} & \frac{3}{5} & 0 & 1 & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{4}{15} & \frac{11}{15} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{11}{15} & \frac{4}{15} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{3}{5} & \frac{2}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{5} & \frac{3}{5} & \frac{3}{5} & \frac{2}{5} \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{7}{15} & \frac{8}{15} & \frac{8}{15} & \frac{7}{15} \\ \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{8}{15} & \frac{7}{15} & \frac{7}{15} & \frac{8}{15} \\ \frac{3}{5} & \frac{2}{5} & \frac{7}{15} & \frac{8}{15} & 1 & 0 & 0 & 1 \\ \frac{2}{5} & \frac{3}{5} & \frac{8}{15} & \frac{7}{15} & 0 & 1 & 1 & 0 \\ \frac{2}{5} & \frac{3}{5} & \frac{8}{15} & \frac{7}{15} & 0 & 1 & 1 & 0 \\ \frac{3}{5} & \frac{2}{5} & \frac{7}{15} & \frac{8}{15} & 1 & 0 & 0 & 1 \end{pmatrix}, \; B B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 & \frac{8}{3} & \frac{8}{3} \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \\ \frac{-8}{3} & \frac{-8}{3} & 0 & 0 & \frac{-8}{3} & \frac{-8}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} =$$

$$\begin{pmatrix} 0 & 0 & \frac{7}{90} & \frac{4}{45} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{4}{45} & \frac{7}{90} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-7}{90} & \frac{-4}{45} & 0 & 0 & \frac{-11}{90} & \frac{-2}{45} & 0 & 0 \\ \frac{-4}{45} & \frac{-7}{90} & 0 & 0 & \frac{-2}{45} & \frac{-11}{90} & 0 & 0 \\ 0 & 0 & \frac{11}{90} & \frac{2}{45} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & \frac{2}{45} & \frac{11}{90} & 0 & 0 & \frac{1}{9} & \frac{1}{18} \\ \frac{-1}{18} & \frac{-1}{9} & 0 & 0 & \frac{-1}{18} & \frac{-1}{9} & 0 & 0 \\ \frac{-1}{9} & \frac{-1}{18} & 0 & 0 & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{12} & \frac{1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{-1}{12} & 0 & 0 & \frac{-1}{12} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{3} & \frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{16}{3} & \frac{16}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{8}{3} & \frac{8}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} & \frac{16}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & \frac{7}{15} & \frac{8}{15} & \frac{3}{5} & \frac{2}{5} & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{8}{15} & \frac{7}{15} & \frac{2}{5} & \frac{3}{5} & \frac{2}{3} & \frac{1}{3} \\ \frac{7}{15} & \frac{8}{15} & 1 & 0 & \frac{11}{15} & \frac{4}{15} & \frac{3}{5} & \frac{2}{5} \\ \frac{8}{15} & \frac{7}{15} & 0 & 1 & \frac{4}{15} & \frac{11}{15} & \frac{2}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{2}{5} & \frac{11}{15} & \frac{4}{15} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{15} & \frac{11}{15} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{3}{5} & \frac{2}{5} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{5} & \frac{3}{5} & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{12}\right)$$

$$T\left(\frac{1}{3}\ \frac{1}{18}\ \frac{1}{5}\ \frac{1}{18}\ \frac{1}{9}\ \frac{1}{6}\ \frac{11}{45}\ \frac{1}{10}\ \frac{1}{5}\ \frac{11}{90}\ \frac{1}{3}\ \frac{7}{90}\ \frac{2}{9}\ \frac{1}{9}\ \frac{1}{15}\ \frac{1}{10}\ \frac{8}{45}\ \frac{7}{45}\ 0\ \frac{1}{6}\right)$$

"IS NM in Vec(K)?", true

$$\text{NM}\left(\frac{16}{3}\ \frac{8}{9}\ \frac{16}{5}\ \frac{8}{9}\ \frac{16}{9}\ \frac{8}{3}\ \frac{176}{45}\ \frac{8}{5}\ \frac{16}{5}\ \frac{88}{45}\ \frac{16}{3}\ \frac{56}{45}\ \frac{32}{9}\ \frac{16}{9}\ \frac{16}{15}\ \frac{8}{5}\ \frac{128}{45}\ \frac{112}{45}\ 0\ \frac{8}{3}\right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{14}{3}\ \frac{10}{9}\ \frac{38}{15}\ \frac{10}{9}\ \frac{14}{9}\ \frac{10}{3}\ \frac{118}{45}\ \frac{34}{15}\ \frac{38}{15}\ \frac{146}{45}\ \frac{14}{3}\ \frac{82}{45}\ \frac{22}{9}\ \frac{14}{9}\ \frac{26}{15}\ \frac{34}{15}\ \frac{94}{45}\ \frac{86}{45}\ \frac{2}{3}\ \frac{10}{3}\right)$$

$$\boldsymbol{\tau} = 32/1\text{ , rank} = 2\text{ , ratio} = 16/1\text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1\text{ , } r' = 1/2\text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 5/36\text{ , min } \boldsymbol{\tau} = 80/9\text{ , } \boldsymbol{\tau}\text{-check is positive? } 208/9$$

$$\text{max r} = 36/5\text{ , } r\text{-check is positive? } 13/18$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 3, 5, 8}, {2, 4, 6, 7}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 4, 6, 8}, {2, 3, 5, 7}}

"PT4" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT5" = {{1, 3, 6, 7}, {2, 4, 5, 8}}

"PT6" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"RG1" = {7, 8}

"RG2" = {5, 6}

"RG3" = {3, 4}

"RG4" = {1, 2}

$$M_c = \begin{pmatrix} \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-4}{9} & \frac{-4}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{20}{9} & \frac{20}{9} & \frac{-8}{9} & \frac{-8}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \\ \frac{-8}{9} & \frac{-8}{9} & \frac{-16}{9} & \frac{-16}{9} & \frac{-8}{9} & \frac{-8}{9} & \frac{32}{9} & \frac{32}{9} \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{31}{36} & \frac{-5}{36} & \frac{59}{180} & \frac{71}{180} & \frac{83}{180} & \frac{47}{180} & \frac{7}{36} & \frac{19}{36} \\ \frac{-5}{36} & \frac{31}{36} & \frac{71}{180} & \frac{59}{180} & \frac{47}{180} & \frac{83}{180} & \frac{19}{36} & \frac{7}{36} \\ \frac{59}{180} & \frac{71}{180} & \frac{31}{36} & \frac{-5}{36} & \frac{107}{180} & \frac{23}{180} & \frac{83}{180} & \frac{47}{180} \\ \frac{71}{180} & \frac{59}{180} & \frac{-5}{36} & \frac{31}{36} & \frac{23}{180} & \frac{107}{180} & \frac{47}{180} & \frac{83}{180} \\ \frac{83}{180} & \frac{47}{180} & \frac{107}{180} & \frac{23}{180} & \frac{31}{36} & \frac{-5}{36} & \frac{7}{36} & \frac{19}{36} \\ \frac{47}{180} & \frac{83}{180} & \frac{23}{180} & \frac{107}{180} & \frac{-5}{36} & \frac{31}{36} & \frac{19}{36} & \frac{7}{36} \\ \frac{7}{36} & \frac{19}{36} & \frac{83}{180} & \frac{47}{180} & \frac{7}{36} & \frac{19}{36} & \frac{31}{36} & \frac{-5}{36} \\ \frac{19}{36} & \frac{7}{36} & \frac{47}{180} & \frac{83}{180} & \frac{19}{36} & \frac{7}{36} & \frac{-5}{36} & \frac{31}{36} \end{pmatrix}$$

$$M_c\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ 1 & 1 & \frac{-2}{5} & \frac{-2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-2}{5} & \frac{-2}{5} & 1 & 1 & \frac{-2}{5} & \frac{-2}{5} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{4} & \frac{-1}{4} & 1 & 1 \end{pmatrix} \quad N_c\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-5}{31} & \frac{59}{155} & \frac{71}{155} & \frac{83}{155} & \frac{47}{155} & \frac{7}{31} & \frac{19}{31} \\ \frac{-5}{31} & 1 & \frac{71}{155} & \frac{59}{155} & \frac{47}{155} & \frac{83}{155} & \frac{19}{31} & \frac{7}{31} \\ \frac{59}{155} & \frac{71}{155} & 1 & \frac{-5}{31} & \frac{107}{155} & \frac{23}{155} & \frac{83}{155} & \frac{47}{155} \\ \frac{71}{155} & \frac{59}{155} & \frac{-5}{31} & 1 & \frac{23}{155} & \frac{107}{155} & \frac{47}{155} & \frac{83}{155} \\ \frac{83}{155} & \frac{47}{155} & \frac{107}{155} & \frac{23}{155} & 1 & \frac{-5}{31} & \frac{7}{31} & \frac{19}{31} \\ \frac{47}{155} & \frac{83}{155} & \frac{23}{155} & \frac{107}{155} & \frac{-5}{31} & 1 & \frac{19}{31} & \frac{7}{31} \\ \frac{7}{31} & \frac{19}{31} & \frac{83}{155} & \frac{47}{155} & \frac{7}{31} & \frac{19}{31} & 1 & \frac{-5}{31} \\ \frac{19}{31} & \frac{7}{31} & \frac{47}{155} & \frac{83}{155} & \frac{19}{31} & \frac{7}{31} & \frac{-5}{31} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 5.333333333, 10.66666667, 7.111111111]

Eigenvalues N_C

[0., 0., 0., 2.888888889, 0.3212986242, 0.7071405096, 1.362246242, 1.609314622]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 3., 2.400000000, 2.600000000]

Eigenvalues N_C–scaled

[0., 0., 0., 3.354838710, 0.3731209831, 0.8211954308, 1.581963379, 1.868881498]

NullSpace M_C

{[0, 0, 1, –1, 0, 0, 0, 0], [0, 0, 0, 0, –1, 1, 0, 0], [1, 0, 0, 1, 1, 0, 1, 0], [1, 0, 0, 1, 1, 0, 0, 1], [–1, 1, 0, 0, 0, 0, 0, 0] }

NullSpace N_C

{[–1, –1, 0, 0, 0, 0, 1, 1], [–1, –1, 1, 1, 0, 0, 0, 0], [–1, –1, 0, 0, 1, 1, 0, 0]}

Eigenvalues M_0

[10.66666667, 5.333333333, 10.66666667, 5.333333333, 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 0.3212986242, 0.7071405096, 1.362246242, 1.609314622]

NullSpace M_0

{[0, 0, 0, 0, –1, 1, 0, 0], [0, 0, 0, 0, 0, 0, –1, 1], [–1, 1, 0, 0, 0, 0, 0, 0], [0, 0, –1, 1, 0, 0, 0, 0]}

NullSpace N_0

{[–1, –1, 0, 0, 1, 1, 0, 0], [–1, –1, 0, 0, 0, 0, 1, 1], [–1, –1, 1, 1, 0, 0, 0, 0]}

Eigenvalues M

[–2.666666667, 2.666666667, –5.333333333, 5.333333333, –2.666666667, 2.666666667, –5.333333333, 5.333333333]

Eigenvalues N

[0., 0., 0., 4., –0.3212986242, –0.7071405096, –1.362246242, –1.609314622]

NullSpace M

{}

NullSpace N

{[−1, −1, 1, 1, 0, 0, 0, 0], [−1, −1, 0, 0, 0, 0, 1, 1], [−1, −1, 0, 0, 1, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 15 & 8 & 7 & 6 & 9 & 10 & 5 \\ 15 & 0 & 7 & 8 & 9 & 6 & 5 & 10 \\ 8 & 7 & 0 & 15 & 4 & 11 & 6 & 9 \\ 7 & 8 & 15 & 0 & 11 & 4 & 9 & 6 \\ 6 & 9 & 4 & 11 & 0 & 15 & 10 & 5 \\ 9 & 6 & 11 & 4 & 15 & 0 & 5 & 10 \\ 10 & 5 & 6 & 9 & 10 & 5 & 0 & 15 \\ 5 & 10 & 9 & 6 & 5 & 10 & 15 & 0 \end{pmatrix}$$