T-Run

[4, 3, 1, 2, 1], [3, 4, 5, 3, 3]

$\tilde{\pi}$ = [3, 1, 4, 2, 2]
$\boldsymbol{\delta}$ = [2, 1, 4, 2, 1]

POSSIBLE RANKS

1 x 12
2 x 6
3 x 4

BASE DETERMINANT 153/1024, .1494140625

NullSpace of Δ

{1, 2, 3, 4, 5}

Nullspace of A

[{3, 5},{1, 2, 4}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[3] v[5] + v[2] v[4] + v[1] v[3]

Degree 3
v[1] v[3] v[5] + v[1] v[3] v[4]

Degree 4
2 v[1] v[2] v[3] v[4] + v[2] v[3] v[4] v[5] + v[1] v[3] v[4] v[5]

Degree 5
2 v[1] v[2] v[3] v[4] v[5]

==

{2, 3, 4}

R: [4, 4, 5, 3, 1]
B: [3, 3, 1, 2, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{-1}{1024}\,(153 + 121s + 88s^2 + 30s^3 + 7s^4 + s^5)\,(-1 + s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 3

B ranking is 2, "vs", 3

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[1] v[3] v[4] v[5]
"B CYCLES", 1 + v[1] v[3]

Eigenvalues

R: [0., −1., 1., 1. I, −1. I]

B: [0., 0., 0., 1., −1.]

NullSpace of R

{[0, 1, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0], [0, 0, 0, 0, 1]}

NullSpace of R^*

{[−1, 1, 0, 0, 0]}

NullSpace of B^*

{[−1, 0, 0, 0, 1], [−1, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 1 & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}\left(3v[1] + v[2] + 4v[3] + 2v[4] + 2v[5]\right)$
degree 2: $\frac{1}{3}\left(3v[1]v[3] + v[2]v[3] + 2v[4]v[5]\right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 4}, {1, 2, 5}}

"PT2" = {{3, 5}, {1, 2, 4}}

"RG1" = {4, 5}

"RG2" = {2, 3}

"RG3" = {1, 3}

$$\pi 2 = [0, 3, 0, 0, 1, 0, 0, 0, 0, 2]$$

supp $\pi 2$ = {2, 5, 10}

$$u2 = [0, 3, 2, 1, 3, 2, 1, 1, 2, 3]$$

supp u2 = {2, 3, 4, 5, 6, 7, 8, 9, 10}

Action of R on ranges, [[3], [1], [1]]
Action of B on ranges, [[2], [3], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} \end{pmatrix}$

RPARTS [2, 1]

BPARTS [1, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[3, 4, 2, 1]

B−BLOCKS,

[4, 3, 4, 3]

with invariant measure, [1, 1, 2, 2]

N by blocks, N − check: true

b_1 = {3, 5}

b_2 = {1, 2, 4}
b_3 = {3, 4}
b_4 = {1, 2, 5}

dim(span of partition vectors), rank(N_0), rank(N):   3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: 3 ⊕ 7/5

$$\mathrm{CLB} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3, 4, 5}}, true

Ω_B in Vec(K)? , {{1, 3}}, true

$$V = \begin{pmatrix} \frac{-1}{4} & \frac{1}{4} & \frac{-1}{3} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{4} & \frac{-1}{3} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{-1}{12} & 0 & \frac{-1}{6} & \frac{1}{2} \\ \frac{1}{4} & \frac{-7}{12} & \frac{1}{3} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{12} & \frac{-1}{3} & \frac{1}{6} & \frac{-1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 4}, {1, 2, 5}}

1, "range", [4, 5], [[5, 5, 4, 4, 5], [4, 4, 5, 5, 4]]

2, "range", [2, 3], [[3, 3, 2, 2, 3], [2, 2, 3, 3, 2]]

3, "range", [1, 3], [[3, 3, 1, 1, 3], [1, 1, 3, 3, 1]]

2, "partition", {{3, 5}, {1, 2, 4}}

1, "range", [4, 5], [[5, 5, 4, 5, 4], [4, 4, 5, 4, 5]]

2, "range", [2, 3], [[3, 3, 2, 3, 2], [2, 2, 3, 2, 3]]

3, "range", [1, 3], [[3, 3, 1, 3, 1], [1, 1, 3, 1, 3]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [2, 3]}, {6, [2, 4]}, {7, [2, 5]}, {8, [3, 4]}, {9, [3, 5]}, {10, [4, 5]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 0 & 3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{2, 5, 10}

$u2 = \begin{pmatrix} 0 & 3 & 2 & 1 & 3 & 2 & 1 & 1 & 2 & 3 \end{pmatrix}$

{2, 3, 4, 5, 6, 7, 8, 9, 10}

picheck $\begin{pmatrix} 3 & 1 & 4 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{4} & \frac{1}{12} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 3 & 1 & 4 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 3 & 1 & 4 & 2 & 2 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix}$$ idem-checks

$$PP_2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{2}{9} \\ 0 & 0 & \frac{2}{3} & \frac{2}{9} & \frac{1}{9} \\ \frac{1}{6} & \frac{1}{18} & \frac{4}{9} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & \frac{2}{9} & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 3 & 1 & 0 & \frac{2}{3} & \frac{4}{3} \\ 3 & 1 & 0 & \frac{2}{3} & \frac{4}{3} \\ 0 & 0 & 4 & \frac{4}{3} & \frac{2}{3} \\ 1 & \frac{1}{3} & \frac{8}{3} & 2 & 0 \\ 2 & \frac{2}{3} & \frac{4}{3} & 0 & 2 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [-1, -1, -2, 2, 2]

$$\ker N_c = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ t-s & -t & -s & s & s \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -2 \end{pmatrix}$

M0 is invertible. det= 8125/5184

$$\ker M_c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} t+s \\ t+s \\ t+s \\ t+s \\ t+s \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = (5)$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 1 & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 3, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & \frac{-13}{6} & 1 & \frac{-37}{36} & \frac{-19}{18} \\ \frac{13}{6} & 0 & 3 & \frac{37}{36} & \frac{19}{18} \\ -1 & -3 & 0 & \frac{-19}{9} & \frac{-37}{18} \\ \frac{37}{36} & \frac{-37}{36} & \frac{19}{9} & 0 & 0 \\ \frac{19}{18} & \frac{-19}{18} & \frac{37}{18} & 0 & 0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0 & \frac{-1}{3} & 0 & \frac{-1}{18} & \frac{-1}{9} \\ \frac{1}{3} & 0 & 0 & \frac{1}{18} & \frac{1}{9} \\ 0 & 0 & 0 & \frac{-2}{9} & \frac{-1}{9} \\ \frac{1}{18} & \frac{-1}{18} & \frac{2}{9} & 0 & 0 \\ \frac{1}{9} & \frac{-1}{9} & \frac{1}{9} & 0 & 0 \end{pmatrix}$$

$$\mathrm{Skew\ Omega} = \begin{pmatrix} 0 & \frac{-1}{6} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{12} \\ \frac{1}{6} & 0 & \frac{1}{4} & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{4} & 0 & \frac{-1}{6} & \frac{-1}{6} \\ \frac{1}{12} & \frac{-1}{12} & \frac{1}{6} & 0 & 0 \\ \frac{1}{12} & \frac{-1}{12} & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{13}{4} & 0 & 3 & 0 & 0 \\ 0 & \frac{13}{12} & 1 & 0 & 0 \\ 3 & 1 & \frac{13}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{13}{6} & 2 \\ 0 & 0 & 0 & 2 & \frac{13}{6} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 6T + 0\Omega$$

$$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{12} & \frac{1}{4} & \frac{1}{4} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{12} & \frac{1}{4} \end{pmatrix}$$

$T\left(\frac{1}{3}\ \ \frac{1}{18}\ \ \frac{1}{6}\ \ 0\ \ \frac{2}{9}\ \ \frac{1}{9}\ \ 0\ \ \frac{1}{6}\ \ \frac{1}{2}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM}\left(2\ \ \frac{1}{3}\ \ 1\ \ 0\ \ \frac{4}{3}\ \ \frac{2}{3}\ \ 0\ \ 1\ \ 3\right)$

"IS MN in Vec(K)?", false

$\mathrm{MN}\left(\frac{11}{5}\ \ \frac{388}{603}\ \ \frac{388}{603}\ \ \frac{-8}{67}\ \ \frac{74}{45}\ \ \frac{34}{45}\ \ \frac{-16}{67}\ \ \frac{124}{67}\ \ \frac{124}{67}\right)$

$$\boldsymbol{\tau} = 13/1 \text{ , rank } = 2 \text{ , ratio } = 13/2 \text{ , } n^2 / r = 25/2$$

$$\boldsymbol{\tau}' = 12/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 13/25$$

$$p^2 = 17/72 \text{ , min } \boldsymbol{\tau} = 425/72 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 511/72$$

$$\text{max r} = 72/17 \text{ , } r\text{-check is positive? } 19/36$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{2} T + 12\Omega$$

There are, 2, partitions and, 3, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 9
out of total no. of elements equal to 12

dim span idems 6 vs no. of idems 6

"PT1" = {{3, 4}, {1, 2, 5}}

"PT2" = {{3, 5}, {1, 2, 4}}

"RG1" = {4, 5}

"RG2" = {2, 3}

"RG3" = {1, 3}

$$M_C = \begin{pmatrix} \frac{27}{16} & \frac{-25}{48} & \frac{11}{12} & \frac{-25}{24} & \frac{-25}{24} \\ \frac{-25}{48} & \frac{131}{144} & \frac{11}{36} & \frac{-25}{72} & \frac{-25}{72} \\ \frac{11}{12} & \frac{11}{36} & \frac{14}{9} & \frac{-25}{18} & \frac{-25}{18} \\ \frac{-25}{24} & \frac{-25}{72} & \frac{-25}{18} & \frac{53}{36} & \frac{47}{36} \\ \frac{-25}{24} & \frac{-25}{72} & \frac{-25}{18} & \frac{47}{36} & \frac{53}{36} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{7}{72} & \frac{31}{72} \\ \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{7}{72} & \frac{31}{72} \\ \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{31}{72} & \frac{7}{72} \\ \frac{7}{72} & \frac{7}{72} & \frac{31}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{31}{72} & \frac{31}{72} & \frac{7}{72} & \frac{-17}{72} & \frac{55}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-25}{81} & \frac{44}{81} & \frac{-50}{81} & \frac{-50}{81} \\ \frac{-75}{131} & 1 & \frac{44}{131} & \frac{-50}{131} & \frac{-50}{131} \\ \frac{33}{56} & \frac{11}{56} & 1 & \frac{-25}{28} & \frac{-25}{28} \\ \frac{-75}{106} & \frac{-25}{106} & \frac{-50}{53} & 1 & \frac{47}{53} \\ \frac{-75}{106} & \frac{-25}{106} & \frac{-50}{53} & \frac{47}{53} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & 1 & \frac{-17}{55} & \frac{7}{55} & \frac{31}{55} \\ 1 & 1 & \frac{-17}{55} & \frac{7}{55} & \frac{31}{55} \\ \frac{-17}{55} & \frac{-17}{55} & 1 & \frac{31}{55} & \frac{7}{55} \\ \frac{7}{55} & \frac{7}{55} & \frac{31}{55} & 1 & \frac{-17}{55} \\ \frac{31}{55} & \frac{31}{55} & \frac{7}{55} & \frac{-17}{55} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{8} & \frac{1}{24} & \frac{-1}{6} & \frac{-1}{36} & \frac{1}{36} \\ \frac{1}{8} & \frac{1}{24} & \frac{-1}{6} & \frac{-1}{36} & \frac{1}{36} \\ \frac{-1}{8} & \frac{-1}{24} & \frac{1}{6} & \frac{1}{36} & \frac{-1}{36} \\ \frac{-1}{24} & \frac{-1}{72} & \frac{1}{18} & \frac{1}{12} & \frac{-1}{12} \\ \frac{1}{24} & \frac{1}{72} & \frac{-1}{18} & \frac{-1}{12} & \frac{1}{12} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{-1}{24} & \frac{1}{24} \\ \frac{1}{24} & \frac{1}{24} & \frac{-1}{24} & \frac{-1}{72} & \frac{1}{72} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{18} & \frac{-1}{18} \\ \frac{-1}{36} & \frac{-1}{36} & \frac{1}{36} & \frac{1}{12} & \frac{-1}{12} \\ \frac{1}{36} & \frac{1}{36} & \frac{-1}{36} & \frac{-1}{12} & \frac{1}{12} \end{pmatrix}$$

$$\text{commutator} = \begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{24} & \frac{-1}{72} & \frac{1}{72} \\ \frac{-1}{12} & 0 & \frac{1}{8} & \frac{1}{72} & \frac{-1}{72} \\ \frac{-1}{24} & \frac{-1}{8} & 0 & \frac{1}{36} & \frac{-1}{36} \\ \frac{1}{72} & \frac{-1}{72} & \frac{-1}{36} & 0 & 0 \\ \frac{-1}{72} & \frac{1}{72} & \frac{1}{36} & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0.1666666667, 5.120240014, 0.2328582901, 1.577457251]

Eigenvalues N_C

[0., 0., 1.924534634, 0.7365275340, 1.158382277]

Eigenvalues M_C–scaled

[0., 0.1132075472, 3.358219074, 0.1659432235, 1.362630156]

Eigenvalues N_C–scaled

[0., 0., 2.519390794, 0.9641814985, 1.516427708]

NullSpace M_C

{[1, 1, 1, 1, 1]}

NullSpace N_c

{[0, 1, 1, -1, -1], [1, -1, 0, 0, 0]}

Eigenvalues M_0

[0.1666666667, 4.166666667, 6.941836498, 0.2153929210,
1.509437247]

Eigenvalues N_0

[0., 0., 2.733824113, 0.7511439170, 1.515031969]

NullSpace M_0

{}

NullSpace N_0

{[1, 0, 1, -1, -1], [-1, 1, 0, 0, 0]}

Eigenvalues M

[0., -2., 2., 3.162277660, -3.162277660]

Eigenvalues N

[0., 0., 2.437798895, -1.676631988, -0.7611669080]

NullSpace M

{[1, -3, 0, 0, 0]}

NullSpace N

{[-1, 1, 0, 0, 0], [1, 0, 1, -1, -1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 3 & 2 & 1 \\ 0 & 0 & 3 & 2 & 1 \\ 3 & 3 & 0 & 1 & 2 \\ 2 & 2 & 1 & 0 & 3 \\ 1 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

{2, 3, 5}

R: [4, 4, 5, 2, 3]
B: [3, 3, 1, 3, 1]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 \\ \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{1}{1024}\left(51 + 34s - 10s^2 - 4s^3 + s^4\right)\left(-1 + s\right)\left(-3 + s\right)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 2

B ranking is 1, "vs", 2

BBAR ranking 1, "vs", 2

"R CYCLES", (1 + v[3] v[5]) (1 + v[2] v[4])
"B CYCLES", 1 + v[1] v[3]

Eigenvalues

R: [0., 1., −1., 1., −1.]

B: [0., 0., 0., 1., −1.]

NullSpace of R

{[1, 0, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0], [0, 0, 0, 1, 0], [0, 0, 0, 0, 1]}

NullSpace of R^*

{[1, −1, 0, 0, 0]}

NullSpace of B^*

{[−1, 0, 0, 1, 0], [0, 0, −1, 0, 1], [−1, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6}$ ($3\,v[1] + v[2] + 4\,v[3] + 2\,v[4] + 2\,v[5]$)

degree 2: $\frac{1}{3}$ ($3\,v[1]v[3] + v[2]v[3] + 2\,v[4]v[5]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 5}, {1, 2, 4}}

"RG1" = {4, 5}

"RG2" = {2, 3}

"RG3" = {1, 3}

$$\pi 2 = [0, 3, 0, 0, 1, 0, 0, 0, 0, 2]$$

supp π2 = {2, 5, 10}

$$u2 = [0, 1, 0, 1, 1, 0, 1, 1, 0, 1]$$

supp u2 = {2, 4, 5, 7, 8, 10}

Action of R on ranges, [[2], [1], [1]]
Action of B on ranges, [[3], [3], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[1, 2]

B−BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N – check: true

$b_1 = \{3, 5\}$
$b_2 = \{1, 2, 4\}$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 6, Shape: $0 \oplus 6/4$

$$\mathrm{CLB} = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 5}, {2, 4}}, true

Ω_B in Vec(K)? , {{1, 3}}, true

$$V = \begin{pmatrix} \frac{-1}{4} & \frac{1}{4} & \frac{-1}{3} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{4} & \frac{-1}{3} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{-1}{12} & 0 & \frac{-1}{6} & \frac{1}{2} \\ \frac{-1}{4} & \frac{7}{12} & \frac{-1}{3} & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{-1}{12} & \frac{1}{3} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{3}{10} & \frac{1}{5} & \frac{3}{10} & \frac{1}{5} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 5}, {1, 2, 4}}

1, "range", [4, 5], [[5, 5, 4, 5, 4], [4, 4, 5, 4, 5]]

2, "range", [2, 3], [[3, 3, 2, 3, 2], [2, 2, 3, 2, 3]]

3, "range", [1, 3], [[3, 3, 1, 3, 1], [1, 1, 3, 1, 3]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [2, 3]}, {6, [2, 4]}, {7, [2, 5]}, {8, [3, 4]}, {9, [3, 5]}, {10, [4, 5]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 0 & 3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

{2, 5, 10}

$u2 = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$

{2, 4, 5, 7, 8, 10}

picheck $\begin{pmatrix} 3 & 1 & 4 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{4} & \frac{1}{12} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 3 & 1 & 4 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 3 & 1 & 4 & 2 & 2 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks   N0−checks

$P_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$ idem−checks   N0−checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix}$$ idem−checks

$$\mathrm{T} = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{6} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix} \quad \mathrm{NM} = \begin{pmatrix} 3 & 1 & 0 & 2 & 0 \\ 3 & 1 & 0 & 2 & 0 \\ 0 & 0 & 4 & 0 & 2 \\ 3 & 1 & 0 & 2 & 0 \\ 0 & 0 & 4 & 0 & 2 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-3, 1, -2, 2, 2]$

$$\ker N_c = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & 0 & -s & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s & 0 & -s \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 2 & 1 & 2 \end{pmatrix}$

M0 is invertible. det= 8125/5184

ker M_c = $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} t+s \\ t+s \\ t+s \\ t+s \\ t+s \end{pmatrix}$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 5 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 2, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & \frac{-13}{6} & 1 & \frac{-13}{12} & -1 \\ \frac{13}{6} & 0 & 3 & \frac{13}{12} & 1 \\ -1 & -3 & 0 & -2 & \frac{-13}{6} \\ \frac{13}{12} & \frac{-13}{12} & 2 & 0 & 0 \\ 1 & -1 & \frac{13}{6} & 0 & 0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0 & \frac{-1}{3} & 0 & \frac{-1}{6} & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{3} \\ \frac{1}{6} & \frac{-1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$

$$\mathrm{Skew\ Omega} = \begin{pmatrix} 0 & \frac{-1}{6} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{12} \\ \frac{1}{6} & 0 & \frac{1}{4} & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{4} & 0 & \frac{-1}{6} & \frac{-1}{6} \\ \frac{1}{12} & \frac{-1}{12} & \frac{1}{6} & 0 & 0 \\ \frac{1}{12} & \frac{-1}{12} & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{13}{4} & 0 & 3 & 0 & 0 \\ 0 & \frac{13}{12} & 1 & 0 & 0 \\ 3 & 1 & \frac{13}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{13}{6} & 2 \\ 0 & 0 & 0 & 2 & \frac{13}{6} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 6T + 0\Omega$$

$$\Omega \left(\frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{12} \quad \frac{1}{4} \right)$$

$T\begin{pmatrix}0 & 0 & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{2}\end{pmatrix}$

"IS NM in Vec(K)?", true

$NM\begin{pmatrix}0 & 0 & 2 & 0 & 1 & 3\end{pmatrix}$

"IS MN in Vec(K)?", false

$MN\begin{pmatrix}0 & 0 & \frac{12}{5} & 0 & \frac{12}{7} & \frac{12}{7}\end{pmatrix}$

$\tau = 13/1$, rank = 2 , ratio = 13/2 , $n^2 / r = 25/2$

$\tau' = 12/1$, $r' = 1/2$, $\tau / n^2 = 13/25$

$p^2 = 17/72$, min $\tau = 425/72$, τ–check is positive? 511/72

max r = 72/17 , r–check is positive? 19/36

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{2} T + 12\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 6
out of total no. of elements equal to 6

dim span idems 3 vs no. of idems 3

"PT1" = {{3, 5}, {1, 2, 4}}

"RG1" = {4, 5}

"RG2" = {2, 3}

"RG3" = {1, 3}

$$M_c = \begin{pmatrix} \frac{27}{16} & \frac{-25}{48} & \frac{11}{12} & \frac{-25}{24} & \frac{-25}{24} \\ \frac{-25}{48} & \frac{131}{144} & \frac{11}{36} & \frac{-25}{72} & \frac{-25}{72} \\ \frac{11}{12} & \frac{11}{36} & \frac{14}{9} & \frac{-25}{18} & \frac{-25}{18} \\ \frac{-25}{24} & \frac{-25}{72} & \frac{-25}{18} & \frac{53}{36} & \frac{47}{36} \\ \frac{-25}{24} & \frac{-25}{72} & \frac{-25}{18} & \frac{47}{36} & \frac{53}{36} \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{55}{72} \\ \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{55}{72} \end{pmatrix}$$

$$M_c\text{-scaled} = \begin{pmatrix} 1 & \frac{-25}{81} & \frac{44}{81} & \frac{-50}{81} & \frac{-50}{81} \\ \frac{-75}{131} & 1 & \frac{44}{131} & \frac{-50}{131} & \frac{-50}{131} \\ \frac{33}{56} & \frac{11}{56} & 1 & \frac{-25}{28} & \frac{-25}{28} \\ \frac{-75}{106} & \frac{-25}{106} & \frac{-50}{53} & 1 & \frac{47}{53} \\ \frac{-75}{106} & \frac{-25}{106} & \frac{-50}{53} & \frac{47}{53} & 1 \end{pmatrix} \quad N_c\text{-scaled} =$$

$$\begin{pmatrix} 1 & 1 & \frac{-17}{55} & 1 & \frac{-17}{55} \\ 1 & 1 & \frac{-17}{55} & 1 & \frac{-17}{55} \\ \frac{-17}{55} & \frac{-17}{55} & 1 & \frac{-17}{55} & 1 \\ 1 & 1 & \frac{-17}{55} & 1 & \frac{-17}{55} \\ \frac{-17}{55} & \frac{-17}{55} & 1 & \frac{-17}{55} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{8} & \frac{1}{24} & \frac{-1}{6} & \frac{1}{12} & \frac{-1}{12} \\ \frac{1}{8} & \frac{1}{24} & \frac{-1}{6} & \frac{1}{12} & \frac{-1}{12} \\ \frac{-1}{8} & \frac{-1}{24} & \frac{1}{6} & \frac{-1}{12} & \frac{1}{12} \\ \frac{1}{8} & \frac{1}{24} & \frac{-1}{6} & \frac{1}{12} & \frac{-1}{12} \\ \frac{-1}{8} & \frac{-1}{24} & \frac{1}{6} & \frac{-1}{12} & \frac{1}{12} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{-1}{8} \\ \frac{1}{24} & \frac{1}{24} & \frac{-1}{24} & \frac{1}{24} & \frac{-1}{24} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} \end{pmatrix}$$

$$\text{commutator} = \begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{24} & \frac{1}{24} & \frac{-1}{24} \\ \frac{-1}{12} & 0 & \frac{1}{8} & \frac{-1}{24} & \frac{1}{24} \\ \frac{-1}{24} & \frac{-1}{8} & 0 & \frac{-1}{12} & \frac{1}{12} \\ \frac{-1}{24} & \frac{1}{24} & \frac{1}{12} & 0 & 0 \\ \frac{1}{24} & \frac{-1}{24} & \frac{-1}{12} & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0.1666666667, 5.120240014, 0.2328582901, 1.577457251]

Eigenvalues N_C

[0., 0., 0., 2.602811177, 1.216633267]

Eigenvalues M_C–scaled

[0., 0.1132075472, 3.358219074, 0.1659432235, 1.362630156]

Eigenvalues N_C–scaled

[0., 0., 0., 3.407316450, 1.592683550]

NullSpace M_C

{[1, 1, 1, 1, 1]}

NullSpace N_c

{[0, 1, 0, −1, 0], [1, 0, 0, −1, 0], [0, 0, −1, 0, 1]}

Eigenvalues M_0

[0.1666666667, 4.166666667, 6.941836498, 0.2153929210,
1.509437247]

Eigenvalues N_0

[0., 0., 0., 3., 2.]

NullSpace M_0

{}

NullSpace N_0

{[0, 0, 1, 0, −1], [0, 1, 0, −1, 0], [1, 0, 0, −1, 0]}

Eigenvalues M

[0., −2., 2., 3.162277660, −3.162277660]

Eigenvalues N

[0., 0., 0., 2.449489743, −2.449489743]

NullSpace M

{[1, −3, 0, 0, 0]}

NullSpace N

{[1, 0, 0, −1, 0], [0, 0, −1, 0, 1], [0, 1, 0, −1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

==

{2, 4, 5}

R: [4, 4, 1, 3, 3]
B: [3, 3, 5, 2, 1]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 \\ \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{1}{1024} (1 + s) (17 + 6s + s^2) (-9 - 4s + s^2) (-1 + s)$

RANK of R is 3

R ranking is 1, "vs", 3

RBAR ranking 1, "vs", 3

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 3

"R CYCLES", 1 + v[1] v[3] v[4]
"B CYCLES", 1 + v[1] v[3] v[5]

Eigenvalues

R: [0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

B: [0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 1], [0, 1, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0]}

NullSpace of R^*

{[-1, 1, 0, 0, 0], [0, 0, 0, -1, 1]}

NullSpace of B^*

{[-1, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 6 & 4 & 2 \\ 0 & 0 & 2 & 0 & 2 \\ 6 & 2 & 0 & 4 & 4 \\ 4 & 0 & 4 & 0 & 0 \\ 2 & 2 & 4 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 5

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3   "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}$ ($3\,v[1] + v[2] + 4\,v[3] + 2\,v[4] + 2\,v[5]$)
degree 2: $\frac{1}{6}$ ($3\,v[1]v[3] + 2\,v[1]v[4] + v[1]v[5] + v[2]v[3] + v[2]v[5] + 2\,v[3]$
$v[4] + 2\,v[3]v[5]$)
degree 3 : $\frac{1}{4}$ ($2\,v[1]v[4] + v[1]v[5] + v[2]v[5]$) ($v[3]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{4, 5}, {1, 2}, {3}}

"RG1" = {2, 3, 5}

"RG2" = {1, 3, 5}

"RG3" = {1, 3, 4}

$$\pi 3 = [0, 0, 0, 2, 1, 0, 0, 1, 0, 0]$$

supp $\pi 3$ = {4, 5, 8}

$$u3 = [0, 0, 0, 1, 1, 0, 1, 1, 0, 0]$$

supp u3 = {4, 5, 7, 8}

Action of R on ranges, [[3], [3], [3]]
Action of B on ranges, [[2], [2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 3, 1]

B–BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N – check: true

$b_1 = \{4, 5\}$
$b_2 = \{1, 2\}$
$b_3 = \{3\}$

dim(span of partition vectors), rank(N_0), rank(N):   3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 8, Shape: $0 \oplus 8/6$

$$\text{CLB} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3, 4}}, true

Ω_B in Vec(K)? , {{1, 3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{4} & \frac{1}{4} & \frac{-1}{3} & \frac{1}{2} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{1}{4} & \frac{-1}{3} & \frac{1}{2} & \frac{-1}{6} \\ \frac{1}{4} & \frac{1}{12} & 0 & \frac{1}{6} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-7}{12} & \frac{1}{3} & \frac{-1}{6} & \frac{1}{6} \\ \frac{-1}{4} & \frac{-1}{12} & \frac{1}{3} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4, 5}, {1, 2}, {3}}

1, "range", [2, 3, 5], [[5, 5, 2, 3, 3], [3, 3, 5, 2, 2], [2, 2, 3, 5, 5]]

2, "range", [1, 3, 5], [[5, 5, 1, 3, 3], [3, 3, 5, 1, 1], [1, 1, 3, 5, 5]]

3, "range", [1, 3, 4], [[4, 4, 1, 3, 3], [3, 3, 4, 1, 1], [1, 1, 3, 4, 4]]

"group has", 3, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

g_1 =   [[1, 3, 2]]

g_2 =   [[1, 2, 3]]

g_3 =   []

linear dimension, 3

"Symmetric?", false

Is Z in Vec(K)? true

$\begin{pmatrix} h[3] & h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

3, "coeff", 1

$$Z[3] = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

EIGS =

$$\begin{pmatrix} 1. & 1. & 1. \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I \\ 1. & -0.5000000000 + 0.8660254040 I & -0.5000000000 - 0.8660254040 I \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 4t^3 + 5t^4 + 7t^5 + 10t^6 + 12t^7 + 15t^8 + 19t^9 + 22t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 3, 4]}, {5, [1, 3, 5]}, {6, [1, 4, 5]}, {7, [2, 3, 4]}, {8, [2, 3, 5]}, {9, [2, 4, 5]}, {10, [3, 4, 5]}

KERNEL HIERARCHY

$$\pi 3 = \begin{pmatrix} 0 & 0 & 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

{4, 5, 8}

$u3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$

{4, 5, 7, 8}

picheck $\begin{pmatrix} 3 & 1 & 4 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{4} & \frac{1}{12} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 0 & 3 & 2 & 1 & 1 & 0 & 1 & 2 & 2 & 0 \end{pmatrix}$

$u2 = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$

picheck $\begin{pmatrix} 6 & 2 & 8 & 4 & 4 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 6 & 2 & 8 & 4 & 4 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 6 & 2 & 8 & 4 & 4 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem−checks   N0−checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$\mathrm{T} = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \mathrm{NM} = \begin{pmatrix} 4 & \frac{4}{3} & \frac{8}{3} & \frac{4}{3} & \frac{4}{3} \\ 4 & \frac{4}{3} & \frac{8}{3} & \frac{4}{3} & \frac{4}{3} \\ 2 & \frac{2}{3} & \frac{16}{3} & \frac{4}{3} & \frac{4}{3} \\ 2 & \frac{2}{3} & \frac{8}{3} & \frac{8}{3} & \frac{8}{3} \\ 2 & \frac{2}{3} & \frac{8}{3} & \frac{8}{3} & \frac{8}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta}$ = [1, −1, 0, 2, −2]

$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} t & -t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -2 & -1 \end{pmatrix}$

M0 is invertible. det= 4475/15552

$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} t+s \\ t+s \\ t+s \\ t+s \\ t+s \end{pmatrix}$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 5 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 4, "vs", 3

$$\mathrm{CNM} = \begin{pmatrix} 0 & \frac{-3}{2} & \frac{2}{3} & \frac{-2}{3} & \frac{-2}{3} \\ \frac{3}{2} & 0 & 2 & \frac{2}{3} & \frac{2}{3} \\ \frac{-2}{3} & -2 & 0 & \frac{-4}{3} & \frac{-4}{3} \\ \frac{2}{3} & \frac{-2}{3} & \frac{4}{3} & 0 & 0 \\ \frac{2}{3} & \frac{-2}{3} & \frac{4}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{-1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0 & \frac{-1}{6} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{12} \\ \frac{1}{6} & 0 & \frac{1}{4} & \frac{1}{12} & \frac{1}{12} \\ \frac{-1}{12} & \frac{-1}{4} & 0 & \frac{-1}{6} & \frac{-1}{6} \\ \frac{1}{12} & \frac{-1}{12} & \frac{1}{6} & 0 & 0 \\ \frac{1}{12} & \frac{-1}{12} & \frac{1}{6} & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{9}{4} & 0 & 2 & \frac{4}{3} & \frac{2}{3} \\ 0 & \frac{3}{4} & \frac{2}{3} & 0 & \frac{2}{3} \\ 2 & \frac{2}{3} & 3 & \frac{4}{3} & \frac{4}{3} \\ \frac{4}{3} & 0 & \frac{4}{3} & \frac{3}{2} & 0 \\ \frac{2}{3} & \frac{2}{3} & \frac{4}{3} & 0 & \frac{3}{2} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = \frac{8}{3} T + 8\Omega$$

$$\Omega \left(\frac{1}{6} \;\; \frac{1}{4} \;\; \frac{1}{6} \;\; \frac{1}{4} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{3} \;\; \frac{1}{12} \;\; \frac{1}{4} \right)$$

$T\left(\frac{1}{2}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{4}\ \ \frac{3}{4}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM}\left(\frac{8}{3}\ \ 2\ \ \frac{4}{3}\ \ 2\ \ \frac{4}{3}\ \ \frac{4}{3}\ \ \frac{8}{3}\ \ \frac{4}{3}\ \ 4\right)$

"IS MN in Vec(K)?", false

$\mathrm{MN}\left(\frac{8}{3}\ \ \frac{8}{7}\ \ \frac{8}{7}\ \ \frac{8}{7}\ \ \frac{8}{7}\ \ \frac{8}{7}\ \ \frac{16}{7}\ \ \frac{8}{3}\ \ \frac{8}{3}\right)$

$\boldsymbol{\tau}$ = 9/1 , rank = 3 , ratio = 3/1 , n^2 / r = 25/3

$\boldsymbol{\tau}'$ = 16/1 , r' = 2/3 , $\boldsymbol{\tau}$ / n^2 = 9/25

p^2 = 17/72 , min $\boldsymbol{\tau}$ = 425/72 , $\boldsymbol{\tau}$–check is positive? 223/72

max r = 72/17 , r–check is positive? 7/24

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 8\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 3

KERNEL HAS LINEAR DIMENSION 9
out of total no. of elements equal to 9

dim span idems 3 vs no. of idems 3

"PT1" = {{4, 5}, {1, 2}, {3}}

"RG1" = {2, 3, 5}

"RG2" = {1, 3, 5}

"RG3" = {1, 3, 4}

$$M_C = \begin{pmatrix} \frac{11}{16} & \frac{-25}{48} & \frac{-1}{12} & \frac{7}{24} & \frac{-3}{8} \\ \frac{-25}{48} & \frac{83}{144} & \frac{-1}{36} & \frac{-25}{72} & \frac{23}{72} \\ \frac{-1}{12} & \frac{-1}{36} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{7}{24} & \frac{-25}{72} & \frac{-1}{18} & \frac{29}{36} & \frac{-25}{36} \\ \frac{-3}{8} & \frac{23}{72} & \frac{-1}{18} & \frac{-25}{36} & \frac{29}{36} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{-17}{72} & \frac{-17}{72} \\ \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{-17}{72} & \frac{-17}{72} \\ \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{-17}{72} \\ \frac{-17}{72} & \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{55}{72} \\ \frac{-17}{72} & \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{55}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-25}{33} & \frac{-4}{33} & \frac{14}{33} & \frac{-6}{11} \\ \frac{-75}{83} & 1 & \frac{-4}{83} & \frac{-50}{83} & \frac{46}{83} \\ \frac{-3}{8} & \frac{-1}{8} & 1 & \frac{-1}{4} & \frac{-1}{4} \\ \frac{21}{58} & \frac{-25}{58} & \frac{-2}{29} & 1 & \frac{-25}{29} \\ \frac{-27}{58} & \frac{23}{58} & \frac{-2}{29} & \frac{-25}{29} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & 1 & \frac{-17}{55} & \frac{-17}{55} & \frac{-17}{55} \\ 1 & 1 & \frac{-17}{55} & \frac{-17}{55} & \frac{-17}{55} \\ \frac{-17}{55} & \frac{-17}{55} & 1 & \frac{-17}{55} & \frac{-17}{55} \\ \frac{-17}{55} & \frac{-17}{55} & \frac{-17}{55} & 1 & 1 \\ \frac{-17}{55} & \frac{-17}{55} & \frac{-17}{55} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{6} & \frac{1}{18} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{6} & \frac{1}{18} & \frac{-1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{12} & \frac{-1}{36} & \frac{2}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{12} & \frac{-1}{36} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{-1}{12} & \frac{-1}{36} & \frac{-1}{9} & \frac{1}{9} & \frac{1}{9} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{-1}{12} \\ \frac{1}{18} & \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} \\ \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} \end{pmatrix}$$

$$\text{commutator} = \begin{pmatrix} 0 & \frac{1}{9} & \frac{1}{36} & \frac{-1}{36} & \frac{-1}{36} \\ \frac{-1}{9} & 0 & \frac{1}{12} & \frac{1}{36} & \frac{1}{36} \\ \frac{-1}{36} & \frac{-1}{12} & 0 & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{36} & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \\ \frac{1}{36} & \frac{-1}{36} & \frac{1}{18} & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0.1612663492, 0.2742589564, 0.6452880361, 2.016408882]

Eigenvalues N_C

[0., 0., 2., 1.403950093, 0.4154943515]

Eigenvalues M_C–scaled

[0., 0.2300365399, 0.8858798966, 1.099270805, 2.784812758]

Eigenvalues N_C–scaled

[0., 0., 2.618181818, 1.837898304, 0.5439198784]

NullSpace M_C

{[1, 1, 1, 1, 1]}

NullSpace N_c

{[1, -1, 0, 0, 0], [0, 0, 0, -1, 1]}

Eigenvalues M_0

[0.1610063683, 0.2420241236, 0.6276324259, 1.956769951, 6.012567129]

Eigenvalues N_0

[1., 0., 0., 2., 2.]

NullSpace M_0

{}

NullSpace N_0

{[0, 0, 0, -1, 1], [-1, 1, 0, 0, 0]}

Eigenvalues M

[0.6043001496, 3.676077028, -0.5735234614, -1.496144794, -2.210708923]

Eigenvalues N

[0., 0., -2., 3.236067977, -1.236067977]

NullSpace M

{}

NullSpace N

{[0, 0, 0, -1, 1], [-1, 1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$