T - Run

[3, 5, 6, 1, 2, 1], [5, 1, 2, 6, 3, 4]

$\tilde{\pi}$ = [3, 3, 3, 1, 3, 2]
$\boldsymbol{\delta}$ = [3, 2, 2, 1, 2, 2]

POSSIBLE RANKS

1 x 15
3 x 5

BASE DETERMINANT 163959/1048576, .1563634872

NullSpace of Δ

{1, 2, 3, 4, 5, 6}

Nullspace of A

` det(A) = ` 1/32

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[4] v[6] + v[2] v[5]

Degree 3
v[2] v[3] v[5] + v[1] v[2] v[3] + v[1] v[2] v[5] + v[1] v[3] v[6]

Degree 4
v[1] v[3] v[5] v[6] + v[2] v[4] v[5] v[6] + v[1] v[2] v[3] v[5] + v[1] v[3] v[4] v[6]

Degree 5
v[2] v[3] v[4] v[5] v[6] + v[1] v[3] v[4] v[5] v[6] + v[1] v[2] v[3] v[4] v[6] + v[1] v[2] v[4] v[5] v[6] + v[1] v[2] v[3] v[5] v[6]

Degree 6
2 v[1] v[2] v[3] v[4] v[5] v[6]

==

{}

R: [3, 5, 6, 1, 2, 1]
B: [5, 1, 2, 6, 3, 4]

TRACE TWO = 1

det AT = $\frac{-5}{32}(1+10t^2+5t^4)(-1+t)$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 6

Level 2 det = $\frac{-3}{33554432}(5246688-600360s-2730676s^2-1031698s^3+339539s^4+322527s^5+17064s^6-27908s^7-1058s^8+1982s^9+4s^{10}-98s^{11}-9s^{12}+3s^{13})(-1+s)$

RANK of R is 5

R ranking is 1, "vs", 5

RBAR ranking 1, "vs", 5

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", (1 + v[1] v[3] v[6]) (1 + v[2] v[5])
"B CYCLES", (1 + v[4] v[6]) (1 + v[1] v[2] v[3] v[5])

Eigenvalues

R: [0., -1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., 1.]

B: [1. I, -1. I, 1., -1., 1., -1.]

NullSpace of R

{[0, 0, 0, 1, 0, 0]}

NullSpace of B

{}

NullSpace of R^*

{[0, 0, 0, 1, 0, -1]}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 21 & 21 & 7 & 21 & 14 \\ 21 & 0 & 21 & 7 & 21 & 14 \\ 21 & 21 & 0 & 7 & 21 & 14 \\ 7 & 7 & 7 & 0 & 7 & 0 \\ 21 & 21 & 21 & 7 & 0 & 14 \\ 14 & 14 & 14 & 0 & 14 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 5

"RANK of the KERNEL is ", 5

"IdemSolvability Check", 3 "Trace mark", 5, "Rank mark", 5, "for kernel rank", 5

degree 1: $\frac{1}{5}$ ($3v[1] + 3v[2] + 3v[3] + v[4] + 3v[5] + 2v[6]$)
degree 2: $\frac{1}{10}$ ($3v[1]v[2] + 3v[1]v[3] + v[1]v[4] + 3v[1]v[5] + 2v[1]v[6] + 3v[2]v[3] + v[2]v[4] + 3v[2]v[5] + 2v[2]v[6] + v[3]v[4] + 3v[3]v[5] + 2v[3]v[6] + v[4]v[5] + 2v[5]v[6]$)
degree 3 : $\frac{1}{30}$ ($3v[1]v[2]v[3] + v[1]v[2]v[4] + 3v[1]v[2]v[5] + 2v[1]v[2]v[6] + v[1]v[3]v[4] + 3v[1]v[3]v[5] + 2v[1]v[3]v[6] + v[1]v[4]v[5] + 2v[1]v[5]v[6] + v[2]v[3]v[4] + 3v[2]v[3]v[5] + 2v[2]v[3]v[6] + v[2]v[4]v[5] + 2v[2]v[5]v[6] + v[3]v[4]v[5] + 2v[3]v[5]v[6]$)
degree 4 : $\frac{1}{15}$ ($v[1]v[2]v[3]v[4] + 3v[1]v[2]v[3]v[5] + 2v[1]v[2]v[3]v[6] + v[1]v[2]v[4]v[5] + 2v[1]v[2]v[5]v[6] + v[1]v[3]v[4]v[5] + 2v[1]v[3]v[5]v[6] + v[2]v[3]v[4]v[5] + 2v[2]v[3]v[5]v[6]$)
degree 5 : $\frac{1}{3}$ ($v[2]$) ($v[3]$) ($v[4] + 2v[6]$) ($v[5]$) ($v[1]$)

Group spectrum $1 + t + t^2 + t^3 + t^4 + t^5$

KERNEL STRUCTURE

"PT1" = {{1}, {5}, {3}, {2}, {4, 6}}

"RG1" = {1, 2, 3, 5, 6}

"RG2" = {1, 2, 3, 4, 5}

$$\pi 5 = [1, 0, 2, 0, 0, 0]$$

supp $\pi 5$ = {1, 3}

$$u5 = [1, 0, 1, 0, 0, 0]$$

supp u5 = {1, 3}

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[5, 4, 1, 2, 3]

B-BLOCKS,

[4, 1, 2, 3, 5]

with invariant measure, [1, 1, 1, 1, 1]

N by blocks, N - check: true

$b_1 = \{1\}$
$b_2 = \{5\}$
$b_3 = \{3\}$
$b_4 = \{2\}$
$b_5 = \{4, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 15 ⊕ 7/5

$$CLB = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3, 6}, {2, 5}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 5}, {4, 6}}, true

$$V = \begin{pmatrix} \frac{2}{25} & \frac{-3}{25} & \frac{7}{25} & \frac{7}{75} & \frac{-13}{25} & \frac{14}{75} \\ \frac{-11}{25} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{75} & \frac{9}{25} & \frac{-2}{75} \\ \frac{3}{25} & \frac{-17}{25} & \frac{-2}{25} & \frac{23}{75} & \frac{-7}{25} & \frac{46}{75} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{-4}{15} & \frac{1}{5} & \frac{-8}{15} \\ \frac{1}{25} & \frac{11}{25} & \frac{-9}{25} & \frac{-3}{25} & \frac{6}{25} & \frac{-6}{25} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{-3}{5} & \frac{1}{5} & \frac{-1}{5} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{2}{9} & \frac{1}{6} & \frac{2}{9} & 0 & \frac{1}{6} & \frac{2}{9} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1}, {5}, {3}, {2}, {4, 6}}

1, "range", [1, 2, 3, 5, 6], [[6, 5, 3, 2, 1, 2], [6, 5, 3, 1, 2, 1], [6, 5, 2, 3, 1, 3], [6, 5, 2, 1, 3, 1], [6, 5, 1, 3, 2, 3], [6, 5, 1, 2, 3, 2], [6, 3, 5, 2, 1, 2], [6, 3, 5, 1, 2, 1], [6, 3, 2, 5, 1, 5], [6, 3, 2, 1, 5, 1], [6, 3, 1, 5, 2, 5], [6, 3, 1, 2, 5, 2], [6, 2, 5, 3, 1, 3], [6, 2, 5, 1, 3, 1], [6, 2, 3, 5, 1, 5], [6, 2, 3, 1, 5, 1], [6, 2, 1, 5, 3, 5], [6, 2, 1, 3, 5, 3], [6, 1, 5, 3, 2, 3], [6, 1, 5, 2, 3, 2], [6, 1, 3, 5, 2, 5], [6, 1, 3, 2, 5, 2], [6, 1, 2, 5, 3, 5], [6, 1, 2, 3, 5, 3], [5, 6, 3, 2, 1, 2], [5, 6, 3, 1, 2, 1], [5, 6, 2, 3, 1, 3], [5, 6, 2, 1, 3, 1], [5, 6, 1, 3, 2, 3], [5, 6, 1, 2, 3, 2], [5, 3, 6, 2, 1, 2], [5, 3, 6, 1, 2, 1], [5, 3, 2, 6, 1, 6], [5, 3, 2, 1, 6, 1], [5, 3, 1, 6, 2, 6], [5, 3, 1, 2, 6, 2], [5, 2, 6, 3, 1, 3], [5, 2, 6, 1, 3, 1], [5, 2, 3, 6, 1, 6], [5, 2, 3, 1, 6, 1], [5, 2, 1, 6, 3, 6], [5, 2, 1, 3, 6, 3], [5, 1, 6, 3, 2, 3], [5, 1, 6, 2, 3, 2], [5, 1, 3, 6, 2, 6], [5, 1, 3, 2, 6, 2], [5, 1, 2, 6, 3, 6], [5, 1, 2, 3, 6, 3], [3, 6, 5, 2, 1, 2], [3, 6, 5, 1, 2, 1], [3, 6, 2, 5, 1, 5], [3, 6, 2, 1, 5, 1], [3, 6, 1, 5, 2, 5], [3, 6, 1, 2, 5, 2], [3, 5, 6, 2, 1, 2], [3, 5, 6, 1, 2, 1], [3, 5, 2, 6, 1, 6], [3, 5, 2, 1, 6, 1], [3, 5, 1, 6, 2, 6], [3, 5, 1, 2, 6, 2], [3, 2, 6, 5, 1, 5], [3, 2, 6, 1, 5, 1], [3, 2, 5, 6, 1, 6], [3, 2, 5, 1, 6, 1], [3, 2, 1, 6, 5, 6], [3, 2, 1, 5, 6, 5], [3, 1, 6, 5, 2, 5], [3, 1, 6, 2, 5, 2], [3, 1, 5, 6, 2, 6], [3, 1, 5, 2, 6, 2], [3, 1, 2, 6, 5, 6], [3, 1, 2, 5, 6, 5], [2, 6, 5, 3, 1, 3], [2, 6, 5, 1, 3, 1], [2, 6, 3, 5, 1, 5], [2, 6, 3, 1, 5, 1], [2, 6, 1, 5, 3, 5], [2, 6, 1, 3, 5, 3], [2, 5, 6, 3, 1, 3], [2, 5, 6, 1, 3, 1], [2, 5, 3, 6, 1, 6], [2, 5, 3, 1, 6, 1], [2, 5, 1, 6, 3, 6], [2, 5, 1, 3, 6, 3], [2, 3, 6, 5, 1, 5], [2, 3, 6, 1, 5, 1], [2, 3, 5, 6, 1, 6], [2, 3, 5, 1, 6, 1], [2, 3, 1, 6, 5, 6], [2, 3, 1, 5, 6, 5], [2, 1, 6, 5, 3, 5], [2, 1, 6, 3, 5, 3], [2, 1, 5, 6, 3, 6], [2, 1, 5, 3, 6, 3], [2, 1, 3, 6, 5, 6], [2, 1, 3, 5, 6, 5], [1, 6, 5, 3, 2, 3], [1, 6, 5, 2, 3, 2], [1, 6, 3, 5, 2, 5], [1, 6, 3, 2, 5, 2], [1, 6, 2, 5, 3, 5], [1, 6, 2, 3, 5, 3], [1, 5, 6, 3, 2, 3], [1, 5, 6, 2, 3, 2], [1, 5, 3, 6, 2, 6], [1, 5, 3, 2, 6, 2], [1, 5, 2, 6, 3, 6], [1, 5, 2, 3, 6, 3], [1, 3, 6, 5, 2, 5], [1, 3, 6, 2, 5, 2], [1, 3, 5, 6, 2, 6], [1, 3, 5, 2, 6, 2], [1, 3, 2, 6, 5, 6], [1, 3, 2, 5, 6, 5], [1, 2, 6, 5, 3, 5], [1, 2, 6, 3, 5, 3], [1, 2, 5, 6, 3, 6], [1, 2, 5, 3, 6, 3], [1, 2, 3, 6, 5, 6], [1, 2, 3, 5, 6, 5]]

2, "range", [1, 2, 3, 4, 5], [[5, 4, 3, 2, 1, 2], [5, 4, 3, 1, 2, 1], [5, 4, 2, 3, 1, 3], [5, 4, 2, 1, 3, 1], [5, 4, 1, 3, 2, 3], [5, 4, 1, 2, 3, 2], [5, 3, 4, 2, 1, 2], [5, 3, 4, 1, 2, 1], [5, 3, 2, 4, 1, 4], [5, 3, 2, 1, 4, 1], [5, 3, 1, 4, 2, 4], [5, 3, 1, 2, 4, 2], [5, 2, 4, 3, 1, 3], [5, 2, 4, 1, 3, 1], [5, 2, 3, 4, 1, 4], [5, 2, 3, 1, 4, 1], [5, 2, 1, 4, 3, 4], [5, 2, 1, 3, 4, 3], [5, 1, 4, 3, 2, 3], [5, 1, 4, 2, 3, 2], [5, 1, 3, 4, 2, 4], [5, 1, 3, 2, 4, 2], [5, 1, 2, 4, 3, 4], [5, 1, 2, 3, 4, 3], [4, 5, 3, 2, 1, 2], [4, 5, 3, 1, 2, 1], [4, 5, 2, 3, 1, 3], [4, 5, 2, 1, 3, 1], [4, 5, 1, 3, 2, 3], [4, 5, 1, 2, 3, 2], [4, 3, 5, 2, 1, 2], [4, 3, 5, 1, 2, 1], [4, 3, 2, 5, 1, 5], [4, 3, 2, 1, 5, 1], [4, 3, 1, 5, 2, 5], [4, 3, 1, 2, 5, 2], [4, 2, 5, 3, 1, 3], [4, 2, 5, 1, 3, 1], [4, 2, 3, 5, 1, 5], [4, 2, 3, 1, 5, 1], [4, 2, 1, 5, 3, 5], [4, 2, 1, 3, 5, 3], [4, 1, 5, 3, 2, 3], [4, 1, 5, 2, 3, 2], [4, 1, 3, 5, 2, 5], [4, 1, 3, 2, 5, 2], [4, 1, 2, 5, 3, 5], [4, 1, 2, 3, 5, 3], [3, 5, 4, 2, 1, 2], [3, 5, 4, 1, 2, 1], [3, 5, 2, 4, 1, 4], [3, 5, 2, 1, 4, 1], [3, 5, 1, 4, 2, 4], [3, 5, 1, 2, 4, 2], [3, 4, 5, 2, 1, 2], [3, 4, 5, 1, 2, 1], [3, 4, 2, 5, 1, 5], [3, 4, 2, 1, 5, 1], [3, 4, 1, 5, 2, 5], [3, 4, 1, 2, 5, 2], [3, 2, 5, 4, 1, 4], [3, 2, 5, 1, 4, 1], [3, 2, 4, 5, 1, 5], [3, 2, 4, 1, 5, 1], [3, 2, 1, 5, 4, 5], [3, 2, 1, 4, 5, 4], [3, 1, 5, 4, 2, 4], [3, 1, 5, 2, 4, 2], [3, 1, 4, 5, 2, 5], [3, 1, 4, 2, 5, 2], [3, 1, 2, 5, 4, 5], [3, 1, 2, 4, 5, 4], [2, 5, 4, 3, 1, 3], [2, 5, 4, 1, 3, 1], [2, 5, 3, 4, 1, 4], [2, 5, 3, 1, 4, 1], [2, 5, 1, 4, 3, 4], [2, 5, 1, 3, 4, 3], [2, 4, 5, 3, 1, 3], [2, 4, 5, 1, 3, 1], [2, 4, 3, 5, 1, 5], [2, 4, 3, 1, 5, 1], [2, 4, 1, 5, 3, 5], [2, 4, 1, 3, 5, 3], [2, 3, 5, 4, 1, 4], [2, 3, 5, 1, 4, 1], [2, 3, 4, 5, 1, 5], [2, 3, 4, 1, 5, 1], [2, 3, 1, 5, 4, 5], [2, 3, 1, 4, 5, 4], [2, 1, 5, 4, 3, 4], [2, 1, 5, 3, 4, 3], [2, 1, 4, 5, 3, 5], [2, 1, 4, 3, 5, 3], [2, 1, 3, 5, 4, 5], [2, 1, 3, 4, 5, 4], [1, 5, 4, 3, 2, 3], [1, 5, 4, 2, 3, 2], [1, 5, 3, 4, 2, 4], [1, 5, 3, 2, 4, 2], [1, 5, 2, 4, 3, 4], [1, 5, 2, 3, 4, 3], [1, 4, 5, 3, 2, 3], [1, 4, 5, 2, 3, 2], [1, 4, 3, 5, 2, 5], [1, 4, 3, 2, 5, 2], [1, 4, 2, 5, 3, 5], [1, 4, 2, 3, 5, 3], [1, 3, 5, 4, 2, 4], [1, 3, 5, 2, 4, 2], [1, 3, 4, 5, 2, 5], [1, 3, 4, 2, 5, 2], [1, 3, 2, 5, 4, 5], [1, 3, 2, 4, 5, 4], [1, 2, 5, 4, 3, 4], [1, 2, 5, 3, 4, 3], [1, 2, 4, 5, 3, 5], [1, 2, 4, 3, 5, 3], [1, 2, 3, 5, 4, 5], [1, 2, 3, 4, 5, 4]]

"group has", 120, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 5, 2, 4]]

$g_2 =$ [[1, 5], [2, 4]]

$g_3 =$ [[1, 5, 3, 2, 4]]

$g_4 =$ [[1, 5], [2, 4, 3]]

$g_5 =$ [[1, 5, 3], [2, 4]]

linear dimension, 17

"Symmetric?", true

Is Z in Vec(K)? true

$\left(72h[1] + 12h[2] \quad -24h[1] - 6h[2] \quad -48h[1] - 12h[2] \quad 6h[2] \quad 6h[2] \quad -48h[1] - 12h[2] \quad 6h[2] \quad 24h[1] \quad 24h[1] \quad 6\right.$

"Basis for Z(G)"

1, "coeff", 24

$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 6

$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. & 1. & 1. & 1. \\ 4. & -1. & -1. & -1. & -1. \end{pmatrix}$

Group spectrum: $1 + t + t^2 + t^3 + t^4 + t^5$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 5t^4 + 7t^5 + 10t^6 + 13t^7 + 18t^8 + 23t^9 + 30t^{10}$

n−choose−rank

{1, [1, 2, 3, 4, 5]}, {2, [1, 2, 3, 4, 6]}, {3, [1, 2, 3, 5, 6]}, {4, [1, 2, 4, 5, 6]}, {5, [1, 3, 4, 5, 6]}, {6, [2, 3, 4, 5, 6]}

KERNEL HIERARCHY

$\pi 5 = \begin{pmatrix} 1 & 0 & 2 & 0 & 0 & 0 \end{pmatrix}$

{1, 3}

$u5 = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$

{1, 3}

picheck $\begin{pmatrix} 3 & 3 & 3 & 1 & 3 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{15} & \frac{1}{5} & \frac{2}{15} \end{pmatrix}$

$\pi 4 = \begin{pmatrix} 1 & 3 & 2 & 1 & 0 & 2 & 1 & 0 & 2 & 0 & 1 & 0 & 2 & 0 & 0 \end{pmatrix}$

$u4 = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} & 0 & \frac{1}{5} & 0 & \frac{1}{5} & 0 & 0 \end{pmatrix}$

picheck $\begin{pmatrix} 12 & 12 & 12 & 4 & 12 & 8 \end{pmatrix}$

$\pi 3 = \begin{pmatrix} 6 & 2 & 6 & 4 & 2 & 6 & 4 & 2 & 0 & 4 & 2 & 6 & 4 & 2 & 0 & 4 & 2 & 0 & 4 & 0 \end{pmatrix}$

$u3 = \begin{pmatrix} \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & 0 & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & 0 & \frac{2}{25} & \frac{2}{25} & 0 & \frac{2}{25} & 0 \end{pmatrix}$

picheck $\begin{pmatrix} 36 & 36 & 36 & 12 & 36 & 24 \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 18 & 18 & 6 & 18 & 12 & 18 & 6 & 18 & 12 & 6 & 18 & 12 & 6 & 0 & 12 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & 0 & \frac{6}{125} \end{pmatrix}$

picheck $\begin{pmatrix} 72 & 72 & 72 & 24 & 72 & 48 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 72 & 72 & 72 & 24 & 72 & 48 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{24}{625} & \frac{24}{625} & \frac{24}{625} & \frac{24}{625} & \frac{24}{625} & \frac{24}{625} \end{pmatrix}$

picheck $\begin{pmatrix} 72 & 72 & 72 & 24 & 72 & 48 \end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem–checks N0–checks

$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$ idem–checks N0–checks

Row Projections

$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix}$ idem–checks

$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{28}{5} & \frac{21}{5} & \frac{21}{5} & \frac{7}{5} & \frac{21}{5} & \frac{14}{5} \\ \frac{21}{5} & \frac{28}{5} & \frac{21}{5} & \frac{7}{5} & \frac{21}{5} & \frac{14}{5} \\ \frac{21}{5} & \frac{21}{5} & \frac{28}{5} & \frac{7}{5} & \frac{21}{5} & \frac{14}{5} \\ \frac{21}{5} & \frac{21}{5} & \frac{21}{5} & \frac{28}{15} & \frac{21}{5} & \frac{56}{15} \\ \frac{21}{5} & \frac{21}{5} & \frac{21}{5} & \frac{7}{5} & \frac{28}{5} & \frac{14}{5} \\ \frac{21}{5} & \frac{21}{5} & \frac{21}{5} & \frac{28}{15} & \frac{21}{5} & \frac{56}{15} \end{pmatrix}$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 0, 0, -1, 0, 1]$

ker $N_C = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & t & 0 & -t \end{pmatrix}$ RB checks

M0 is invertible. det= 64/15625

$$\ker M_c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = (6)$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 6, "vs", 5

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & 0 & \frac{-14}{15} & 0 & \frac{-7}{15} \\ 0 & 0 & 0 & \frac{-14}{15} & 0 & \frac{-7}{15} \\ 0 & 0 & 0 & \frac{-14}{15} & 0 & \frac{-7}{15} \\ \frac{14}{15} & \frac{14}{15} & \frac{14}{15} & 0 & \frac{14}{15} & \frac{8}{15} \\ 0 & 0 & 0 & \frac{-14}{15} & 0 & \frac{-7}{15} \\ \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{-8}{15} & \frac{7}{15} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{3} & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & 0 & 0 & \frac{-2}{15} & 0 & \frac{-1}{15} \\ 0 & 0 & 0 & \frac{-2}{15} & 0 & \frac{-1}{15} \\ 0 & 0 & 0 & \frac{-2}{15} & 0 & \frac{-1}{15} \\ \frac{2}{15} & \frac{2}{15} & \frac{2}{15} & 0 & \frac{2}{15} & \frac{1}{15} \\ 0 & 0 & 0 & \frac{-2}{15} & 0 & \frac{-1}{15} \\ \frac{1}{15} & \frac{1}{15} & \frac{1}{15} & \frac{-1}{15} & \frac{1}{15} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{5} & \frac{7}{5} & \frac{7}{5} & \frac{7}{15} & \frac{7}{5} & \frac{14}{15} \\ \frac{7}{5} & \frac{8}{5} & \frac{7}{5} & \frac{7}{15} & \frac{7}{5} & \frac{14}{15} \\ \frac{7}{5} & \frac{7}{5} & \frac{8}{5} & \frac{7}{15} & \frac{7}{5} & \frac{14}{15} \\ \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{8}{15} & \frac{7}{15} & 0 \\ \frac{7}{5} & \frac{7}{5} & \frac{7}{5} & \frac{7}{15} & \frac{8}{5} & \frac{14}{15} \\ \frac{14}{15} & \frac{14}{15} & \frac{14}{15} & 0 & \frac{14}{15} & \frac{16}{15} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = \frac{7}{5} T + 21\Omega$$

$$\Omega \begin{pmatrix} \frac{1}{15} & \frac{8}{5} & \frac{1}{15} & \frac{17}{15} & \frac{7}{15} & \frac{1}{5} & \frac{17}{15} & \frac{1}{15} & \frac{2}{3} & \frac{1}{5} & \frac{1}{5} & \frac{2}{3} & \frac{1}{15} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{2}{15} & \frac{1}{5} & \frac{1}{15} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

$$T \begin{pmatrix} 0 & 2 & \frac{1}{3} & 1 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \begin{pmatrix} \frac{7}{5} & \frac{182}{5} & \frac{28}{15} & \frac{126}{5} & \frac{49}{5} & \frac{21}{5} & \frac{133}{5} & \frac{7}{5} & \frac{77}{5} & \frac{21}{5} & \frac{21}{5} & \frac{77}{5} & \frac{7}{5} & \frac{21}{5} & \frac{28}{5} & \frac{21}{5} & \frac{14}{5} & \frac{21}{5} & \frac{7}{5} & \frac{21}{5} & \frac{21}{5} & \frac{28}{5} \end{pmatrix}$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{105}{58} \quad \frac{10367}{290} \quad \frac{469}{145} \quad \frac{3563}{145} \quad \frac{273}{29} \quad \frac{105}{29} \quad \frac{7847}{290} \quad \frac{105}{58} \quad \frac{4501}{290} \quad \frac{231}{58} \quad \frac{231}{58} \quad \frac{4501}{290} \quad \frac{105}{58} \quad \frac{231}{58} \quad \frac{1561}{290} \quad \frac{231}{58} \quad \frac{105}{58} \quad \frac{231}{58} \quad \frac{105}{58} \right.$$

$$\boldsymbol{\tau} = 8/1 \text{ , rank} = 5 \text{ , ratio} = 8/5 \text{ , } n^2 / r = 36/5$$

$$\boldsymbol{\tau}' = 28/1 \text{ , } r' = 4/5 \text{ , } \boldsymbol{\tau} / n^2 = 2/9$$

$$p^2 = 41/225 \text{ , min } \boldsymbol{\tau} = 164/25 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 36/25$$

$$\text{max r} = 225/41 \text{ , } r\text{-check is positive? } 4/45$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{5} T + 7\Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 120

KERNEL HAS LINEAR DIMENSION 22
out of total no. of elements equal to 240

dim span idems 2 vs no. of idems 2

"PT1" = {{1}, {5}, {3}, {2}, {4, 6}}

"RG1" = {1, 2, 3, 5, 6}

"RG2" = {1, 2, 3, 4, 5}

$$M_C = \begin{pmatrix} \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{75} & \frac{-1}{25} & \frac{-2}{75} \\ \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{75} & \frac{-1}{25} & \frac{-2}{75} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{75} & \frac{-1}{25} & \frac{-2}{75} \\ \frac{-1}{75} & \frac{-1}{75} & \frac{-1}{75} & \frac{28}{75} & \frac{-1}{75} & \frac{-8}{25} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{75} & \frac{4}{25} & \frac{-2}{75} \\ \frac{-2}{75} & \frac{-2}{75} & \frac{-2}{75} & \frac{-8}{25} & \frac{-2}{75} & \frac{32}{75} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{184}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} \\ \frac{-41}{225} & \frac{184}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} \\ \frac{-41}{225} & \frac{-41}{225} & \frac{184}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} \\ \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{184}{225} & \frac{-41}{225} & \frac{184}{225} \\ \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{184}{225} & \frac{-41}{225} \\ \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{184}{225} & \frac{-41}{225} & \frac{184}{225} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{12} & \frac{-1}{4} & \frac{-1}{6} \\ \frac{-1}{4} & 1 & \frac{-1}{4} & \frac{-1}{12} & \frac{-1}{4} & \frac{-1}{6} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{12} & \frac{-1}{4} & \frac{-1}{6} \\ \frac{-1}{28} & \frac{-1}{28} & \frac{-1}{28} & 1 & \frac{-1}{28} & \frac{-6}{7} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{12} & 1 & \frac{-1}{6} \\ \frac{-1}{16} & \frac{-1}{16} & \frac{-1}{16} & \frac{-3}{4} & \frac{-1}{16} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} \\ \frac{-41}{184} & 1 & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} \\ \frac{-41}{184} & \frac{-41}{184} & 1 & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} \\ \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & 1 & \frac{-41}{184} & 1 \\ \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & 1 & \frac{-41}{184} \\ \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & 1 & \frac{-41}{184} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{75} & \frac{-1}{25} & \frac{-2}{75} \\ \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{75} & \frac{-1}{25} & \frac{-2}{75} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{75} & \frac{-1}{25} & \frac{-2}{75} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{4}{75} & \frac{-1}{25} & \frac{8}{75} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{75} & \frac{4}{25} & \frac{-2}{75} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{4}{75} & \frac{-1}{25} & \frac{8}{75} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{-1}{75} & \frac{-1}{75} & \frac{-1}{75} & \frac{4}{75} & \frac{-1}{75} & \frac{4}{75} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} \\ \frac{-2}{75} & \frac{-2}{75} & \frac{-2}{75} & \frac{8}{75} & \frac{-2}{75} & \frac{8}{75} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & \frac{-2}{75} & 0 & \frac{-1}{75} \\ 0 & 0 & 0 & \frac{-2}{75} & 0 & \frac{-1}{75} \\ 0 & 0 & 0 & \frac{-2}{75} & 0 & \frac{-1}{75} \\ \frac{2}{75} & \frac{2}{75} & \frac{2}{75} & 0 & \frac{2}{75} & \frac{-4}{75} \\ 0 & 0 & 0 & \frac{-2}{75} & 0 & \frac{-1}{75} \\ \frac{1}{75} & \frac{1}{75} & \frac{1}{75} & \frac{4}{75} & \frac{1}{75} & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0.7217725413, 0.1182274587, 0.2000000000, 0.2000000000, 0.2000000000]

Eigenvalues N_C

[0., 1.808357699, 0.0983089673, 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 1.804745225, 0.4452547747, 1.250000000, 1.250000000, 1.250000000]

Eigenvalues N_C–scaled

[0., 2.211306970, 0.120214770, 1.222826087, 1.222826087, 1.222826087]

NullSpace M_C

{[1, 1, 1, 1, 1, 1]}

NullSpace N_C

{[0, 0, 0, −1, 0, 1]}

Eigenvalues M_0

[6.576546381, 0.1089619612, 0.7144916588, 0.2000000000, 0.2000000000, 0.2000000000]

Eigenvalues N_0

[0., 2., 1., 1., 1., 1.]

NullSpace M_0

{}

NullSpace N_0

{[0, 0, 0, 1, 0, −1]}

Eigenvalues M

[0., 5.060668092, −0.860668092, −1.400000000, −1.400000000, −1.400000000]

Eigenvalues N

[0., 4.701562118, −1.701562118, −1., −1., −1.]

NullSpace M

{[0, 0, 0, −2, 0, 1]}

NullSpace N

{[0, 0, 0, 1, 0, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

==

20, [1, −1, 1, −1, −1, 1]