

T-Run

[3, 4, 4, 3, 2, 5], [6, 1, 2, 6, 1, 3]

$$\tilde{\pi} = [3, 4, 6, 5, 2, 4]$$

$$\delta = [2, 2, 3, 2, 1, 2]$$

POSSIBLE RANKS

1 x 24

2 x 12

3 x 8

4 x 6

BASE DETERMINANT 55/256, .2148437500

NullSpace of Δ

{3, 5, 6}, {1, 2, 4}

Nullspace of A

[[5, 6],[3]]

STRATIFIED CYCLE COVERS

Degree 0

1

Degree 1

0

Degree 2

v[3] v[4]

Degree 3

v[3] v[4] v[6] + v[2] v[3] v[4] + v[1] v[2] v[3] + v[1] v[6] v[5]

Degree 4

v[2] v[3] v[4] v[6] + v[2] v[4] v[6] v[5] + v[1] v[2] v[6] v[5] + v[1] v[2] v[3] v[6]

Degree 5

2 v[1] v[3] v[4] v[6] v[5]

Degree 6

4 v[1] v[2] v[3] v[4] v[6] v[5]

=====

{5}

R: [3, 4, 4, 3, 1, 5]

B: [6, 1, 2, 6, 2, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

$$\text{Level 2 det} = \frac{3}{1024} (-1 + s) (-220 - 55s - 58s^2 + 12s^3 - 2s^4 + 3s^5)$$

RANK of R is 4

R ranking is 3, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[3] v[4]

"B CYCLES", 1 + v[1] v[2] v[3] v[6]

Eigenvalues

R: [0., 0., 0., 0., 1., -1.]

B: [0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0], [0, 0, 0, 1, 0, 0]}

NullSpace of R*

{[0, -1, 1, 0, 0, 0], [-1, 0, 0, 1, 0, 0]}

NullSpace of B*

{[0, 0, 1, 0, -1, 0], [-1, 0, 0, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12 \\ 9 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 9 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6} (3 v[1] + 4 v[2] + 6 v[3] + 5 v[4] + 2 v[5] + 4 v[6])$

degree 2: $\frac{1}{3} (3 v[1]v[3] + 4 v[2]v[6] + 3 v[3]v[4] + 2 v[4]v[5])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{2, 3, 5}, {1, 4, 6}}

"PT2" = {{1, 2, 4}, {3, 5, 6}}

"RG1" = {2, 6}

"RG2" = {4, 5}

"RG3" = {3, 4}

"RG4" = {1, 3}

$$\pi_2 = [0, 3, 0, 0, 0, 0, 0, 0, 4, 3, 0, 0, 2, 0, 0]$$

supp $\pi_2 = \{2, 9, 10, 13\}$

$$u_2 = [2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2]$$

supp $u_2 = \{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15\}$

Action of R on ranges, [[2], [4], [3], [3]]

Action of B on ranges, [[4], [1], [1], [1]]

$$\beta = \left(\frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{4} \right)$$

RPARTS [1, 1]

BPARTS [2, 1]

$$\alpha = \left(\frac{2}{3} \quad \frac{1}{3} \right)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 2, 3]

B-BLOCKS,

[2, 4, 1, 3]

with invariant measure, [1, 2, 2, 1]

N by blocks, N - check: true

$b_1 = \{1, 2, 4\}$

$b_2 = \{2, 3, 5\}$

$b_3 = \{1, 4, 6\}$

$b_4 = \{3, 5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 14, Shape: 3 ⊕ 11/9

$$CLB = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 4}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 6}}, true

$$V = \begin{pmatrix} -\frac{1}{16} & \frac{1}{12} & \frac{3}{8} & \frac{11}{48} & -\frac{5}{24} & -\frac{5}{12} \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ -\frac{3}{16} & -\frac{5}{12} & \frac{1}{8} & \frac{17}{48} & \frac{1}{24} & \frac{1}{12} \\ -\frac{1}{16} & \frac{1}{12} & \frac{3}{8} & \frac{11}{48} & -\frac{5}{24} & -\frac{5}{12} \\ \frac{5}{16} & -\frac{5}{12} & \frac{1}{8} & -\frac{7}{48} & \frac{1}{24} & \frac{1}{12} \\ \frac{1}{4} & 0 & -\frac{1}{2} & -\frac{1}{4} & \frac{1}{2} & 0 \end{pmatrix}$$

Omega-checks true, true

$$\pi_R = \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \text{ vs } \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(\frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ 0 \ 0 \ \frac{1}{4}\right) \text{ vs } \left(\frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ 0 \ 0 \ \frac{1}{4}\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{2, 3, 5}, {1, 4, 6}}

1, "range", [2, 6], [[6, 2, 2, 6, 2, 6], [2, 6, 6, 2, 6, 2]]

2, "range", [4, 5], [[5, 4, 4, 5, 4, 5], [4, 5, 5, 4, 5, 4]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 3], [1, 3, 3, 1, 3, 1]]

2, "partition", {{1, 2, 4}, {3, 5, 6}}

1, "range", [2, 6], [[6, 6, 2, 6, 2, 2], [2, 2, 6, 2, 6, 6]]

2, "range", [4, 5], [[5, 5, 4, 5, 4, 4], [4, 4, 5, 4, 5, 5]]

3, "range", [3, 4], [[4, 4, 3, 4, 3, 3], [3, 3, 4, 3, 4, 4]]

4, "range", [1, 3], [[3, 3, 1, 3, 1, 1], [1, 1, 3, 1, 3, 3]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 = []$

$g_2 = [[1, 2]]$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

(h[1] h[2])

"Basis for Z(G)"

1, "coeff", 1

$$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$$\pi_2 = (0 \ 3 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 4 \ 3 \ 0 \ 0 \ 2 \ 0 \ 0)$$

{2, 9, 10, 13}

$$u_2 = (2 \ 3 \ 0 \ 3 \ 1 \ 1 \ 2 \ 1 \ 3 \ 3 \ 0 \ 2 \ 3 \ 1 \ 2)$$

{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15}

picheck (3 4 6 5 2 4)

$$\pi = \left(\frac{1}{8} \ \frac{1}{6} \ \frac{1}{4} \ \frac{5}{24} \ \frac{1}{12} \ \frac{1}{6} \right)$$

$$\pi_1 = (3 \ 4 \ 6 \ 5 \ 2 \ 4)$$

$$u_1 = \left(\frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \right)$$

picheck (3 4 6 5 2 4)

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{9} & 0 & \frac{5}{12} & 0 & \frac{2}{9} \\ \frac{1}{12} & \frac{1}{3} & \frac{1}{3} & \frac{5}{36} & \frac{1}{9} & 0 \\ 0 & \frac{2}{9} & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{9} \\ \frac{1}{4} & \frac{1}{9} & 0 & \frac{5}{12} & 0 & \frac{2}{9} \\ 0 & \frac{2}{9} & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{9} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{5}{18} & \frac{1}{18} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{9}{4} & 1 & 0 & \frac{15}{4} & 0 & 2 \\ \frac{3}{4} & 3 & 3 & \frac{5}{4} & 1 & 0 \\ 0 & 2 & \frac{9}{2} & 0 & \frac{3}{2} & 1 \\ \frac{9}{4} & 1 & 0 & \frac{15}{4} & 0 & 2 \\ 0 & 2 & \frac{9}{2} & 0 & \frac{3}{2} & 1 \\ \frac{3}{2} & 0 & \frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [-1, -4, 2, 5, 2, -4]$$

$$\ker N_C = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} -t+s & t & -t+s & -s & -s & t \\ -t+s & t & -t+s & -s & -s & t \\ -s & 0 & 0 & s & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\pi\Delta \text{ via } \ker N_C \quad (-1 \ 5 \ 2)$$

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \\ 1 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} s & -t \\ -t-s & 0 \\ -s & t \\ s & -t \\ -s & t \\ t+s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -t & t+s & t \\ 0 & 0 & t+s \\ t & 0 & s \\ -t & t+s & t \\ t & 0 & s \\ 0 & t+s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^\dagger = (0 \ 3 \ 3)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ \frac{2}{3} & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ 1 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & \frac{3}{4} & \frac{9}{4} & \frac{3}{2} & \frac{-3}{4} & \frac{3}{4} \\ \frac{-3}{4} & 0 & \frac{3}{2} & \frac{3}{4} & \frac{-3}{2} & 0 \\ \frac{-9}{4} & \frac{-3}{2} & 0 & \frac{-3}{4} & -3 & \frac{-3}{2} \\ \frac{-3}{2} & \frac{-3}{4} & \frac{3}{4} & 0 & \frac{-9}{4} & \frac{-3}{4} \\ \frac{3}{4} & \frac{3}{2} & 3 & \frac{9}{4} & 0 & \frac{3}{2} \\ \frac{-3}{4} & 0 & \frac{3}{2} & \frac{3}{4} & \frac{-3}{2} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{1}{36} & 0 & \frac{1}{6} & 0 & \frac{1}{18} \\ \frac{-1}{36} & 0 & \frac{1}{9} & \frac{1}{36} & \frac{-1}{9} & 0 \\ 0 & \frac{-1}{9} & 0 & 0 & \frac{-1}{3} & \frac{-1}{18} \\ \frac{-1}{6} & \frac{-1}{36} & 0 & 0 & 0 & \frac{-1}{18} \\ 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & \frac{1}{18} \\ \frac{-1}{18} & 0 & \frac{1}{18} & \frac{1}{18} & \frac{-1}{18} & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{24} & \frac{1}{8} & \frac{1}{12} & \frac{-1}{24} & \frac{1}{24} \\ \frac{-1}{24} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{-1}{12} & 0 \\ \frac{-1}{8} & \frac{-1}{12} & 0 & \frac{-1}{24} & \frac{-1}{6} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{-1}{24} & \frac{1}{24} & 0 & \frac{-1}{8} & \frac{-1}{24} \\ \frac{1}{24} & \frac{1}{12} & \frac{1}{6} & \frac{1}{8} & 0 & \frac{1}{12} \\ \frac{-1}{24} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{9}{4} & 0 & \frac{9}{4} & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 3 \\ \frac{9}{4} & 0 & \frac{9}{2} & \frac{9}{4} & 0 & 0 \\ 0 & 0 & \frac{9}{4} & \frac{15}{4} & \frac{3}{2} & 0 \\ 0 & 0 & 0 & \frac{3}{2} & \frac{3}{2} & 0 \\ 0 & 3 & 0 & 0 & 0 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$$\Omega \left(\frac{1}{4} \quad \frac{1}{8} \quad \frac{5}{24} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{8} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{5}{24} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{8} \right)$$

$$T \left(\frac{1}{2} \quad 0 \quad \frac{5}{36} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{12} \quad \frac{2}{9} \quad 0 \quad \frac{5}{12} \quad 0 \quad \frac{1}{9} \quad \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(\frac{9}{2} \quad 0 \quad \frac{5}{4} \quad 3 \quad 3 \quad \frac{3}{4} \quad 2 \quad 0 \quad \frac{15}{4} \quad 0 \quad 1 \quad \frac{9}{4} \right)$$

"IS MN in Vec(K)?", false

$$MN (3 \quad 0 \quad 1 \quad 2 \quad 3 \quad 1 \quad 2 \quad 0 \quad 3 \quad 0 \quad 1 \quad 3)$$

$$\tau = 18/1, \text{ rank} = 2, \text{ ratio} = 9/1, n^2/r = 18/1$$

$$\tau' = 18/1, r' = 1/2, \tau/n^2 = 1/2$$

$$p^2 = 53/288, \text{ min } \tau = 53/8, \tau\text{-check is positive? } 91/8$$

$$\text{max } r = 288/53, r\text{-check is positive? } 91/144$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

$$\text{"PT1"} = \{\{2, 3, 5\}, \{1, 4, 6\}\}$$

$$\text{"PT2"} = \{\{1, 2, 4\}, \{3, 5, 6\}\}$$

$$\text{"RG1"} = \{2, 6\}$$

$$\text{"RG2"} = \{4, 5\}$$

$$\text{"RG3"} = \{3, 4\}$$

"RG4" = {1, 3}

$$M_C = \begin{pmatrix} \frac{27}{16} & \frac{-3}{4} & \frac{9}{8} & \frac{-15}{16} & \frac{-3}{8} & \frac{-3}{4} \\ \frac{-3}{4} & 2 & \frac{-3}{2} & \frac{-5}{4} & \frac{-1}{2} & 2 \\ \frac{9}{8} & \frac{-3}{2} & \frac{9}{4} & \frac{3}{8} & \frac{-3}{4} & \frac{-3}{2} \\ \frac{-15}{16} & \frac{-5}{4} & \frac{3}{8} & \frac{35}{16} & \frac{7}{8} & \frac{-5}{4} \\ \frac{-3}{8} & \frac{-1}{2} & \frac{-3}{4} & \frac{7}{8} & \frac{5}{4} & \frac{-1}{2} \\ \frac{-3}{4} & 2 & \frac{-3}{2} & \frac{-5}{4} & \frac{-1}{2} & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{235}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} \\ \frac{43}{288} & \frac{235}{288} & \frac{139}{288} & \frac{43}{288} & \frac{139}{288} & \frac{-53}{288} \\ \frac{-53}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} \\ \frac{235}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} \\ \frac{-53}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} \\ \frac{139}{288} & \frac{-53}{288} & \frac{43}{288} & \frac{139}{288} & \frac{43}{288} & \frac{235}{288} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-4}{9} & \frac{2}{3} & \frac{-5}{9} & \frac{-2}{9} & \frac{-4}{9} \\ \frac{-3}{8} & 1 & \frac{-3}{4} & \frac{-5}{8} & \frac{-1}{4} & 1 \\ \frac{1}{2} & \frac{-2}{3} & 1 & \frac{1}{6} & \frac{-1}{3} & \frac{-2}{3} \\ \frac{-3}{7} & \frac{-4}{7} & \frac{6}{35} & 1 & \frac{2}{5} & \frac{-4}{7} \\ \frac{-3}{10} & \frac{-2}{5} & \frac{-3}{5} & \frac{7}{10} & 1 & \frac{-2}{5} \\ \frac{-3}{8} & 1 & \frac{-3}{4} & \frac{-5}{8} & \frac{-1}{4} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{43}{235} & \frac{-53}{235} & 1 & \frac{-53}{235} & \frac{139}{235} \\ \frac{43}{235} & 1 & \frac{139}{235} & \frac{43}{235} & \frac{139}{235} & \frac{-53}{235} \\ \frac{-53}{235} & \frac{139}{235} & 1 & \frac{-53}{235} & 1 & \frac{43}{235} \\ 1 & \frac{43}{235} & \frac{-53}{235} & 1 & \frac{-53}{235} & \frac{139}{235} \\ \frac{-53}{235} & \frac{139}{235} & 1 & \frac{-53}{235} & 1 & \frac{43}{235} \\ \frac{139}{235} & \frac{-53}{235} & \frac{43}{235} & \frac{139}{235} & \frac{43}{235} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 6.410841895, 1.289330471, 3.674827635]

Eigenvalues N_C

[0., 0., 0., 1.895833333, 2.187184271, 0.8128157289]

Eigenvalues M_C -scaled

[0., 0., 0., 3.140508746, 0.7826632797, 2.076827974]

Eigenvalues N_C -scaled

[0., 0., 0., 2.323404255, 2.680464127, 0.9961316167]

NullSpace M_C

{[0, 1, 1, 0, 1, 0], [0, 0, 1, 0, 1, 1], [1, 0, -1, 1, -1, 0]}

NullSpace N_C

{[0, 0, -1, 0, 1, 0], [-1, 1, -1, 0, 0, 1], [-1, 0, 0, 1, 0, 0]}

Eigenvalues M_0

[0., 0., 6., 7.188657443, 1.155932192, 3.655410366]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[-1, 0, 1, -1, 1, 0], [0, -1, 0, 0, 0, 1]}

NullSpace N_0

{[1, -1, 1, 0, 0, -1], [-1, 0, 0, 1, 0, 0], [1, -1, 0, 0, 1, -1]}

Eigenvalues M

[-3., 3., -3.372461083, 3.372461083, -1.000752839, 1.000752839]

Eigenvalues N

[0., 0., 0., 3., -0.8128157289, -2.187184271]

NullSpace M

{}

NullSpace N

{[1, -1, 1, 0, 0, -1], [-1, 0, 0, 1, 0, 0], [1, -1, 0, 0, 1, -1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 3 & 0 & 3 & 1 \\ 2 & 0 & 1 & 2 & 1 & 3 \\ 3 & 1 & 0 & 3 & 0 & 2 \\ 0 & 2 & 3 & 0 & 3 & 1 \\ 3 & 1 & 0 & 3 & 0 & 2 \\ 1 & 3 & 2 & 1 & 2 & 0 \end{pmatrix}$$

=====

{2, 5}

R: [3, 1, 4, 3, 1, 5]

B: [6, 4, 2, 6, 2, 3]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

$$\text{Level 2 det} = \frac{-3}{1024} (-1 + s) (220 + 17s + 46s^2 + 36s^3 - 2s^4 + 3s^5)$$

RANK of R is 4

R ranking is 3, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", $1 + v[3] v[4]$

"B CYCLES", $1 + v[2] v[3] v[4] v[6]$

Eigenvalues

R: [0., 0., 0., 0., 1., -1.]

B: [0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]}

NullSpace of B

{[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0]}

NullSpace of R^*

{[-1, 0, 0, 1, 0, 0], [0, -1, 0, 0, 1, 0]}

NullSpace of B^*

{[-1, 0, 0, 1, 0, 0], [0, 0, -1, 0, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 3 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12 \\ 3 & 0 & 0 & 15 & 0 & 0 \\ 0 & 0 & 15 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6} (3 v[1] + 4 v[2] + 6 v[3] + 5 v[4] + 2 v[5] + 4 v[6])$

degree 2: $\frac{1}{3} (v[1]v[3] + 2 v[1]v[5] + 4 v[2]v[6] + 5 v[3]v[4])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{2, 3, 5}, {1, 4, 6}}

"PT2" = {{1, 2, 4}, {3, 5, 6}}

"RG1" = {2, 6}

"RG2" = {3, 4}

"RG3" = {1, 5}

"RG4" = {1, 3}

$$\pi_2 = [0, 1, 0, 2, 0, 0, 0, 0, 4, 5, 0, 0, 0, 0, 0]$$

supp $\pi_2 = \{2, 4, 9, 10\}$

$$u_2 = [2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2]$$

supp $u_2 = \{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15\}$

Action of R on ranges, [[3], [2], [4], [2]]

Action of B on ranges, [[2], [1], [1], [1]]

$$\beta = \begin{pmatrix} \frac{1}{3} & \frac{5}{12} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

RPARTS [1, 1]

BPARTS [2, 1]

$$\alpha = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 2, 3]

B-BLOCKS,

[2, 4, 1, 3]

with invariant measure, [1, 2, 2, 1]

N by blocks, N - check: true

$b_1 = \{1, 2, 4\}$

$b_2 = \{2, 3, 5\}$

$b_3 = \{1, 4, 6\}$

$b_4 = \{3, 5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 14, Shape: 3 ⊕ 11/9

$$CLB = \begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 4}}, true

Ω_B in Vec(K)? , {{2, 3, 4, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{16} & \frac{1}{12} & \frac{3}{8} & \frac{11}{48} & \frac{-5}{24} & \frac{-5}{12} \\ \frac{1}{2} & 0 & 0 & \frac{-1}{2} & 0 & 0 \\ \frac{-3}{16} & \frac{-5}{12} & \frac{1}{8} & \frac{17}{48} & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{16} & \frac{1}{12} & \frac{3}{8} & \frac{11}{48} & \frac{-5}{24} & \frac{-5}{12} \\ \frac{5}{16} & \frac{-5}{12} & \frac{1}{8} & \frac{-7}{48} & \frac{1}{24} & \frac{1}{12} \\ \frac{1}{4} & 0 & \frac{-1}{2} & \frac{-1}{4} & \frac{1}{2} & 0 \end{pmatrix}$$

Omega-checks true, true

$$\pi_R = \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \text{ vs } \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(0 \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ 0 \ \frac{1}{4}\right) \text{ vs } \left(0 \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ 0 \ \frac{1}{4}\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{2, 3, 5}, {1, 4, 6}}

1, "range", [2, 6], [[6, 2, 2, 6, 2, 6], [2, 6, 6, 2, 6, 2]]

2, "range", [3, 4], [[4, 3, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3]]

3, "range", [1, 5], [[5, 1, 1, 5, 1, 5], [1, 5, 5, 1, 5, 1]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 3], [1, 3, 3, 1, 3, 1]]

2, "partition", {{1, 2, 4}, {3, 5, 6}}

1, "range", [2, 6], [[6, 6, 2, 6, 2, 2], [2, 2, 6, 2, 6, 6]]

2, "range", [3, 4], [[4, 4, 3, 4, 3, 3], [3, 3, 4, 3, 4, 4]]

3, "range", [1, 5], [[5, 5, 1, 5, 1, 1], [1, 1, 5, 1, 5, 5]]

4, "range", [1, 3], [[3, 3, 1, 3, 1, 1], [1, 1, 3, 1, 3, 3]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 = []$

$g_2 = [[1, 2]]$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

(h[1] h[2])

"Basis for Z(G)"

1, "coeff", 1

$$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$$\pi_2 = (0 \ 1 \ 0 \ 2 \ 0 \ 0 \ 0 \ 0 \ 0 \ 4 \ 5 \ 0 \ 0 \ 0 \ 0 \ 0)$$

{2, 4, 9, 10}

$$u_2 = (2 \ 3 \ 0 \ 3 \ 1 \ 1 \ 2 \ 1 \ 3 \ 3 \ 0 \ 2 \ 3 \ 1 \ 2)$$

{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15}

picheck (3 4 6 5 2 4)

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{6} & \frac{1}{4} & \frac{5}{24} & \frac{1}{12} & \frac{1}{6} \end{pmatrix}$$

$$\pi_1 = (3 \ 4 \ 6 \ 5 \ 2 \ 4)$$

$$u_1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

picheck (3 4 6 5 2 4)

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{9} & 0 & \frac{5}{12} & 0 & \frac{2}{9} \\ \frac{1}{12} & \frac{1}{3} & \frac{1}{3} & \frac{5}{36} & \frac{1}{9} & 0 \\ 0 & \frac{2}{9} & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{9} \\ \frac{1}{4} & \frac{1}{9} & 0 & \frac{5}{12} & 0 & \frac{2}{9} \\ 0 & \frac{2}{9} & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{9} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{5}{18} & \frac{1}{18} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{9}{4} & 1 & 0 & \frac{15}{4} & 0 & 2 \\ \frac{3}{4} & 3 & 3 & \frac{5}{4} & 1 & 0 \\ 0 & 2 & \frac{9}{2} & 0 & \frac{3}{2} & 1 \\ \frac{9}{4} & 1 & 0 & \frac{15}{4} & 0 & 2 \\ 0 & 2 & \frac{9}{2} & 0 & \frac{3}{2} & 1 \\ \frac{3}{2} & 0 & \frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [3, -4, 2, 1, 2, -4]$$

$$\ker N_C = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ s & 0 & 0 & -s & 0 & 0 \\ s & -t & -s+t & -s+t & s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\pi\Delta \text{ via } \ker N_C \ (3 \ 2 \ -4)$$

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \\ 1 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} s & -t \\ -s-t & 0 \\ -s & t \\ s & -t \\ -s & t \\ t+s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t & s & 0 \\ 0 & 0 & t+s \\ -t & t & t+s \\ t & s & 0 \\ -t & t & t+s \\ 0 & t+s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\pi\pi_X^\dagger = (0 \ 3 \ 3)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ \frac{2}{3} & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ 1 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & \frac{3}{4} & \frac{9}{4} & \frac{3}{2} & \frac{-3}{4} & \frac{3}{4} \\ \frac{-3}{4} & 0 & \frac{3}{2} & \frac{3}{4} & \frac{-3}{2} & 0 \\ \frac{-9}{4} & \frac{-3}{2} & 0 & \frac{-3}{4} & -3 & \frac{-3}{2} \\ \frac{-3}{2} & \frac{-3}{4} & \frac{3}{4} & 0 & \frac{-9}{4} & \frac{-3}{4} \\ \frac{3}{4} & \frac{3}{2} & 3 & \frac{9}{4} & 0 & \frac{3}{2} \\ \frac{-3}{4} & 0 & \frac{3}{2} & \frac{3}{4} & \frac{-3}{2} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{1}{36} & 0 & \frac{1}{6} & 0 & \frac{1}{18} \\ \frac{-1}{36} & 0 & \frac{1}{9} & \frac{1}{36} & \frac{-1}{9} & 0 \\ 0 & \frac{-1}{9} & 0 & 0 & \frac{-1}{3} & \frac{-1}{18} \\ \frac{-1}{6} & \frac{-1}{36} & 0 & 0 & 0 & \frac{-1}{18} \\ 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & \frac{1}{18} \\ \frac{-1}{18} & 0 & \frac{1}{18} & \frac{1}{18} & \frac{-1}{18} & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{24} & \frac{1}{8} & \frac{1}{12} & \frac{-1}{24} & \frac{1}{24} \\ \frac{-1}{24} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{-1}{12} & 0 \\ \frac{-1}{8} & \frac{-1}{12} & 0 & \frac{-1}{24} & \frac{-1}{6} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{-1}{24} & \frac{1}{24} & 0 & \frac{-1}{8} & \frac{-1}{24} \\ \frac{1}{24} & \frac{1}{12} & \frac{1}{6} & \frac{1}{8} & 0 & \frac{1}{12} \\ \frac{-1}{24} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{9}{4} & 0 & \frac{3}{4} & 0 & \frac{3}{2} & 0 \\ 0 & 3 & 0 & 0 & 0 & 3 \\ \frac{3}{4} & 0 & \frac{9}{2} & \frac{15}{4} & 0 & 0 \\ 0 & 0 & \frac{15}{4} & \frac{15}{4} & 0 & 0 \\ \frac{3}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 3 & 0 & 0 & 0 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$$\Omega \left(\frac{1}{24} \quad \frac{1}{24} \quad 0 \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{8} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{5}{24} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{8} \right)$$

$$T \left(\frac{1}{12} \quad 0 \quad \frac{1}{18} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{12} \quad \frac{2}{9} \quad 0 \quad \frac{5}{12} \quad 0 \quad \frac{1}{9} \quad \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(\frac{3}{4} \quad 0 \quad \frac{1}{2} \quad 3 \quad 3 \quad \frac{3}{4} \quad 2 \quad 0 \quad \frac{15}{4} \quad 0 \quad 1 \quad \frac{9}{4} \right)$$

"IS MN in Vec(K)?", false

$$MN (0 \quad 0 \quad 0 \quad 2 \quad 3 \quad 1 \quad 2 \quad 0 \quad 3 \quad 0 \quad 1 \quad 3)$$

$$\tau = 18/1, \text{ rank} = 2, \text{ ratio} = 9/1, n^2/r = 18/1$$

$$\tau' = 18/1, r' = 1/2, \tau/n^2 = 1/2$$

$$p^2 = 53/288, \text{ min } \tau = 53/8, \tau\text{-check is positive? } 91/8$$

$$\text{max } r = 288/53, r\text{-check is positive? } 91/144$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

$$\text{"PT1"} = \{\{2, 3, 5\}, \{1, 4, 6\}\}$$

$$\text{"PT2"} = \{\{1, 2, 4\}, \{3, 5, 6\}\}$$

$$\text{"RG1"} = \{2, 6\}$$

$$\text{"RG2"} = \{3, 4\}$$

$$\text{"RG3"} = \{1, 5\}$$

"RG4" = {1, 3}

$$M_C = \begin{pmatrix} \frac{27}{16} & \frac{-3}{4} & \frac{-3}{8} & \frac{-15}{16} & \frac{9}{8} & \frac{-3}{4} \\ \frac{-3}{4} & 2 & \frac{-3}{2} & \frac{-5}{4} & \frac{-1}{2} & 2 \\ \frac{-3}{8} & \frac{-3}{2} & \frac{9}{4} & \frac{15}{8} & \frac{-3}{4} & \frac{-3}{2} \\ \frac{-15}{16} & \frac{-5}{4} & \frac{15}{8} & \frac{35}{16} & \frac{-5}{8} & \frac{-5}{4} \\ \frac{9}{8} & \frac{-1}{2} & \frac{-3}{4} & \frac{-5}{8} & \frac{5}{4} & \frac{-1}{2} \\ \frac{-3}{4} & 2 & \frac{-3}{2} & \frac{-5}{4} & \frac{-1}{2} & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{235}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} \\ \frac{43}{288} & \frac{235}{288} & \frac{139}{288} & \frac{43}{288} & \frac{139}{288} & \frac{-53}{288} \\ \frac{-53}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} \\ \frac{235}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} \\ \frac{-53}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} \\ \frac{139}{288} & \frac{-53}{288} & \frac{43}{288} & \frac{139}{288} & \frac{43}{288} & \frac{235}{288} \end{pmatrix}$$

$$M_{C\text{-scaled}} = \begin{pmatrix} 1 & \frac{-4}{9} & \frac{-2}{9} & \frac{-5}{9} & \frac{2}{3} & \frac{-4}{9} \\ \frac{-3}{8} & 1 & \frac{-3}{4} & \frac{-5}{8} & \frac{-1}{4} & 1 \\ \frac{-1}{6} & \frac{-2}{3} & 1 & \frac{5}{6} & \frac{-1}{3} & \frac{-2}{3} \\ \frac{-3}{7} & \frac{-4}{7} & \frac{6}{7} & 1 & \frac{-2}{7} & \frac{-4}{7} \\ \frac{9}{10} & \frac{-2}{5} & \frac{-3}{5} & \frac{-1}{2} & 1 & \frac{-2}{5} \\ \frac{-3}{8} & 1 & \frac{-3}{4} & \frac{-5}{8} & \frac{-1}{4} & 1 \end{pmatrix} \quad N_{C\text{-scaled}} = \begin{pmatrix} 1 & \frac{43}{235} & \frac{-53}{235} & 1 & \frac{-53}{235} & \frac{139}{235} \\ \frac{43}{235} & 1 & \frac{139}{235} & \frac{43}{235} & \frac{139}{235} & \frac{-53}{235} \\ \frac{-53}{235} & \frac{139}{235} & 1 & \frac{-53}{235} & 1 & \frac{43}{235} \\ 1 & \frac{43}{235} & \frac{-53}{235} & 1 & \frac{-53}{235} & \frac{139}{235} \\ \frac{-53}{235} & \frac{139}{235} & 1 & \frac{-53}{235} & 1 & \frac{43}{235} \\ \frac{139}{235} & \frac{-53}{235} & \frac{43}{235} & \frac{139}{235} & \frac{43}{235} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 6.811196768, 0.629770625, 3.934032609]

Eigenvalues N_C

[0., 0., 0., 1.895833333, 2.187184271, 0.8128157289]

Eigenvalues M_C -scaled

[0., 0., 0., 3.236783071, 0.365404790, 2.397812138]

Eigenvalues N_C -scaled

[0., 0., 0., 2.323404255, 2.680464127, 0.9961316167]

NullSpace M_C

{[1, 1, 0, 1, 0, 0], [-1, 0, 1, -1, 1, 0], [1, 0, 0, 1, 0, 1]}

NullSpace N_C

{[0, 1, -1, -1, 0, 1], [1, 0, 0, -1, 0, 0], [0, 0, -1, 0, 1, 0]}

Eigenvalues M_0

[0., 0., 6., 7.951833037, 0.618821322, 3.429345642]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[1, 0, -1, 1, -1, 0], [0, -1, 0, 0, 0, 1]}

NullSpace N_0

{[0, 0, -1, 0, 1, 0], [-1, 1, -1, 0, 0, 1], [-1, 0, 0, 1, 0, 0]}

Eigenvalues M

[-3., 3., -3.837504551, 3.837504551, -1.465796307, 1.465796307]

Eigenvalues N

[0., 0., 0., 3., -0.8128157289, -2.187184271]

NullSpace M

{}

NullSpace N

{[-1, 1, -1, 0, 0, 1], [-1, 0, 0, 1, 0, 0], [0, 0, -1, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 3 & 0 & 3 & 1 \\ 2 & 0 & 1 & 2 & 1 & 3 \\ 3 & 1 & 0 & 3 & 0 & 2 \\ 0 & 2 & 3 & 0 & 3 & 1 \\ 3 & 1 & 0 & 3 & 0 & 2 \\ 1 & 3 & 2 & 1 & 2 & 0 \end{pmatrix}$$

=====

{5, 6}

R: [3, 4, 4, 3, 1, 3]
B: [6, 1, 2, 6, 2, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

Level 2 det = $\frac{3}{1024} (-220 + 61s + 9s^2 - 13s^3 + 3s^4) (-1 + s) (1 + s)$

RANK of R is 3

R ranking is 2, "vs", 3

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[3] v[4]

"B CYCLES", 1 + v[1] v[2] v[6] v[5]

Eigenvalues

R: [0., 0., 0., 0., 1., -1.]

B: [0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0]}

NullSpace of R*

{[-1, 0, 0, 0, 0, 1], [-1, 0, 0, 1, 0, 0], [0, -1, 1, 0, 0, 0]}

NullSpace of B*

{[-1, 0, 0, 1, 0, 0], [0, 0, -1, 0, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 3 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12 \\ 3 & 0 & 0 & 15 & 0 & 0 \\ 0 & 0 & 15 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6} (3 v[1] + 4 v[2] + 6 v[3] + 5 v[4] + 2 v[5] + 4 v[6])$

degree 2: $\frac{1}{3} (v[1]v[3] + 2 v[1]v[5] + 4 v[2]v[6] + 5 v[3]v[4])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{2, 3, 5}, {1, 4, 6}}

"PT2" = {{1, 2, 4}, {3, 5, 6}}

"RG1" = {2, 6}

"RG2" = {3, 4}

"RG3" = {1, 5}

"RG4" = {1, 3}

$$\pi_2 = [0, 1, 0, 2, 0, 0, 0, 0, 4, 5, 0, 0, 0, 0, 0]$$

supp $\pi_2 = \{2, 4, 9, 10\}$

$$u_2 = [2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2]$$

supp $u_2 = \{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15\}$

Action of R on ranges, [[2], [2], [4], [2]]

Action of B on ranges, [[3], [1], [1], [1]]

$$\beta = \begin{pmatrix} \frac{1}{3} & \frac{5}{12} & \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

RPARTS [1, 1]

BPARTS [2, 1]

$$\alpha = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 2, 3]

B-BLOCKS,

[2, 4, 1, 3]

with invariant measure, [1, 2, 2, 1]

N by blocks, N - check: true

$b_1 = \{1, 2, 4\}$

$b_2 = \{2, 3, 5\}$

$b_3 = \{1, 4, 6\}$

$b_4 = \{3, 5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: 3 ⊕ 10/8

$$CLB = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 4}}, true

Ω_B in Vec(K)? , {{1, 2, 5, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{16} & \frac{1}{12} & \frac{3}{8} & \frac{11}{48} & \frac{-5}{24} & \frac{-5}{12} \\ \frac{-1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{-3}{16} & \frac{-5}{12} & \frac{1}{8} & \frac{17}{48} & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{16} & \frac{1}{12} & \frac{3}{8} & \frac{11}{48} & \frac{-5}{24} & \frac{-5}{12} \\ \frac{5}{16} & \frac{-5}{12} & \frac{1}{8} & \frac{-7}{48} & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{4} & 0 & \frac{1}{2} & \frac{1}{4} & \frac{-1}{2} & 0 \end{pmatrix}$$

Omega-checks true, true

$$\pi_R = \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \text{ vs } \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(\frac{1}{4} \ \frac{1}{4} \ 0 \ 0 \ \frac{1}{4} \ \frac{1}{4}\right) \text{ vs } \left(\frac{1}{4} \ \frac{1}{4} \ 0 \ 0 \ \frac{1}{4} \ \frac{1}{4}\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{2, 3, 5}, {1, 4, 6}}

1, "range", [2, 6], [[6, 2, 2, 6, 2, 6], [2, 6, 6, 2, 6, 2]]

2, "range", [3, 4], [[4, 3, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3]]

3, "range", [1, 5], [[5, 1, 1, 5, 1, 5], [1, 5, 5, 1, 5, 1]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 3], [1, 3, 3, 1, 3, 1]]

2, "partition", {{1, 2, 4}, {3, 5, 6}}

1, "range", [2, 6], [[6, 6, 2, 6, 2, 2], [2, 2, 6, 2, 6, 6]]

2, "range", [3, 4], [[4, 4, 3, 4, 3, 3], [3, 3, 4, 3, 4, 4]]

3, "range", [1, 5], [[5, 5, 1, 5, 1, 1], [1, 1, 5, 1, 5, 5]]

4, "range", [1, 3], [[3, 3, 1, 3, 1, 1], [1, 1, 3, 1, 3, 3]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 = []$

$g_2 = [[1, 2]]$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

(h[1] h[2])

"Basis for Z(G)"

1, "coeff", 1

$$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$$\pi_2 = (0 \ 1 \ 0 \ 2 \ 0 \ 0 \ 0 \ 0 \ 0 \ 4 \ 5 \ 0 \ 0 \ 0 \ 0 \ 0)$$

{2, 4, 9, 10}

$$u_2 = (2 \ 3 \ 0 \ 3 \ 1 \ 1 \ 2 \ 1 \ 3 \ 3 \ 0 \ 2 \ 3 \ 1 \ 2)$$

{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15}

picheck (3 4 6 5 2 4)

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{6} & \frac{1}{4} & \frac{5}{24} & \frac{1}{12} & \frac{1}{6} \end{pmatrix}$$

$$\pi_1 = (3 \ 4 \ 6 \ 5 \ 2 \ 4)$$

$$u_1 = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{pmatrix}$$

picheck (3 4 6 5 2 4)

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{9} & 0 & \frac{5}{12} & 0 & \frac{2}{9} \\ \frac{1}{12} & \frac{1}{3} & \frac{1}{3} & \frac{5}{36} & \frac{1}{9} & 0 \\ 0 & \frac{2}{9} & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{9} \\ \frac{1}{4} & \frac{1}{9} & 0 & \frac{5}{12} & 0 & \frac{2}{9} \\ 0 & \frac{2}{9} & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{9} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{5}{18} & \frac{1}{18} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{9}{4} & 1 & 0 & \frac{15}{4} & 0 & 2 \\ \frac{3}{4} & 3 & 3 & \frac{5}{4} & 1 & 0 \\ 0 & 2 & \frac{9}{2} & 0 & \frac{3}{2} & 1 \\ \frac{9}{4} & 1 & 0 & \frac{15}{4} & 0 & 2 \\ 0 & 2 & \frac{9}{2} & 0 & \frac{3}{2} & 1 \\ \frac{3}{2} & 0 & \frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [-1, -4, 6, 5, -2, -4]$$

$$\ker N_C = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -t+s & t & 0 & -s & -t & t \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -t & t & 0 & 0 & -t & t \end{pmatrix} \quad \text{RB checks}$$

$$\pi\Delta \text{ via } \ker N_C \quad (-2 \quad 5 \quad 6)$$

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} s & t \\ -t-s & 0 \\ -s & -t \\ s & t \\ -s & -t \\ s+t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & s & t \\ s+t & -t-s & s+t \\ s+t & -s & s \\ 0 & s & t \\ s+t & -s & s \\ 0 & s+t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\pi\pi_X^\dagger = (3 \quad 0 \quad 3)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ \frac{2}{3} & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ 1 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & \frac{3}{4} & \frac{9}{4} & \frac{3}{2} & \frac{-3}{4} & \frac{3}{4} \\ \frac{-3}{4} & 0 & \frac{3}{2} & \frac{3}{4} & \frac{-3}{2} & 0 \\ \frac{-9}{4} & \frac{-3}{2} & 0 & \frac{-3}{4} & -3 & \frac{-3}{2} \\ \frac{-3}{2} & \frac{-3}{4} & \frac{3}{4} & 0 & \frac{-9}{4} & \frac{-3}{4} \\ \frac{3}{4} & \frac{3}{2} & 3 & \frac{9}{4} & 0 & \frac{3}{2} \\ \frac{-3}{4} & 0 & \frac{3}{2} & \frac{3}{4} & \frac{-3}{2} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{1}{36} & 0 & \frac{1}{6} & 0 & \frac{1}{18} \\ \frac{-1}{36} & 0 & \frac{1}{9} & \frac{1}{36} & \frac{-1}{9} & 0 \\ 0 & \frac{-1}{9} & 0 & 0 & \frac{-1}{3} & \frac{-1}{18} \\ \frac{-1}{6} & \frac{-1}{36} & 0 & 0 & 0 & \frac{-1}{18} \\ 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & \frac{1}{18} \\ \frac{-1}{18} & 0 & \frac{1}{18} & \frac{1}{18} & \frac{-1}{18} & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{24} & \frac{1}{8} & \frac{1}{12} & \frac{-1}{24} & \frac{1}{24} \\ \frac{-1}{24} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{-1}{12} & 0 \\ \frac{-1}{8} & \frac{-1}{12} & 0 & \frac{-1}{24} & \frac{-1}{6} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{-1}{24} & \frac{1}{24} & 0 & \frac{-1}{8} & \frac{-1}{24} \\ \frac{1}{24} & \frac{1}{12} & \frac{1}{6} & \frac{1}{8} & 0 & \frac{1}{12} \\ \frac{-1}{24} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{9}{4} & 0 & \frac{3}{4} & 0 & \frac{3}{2} & 0 \\ 0 & 3 & 0 & 0 & 0 & 3 \\ \frac{3}{4} & 0 & \frac{9}{2} & \frac{15}{4} & 0 & 0 \\ 0 & 0 & \frac{15}{4} & \frac{15}{4} & 0 & 0 \\ \frac{3}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 3 & 0 & 0 & 0 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$$\Omega \left(\frac{1}{24} \quad \frac{1}{24} \quad 0 \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{8} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{5}{24} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{8} \right)$$

$$T \left(\frac{1}{12} \quad 0 \quad \frac{1}{18} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{12} \quad \frac{2}{9} \quad 0 \quad \frac{5}{12} \quad 0 \quad \frac{1}{9} \quad \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(\frac{3}{4} \quad 0 \quad \frac{1}{2} \quad 3 \quad 3 \quad \frac{3}{4} \quad 2 \quad 0 \quad \frac{15}{4} \quad 0 \quad 1 \quad \frac{9}{4} \right)$$

"IS MN in Vec(K)?", false

$$MN (0 \quad 0 \quad 0 \quad 2 \quad 3 \quad 1 \quad 2 \quad 0 \quad 3 \quad 0 \quad 1 \quad 3)$$

$$\tau = 18/1, \text{ rank} = 2, \text{ ratio} = 9/1, n^2/r = 18/1$$

$$\tau' = 18/1, r' = 1/2, \tau/n^2 = 1/2$$

$$p^2 = 53/288, \text{ min } \tau = 53/8, \tau\text{-check is positive? } 91/8$$

$$\text{max } r = 288/53, r\text{-check is positive? } 91/144$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

$$\text{"PT1"} = \{\{2, 3, 5\}, \{1, 4, 6\}\}$$

$$\text{"PT2"} = \{\{1, 2, 4\}, \{3, 5, 6\}\}$$

$$\text{"RG1"} = \{2, 6\}$$

$$\text{"RG2"} = \{3, 4\}$$

$$\text{"RG3"} = \{1, 5\}$$

"RG4" = {1, 3}

$$M_C = \begin{pmatrix} \frac{27}{16} & \frac{-3}{4} & \frac{-3}{8} & \frac{-15}{16} & \frac{9}{8} & \frac{-3}{4} \\ \frac{-3}{4} & 2 & \frac{-3}{2} & \frac{-5}{4} & \frac{-1}{2} & 2 \\ \frac{-3}{8} & \frac{-3}{2} & \frac{9}{4} & \frac{15}{8} & \frac{-3}{4} & \frac{-3}{2} \\ \frac{-15}{16} & \frac{-5}{4} & \frac{15}{8} & \frac{35}{16} & \frac{-5}{8} & \frac{-5}{4} \\ \frac{9}{8} & \frac{-1}{2} & \frac{-3}{4} & \frac{-5}{8} & \frac{5}{4} & \frac{-1}{2} \\ \frac{-3}{4} & 2 & \frac{-3}{2} & \frac{-5}{4} & \frac{-1}{2} & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{235}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} \\ \frac{43}{288} & \frac{235}{288} & \frac{139}{288} & \frac{43}{288} & \frac{139}{288} & \frac{-53}{288} \\ \frac{-53}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} \\ \frac{235}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} \\ \frac{-53}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} \\ \frac{139}{288} & \frac{-53}{288} & \frac{43}{288} & \frac{139}{288} & \frac{43}{288} & \frac{235}{288} \end{pmatrix}$$

$$M_{C\text{-scaled}} = \begin{pmatrix} 1 & \frac{-4}{9} & \frac{-2}{9} & \frac{-5}{9} & \frac{2}{3} & \frac{-4}{9} \\ \frac{-3}{8} & 1 & \frac{-3}{4} & \frac{-5}{8} & \frac{-1}{4} & 1 \\ \frac{-1}{6} & \frac{-2}{3} & 1 & \frac{5}{6} & \frac{-1}{3} & \frac{-2}{3} \\ \frac{-3}{7} & \frac{-4}{7} & \frac{6}{7} & 1 & \frac{-2}{7} & \frac{-4}{7} \\ \frac{9}{10} & \frac{-2}{5} & \frac{-3}{5} & \frac{-1}{2} & 1 & \frac{-2}{5} \\ \frac{-3}{8} & 1 & \frac{-3}{4} & \frac{-5}{8} & \frac{-1}{4} & 1 \end{pmatrix} \quad N_{C\text{-scaled}} = \begin{pmatrix} 1 & \frac{43}{235} & \frac{-53}{235} & 1 & \frac{-53}{235} & \frac{139}{235} \\ \frac{43}{235} & 1 & \frac{139}{235} & \frac{43}{235} & \frac{139}{235} & \frac{-53}{235} \\ \frac{-53}{235} & \frac{139}{235} & 1 & \frac{-53}{235} & 1 & \frac{43}{235} \\ 1 & \frac{43}{235} & \frac{-53}{235} & 1 & \frac{-53}{235} & \frac{139}{235} \\ \frac{-53}{235} & \frac{139}{235} & 1 & \frac{-53}{235} & 1 & \frac{43}{235} \\ \frac{139}{235} & \frac{-53}{235} & \frac{43}{235} & \frac{139}{235} & \frac{43}{235} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 6.811196768, 0.629770625, 3.934032609]

Eigenvalues N_C

[0., 0., 0., 1.895833333, 2.187184271, 0.8128157289]

Eigenvalues M_C -scaled

[0., 0., 0., 3.236783071, 0.365404790, 2.397812138]

Eigenvalues N_C -scaled

[0., 0., 0., 2.323404255, 2.680464127, 0.9961316167]

NullSpace M_C

{[1, 1, 0, 1, 0, 0], [-1, 0, 1, -1, 1, 0], [1, 0, 0, 1, 0, 1]}

NullSpace N_C

{[-1, 1, -1, 0, 0, 1], [-1, 0, 0, 1, 0, 0], [0, 0, -1, 0, 1, 0]}

Eigenvalues M_0

[0., 0., 6., 7.951833037, 0.618821322, 3.429345642]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[0, -1, 0, 0, 0, 1], [-1, 0, 1, -1, 1, 0]}

NullSpace N_0

{[0, 0, -1, 0, 1, 0], [-1, 1, -1, 0, 0, 1], [-1, 0, 0, 1, 0, 0]}

Eigenvalues M

[-3., 3., -3.837504551, 3.837504551, -1.465796307, 1.465796307]

Eigenvalues N

[0., 0., 0., 3., -0.8128157289, -2.187184271]

NullSpace M

{}

NullSpace N

{[1, -1, 0, 0, 1, -1], [-1, 0, 0, 1, 0, 0], [1, -1, 1, 0, 0, -1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 3 & 0 & 3 & 1 \\ 2 & 0 & 1 & 2 & 1 & 3 \\ 3 & 1 & 0 & 3 & 0 & 2 \\ 0 & 2 & 3 & 0 & 3 & 1 \\ 3 & 1 & 0 & 3 & 0 & 2 \\ 1 & 3 & 2 & 1 & 2 & 0 \end{pmatrix}$$

=====

20, [1, -1, 1, -1, -1, 1]

=====

{2, 5, 6}

R: [3, 1, 4, 3, 1, 3]

B: [6, 4, 2, 6, 2, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 5

$$\text{Level 2 det} = \frac{-3}{1024} (220 + 121s + 14s^2 - 28s^3 - 10s^4 + 3s^5) (-1 + s)$$

RANK of R is 3

R ranking is 2, "vs", 3

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 2, "vs", 4

"R CYCLES", 1 + v[3] v[4]

"B CYCLES", 1 + v[2] v[4] v[6] v[5]

Eigenvalues

R: [0., 0., 0., 0., 1., -1.]

B: [0., 0., -1., 1., 1. I, -1. I]

NullSpace of R

{[0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]}

NullSpace of B

{[1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0]}

NullSpace of R*

{[0, 1, 0, 0, -1, 0], [1, 0, 0, 0, 0, -1], [0, 0, 0, 1, 0, -1]}

NullSpace of B*

{[-1, 0, 0, 1, 0, 0], [0, 0, -1, 0, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12 \\ 9 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 9 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 6

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{6} (3 v[1] + 4 v[2] + 6 v[3] + 5 v[4] + 2 v[5] + 4 v[6])$

degree 2: $\frac{1}{3} (3 v[1]v[3] + 4 v[2]v[6] + 3 v[3]v[4] + 2 v[4]v[5])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{2, 3, 5}, {1, 4, 6}}

"PT2" = {{1, 2, 4}, {3, 5, 6}}

"RG1" = {2, 6}

"RG2" = {4, 5}

"RG3" = {3, 4}

"RG4" = {1, 3}

$$\pi_2 = [0, 3, 0, 0, 0, 0, 0, 0, 4, 3, 0, 0, 2, 0, 0]$$

supp $\pi_2 = \{2, 9, 10, 13\}$

$$u_2 = [2, 3, 0, 3, 1, 1, 2, 1, 3, 3, 0, 2, 3, 1, 2]$$

supp $u_2 = \{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15\}$

Action of R on ranges, [[4], [4], [3], [3]]

Action of B on ranges, [[2], [1], [1], [1]]

$$\beta = \left(\frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{4} \right)$$

RPARTS [1, 1]

BPARTS [2, 1]

$$\alpha = \left(\frac{2}{3} \quad \frac{1}{3} \right)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 2, 3]

B-BLOCKS,

[2, 4, 1, 3]

with invariant measure, [1, 2, 2, 1]

N by blocks, N - check: true

$b_1 = \{1, 2, 4\}$

$b_2 = \{2, 3, 5\}$

$b_3 = \{1, 4, 6\}$

$b_4 = \{3, 5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 12, Shape: 3 ⊕ 9/7

$$CLB = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)?, {{3, 4}}, true

Ω_B in Vec(K)?, {{2, 4, 5, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{16} & \frac{1}{12} & \frac{3}{8} & \frac{11}{48} & \frac{-5}{24} & \frac{-5}{12} \\ \frac{1}{2} & 0 & 0 & \frac{-1}{2} & 0 & 0 \\ \frac{-3}{16} & \frac{-5}{12} & \frac{1}{8} & \frac{17}{48} & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{16} & \frac{1}{12} & \frac{3}{8} & \frac{11}{48} & \frac{-5}{24} & \frac{-5}{12} \\ \frac{5}{16} & \frac{-5}{12} & \frac{1}{8} & \frac{-7}{48} & \frac{1}{24} & \frac{1}{12} \\ \frac{-1}{4} & 0 & \frac{1}{2} & \frac{1}{4} & \frac{-1}{2} & 0 \end{pmatrix}$$

Omega-checks true, true

$$\pi_R = \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \text{ vs } \left(0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(0 \ \frac{1}{4} \ 0 \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}\right) \text{ vs } \left(0 \ \frac{1}{4} \ 0 \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{2, 3, 5}, {1, 4, 6}}

1, "range", [2, 6], [[6, 2, 2, 6, 2, 6], [2, 6, 6, 2, 6, 2]]

2, "range", [4, 5], [[5, 4, 4, 5, 4, 5], [4, 5, 5, 4, 5, 4]]

3, "range", [3, 4], [[4, 3, 3, 4, 3, 4], [3, 4, 4, 3, 4, 3]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 3], [1, 3, 3, 1, 3, 1]]

2, "partition", {{1, 2, 4}, {3, 5, 6}}

1, "range", [2, 6], [[6, 6, 2, 6, 2, 2], [2, 2, 6, 2, 6, 6]]

2, "range", [4, 5], [[5, 5, 4, 5, 4, 4], [4, 4, 5, 4, 5, 5]]

3, "range", [3, 4], [[4, 4, 3, 4, 3, 3], [3, 3, 4, 3, 4, 4]]

4, "range", [1, 3], [[3, 3, 1, 3, 1, 1], [1, 1, 3, 1, 3, 3]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 = []$

$g_2 = [[1, 2]]$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

(h[1] h[2])

"Basis for Z(G)"

1, "coeff", 1

$$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$$\pi_2 = (0 \ 3 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 4 \ 3 \ 0 \ 0 \ 2 \ 0 \ 0)$$

{2, 9, 10, 13}

$$u_2 = (2 \ 3 \ 0 \ 3 \ 1 \ 1 \ 2 \ 1 \ 3 \ 3 \ 0 \ 2 \ 3 \ 1 \ 2)$$

{1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15}

picheck (3 4 6 5 2 4)

$$\pi = \left(\frac{1}{8} \ \frac{1}{6} \ \frac{1}{4} \ \frac{5}{24} \ \frac{1}{12} \ \frac{1}{6} \right)$$

$$\pi_1 = (3 \ 4 \ 6 \ 5 \ 2 \ 4)$$

$$u_1 = \left(\frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \right)$$

picheck (3 4 6 5 2 4)

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{5}{12} & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{3} & 0 & \frac{5}{12} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{9} & 0 & \frac{5}{12} & 0 & \frac{2}{9} \\ \frac{1}{12} & \frac{1}{3} & \frac{1}{3} & \frac{5}{36} & \frac{1}{9} & 0 \\ 0 & \frac{2}{9} & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{9} \\ \frac{1}{4} & \frac{1}{9} & 0 & \frac{5}{12} & 0 & \frac{2}{9} \\ 0 & \frac{2}{9} & \frac{1}{2} & 0 & \frac{1}{6} & \frac{1}{9} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{5}{18} & \frac{1}{18} & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{9}{4} & 1 & 0 & \frac{15}{4} & 0 & 2 \\ \frac{3}{4} & 3 & 3 & \frac{5}{4} & 1 & 0 \\ 0 & 2 & \frac{9}{2} & 0 & \frac{3}{2} & 1 \\ \frac{9}{4} & 1 & 0 & \frac{15}{4} & 0 & 2 \\ 0 & 2 & \frac{9}{2} & 0 & \frac{3}{2} & 1 \\ \frac{3}{2} & 0 & \frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [3, -4, 6, 1, -2, -4]$$

$$\ker N_C = \begin{pmatrix} 0 & 0 & -1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & -s & 0 & 0 \\ s & -t & 0 & t-s & t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\pi\Delta \text{ via } \ker N_C \quad (-2 \quad -4 \quad 1)$$

$$\ker M_0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} s & t \\ -s-t & 0 \\ -s & -t \\ s & t \\ -s & -t \\ s+t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & s & t \\ s+t & 0 & 0 \\ s+t & t & -t \\ 0 & s & t \\ s+t & t & -t \\ 0 & s+t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^\dagger = (3 \quad 3 \quad 0)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ \frac{2}{3} & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ 1 & \frac{2}{3} & 0 & 1 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 1 & 0 & 1 & \frac{2}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & \frac{3}{4} & \frac{9}{4} & \frac{3}{2} & \frac{-3}{4} & \frac{3}{4} \\ \frac{-3}{4} & 0 & \frac{3}{2} & \frac{3}{4} & \frac{-3}{2} & 0 \\ \frac{-9}{4} & \frac{-3}{2} & 0 & \frac{-3}{4} & -3 & \frac{-3}{2} \\ \frac{-3}{2} & \frac{-3}{4} & \frac{3}{4} & 0 & \frac{-9}{4} & \frac{-3}{4} \\ \frac{3}{4} & \frac{3}{2} & 3 & \frac{9}{4} & 0 & \frac{3}{2} \\ \frac{-3}{4} & 0 & \frac{3}{2} & \frac{3}{4} & \frac{-3}{2} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{1}{36} & 0 & \frac{1}{6} & 0 & \frac{1}{18} \\ \frac{-1}{36} & 0 & \frac{1}{9} & \frac{1}{36} & \frac{-1}{9} & 0 \\ 0 & \frac{-1}{9} & 0 & 0 & \frac{-1}{3} & \frac{-1}{18} \\ \frac{-1}{6} & \frac{-1}{36} & 0 & 0 & 0 & \frac{-1}{18} \\ 0 & \frac{1}{9} & \frac{1}{3} & 0 & 0 & \frac{1}{18} \\ \frac{-1}{18} & 0 & \frac{1}{18} & \frac{1}{18} & \frac{-1}{18} & 0 \end{pmatrix} \quad \text{Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{24} & \frac{1}{8} & \frac{1}{12} & \frac{-1}{24} & \frac{1}{24} \\ \frac{-1}{24} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{-1}{12} & 0 \\ \frac{-1}{8} & \frac{-1}{12} & 0 & \frac{-1}{24} & \frac{-1}{6} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{-1}{24} & \frac{1}{24} & 0 & \frac{-1}{8} & \frac{-1}{24} \\ \frac{1}{24} & \frac{1}{12} & \frac{1}{6} & \frac{1}{8} & 0 & \frac{1}{12} \\ \frac{-1}{24} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{9}{4} & 0 & \frac{9}{4} & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 3 \\ \frac{9}{4} & 0 & \frac{9}{2} & \frac{9}{4} & 0 & 0 \\ 0 & 0 & \frac{9}{4} & \frac{15}{4} & \frac{3}{2} & 0 \\ 0 & 0 & 0 & \frac{3}{2} & \frac{3}{2} & 0 \\ 0 & 3 & 0 & 0 & 0 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ 1 & \frac{1}{3} & 0 & 1 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 9T + 0\Omega$$

$$\Omega \left(\frac{1}{4} \quad \frac{1}{8} \quad \frac{5}{24} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{8} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{5}{24} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{8} \right)$$

$$T \left(\frac{1}{2} \quad 0 \quad \frac{5}{36} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{12} \quad \frac{2}{9} \quad 0 \quad \frac{5}{12} \quad 0 \quad \frac{1}{9} \quad \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(\frac{9}{2} \quad 0 \quad \frac{5}{4} \quad 3 \quad 3 \quad \frac{3}{4} \quad 2 \quad 0 \quad \frac{15}{4} \quad 0 \quad 1 \quad \frac{9}{4} \right)$$

"IS MN in Vec(K)?", false

$$MN (3 \quad 0 \quad 1 \quad 2 \quad 3 \quad 1 \quad 2 \quad 0 \quad 3 \quad 0 \quad 1 \quad 3)$$

$$\tau = 18/1, \text{ rank} = 2, \text{ ratio} = 9/1, n^2/r = 18/1$$

$$\tau' = 18/1, r' = 1/2, \tau/n^2 = 1/2$$

$$p^2 = 53/288, \text{ min } \tau = 53/8, \tau\text{-check is positive? } 91/8$$

$$\text{max } r = 288/53, r\text{-check is positive? } 91/144$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

$$\text{"PT1"} = \{\{2, 3, 5\}, \{1, 4, 6\}\}$$

$$\text{"PT2"} = \{\{1, 2, 4\}, \{3, 5, 6\}\}$$

$$\text{"RG1"} = \{2, 6\}$$

$$\text{"RG2"} = \{4, 5\}$$

$$\text{"RG3"} = \{3, 4\}$$

"RG4" = {1, 3}

$$M_C = \begin{pmatrix} \frac{27}{16} & \frac{-3}{4} & \frac{9}{8} & \frac{-15}{16} & \frac{-3}{8} & \frac{-3}{4} \\ \frac{-3}{4} & 2 & \frac{-3}{2} & \frac{-5}{4} & \frac{-1}{2} & 2 \\ \frac{9}{8} & \frac{-3}{2} & \frac{9}{4} & \frac{3}{8} & \frac{-3}{4} & \frac{-3}{2} \\ \frac{-15}{16} & \frac{-5}{4} & \frac{3}{8} & \frac{35}{16} & \frac{7}{8} & \frac{-5}{4} \\ \frac{-3}{8} & \frac{-1}{2} & \frac{-3}{4} & \frac{7}{8} & \frac{5}{4} & \frac{-1}{2} \\ \frac{-3}{4} & 2 & \frac{-3}{2} & \frac{-5}{4} & \frac{-1}{2} & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{235}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} \\ \frac{43}{288} & \frac{235}{288} & \frac{139}{288} & \frac{43}{288} & \frac{139}{288} & \frac{-53}{288} \\ \frac{-53}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} \\ \frac{235}{288} & \frac{43}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{139}{288} \\ \frac{-53}{288} & \frac{139}{288} & \frac{235}{288} & \frac{-53}{288} & \frac{235}{288} & \frac{43}{288} \\ \frac{139}{288} & \frac{-53}{288} & \frac{43}{288} & \frac{139}{288} & \frac{43}{288} & \frac{235}{288} \end{pmatrix}$$

$$M_{C\text{-scaled}} = \begin{pmatrix} 1 & \frac{-4}{9} & \frac{2}{3} & \frac{-5}{9} & \frac{-2}{9} & \frac{-4}{9} \\ \frac{-3}{8} & 1 & \frac{-3}{4} & \frac{-5}{8} & \frac{-1}{4} & 1 \\ \frac{1}{2} & \frac{-2}{3} & 1 & \frac{1}{6} & \frac{-1}{3} & \frac{-2}{3} \\ \frac{-3}{7} & \frac{-4}{7} & \frac{6}{35} & 1 & \frac{2}{5} & \frac{-4}{7} \\ \frac{-3}{10} & \frac{-2}{5} & \frac{-3}{5} & \frac{7}{10} & 1 & \frac{-2}{5} \\ \frac{-3}{8} & 1 & \frac{-3}{4} & \frac{-5}{8} & \frac{-1}{4} & 1 \end{pmatrix} \quad N_{C\text{-scaled}} = \begin{pmatrix} 1 & \frac{43}{235} & \frac{-53}{235} & 1 & \frac{-53}{235} & \frac{139}{235} \\ \frac{43}{235} & 1 & \frac{139}{235} & \frac{43}{235} & \frac{139}{235} & \frac{-53}{235} \\ \frac{-53}{235} & \frac{139}{235} & 1 & \frac{-53}{235} & 1 & \frac{43}{235} \\ 1 & \frac{43}{235} & \frac{-53}{235} & 1 & \frac{-53}{235} & \frac{139}{235} \\ \frac{-53}{235} & \frac{139}{235} & 1 & \frac{-53}{235} & 1 & \frac{43}{235} \\ \frac{139}{235} & \frac{-53}{235} & \frac{43}{235} & \frac{139}{235} & \frac{43}{235} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 6.410841895, 1.289330471, 3.674827635]

Eigenvalues N_C

[0., 0., 0., 1.895833333, 2.187184271, 0.8128157289]

Eigenvalues $M_{C\text{-scaled}}$

[0., 0., 0., 3.140508746, 0.7826632797, 2.076827974]

Eigenvalues $N_{C\text{-scaled}}$

[0., 0., 0., 2.323404255, 2.680464127, 0.9961316167]

NullSpace M_C

{[0, 1, 0, 0, 0, -1], [1, 0, 0, 1, 0, 1], [0, 0, 1, 0, 1, 1]}

NullSpace N_C

{[0, 0, -1, 0, 1, 0], [-1, 0, 0, 1, 0, 0], [-1, 1, -1, 0, 0, 1]}

Eigenvalues M_0

[0., 0., 6., 7.188657443, 1.155932192, 3.655410366]

Eigenvalues N_0

[0., 0., 0., 3., 2.187184271, 0.8128157289]

NullSpace M_0

{[0, 1, 0, 0, 0, -1], [1, 0, -1, 1, -1, 0]}

NullSpace N_0

{[-1, 0, 0, 1, 0, 0], [1, -1, 0, 0, 1, -1], [1, -1, 1, 0, 0, -1]}

Eigenvalues M

[-3., 3., -3.372461083, 3.372461083, -1.000752839, 1.000752839]

Eigenvalues N

[0., 0., 0., 3., -0.8128157289, -2.187184271]

NullSpace M

{}

NullSpace N

{[0, -1, 1, 1, 0, -1], [0, -1, 0, 1, 1, -1], [1, 0, 0, -1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 3 & 0 & 3 & 1 \\ 2 & 0 & 1 & 2 & 1 & 3 \\ 3 & 1 & 0 & 3 & 0 & 2 \\ 0 & 2 & 3 & 0 & 3 & 1 \\ 3 & 1 & 0 & 3 & 0 & 2 \\ 1 & 3 & 2 & 1 & 2 & 0 \end{pmatrix}$$