

T-Run

[3, 4, 1, 3, 2], [5, 5, 5, 5, 4]

$$\tilde{\tau} = [1, 2, 2, 3, 4]$$

$$\delta = [1, 1, 2, 2, 4]$$

POSSIBLE RANKS

$$1 \times 12$$

$$2 \times 6$$

$$3 \times 4$$

BASE DETERMINANT 153/1024, .1494140625

NullSpace of Δ

{1, 2, 3, 4, 5}

Nullspace of A

[[2, 5],[1, 3, 4]]

STRATIFIED CYCLE COVERS

Degree 0

1

Degree 1

0

Degree 2

$$v[5] v[4] + v[1] v[3] + v[2] v[5]$$

Degree 3

$$v[2] v[5] v[4] + v[3] v[5] v[4]$$

Degree 4

$$v[2] v[3] v[5] v[4] + 2 v[1] v[3] v[5] v[4] + v[1] v[2] v[3] v[5]$$

Degree 5

$$2 v[1] v[2] v[3] v[5] v[4]$$

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{2}

R: [3, 5, 1, 3, 2]

B: [5, 4, 5, 5, 4]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

$$\text{Level 2 det} = \frac{1}{1024} (-3 + s) (-1 + s) (51 + 34s - 10s^2 - 4s^3 + s^4)$$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 2

B ranking is 1, "vs", 2

BBAR ranking 1, "vs", 2

"R CYCLES", $(1 + v[1] v[3]) (1 + v[2] v[5])$

"B CYCLES", $1 + v[5] v[4]$

Eigenvalues

R: [0., 1., -1., 1., -1.]

B: [0., 0., 0., 1., -1.]

NullSpace of R

{[0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 1, 0, 0], [0, 1, 0, 0, 0], [1, 0, 0, 0, 0]}

NullSpace of R^*

{[1, 0, 0, -1, 0]}

NullSpace of B^*

{[0, 0, 1, -1, 0], [0, 1, 0, 0, -1], [1, 0, 0, -1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \\ 1 & 0 & 0 & 3 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{4} (v[1] + 2 v[2] + 2 v[3] + 3 v[4] + 4 v[5])$

degree 2: $\frac{1}{2} (v[1]v[5] + 2 v[2]v[3] + 3 v[5]v[4])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{2, 5}, {1, 3, 4}}

"RG1" = {4, 5}

"RG2" = {2, 3}

$$"RG3" = \{1, 5\}$$

$$\pi_2 = [0, 0, 0, 1, 2, 0, 0, 0, 0, 3]$$

$$\text{supp } \pi_2 = \{4, 5, 10\}$$

$$u_2 = [1, 0, 0, 1, 1, 1, 0, 0, 1, 1]$$

$$\text{supp } u_2 = \{1, 4, 5, 6, 9, 10\}$$

Action of R on ranges, [[2], [3], [2]]

Action of B on ranges, [[1], [1], [1]]

$$\beta = \left(\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{6} \right)$$

RPARTS [1]

BPARTS [1]

$$\alpha = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2]

B-BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N - check: true

$$b_1 = \{2, 5\}$$

$$b_2 = \{1, 3, 4\}$$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 6, Shape: 0 ⊕ 6/4

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3}, {2, 5}}, true

Ω_B in Vec(K)? , {{4, 5}}, true

$$V = \begin{pmatrix} \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{4} & \frac{-1}{3} \\ \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{3} \\ \frac{7}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{-1}{4} & \frac{-1}{3} \\ \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{4} & \frac{-1}{3} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{4} & 0 \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$$\pi_R = \begin{pmatrix} \frac{3}{10} & \frac{1}{5} & \frac{3}{10} & 0 & \frac{1}{5} \end{pmatrix} \text{ vs } (0 \ 0 \ 0 \ 0 \ 0) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \text{ vs } \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{2, 5}, {1, 3, 4}}

1, "range", [4, 5], [[5, 4, 5, 5, 4], [4, 5, 4, 4, 5]]

2, "range", [2, 3], [[3, 2, 3, 3, 2], [2, 3, 2, 2, 3]]

3, "range", [1, 5], [[5, 1, 5, 5, 1], [1, 5, 1, 1, 5]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 = [[1, 2]]$

$g_2 = []$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
(h[2] h[1])

"Basis for Z(G)"

1, "coeff", 1

$$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [2, 3]}, {6, [2, 4]}, {7, [2, 5]},
{8, [3, 4]}, {9, [3, 5]}, {10, [4, 5]}

KERNEL HIERARCHY

$$\pi_2 = (0 \ 0 \ 0 \ 1 \ 2 \ 0 \ 0 \ 0 \ 0 \ 3)$$

{4, 5, 10}

$$u_2 = (1 \ 0 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1)$$

{1, 4, 5, 6, 9, 10}

picheck (1 2 2 3 4)

$$\pi = \left(\frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{3} \right)$$

$$\pi 1 = (1 \quad 2 \quad 2 \quad 3 \quad 4)$$

$$u1 = \left(\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \right)$$

picheck (1 2 2 3 4)

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 2 & 0 & 0 & 4 \\ 1 & 0 & 2 & 3 & 0 \\ 1 & 0 & 2 & 3 & 0 \\ 0 & 2 & 0 & 0 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [1, 2, 2, -3, -2]$$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & s & 0 & 0 & -s \\ s & 0 & -s & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\pi\Delta \text{ via ker NC } (-3 \quad -2 \quad 2)$$

M0 is invertible. det= 8125/5184

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (5)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 2, "vs", 2

$$CNM = \begin{pmatrix} 0 & 1 & \frac{13}{12} & \frac{13}{6} & 3 \\ -1 & 0 & 0 & 1 & \frac{13}{6} \\ \frac{-13}{12} & 0 & 0 & \frac{13}{12} & 2 \\ \frac{-13}{6} & -1 & \frac{-13}{12} & 0 & 1 \\ -3 & \frac{-13}{6} & -2 & -1 & 0 \end{pmatrix} \text{ Skew T} = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{-1}{6} & 0 & 0 & \frac{1}{6} & 0 \\ \frac{-1}{3} & 0 & \frac{-1}{6} & 0 & 0 \\ 0 & \frac{-1}{3} & 0 & 0 & 0 \end{pmatrix}$$

$$\text{Skew Omega} = \begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{4} \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & 0 & \frac{1}{12} \\ \frac{-1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{13}{12} & 0 & 0 & 0 & 1 \\ 0 & \frac{13}{6} & 2 & 0 & 0 \\ 0 & 2 & \frac{13}{6} & 0 & 0 \\ 0 & 0 & 0 & \frac{13}{4} & 3 \\ 1 & 0 & 0 & 3 & \frac{13}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 6T + 0\Omega$$

$$\Omega \left(\frac{1}{12} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \right)$$

$$T \left(0 \quad 0 \quad \frac{1}{2} \quad \frac{1}{3} \quad 0 \quad \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(0 \quad 0 \quad 3 \quad 2 \quad 0 \quad 1 \right)$$

"IS MN in Vec(K)?", false

$$MN \left(0 \quad 0 \quad \frac{12}{7} \quad \frac{12}{5} \quad 0 \quad \frac{12}{7} \right)$$

$$\tau = 13/1, \text{ rank} = 2, \text{ ratio} = 13/2, n^2 / r = 25/2$$

$$\tau' = 12/1, r' = 1/2, \tau / n^2 = 13/25$$

$$p^2 = 17/72, \text{ min } \tau = 425/72, \tau\text{-check is positive? } 511/72$$

$$\text{max } r = 72/17, r\text{-check is positive? } 19/36$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{2} T + 12\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 6
out of total no. of elements equal to 6

dim span idems 3 vs no. of idems 3

$$\text{"PT1"} = \{\{2, 5\}, \{1, 3, 4\}\}$$

$$\text{"RG1"} = \{4, 5\}$$

$$\text{"RG2"} = \{2, 3\}$$

$$\text{"RG3"} = \{1, 5\}$$

$$M_c = \begin{pmatrix} \frac{131}{144} & \frac{-25}{72} & \frac{-25}{72} & \frac{-25}{48} & \frac{11}{36} \\ \frac{-25}{72} & \frac{53}{36} & \frac{47}{36} & \frac{-25}{24} & \frac{-25}{18} \\ \frac{-25}{72} & \frac{47}{36} & \frac{53}{36} & \frac{-25}{24} & \frac{-25}{18} \\ \frac{-25}{48} & \frac{-25}{24} & \frac{-25}{24} & \frac{27}{16} & \frac{11}{12} \\ \frac{11}{36} & \frac{-25}{18} & \frac{-25}{18} & \frac{11}{12} & \frac{14}{9} \end{pmatrix} \quad N_c = \begin{pmatrix} \frac{55}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{-17}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} \\ \frac{55}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{55}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{-17}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-50}{131} & \frac{-50}{131} & \frac{-75}{131} & \frac{44}{131} \\ \frac{-25}{106} & 1 & \frac{47}{53} & \frac{-75}{106} & \frac{-50}{53} \\ \frac{-25}{106} & \frac{47}{53} & 1 & \frac{-75}{106} & \frac{-50}{53} \\ \frac{-25}{81} & \frac{-50}{81} & \frac{-50}{81} & 1 & \frac{44}{81} \\ \frac{11}{56} & \frac{-25}{28} & \frac{-25}{28} & \frac{33}{56} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-17}{55} & 1 & 1 & \frac{-17}{55} \\ \frac{-17}{55} & 1 & \frac{-17}{55} & \frac{-17}{55} & 1 \\ 1 & \frac{-17}{55} & 1 & 1 & \frac{-17}{55} \\ 1 & \frac{-17}{55} & 1 & 1 & \frac{-17}{55} \\ \frac{-17}{55} & 1 & \frac{-17}{55} & \frac{-17}{55} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{24} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{8} & \frac{-1}{6} \\ \frac{-1}{24} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{8} & \frac{1}{6} \\ \frac{1}{24} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{8} & \frac{-1}{6} \\ \frac{1}{24} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{8} & \frac{-1}{6} \\ \frac{-1}{24} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{8} & \frac{1}{6} \end{pmatrix}$$

$$M_C N_C = \begin{pmatrix} \frac{1}{24} & \frac{-1}{24} & \frac{1}{24} & \frac{1}{24} & \frac{-1}{24} \\ \frac{-1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{-1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} \\ \frac{1}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{-1}{8} \\ \frac{-1}{6} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\text{commutator} = \begin{pmatrix} 0 & \frac{1}{24} & \frac{-1}{24} & \frac{-1}{12} & \frac{1}{8} \\ \frac{-1}{24} & 0 & 0 & \frac{1}{24} & \frac{-1}{12} \\ \frac{1}{24} & 0 & 0 & \frac{-1}{24} & \frac{1}{12} \\ \frac{1}{12} & \frac{-1}{24} & \frac{1}{24} & 0 & \frac{1}{24} \\ \frac{-1}{8} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{24} & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0.1666666667, 5.120240014, 0.2328582901, 1.577457251]

Eigenvalues N_C

[0., 0., 0., 2.602811177, 1.216633267]

Eigenvalues M_C -scaled

[0., 0.1132075472, 3.358219074, 0.1659432235, 1.362630156]

Eigenvalues N_C -scaled

[0., 0., 0., 3.407316450, 1.592683550]

NullSpace M_C

{[1, 1, 1, 1, 1]}

NullSpace N_C

{[0, 0, 1, -1, 0], [0, 1, 0, 0, -1], [1, 0, 0, -1, 0]}

Eigenvalues M_0

[0.1666666667, 4.166666667, 6.941836498, 0.2153929210, 1.509437247]

Eigenvalues N_0

[0., 0., 0., 3., 2.]

NullSpace M_0

{}

NullSpace N_0

{[0, -1, 0, 0, 1], [-1, 0, 0, 1, 0], [-1, 0, 1, 0, 0]}

Eigenvalues M

[0., -2., 2., 3.162277660, -3.162277660]

Eigenvalues N

[0., 0., 0., 2.449489743, -2.449489743]

NullSpace M

{[-3, 0, 0, 1, 0]}

NullSpace N

{[0, 1, 0, 0, -1], [0, 0, 1, -1, 0], [1, 0, 0, -1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

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{3}

R: [3, 4, 5, 3, 2]

B: [5, 5, 1, 5, 4]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

$$\text{Level 2 det} = \frac{-1}{1024} (153 + 121s + 88s^2 + 30s^3 + 7s^4 + s^5) (-1 + s)$$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 3

B ranking is 2, "vs", 3

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[2] v[3] v[5] v[4]

"B CYCLES", 1 + v[5] v[4]

Eigenvalues

R: [0., -1., 1., 1. I, -1. I]

B: [0., 0., 0., 1., -1.]

NullSpace of R

{[1, 0, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0], [0, 0, 1, 0, 0]}

NullSpace of R^*

{[-1, 0, 0, 1, 0]}

NullSpace of B^*

{[-1, 0, 0, 1, 0], [-1, 1, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \\ 1 & 0 & 0 & 3 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \\ 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{4} (v[1] + 2v[2] + 2v[3] + 3v[4] + 4v[5])$

degree 2: $\frac{1}{2} (v[1]v[5] + 2v[2]v[3] + 3v[5]v[4])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{2, 5}, {1, 3, 4}}

"PT2" = {{3, 5}, {1, 2, 4}}

$$\text{"RG1"} = \{4, 5\}$$

$$\text{"RG2"} = \{2, 3\}$$

$$\text{"RG3"} = \{1, 5\}$$

$$\pi_2 = [0, 0, 0, 1, 2, 0, 0, 0, 0, 3]$$

$$\text{supp } \pi_2 = \{4, 5, 10\}$$

$$u_2 = [1, 2, 0, 3, 3, 1, 2, 2, 1, 3]$$

$$\text{supp } u_2 = \{1, 2, 4, 5, 6, 7, 8, 9, 10\}$$

Action of R on ranges, [[2], [1], [2]]

Action of B on ranges, [[1], [3], [1]]

$$\beta = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$$

RPARTS [2, 1]

BPARTS [2, 2]

$$\alpha = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[4, 3, 1, 2]

B-BLOCKS,

[2, 1, 2, 1]

with invariant measure, [2, 2, 1, 1]

N by blocks, N - check: true

$$b_1 = \{3, 5\}$$

$$b_2 = \{1, 2, 4\}$$

$$b_3 = \{2, 5\}$$

$$b_4 = \{1, 3, 4\}$$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: $3 \oplus 7/5$

$$CLB = \begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3, 4, 5}}, true

Ω_B in Vec(K)? , {{4, 5}}, true

$$V = \begin{pmatrix} \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{4} & \frac{-1}{3} \\ \frac{1}{12} & \frac{-1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{-1}{3} \\ \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{4} & \frac{-1}{3} \\ \frac{-1}{12} & \frac{1}{2} & \frac{-1}{6} & \frac{-1}{4} & 0 \end{pmatrix}$$

Omega-checks true, true

$$\pi_R = \left(0 \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4}\right) \text{ vs } \left(0 \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4}\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(0 \quad 0 \quad 0 \quad \frac{1}{2} \quad \frac{1}{2}\right) \text{ vs } \left(0 \quad 0 \quad 0 \quad \frac{1}{2} \quad \frac{1}{2}\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{2, 5}, {1, 3, 4}}

1, "range", [4, 5], [[5, 4, 5, 5, 4], [4, 5, 4, 4, 5]]

2, "range", [2, 3], [[3, 2, 3, 3, 2], [2, 3, 2, 2, 3]]

3, "range", [1, 5], [[5, 1, 5, 5, 1], [1, 5, 1, 1, 5]]

2, "partition", {{3, 5}, {1, 2, 4}}

1, "range", [4, 5], [[5, 5, 4, 5, 4], [4, 4, 5, 4, 5]]

2, "range", [2, 3], [[3, 3, 2, 3, 2], [2, 2, 3, 2, 3]]

3, "range", [1, 5], [[5, 5, 1, 5, 1], [1, 1, 5, 1, 5]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 = [[1, 2]]$

$$g_2 = []$$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

(h[2] h[1])

"Basis for Z(G)"

1, "coeff", 1

$$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [2, 3]}, {6, [2, 4]}, {7, [2, 5]},
 {8, [3, 4]}, {9, [3, 5]}, {10, [4, 5]}

KERNEL HIERARCHY

$$\pi_2 = (0 \ 0 \ 0 \ 1 \ 2 \ 0 \ 0 \ 0 \ 0 \ 3)$$

{4, 5, 10}

$$u_2 = (1 \ 2 \ 0 \ 3 \ 3 \ 1 \ 2 \ 2 \ 1 \ 3)$$

{1, 2, 4, 5, 6, 7, 8, 9, 10}

picheck (1 2 2 3 4)

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{3} \right)$$

$$\pi_1 = (1 \ 2 \ 2 \ 3 \ 4)$$

$$u_1 = \left(\frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \right)$$

picheck (1 2 2 3 4)

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{2} & 0 \\ \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{2}{9} \\ \frac{1}{18} & 0 & \frac{1}{3} & \frac{1}{6} & \frac{4}{9} \\ \frac{1}{6} & \frac{2}{9} & \frac{1}{9} & \frac{1}{2} & 0 \\ 0 & \frac{1}{9} & \frac{2}{9} & 0 & \frac{2}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 1 & \frac{4}{3} & \frac{2}{3} & 3 & 0 \\ \frac{2}{3} & 2 & 0 & 2 & \frac{4}{3} \\ \frac{1}{3} & 0 & 2 & 1 & \frac{8}{3} \\ 1 & \frac{4}{3} & \frac{2}{3} & 3 & 0 \\ 0 & \frac{2}{3} & \frac{4}{3} & 0 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [-1, 2, 2, -1, -2]$$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ t & -s & -s & s-t & s \end{pmatrix} \quad \text{RB checks}$$

$$\pi\Delta \text{ via } \ker NC \begin{pmatrix} -1 & 2 \end{pmatrix}$$

M0 is invertible. det= 8125/5184

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^\dagger = (5)$$

$$RN_0R^* = \begin{pmatrix} 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & 0 & 1 & \frac{2}{3} & \frac{1}{3} \\ 1 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix}, \quad RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}, \quad BB^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 3, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & \frac{19}{18} & \frac{37}{36} & \frac{13}{6} & 3 \\ \frac{-19}{18} & 0 & 0 & \frac{19}{18} & \frac{37}{18} \\ \frac{-37}{36} & 0 & 0 & \frac{37}{36} & \frac{19}{9} \\ \frac{-13}{6} & \frac{-19}{18} & \frac{-37}{36} & 0 & 1 \\ -3 & \frac{-37}{18} & \frac{-19}{9} & -1 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & \frac{1}{9} & \frac{1}{18} & \frac{1}{3} & 0 \\ \frac{-1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} \\ \frac{-1}{18} & 0 & 0 & \frac{1}{18} & \frac{2}{9} \\ \frac{-1}{3} & \frac{-1}{9} & \frac{-1}{18} & 0 & 0 \\ 0 & \frac{-1}{9} & \frac{-2}{9} & 0 & 0 \end{pmatrix}$$

$$\text{Skew Omega} = \begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{4} \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & 0 & \frac{1}{12} \\ \frac{-1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{13}{12} & 0 & 0 & 0 & 1 \\ 0 & \frac{13}{6} & 2 & 0 & 0 \\ 0 & 2 & \frac{13}{6} & 0 & 0 \\ 0 & 0 & 0 & \frac{13}{4} & 3 \\ 1 & 0 & 0 & 3 & \frac{13}{3} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 1 & \frac{2}{3} & \frac{1}{3} & 1 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 6T + 0\Omega$$

$$\Omega \left(\frac{1}{12} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \right)$$

$$T \left(\frac{1}{18} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{9} \quad 0 \quad \frac{1}{2} \quad \frac{1}{9} \quad \frac{2}{9} \quad \frac{1}{6} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(\frac{1}{3} \quad 2 \quad 2 \quad \frac{2}{3} \quad 0 \quad 3 \quad \frac{2}{3} \quad \frac{4}{3} \quad 1 \right)$$

"IS MN in Vec(K)?", false

$$MN \left(\frac{388}{603} \quad \frac{656}{603} \quad \frac{11}{5} \quad \frac{656}{603} \quad \frac{-16}{67} \quad \frac{124}{67} \quad \frac{34}{45} \quad \frac{74}{45} \quad \frac{124}{67} \right)$$

$$\tau = 13/1, \text{ rank} = 2, \text{ ratio} = 13/2, n^2 / r = 25/2$$

$$\tau' = 12/1, r' = 1/2, \tau / n^2 = 13/25$$

$$p^2 = 17/72, \text{ min } \tau = 425/72, \tau\text{-check is positive? } 511/72$$

$$\text{max } r = 72/17, r\text{-check is positive? } 19/36$$

IS NOMO a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{2} T + 12 \Omega$$

There are, 2, partitions and, 3, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 9
out of total no. of elements equal to 12

dim span idems 6 vs no. of idems 6

$$\text{"PT1"} = \{\{2, 5\}, \{1, 3, 4\}\}$$

$$\text{"PT2"} = \{\{3, 5\}, \{1, 2, 4\}\}$$

$$\text{"RG1"} = \{4, 5\}$$

$$\text{"RG2"} = \{2, 3\}$$

$$\text{"RG3"} = \{1, 5\}$$

$$M_C = \begin{pmatrix} \frac{131}{144} & \frac{-25}{72} & \frac{-25}{72} & \frac{-25}{48} & \frac{11}{36} \\ \frac{-25}{72} & \frac{53}{36} & \frac{47}{36} & \frac{-25}{24} & \frac{-25}{18} \\ \frac{-25}{72} & \frac{47}{36} & \frac{53}{36} & \frac{-25}{24} & \frac{-25}{18} \\ \frac{-25}{48} & \frac{-25}{24} & \frac{-25}{24} & \frac{27}{16} & \frac{11}{12} \\ \frac{11}{36} & \frac{-25}{18} & \frac{-25}{18} & \frac{11}{12} & \frac{14}{9} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{55}{72} & \frac{31}{72} & \frac{7}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{31}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{31}{72} & \frac{7}{72} \\ \frac{7}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{7}{72} & \frac{31}{72} \\ \frac{55}{72} & \frac{31}{72} & \frac{7}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{-17}{72} & \frac{7}{72} & \frac{31}{72} & \frac{-17}{72} & \frac{55}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-50}{131} & \frac{-50}{131} & \frac{-75}{131} & \frac{44}{131} \\ \frac{-25}{106} & 1 & \frac{47}{53} & \frac{-75}{106} & \frac{-50}{53} \\ \frac{-25}{106} & \frac{47}{53} & 1 & \frac{-75}{106} & \frac{-50}{53} \\ \frac{-25}{81} & \frac{-50}{81} & \frac{-50}{81} & 1 & \frac{44}{81} \\ \frac{11}{56} & \frac{-25}{28} & \frac{-25}{28} & \frac{33}{56} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{31}{55} & \frac{7}{55} & 1 & \frac{-17}{55} \\ \frac{31}{55} & 1 & \frac{-17}{55} & \frac{31}{55} & \frac{7}{55} \\ \frac{7}{55} & \frac{-17}{55} & 1 & \frac{7}{55} & \frac{31}{55} \\ 1 & \frac{31}{55} & \frac{7}{55} & 1 & \frac{-17}{55} \\ \frac{-17}{55} & \frac{7}{55} & \frac{31}{55} & \frac{-17}{55} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{24} & \frac{1}{36} & \frac{-1}{36} & \frac{1}{8} & \frac{-1}{6} \\ \frac{1}{72} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{24} & \frac{-1}{18} \\ \frac{-1}{72} & \frac{-1}{12} & \frac{1}{12} & \frac{-1}{24} & \frac{1}{18} \\ \frac{1}{24} & \frac{1}{36} & \frac{-1}{36} & \frac{1}{8} & \frac{-1}{6} \\ \frac{-1}{24} & \frac{-1}{36} & \frac{1}{36} & \frac{-1}{8} & \frac{1}{6} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{24} & \frac{1}{72} & \frac{-1}{72} & \frac{1}{24} & \frac{-1}{24} \\ \frac{1}{36} & \frac{1}{12} & \frac{-1}{12} & \frac{1}{36} & \frac{-1}{36} \\ \frac{-1}{36} & \frac{-1}{12} & \frac{1}{12} & \frac{-1}{36} & \frac{1}{36} \\ \frac{1}{8} & \frac{1}{24} & \frac{-1}{24} & \frac{1}{8} & \frac{-1}{8} \\ \frac{-1}{6} & \frac{-1}{18} & \frac{1}{18} & \frac{-1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\text{commutator} = \begin{pmatrix} 0 & \frac{-1}{72} & \frac{1}{72} & \frac{-1}{12} & \frac{1}{8} \\ \frac{1}{72} & 0 & 0 & \frac{-1}{72} & \frac{1}{36} \\ \frac{-1}{72} & 0 & 0 & \frac{1}{72} & \frac{-1}{36} \\ \frac{1}{12} & \frac{1}{72} & \frac{-1}{72} & 0 & \frac{1}{24} \\ \frac{-1}{8} & \frac{-1}{36} & \frac{1}{36} & \frac{-1}{24} & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0.1666666667, 5.120240014, 0.2328582901, 1.577457251]

Eigenvalues N_C

[0., 0., 1.924534634, 0.7365275340, 1.158382277]

Eigenvalues M_C -scaled

[0., 0.1132075472, 3.358219074, 0.1659432235, 1.362630156]

Eigenvalues N_C -scaled

[0., 0., 2.519390794, 0.9641814985, 1.516427708]

NullSpace M_C

{{[1, 1, 1, 1, 1]}}

NullSpace N_C

{[-1, 1, 1, 0, -1], [-1, 0, 0, 1, 0]}

Eigenvalues M_0

[0.1666666667, 4.166666667, 6.941836498, 0.2153929210,
1.509437247]

Eigenvalues N_0

[0., 0., 2.733824113, 0.7511439170, 1.515031969]

NullSpace M_0

{}

NullSpace N_0

{[1, 0, 0, -1, 0], [0, 1, 1, -1, -1]}

Eigenvalues M

[0., -2., 2., 3.162277660, -3.162277660]

Eigenvalues N

[0., 0., 2.437798895, -1.676631988, -0.7611669080]

NullSpace M

{[-3, 0, 0, 1, 0]}

NullSpace N

{[0, 1, 1, -1, -1], [1, 0, 0, -1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 0 & 3 \\ 1 & 0 & 3 & 1 & 2 \\ 2 & 3 & 0 & 2 & 1 \\ 0 & 1 & 2 & 0 & 3 \\ 3 & 2 & 1 & 3 & 0 \end{pmatrix}$$

=====

{2, 3, 5}

R: [3, 5, 5, 3, 4]

B: [5, 4, 1, 5, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & \frac{5}{6} & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

$$\text{Level 2 det} = \frac{1}{1024} (17 + 6s + s^2) (1 + s) (-1 + s) (-9 - 4s + s^2)$$

RANK of R is 3

R ranking is 1, "vs", 3

RBAR ranking 1, "vs", 3

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 3

"R CYCLES", 1 + v[3] v[5] v[4]

"B CYCLES", 1 + v[2] v[5] v[4]

Eigenvalues

R: [0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

B: [0., 0., 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I]

NullSpace of R

{[1, 0, 0, 0, 0], [0, 1, 0, 0, 0]}

NullSpace of B

{[0, 0, 1, 0, 0]}

NullSpace of R^*

{[0, -1, 1, 0, 0], [-1, 0, 0, 1, 0]}

NullSpace of B^*

{[-1, 0, 0, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 2 & 0 & 0 & 2 \\ 2 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 4 & 4 \\ 0 & 2 & 4 & 0 & 6 \\ 2 & 4 & 4 & 6 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 5

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 3

$$\text{degree 1: } \frac{1}{4} (v[1] + 2v[2] + 2v[3] + 3v[4] + 4v[5])$$

$$\text{degree 2: } \frac{1}{4} (v[1]v[2] + v[1]v[5] + v[2]v[4] + 2v[2]v[5] + 2v[3]v[4] + 2v[3]v[5] + 3v[5]v[4])$$

$$\text{degree 3 : } \frac{1}{4} (v[1]v[2] + v[2]v[4] + 2v[3]v[4]) (v[5])$$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

$$\text{"PT1"} = \{\{1, 4\}, \{5\}, \{2, 3\}\}$$

$$\text{"RG1"} = \{3, 4, 5\}$$

$$\text{"RG2"} = \{2, 4, 5\}$$

$$\text{"RG3"} = \{1, 2, 5\}$$

$$\pi_3 = [0, 0, 1, 0, 0, 0, 0, 0, 1, 2]$$

$$\text{supp } \pi_3 = \{3, 9, 10\}$$

$$u_3 = [0, 0, 1, 0, 1, 0, 0, 0, 1, 1]$$

$$\text{supp } u_3 = \{3, 5, 9, 10\}$$

Action of R on ranges, [[1], [1], [1]]

Action of B on ranges, [[3], [2], [2]]

$$\beta = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$$\alpha = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 3, 1]

B-BLOCKS,

[3, 1, 2]

with invariant measure, [1, 1, 1]

N by blocks, N - check: true

$$b_1 = \{1, 4\}$$

$$b_2 = \{5\}$$

$$b_3 = \{2, 3\}$$

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

LIE STRUCTURE

Dimension of Lie algebra: 8, Shape: $0 \oplus 8/6$

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 4, 5}}, true

Ω_B in Vec(K)? , {{2, 4, 5}}, true

$$V = \begin{pmatrix} \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{4} & \frac{-1}{3} \\ \frac{-1}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{-1}{4} & \frac{1}{3} \\ \frac{-7}{12} & \frac{1}{6} & \frac{-1}{6} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{4} & \frac{-1}{6} & \frac{1}{2} & \frac{-1}{4} & \frac{-1}{3} \\ \frac{1}{12} & \frac{-1}{2} & \frac{1}{6} & \frac{1}{4} & 0 \end{pmatrix}$$

Omega-checks true, true

$$\pi_R = \left(0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}\right) \text{ vs } \left(0 \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}\right) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \left(0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3}\right) \text{ vs } \left(0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3}\right) \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 4}, {5}, {2, 3}}

1, "range", [3, 4, 5], [[5, 4, 4, 5, 3], [4, 3, 3, 4, 5], [3, 5, 5, 3, 4]]

2, "range", [2, 4, 5], [[5, 4, 4, 5, 2], [4, 2, 2, 4, 5], [2, 5, 5, 2, 4]]

3, "range", [1, 2, 5], [[5, 1, 1, 5, 2], [2, 5, 5, 2, 1], [1, 2, 2, 1, 5]]

"group has", 3, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$g_1 = [[1, 2, 3]]$

$g_2 = []$

$g_3 = [[1, 3, 2]]$

linear dimension, 3

"Symmetric?", false

Is Z in Vec(K)? true

$(h[2] \ h[1] \ h[3])$

"Basis for Z(G)"

1, "coeff", 1

$Z[1] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[2] = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

3, "coeff", 1

$$Z[3] = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

1, 3, true

2, 3, true

EIGS =

$$\begin{pmatrix} 1. & & 1. & & & & 1. \\ 1. & -0.5000000000 + 0.8660254040i & & -0.5000000000 - 0.8660254040i & & & \\ 1. & -0.5000000000 + 0.8660254040i & & -0.5000000000 - 0.8660254040i & & & \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 4t^3 + 5t^4 + 7t^5 + 10t^6 + 12t^7 + 15t^8 + 19t^9 + 22t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 3, 4]}, {5, [1, 3, 5]}, {6, [1, 4, 5]}, {7, [2, 3, 4]}, {8, [2, 3, 5]}, {9, [2, 4, 5]}, {10, [3, 4, 5]}

KERNEL HIERARCHY

$$\pi_3 = (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 2)$$

{3, 9, 10}

$$u3 = (0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1)$$

{3, 5, 9, 10}

$$\text{picheck } (1 \ 2 \ 2 \ 3 \ 4)$$

$$\pi = \left(\frac{1}{12} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{4} \ \frac{1}{3} \right)$$

$$\pi2 = (1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 2 \ 2 \ 2 \ 3)$$

$$u2 = \left(\frac{1}{3} \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \ \frac{1}{3} \right)$$

$$\text{picheck } (2 \ 4 \ 4 \ 6 \ 8)$$

$$\pi1 = (2 \ 4 \ 4 \ 6 \ 8)$$

$$u1 = \left(\frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \ \frac{2}{9} \right)$$

$$\text{picheck } (2 \ 4 \ 4 \ 6 \ 8)$$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 4 & \frac{8}{3} \\ \frac{2}{3} & \frac{8}{3} & \frac{8}{3} & 2 & \frac{8}{3} \\ \frac{2}{3} & \frac{8}{3} & \frac{8}{3} & 2 & \frac{8}{3} \\ \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 4 & \frac{8}{3} \\ \frac{2}{3} & \frac{4}{3} & \frac{4}{3} & 2 & \frac{16}{3} \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [-1, -2, 2, 1, 0]$$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} t & 0 & 0 & -t & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\pi\Delta \text{ via ker NC } (2 \ 1)$$

M0 is invertible. det= 4475/15552

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (5)$$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 4, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & \frac{2}{3} & \frac{2}{3} & \frac{3}{2} & 2 \\ \frac{-2}{3} & 0 & 0 & \frac{2}{3} & \frac{4}{3} \\ \frac{-2}{3} & 0 & 0 & \frac{2}{3} & \frac{4}{3} \\ \frac{-3}{2} & \frac{-2}{3} & \frac{-2}{3} & 0 & \frac{2}{3} \\ -2 & \frac{-4}{3} & \frac{-4}{3} & \frac{-2}{3} & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{4} \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{12} & 0 & 0 & \frac{1}{12} & \frac{1}{6} \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{12} & 0 & \frac{1}{12} \\ \frac{-1}{4} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{12} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{3}{4} & \frac{2}{3} & 0 & 0 & \frac{2}{3} \\ \frac{2}{3} & \frac{3}{2} & 0 & \frac{2}{3} & \frac{4}{3} \\ 0 & 0 & \frac{3}{2} & \frac{4}{3} & \frac{4}{3} \\ 0 & \frac{2}{3} & \frac{4}{3} & \frac{9}{4} & 2 \\ \frac{2}{3} & \frac{4}{3} & \frac{4}{3} & 2 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = \frac{8}{3} T + 8\Omega$$

$$\Omega \left(\frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{12} \right)$$

$$T \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

"IS NM in Vec(K)?", true

$$NM \begin{pmatrix} \frac{4}{3} & \frac{2}{3} & \frac{8}{3} & \frac{2}{3} & \frac{8}{3} & 4 & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} \end{pmatrix}$$

"IS MN in Vec(K)?", false

$$MN \begin{pmatrix} \frac{8}{7} & \frac{8}{7} & \frac{8}{3} & \frac{8}{7} & \frac{16}{7} & \frac{8}{3} & \frac{8}{7} & \frac{8}{7} & \frac{8}{3} \end{pmatrix}$$

$$\tau = 9/1, \text{ rank} = 3, \text{ ratio} = 3/1, n^2 / r = 25/3$$

$$\tau' = 16/1, r' = 2/3, \tau / n^2 = 9/25$$

$$p^2 = 17/72, \text{ min } \tau = 425/72, \tau\text{-check is positive? } 223/72$$

$$\text{max } r = 72/17, r\text{-check is positive? } 7/24$$

IS NOMO a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{3} T + 8\Omega$$

There are, 1, partitions and, 3, ranges, with a group size of, 3

KERNEL HAS LINEAR DIMENSION 9
out of total no. of elements equal to 9

dim span idems 3 vs no. of idems 3

$$\text{"PT1"} = \{\{1, 4\}, \{5\}, \{2, 3\}\}$$

$$\text{"RG1"} = \{3, 4, 5\}$$

$$\text{"RG2"} = \{2, 4, 5\}$$

$$\text{"RG3"} = \{1, 2, 5\}$$

$$M_C = \begin{pmatrix} \frac{83}{144} & \frac{23}{72} & \frac{-25}{72} & \frac{-25}{48} & \frac{-1}{36} \\ \frac{23}{72} & \frac{29}{36} & \frac{-25}{36} & \frac{-3}{8} & \frac{-1}{18} \\ \frac{-25}{72} & \frac{-25}{36} & \frac{29}{36} & \frac{7}{24} & \frac{-1}{18} \\ \frac{-25}{48} & \frac{-3}{8} & \frac{7}{24} & \frac{11}{16} & \frac{-1}{12} \\ \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{12} & \frac{2}{9} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{55}{72} & \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{-17}{72} & \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{-17}{72} \\ \frac{-17}{72} & \frac{55}{72} & \frac{55}{72} & \frac{-17}{72} & \frac{-17}{72} \\ \frac{55}{72} & \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} & \frac{-17}{72} \\ \frac{-17}{72} & \frac{-17}{72} & \frac{-17}{72} & \frac{-17}{72} & \frac{55}{72} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{46}{83} & \frac{-50}{83} & \frac{-75}{83} & \frac{-4}{83} \\ \frac{23}{58} & 1 & \frac{-25}{29} & \frac{-27}{58} & \frac{-2}{29} \\ \frac{-25}{58} & \frac{-25}{29} & 1 & \frac{21}{58} & \frac{-2}{29} \\ \frac{-25}{33} & \frac{-6}{11} & \frac{14}{33} & 1 & \frac{-4}{33} \\ \frac{-1}{8} & \frac{-1}{4} & \frac{-1}{4} & \frac{-3}{8} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-17}{55} & \frac{-17}{55} & 1 & \frac{-17}{55} \\ \frac{-17}{55} & 1 & 1 & \frac{-17}{55} & \frac{-17}{55} \\ \frac{-17}{55} & 1 & 1 & \frac{-17}{55} & \frac{-17}{55} \\ 1 & \frac{-17}{55} & \frac{-17}{55} & 1 & \frac{-17}{55} \\ \frac{-17}{55} & \frac{-17}{55} & \frac{-17}{55} & \frac{-17}{55} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{6} & \frac{-1}{9} \\ \frac{-1}{36} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{12} & \frac{-1}{9} \\ \frac{-1}{36} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{12} & \frac{-1}{9} \\ \frac{1}{18} & \frac{-1}{18} & \frac{-1}{18} & \frac{1}{6} & \frac{-1}{9} \\ \frac{-1}{36} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{12} & \frac{2}{9} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{18} & \frac{-1}{36} & \frac{-1}{36} & \frac{1}{18} & \frac{-1}{36} \\ \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{-1}{18} & \frac{1}{9} & \frac{1}{9} & \frac{-1}{18} & \frac{-1}{18} \\ \frac{1}{6} & \frac{-1}{12} & \frac{-1}{12} & \frac{1}{6} & \frac{-1}{12} \\ \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} \end{pmatrix}$$

$$\text{commutator} = \begin{pmatrix} 0 & \frac{1}{36} & \frac{1}{36} & \frac{-1}{9} & \frac{1}{12} \\ \frac{-1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{18} \\ \frac{-1}{36} & 0 & 0 & \frac{1}{36} & \frac{1}{18} \\ \frac{1}{9} & \frac{-1}{36} & \frac{-1}{36} & 0 & \frac{1}{36} \\ \frac{-1}{12} & \frac{-1}{18} & \frac{-1}{18} & \frac{-1}{36} & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0.1612663492, 0.2742589564, 0.6452880361, 2.016408882]

Eigenvalues N_C

[0., 0., 2., 1.403950093, 0.4154943515]

Eigenvalues M_C -scaled

[0., 0.2300365399, 0.8858798966, 1.099270805, 2.784812758]

Eigenvalues N_C -scaled

[0., 0., 2.618181818, 1.837898304, 0.5439198784]

NullSpace M_C

{[1, 1, 1, 1, 1]}

NullSpace N_C

{[-1, 0, 0, 1, 0], [0, -1, 1, 0, 0]}

Eigenvalues M_0

[0.1610063683, 0.2420241236, 0.6276324259, 1.956769951, 6.012567129]

Eigenvalues N_0

[1., 0., 0., 2., 2.]

NullSpace M_0

{}

NullSpace N_0

{[-1, 0, 0, 1, 0], [0, 1, -1, 0, 0]}

Eigenvalues M

[0.6043001496, 3.676077028, -0.5735234614, -1.496144794, -2.210708923]

Eigenvalues N

[0., 0., -2., 3.236067977, -1.236067977]

NullSpace M

{}

NullSpace N

{[0, -1, 1, 0, 0], [-1, 0, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$