T-Run

[3, 3, 1, 1, 7, 7, 5, 5], [6, 8, 8, 6, 2, 4, 4,

$\tilde{\pi}$ = [1, 1, 1, 1, 1, 1, 1, 1]
$\boldsymbol{\delta}$ = [2, 2, 2, 2, 2, 2, 2, 2]

POSSIBLE RANKS

1 x 8
2 x 4

BASE DETERMINANT 4236243/134217728, .3156246990e-1

NullSpace of Δ

{2, 4, 5, 7}, {1, 3, 6, 8}

Nullspace of A

[{2, 4},{5, 7}] `,` [{1, 3},{6, 8}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[2] v[8] + v[1] v[3] + v[5] v[7] + v[4] v[6]

Degree 3
v[4] v[6] v[7] + v[2] v[5] v[8] + v[1] v[4] v[6] + v[2] v[3] v[8]

Degree 4
v[2] v[3] v[5] v[8] + v[1] v[4] v[6] v[7] + v[2] v[5] v[8] v[7] + v[1] v[2] v[3] v[8] + v[1] v[3] v[5] v[7] + v[2] v[4] v[6] v[8] + v[1] v[3] v[4] v[6] + v[4] v[5] v[6] v[7]

Degree 5
v[1] v[3] v[4] v[6] v[7] + v[1] v[2] v[4] v[6] v[8] + v[2] v[4] v[5] v[6] v[8] + v[2] v[3] v[4] v[6] v[8] + v[1] v[4] v[5] v[6] v[7] + v[2] v[4] v[6] v[8] v[7] + v[2] v[3] v[5] v[8] v[7] + v[1] v[2] v[3] v[5] v[8]

Degree 6
2 v[1] v[2] v[3] v[4] v[6] v[8] + v[1] v[2] v[4] v[6] v[8] v[7] + v[2] v[3] v[4] v[5] v[6] v[8] + v[1] v[2] v[3] v[5] v[8] v[7] + v[2] v[3] v[4] v[6] v[8] v[7] + v[1] v[3] v[4] v[5] v[8] v[7] + v[1] v[2] v[4] v[5] v[6] v[8] + v[1] v[3] v[4] v[5] v[6] v[7] + v[1] v[2] v[3] v[5] v[6] v[7] + 2 v[2] v[4] v[5] v[6] v[8] v[7]

Degree 7
2 v[2] v[3] v[4] v[5] v[6] v[8] v[7] + 2 v[1] v[2] v[3] v[4] v[6] v[8] v[7] + 2 v[1] v[2] v[3] v[4] v[5] v[6] v[8] + 2 v[1] v[2] v[4] v[5] v[6] v[8] v[7]

Degree 8
4 v[1] v[2] v[3] v[4] v[5] v[6] v[8] v[7]

==

{}

R: [3, 3, 1, 1, 7, 7, 5, 5]
B: [6, 8, 8, 6, 2, 4, 4, 2]

TRACE TWO = 3

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{1}{134217728}(33-2s^2+s^4)(-31-2s^2+s^4)(101+48s+18s^2-8s^3+s^4)(-1+s)^3(1+s)(41+12s-18s^2-4s^3+s^4)$

RANK of R is 4

R ranking is 1, "vs", 4

RBAR ranking 1, "vs", 4

RANK of B is 4

B ranking is 1, "vs", 4

BBAR ranking 1, "vs", 4

"R CYCLES", (1 + v[1] v[3]) (1 + v[5] v[7])
"B CYCLES", (1 + v[2] v[8]) (1 + v[4] v[6])

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, 0, 0, 1, −1], [0, 0, 0, 0, 1, −1, 0, 0], [0, 0, 1, −1, 0, 0, 0, 0], [1, −1, 0, 0, 0, 0, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, 0, 1, −1, 0], [0, 1, −1, 0, 0, 0, 0, 0], [1, 0, 0, −1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, −1]}

FIXED POINTS DIMENSION 3

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 & 2 \\ 2 & 0 & 0 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 & 2 \\ 2 & 0 & 2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{2} & 1 & \frac{1}{2} & 1 & 1 & 1 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 1 & 1 & 1 & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ 1 & 1 & 1 & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 1 \\ 1 & 1 & 1 & 1 & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 1 & 1 & 1 & \frac{1}{2} & 1 & \frac{1}{2} & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 8

"RANK of the KERNEL is ", 4

"IdemSolvability Check", 1 "Trace mark", 0, "Rank mark", 4, "for kernel rank", 4

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[3] + v[1]v[5] + v[1]v[7] + v[2]v[4] + v[2]v[6] + v[2]v[8] + v[3]v[5] + v[3]v[7] + v[4]v[6] + v[4]v[8] + v[5]v[7] + v[6]v[8]$)
degree 3 : $\frac{1}{8}$ ($v[1]v[3]v[5] + v[1]v[3]v[7] + v[1]v[5]v[7] + v[2]v[4]v[6] + v[2]v[4]v[8] + v[2]v[6]v[8] + v[3]v[5]v[7] + v[4]v[6]v[8]$)
degree 4 : $\frac{1}{2}$ ($v[1]v[3]v[5]v[7] + v[2]v[4]v[6]v[8]$)

Group spectrum $1 + t + 3t^2 + t^3 + t^4$

KERNEL STRUCTURE

"PT1" = {{1, 4}, {5, 8}, {6, 7}, {2, 3}}

"PT2" = {{1, 2}, {5, 6}, {7, 8}, {3, 4}}

"RG1" = {2, 4, 6, 8}

"RG2" = {1, 3, 5, 7}

$\pi 4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 4$ = {21, 50}

$u4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 2, 1, 1, 2, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 2, 1, 1, 2, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp u4 = {10, 11, 14, 15, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 56, 57, 60, 61}

Action of R on ranges, [[2], [2]]
Action of B on ranges, [[1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [2, 2]

BPARTS [1, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[8, 8, 7, 6, 2, 7, 6, 2]

B−BLOCKS,

[4, 3, 5, 1, 3, 1, 5, 4]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

b_1 = {1, 4}
b_2 = {1, 2}
b_3 = {5, 8}
b_4 = {6, 7}
b_5 = {2, 3}
b_6 = {5, 6}

$b_7 = \{7, 8\}$
$b_8 = \{3, 4\}$

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & h[1] & 0 & h[3] & 0 & h[4] & 0 \\ 0 & h[2] & 0 & h[1] & 0 & h[3] & 0 & h[4] \\ h[1] & 0 & h[2] & 0 & h[4] & 0 & h[3] & 0 \\ 0 & h[1] & 0 & h[2] & 0 & h[4] & 0 & h[3] \\ h[3] & 0 & h[4] & 0 & h[2] & 0 & h[1] & 0 \\ 0 & h[3] & 0 & h[4] & 0 & h[2] & 0 & h[1] \\ h[4] & 0 & h[3] & 0 & h[1] & 0 & h[2] & 0 \\ 0 & h[4] & 0 & h[3] & 0 & h[1] & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 10, Shape: $6 \oplus 4/2$

$$\text{CLB} = \begin{pmatrix} -1 & 1 & 1 & -1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 7}, {1, 3}}, true

Ω_B in Vec(K)? , {{4, 6}, {2, 8}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 4}, {5, 8}, {6, 7}, {2, 3}}

1, "range", [2, 4, 6, 8], [[8, 6, 6, 8, 4, 2, 2, 4], [6, 8, 8, 6, 2, 4, 4, 2], [4, 2, 2, 4, 8, 6, 6, 8], [2, 4, 4, 2, 6, 8, 8, 6]]

2, "range", [1, 3, 5, 7], [[7, 5, 5, 7, 3, 1, 1, 3], [5, 7, 7, 5, 1, 3, 3, 1], [3, 1, 1, 3, 7, 5, 5, 7], [1, 3, 3, 1, 5, 7, 7, 5]]

2, "partition", {{1, 2}, {5, 6}, {7, 8}, {3, 4}}

1, "range", [2, 4, 6, 8], [[8, 8, 6, 6, 4, 4, 2, 2], [6, 6, 8, 8, 2, 2, 4, 4], [4, 4, 2, 2, 8, 8, 6, 6], [2, 2, 4, 4, 6, 6, 8, 8]]

2, "range", [1, 3, 5, 7], [[7, 7, 5, 5, 3, 3, 1, 1], [5, 5, 7, 7, 1, 1, 3, 3], [3, 3, 1, 1, 7, 7, 5, 5], [1, 1, 3, 3, 5, 5, 7, 7]]

"group has", 4, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 3], [2, 4]]

$g_2 =$ [[1, 4], [2, 3]]

$g_3 =$ []

$g_4 =$ [[1, 2], [3, 4]]

linear dimension, 4

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[3] & h[4] & h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

4, "coeff", 1

$$Z[\ 4\ ] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
2, 3, true
2, 4, true
3, 4, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. \\ 1. & -1. & 1. & -1. \\ 1. & -1. & 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 4 & 0 \\ 2 & 2 & 6 & 2 \\ 0 & 0 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 3t^2 + t^3 + t^4$

Molien Series to order 10: $1 + t + 4t^2 + 5t^3 + 11t^4 + 14t^5 + 24t^6 + 30t^7 + 45t^8 + 55t^9 + 76t^{10}$

n-choose-rank

{1, [1, 2, 3, 4]}, {2, [1, 2, 3, 5]}, {3, [1, 2, 3, 6]}, {4, [1, 2, 3, 7]}, {5, [1, 2, 3, 8]}, {6, [1, 2, 4, 5]}, {7, [1, 2, 4, 6]}, {8, [1, 2, 4, 7]}, {9, [1, 2, 4, 8]}, {[1, 2, 5, 6], 10}, {11, [1, 2, 5, 7]}, {12, [1, 2, 5, 8]}, {13, [1, 2, 6, 7]}, {14, [1, 2, 6, 8]}, {15, [1, 2, 7, 8]}, {16, [1, 3, 4, 5]}, {17, [1, 3, 4, 6]}, {18, [1, 3, 4, 7]}, {19, [1, 3, 4, 8]}, {20, [1, 3, 5, 6]}, {21, [1, 3, 5, 7]}, {22, [1, 3, 5, 8]}, {23, [1, 3, 6, 7]}, {24, [1, 3, 6, 8]}, {25, [1, 3, 7, 8]}, {26, [1, 4, 5, 6]}, {27, [1, 4, 5, 7]}, {28, [1, 4, 5, 8]}, {29, [1, 4, 6, 7]}, {30, [1, 4, 6, 8]}, {31, [1, 4, 7, 8]}, {32, [1, 5, 6, 7]}, {33, [1, 5, 6, 8]}, {34, [1, 5, 7, 8]}, {35, [1, 6, 7, 8]}, {36, [2, 3, 4, 5]}, {37, [2, 3, 4, 6]}, {38, [2, 3, 4, 7]}, {39, [2, 3, 4, 8]}, {40, [2, 3, 5, 6]}, {41, [2, 3, 5, 7]}, {42, [2, 3, 5, 8]}, {43, [2, 3, 6, 7]}, {44, [2, 3, 6, 8]}, {45, [2, 3, 7, 8]}, {46, [2, 4, 5, 6]}, {47, [2, 4, 5, 7]}, {48, [2, 4, 5, 8]}, {49, [2, 4, 6, 7]}, {50, [2, 4, 6, 8]}, {51, [2, 4, 7, 8]}, {52, [2, 5, 6, 7]}, {53, [2, 5, 6, 8]}, {54, [2, 5, 7, 8]}, {55, [2, 6, 7, 8]}, {56, [3, 4, 5, 6]}, {57, [3, 4, 5, 7]}, {58, [3, 4, 5, 8]}, {59, [3, 4, 6, 7]}, {60, [3, 4, 6, 8]}, {61, [3, 4, 7, 8]}, {62, [3, 5, 6, 7]}, {63, [3, 5, 6, 8]}, {64, [3, 5, 7, 8]}, {65, [3, 6, 7, 8]}, {66, [4, 5, 6, 7]}, {67, [4, 5, 6, 8]}, {68, [4, 5, 7, 8]}, {69, [4, 6, 7, 8]}, {70, [5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 4 =$

$(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0$

{21, 50}

$u4 =$
$(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 2\ 1\ 1\ 2\ 1\ 0\ 1\ 1$

{10, 11, 14, 15, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 56, 57, 60, 61}

picheck $(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1)$

$\pi = \left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$\pi 3 =$
$(0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0$

$u3 =$
$\left(0\ 0\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ 0\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{2}\ \frac{1}{4}\ 0\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{2}\right.$

picheck $(3\ 3\ 3\ 3\ 3\ 3\ 3\ 3)$

$\pi 2 =$
$(0\ 2\ 0\ 2\ 0\ 2\ 0\ 0\ 2\ 0\ 2\ 0\ 2\ 0\ 2\ 0\ 2\ 0\ 0\ 2\ 0\ 2\ 0\ 2\ 0\ 0\ 2\ 0$

$u2 =$
$\left(\frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\right.$

picheck $(6\ 6\ 6\ 6\ 6\ 6\ 6\ 6)$

$\pi 1 = (6\ 6\ 6\ 6\ 6\ 6\ 6\ 6)$

$u1 = \left(\frac{3}{16}\ \frac{3}{16}\ \frac{3}{16}\ \frac{3}{16}\ \frac{3}{16}\ \frac{3}{16}\ \frac{3}{16}\ \frac{3}{16}\right)$

picheck $(6\ 6\ 6\ 6\ 6\ 6\ 6\ 6)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 6 & 5 & 4 & 5 & 4 & 4 & 4 & 4 \\ 5 & 6 & 5 & 4 & 4 & 4 & 4 & 4 \\ 4 & 5 & 6 & 5 & 4 & 4 & 4 & 4 \\ 5 & 4 & 5 & 6 & 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 & 6 & 5 & 4 & 5 \\ 4 & 4 & 4 & 4 & 5 & 6 & 5 & 4 \\ 4 & 4 & 4 & 4 & 4 & 5 & 6 & 5 \\ 4 & 4 & 4 & 4 & 5 & 4 & 5 & 6 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, 1, -1, 1, -1, 1, -1]$

$$\ker N_c = \begin{pmatrix} 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ RB

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & 0 & t & 0 & 0 \\ 0 & s & -t & -t & 0 & -t \\ -s & -s & -t & -t & -s & -t \\ -s & -s & 0 & t & -s & 0 \\ s & 0 & t & 0 & 0 & 0 \\ s & 0 & 0 & 0 & 0 & t \\ 0 & 0 & 0 & 0 & s & t \\ 0 & 0 & t & 0 & s & 0 \end{pmatrix}$$ RB checks

$$\ker M_c = \begin{pmatrix} -1 & 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & 0 & 0 & t & 0 \\ 0 & t & s & 0 & 0 & 0 & 0 \\ -s & s+t & -s & s & -s & s & s \\ -s & s & -s & s & -s & s+t & s \\ s & 0 & 0 & 0 & 0 & 0 & t \\ s & 0 & 0 & t & 0 & 0 & 0 \\ 0 & 0 & 0 & t & s & 0 & 0 \\ 0 & 0 & 0 & 0 & s & 0 & t \end{pmatrix} \quad \mathrm{RB}$$

checks

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 2 & 0 & 2 & 0 & 2 & 2 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 4

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew}$$

$$\mathrm{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2&0&2&0&2&0&2&0\\ 0&2&0&2&0&2&0&2\\ 2&0&2&0&2&0&2&0\\ 0&2&0&2&0&2&0&2\\ 2&0&2&0&2&0&2&0\\ 0&2&0&2&0&2&0&2\\ 2&0&2&0&2&0&2&0\\ 0&2&0&2&0&2&0&2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{1}{2}&0&\frac{1}{2}&0&0&0&0\\ \frac{1}{2}&1&\frac{1}{2}&0&0&0&0&0\\ 0&\frac{1}{2}&1&\frac{1}{2}&0&0&0&0\\ \frac{1}{2}&0&\frac{1}{2}&1&0&0&0&0\\ 0&0&0&0&1&\frac{1}{2}&0&\frac{1}{2}\\ 0&0&0&0&\frac{1}{2}&1&\frac{1}{2}&0\\ 0&0&0&0&0&\frac{1}{2}&1&\frac{1}{2}\\ 0&0&0&0&\frac{1}{2}&0&\frac{1}{2}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 4T + 32\Omega$$

$\Omega \left(\frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8}\right)$

$T \left(0 \;\; 0 \;\; \frac{1}{2} \;\; \frac{1}{4} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{4} \;\; 0 \;\; \frac{1}{4} \;\; \frac{1}{2}\right)$

"IS NM in Vec(K)?", true

NM $(4 \;\; 4 \;\; 6 \;\; 5 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 5 \;\; 4 \;\; 5 \;\; 6)$

"IS MN in Vec(K)?", true

MN $(4 \;\; 4 \;\; 6 \;\; 5 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 5 \;\; 4 \;\; 5 \;\; 6)$

$\boldsymbol{\tau}$ = 16/1 , rank = 4 , ratio = 4/1 , n^2 / r = 16/1

$\boldsymbol{\tau}'$ = 48/1 , r' = 3/4 , $\boldsymbol{\tau} / n^2$ = 1/4

p^2 = 1/8 , min $\boldsymbol{\tau}$ = 8/1 , $\boldsymbol{\tau}$–check is positive? 8/1

max r = 8/1 , r–check is positive? 1/2

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 16\Omega$$

There are, 2, partitions and, 2, ranges, with a group size of, 4

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 4}, {5, 8}, {6, 7}, {2, 3}}

"PT2" = {{1, 2}, {5, 6}, {7, 8}, {3, 4}}

"RG1" = {2, 4, 6, 8}

"RG2" = {1, 3, 5, 7}

$$M_C = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_C

[2., 0., 0., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_C–scaled

[2.285714286, 0., 0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 0, 1, 0, 0, 1, 0], [0, 0, 0, −1, 0, 1, 0, 0], [0, 0, 0, 1, 1, 0, 0, 0], [0, 0, 1, 1, 0, 0, 0, 0], [0, 1, 0, −1, 0, 0, 0, 0], [1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, −1, 0, 0, 0, 1]}

NullSpace N_C

{[0, 0, 0, 0, 1, −1, 1, −1], [1, −1, 1, −1, 0, 0, 0, 0]}

Eigenvalues M_0

[8., 8., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 2., 2., 1., 1., 1., 1.]

NullSpace M_0

{[0, 1, 0, 0, 0, 0, 0, −1], [0, 0, 0, 1, 0, 0, 0, −1], [1, 0, 0, 0, 0, 0, −1, 0], [0, 0, 1, 0, 0, 0, −1, 0], [0, 0, 0, 0, 1, 0, −1, 0], [0, 0, 0, 0, 0, 1, 0, −1]}

NullSpace N_0

{[−1, 1, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, −1, 1]}

Eigenvalues M

[6., 6., −2., −2., −2., −2., −2., −2.]

Eigenvalues N

[6., −2., 0., 0., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[−1, 1, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, −1, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2 : \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$1, 3 : \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$2, 3 : \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

==

{2, 4}

R: [3, 8, 1, 6, 7, 7, 5, 5]
B: [6, 3, 8, 1, 2, 4, 4, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 7

Level 2 det = $\frac{1}{67108864}(-1+s)(4141+2121s+811s^2+65s^3-67s^4+43s^5+41s^6+11s^7+2s^8)(-1023-33s-82s^2+98s^3+32s^4-14s^6-2s^7-s^8+s^9)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[1] v[3]) (1 + v[5] v[7])
"B CYCLES", (1 + v[1] v[4] v[6]) (1 + v[2] v[3] v[8])

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 0., 0.]

NullSpace of R

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of R^*

{[0, 0, 0, 0, 0, 0, 1, −1], [0, 0, 0, 0, −1, 1, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 0, 0, 1], [0, 0, 0, 0, 0, −1, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 1 "Trace mark", 2, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8] \right)$

degree 2: $\frac{1}{4}$ ($v[1]v[3] + v[2]v[4] + v[5]v[7] + v[6]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 4, 6, 7}, {1, 2, 5, 8}}

"PT2" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT3" = {{2, 3, 6, 7}, {1, 4, 5, 8}}

"PT4" = {{2, 3, 5, 6}, {1, 4, 7, 8}}

"PT5" = {{2, 3, 5, 8}, {1, 4, 6, 7}}

"PT6" = {{2, 3, 7, 8}, {1, 4, 5, 6}}

"PT7" = {{3, 4, 5, 8}, {1, 2, 6, 7}}

"PT8" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"RG1" = {6, 8}

"RG2" = {5, 7}

"RG3" = {2, 4}

"RG4" = {1, 3}

$\pi 2$ = [0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]

supp $\pi 2$ = {2, 9, 24, 27}

$u2$ = [1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[2], [2], [1], [4]]
Action of B on ranges, [[3], [3], [4], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [6, 4, 6, 4, 2, 8, 2, 8]

BPARTS [7, 7, 1, 1, 5, 5, 3, 3]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[10, D, B, D, 10, C, B, C, 8, 3, 7, 6, 6, 8, 3, 7]

B−BLOCKS,

[1, E, F, F, 2, 2, E, 1, 4, A, 4, 9, A, 5, 9, 5]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{2, 3, 5, 8\}$
$b_2 = \{2, 3, 6, 7\}$
$b_3 = \{3, 4, 5, 6\}$
$b_4 = \{3, 4, 6, 7\}$
$b_5 = \{3, 4, 5, 8\}$
$b_6 = \{3, 4, 7, 8\}$
$b_7 = \{2, 3, 5, 6\}$
$b_8 = \{2, 3, 7, 8\}$
$b_9 = \{1, 4, 5, 8\}$
$b_{10} = \{1, 4, 6, 7\}$
$b_{11} = \{1, 4, 7, 8\}$
$b_{12} = \{1, 2, 5, 6\}$
$b_{13} = \{1, 4, 5, 6\}$
$b_{14} = \{1, 2, 5, 8\}$
$b_{15} = \{1, 2, 6, 7\}$
$b_{16} = \{1, 2, 7, 8\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & h[2] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[1] & 0 & h[2] & 0 & 0 & 0 & 0 \\ h[2] & 0 & h[1] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[2] & 0 & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & 0 & h[2] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[1] & 0 & h[2] \\ 0 & 0 & 0 & 0 & h[2] & 0 & h[1] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[2] & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 25, Shape: $18 \oplus 7/5$

$$\text{CLB} = \begin{pmatrix} -1 & 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 7}, {1, 3}}, false

Ω_B in Vec(K)? , {{2, 3, 8}, {1, 4, 6}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} \end{pmatrix}$$

Omega−checks true, true

"I−V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{8} & 0 & \frac{1}{8} & 0 & \frac{3}{8} & 0 & \frac{3}{8} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & 0 & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 4, 6, 7}, {1, 2, 5, 8}}

1, "range", [6, 8], [[8, 8, 6, 6, 8, 6, 6, 8], [6, 6, 8, 8, 6, 8, 8, 6]]

2, "range", [5, 7], [[7, 7, 5, 5, 7, 5, 5, 7], [5, 5, 7, 7, 5, 7, 7, 5]]

3, "range", [2, 4], [[4, 4, 2, 2, 4, 2, 2, 4], [2, 2, 4, 4, 2, 4, 4, 2]]

4, "range", [1, 3], [[3, 3, 1, 1, 3, 1, 1, 3], [1, 1, 3, 3, 1, 3, 3, 1]]

2, "partition", {{3, 4, 5, 6}, {1, 2, 7, 8}}

1, "range", [6, 8], [[8, 8, 6, 6, 6, 6, 8, 8], [6, 6, 8, 8, 8, 8, 6, 6]]

2, "range", [5, 7], [[7, 7, 5, 5, 5, 5, 7, 7], [5, 5, 7, 7, 7, 7, 5, 5]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 2, 4, 4], [2, 2, 4, 4, 4, 4, 2, 2]]

4, "range", [1, 3], [[3, 3, 1, 1, 1, 1, 3, 3], [1, 1, 3, 3, 3, 3, 1, 1]]

3, "partition", {{2, 3, 6, 7}, {1, 4, 5, 8}}

1, "range", [6, 8], [[8, 6, 6, 8, 8, 6, 6, 8], [6, 8, 8, 6, 6, 8, 8, 6]]

2, "range", [5, 7], [[7, 5, 5, 7, 7, 5, 5, 7], [5, 7, 7, 5, 5, 7, 7, 5]]

3, "range", [2, 4], [[4, 2, 2, 4, 4, 2, 2, 4], [2, 4, 4, 2, 2, 4, 4, 2]]

4, "range", [1, 3], [[3, 1, 1, 3, 3, 1, 1, 3], [1, 3, 3, 1, 1, 3, 3, 1]]

4, "partition", {{2, 3, 5, 6}, {1, 4, 7, 8}}

1, "range", [6, 8], [[8, 6, 6, 8, 6, 6, 8, 8], [6, 8, 8, 6, 8, 8, 6, 6]]

2, "range", [5, 7], [[7, 5, 5, 7, 5, 5, 7, 7], [5, 7, 7, 5, 7, 7, 5, 5]]

3, "range", [2, 4], [[4, 2, 2, 4, 2, 2, 4, 4], [2, 4, 4, 2, 4, 4, 2, 2]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 1, 3, 3], [1, 3, 3, 1, 3, 3, 1, 1]]

5, "partition", {{2, 3, 5, 8}, {1, 4, 6, 7}}

1, "range", [6, 8], [[8, 6, 6, 8, 6, 8, 8, 6], [6, 8, 8, 6, 8, 6, 6, 8]]

2, "range", [5, 7], [[7, 5, 5, 7, 5, 7, 7, 5], [5, 7, 7, 5, 7, 5, 5, 7]]

3, "range", [2, 4], [[4, 2, 2, 4, 2, 4, 4, 2], [2, 4, 4, 2, 4, 2, 2, 4]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 3, 3, 1], [1, 3, 3, 1, 3, 1, 1, 3]]

6, "partition", {{2, 3, 7, 8}, {1, 4, 5, 6}}

1, "range", [6, 8], [[8, 6, 6, 8, 8, 8, 6, 6], [6, 8, 8, 6, 6, 6, 8, 8]]

2, "range", [5, 7], [[7, 5, 5, 7, 7, 7, 5, 5], [5, 7, 7, 5, 5, 5, 7, 7]]

3, "range", [2, 4], [[4, 2, 2, 4, 4, 4, 2, 2], [2, 4, 4, 2, 2, 2, 4, 4]]

4, "range", [1, 3], [[3, 1, 1, 3, 3, 3, 1, 1], [1, 3, 3, 1, 1, 1, 3, 3]]

7, "partition", {{3, 4, 5, 8}, {1, 2, 6, 7}}

1, "range", [6, 8], [[8, 8, 6, 6, 6, 8, 8, 6], [6, 6, 8, 8, 8, 6, 6, 8]]

2, "range", [5, 7], [[7, 7, 5, 5, 5, 7, 7, 5], [5, 5, 7, 7, 7, 5, 5, 7]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 4, 4, 2], [2, 2, 4, 4, 4, 2, 2, 4]]

4, "range", [1, 3], [[3, 3, 1, 1, 1, 3, 3, 1], [1, 1, 3, 3, 3, 1, 1, 3]]

8, "partition", {{3, 4, 7, 8}, {1, 2, 5, 6}}

1, "range", [6, 8], [[8, 8, 6, 6, 8, 8, 6, 6], [6, 6, 8, 8, 6, 6, 8, 8]]

2, "range", [5, 7], [[7, 7, 5, 5, 7, 7, 5, 5], [5, 5, 7, 7, 5, 5, 7, 7]]

3, "range", [2, 4], [[4, 4, 2, 2, 4, 4, 2, 2], [2, 2, 4, 4, 2, 2, 4, 4]]

4, "range", [1, 3], [[3, 3, 1, 1, 3, 3, 1, 1], [1, 1, 3, 3, 1, 1, 3, 3]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$

{2, 9, 24, 27}

$u2 =$
$\begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \end{pmatrix}$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_7 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_8 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 2 & 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 0 & 2 & 2 & 2 & 2 \\ 0 & 2 & 4 & 2 & 2 & 2 & 2 & 2 \\ 2 & 0 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 2 & 4 & 2 & 0 \\ 2 & 2 & 2 & 2 & 0 & 2 & 4 & 2 \\ 2 & 2 & 2 & 2 & 2 & 0 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, -1, 0, -1, 1, 0, 1, 0]$

$$\ker N_c = \begin{pmatrix} -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -s+t & 0 & -s+t & 0 & 0 & -t+s & 0 & -t+s \\ -s & t & -s & t & s & -t & s & -t \\ -s & t & -s & t & s & -t & s & -t \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & 0 & -t \\ 0 & t & 0 & s \\ 0 & -s & 0 & t \\ 0 & -t & 0 & -s \\ s & 0 & -t & 0 \\ s & 0 & t & 0 \\ -s & 0 & t & 0 \\ -s & 0 & -t & 0 \end{pmatrix}$$ RB checks

$$\ker M_C = \begin{pmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} s & t & 0 & 0 & 0 \\ t & 0 & 0 & s & 0 \\ -s & s & 0 & s+t & 0 \\ -t & s+t & 0 & t & 0 \\ 0 & t & s & t & -t \\ 0 & 0 & s & 0 & t \\ 0 & s & -s & s & t \\ 0 & s+t & -s & s+t & -t \end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 0 & 4 & 0 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\,T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew}$$

$$\mathrm{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&4&0&0&0&0&0\\ 0&4&0&4&0&0&0&0\\ 4&0&4&0&0&0&0&0\\ 0&4&0&4&0&0&0&0\\ 0&0&0&0&4&0&4&0\\ 0&0&0&0&0&4&0&4\\ 0&0&0&0&4&0&4&0\\ 0&0&0&0&0&4&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{1}{2}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&1&\frac{1}{2}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 0&\frac{1}{2}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&0&\frac{1}{2}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{2}&0&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{2}&0\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{2}&1&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{2}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ 0\ \frac{1}{8}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

NM $(4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 2\ 2\ 0\ 2\ 4)$

"IS MN in Vec(K)?", true

MN $(4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 2\ 2\ 0\ 2\ 4)$

$\boldsymbol{\tau}$ = 32/1 , rank = 2 , ratio = 16/1 , n^2 / r = 32/1

$\boldsymbol{\tau}'$ = 32/1 , r' = 1/2 , $\boldsymbol{\tau} / n^2$ = 1/2

p^2 = 1/8 , min $\boldsymbol{\tau}$ = 8/1 , $\boldsymbol{\tau}$−check is positive? 24/1

max r = 8/1 , r−check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 8, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 64

dim span idems 16 vs no. of idems 32

"PT1" = {{3, 4, 6, 7}, {1, 2, 5, 8}}

"PT2" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT3" = {{2, 3, 6, 7}, {1, 4, 5, 8}}

"PT4" = {{2, 3, 5, 6}, {1, 4, 7, 8}}

"PT5" = {{2, 3, 5, 8}, {1, 4, 6, 7}}

"PT6" = {{2, 3, 7, 8}, {1, 4, 5, 6}}

"PT7" = {{3, 4, 5, 8}, {1, 2, 6, 7}}

"PT8" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"RG1" = {6, 8}

"RG2" = {5, 7}

"RG3" = {2, 4}

"RG4" = {1, 3}

$$M_C = \begin{pmatrix} 3 & -1 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 3 & -1 & -1 & -1 & -1 \\ 3 & -1 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & -1 & 3 \\ -1 & -1 & -1 & -1 & 3 & -1 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & -1 & 3 \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[3., 0., 0., 0., 1., 1., 1., 1.]

Eigenvalues M_C-scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C-scaled

[3.428571429, 0., 0., 0., 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 1, 1, 0, 0, 1, 1], [0, 1, 0, −1, 0, 0, 0, 0], [1, 0, 0, 1, 0, 0, 1, 1], [0, 0, 0, 0, 0, 1, 0, −1], [0, 0, 0, 0, 1, 0, −1, 0]}

NullSpace N_c

{[0, −1, 0, −1, 0, 1, 0, 1], [0, −1, 0, −1, 1, 0, 1, 0], [1, −1, 1, −1, 0, 0, 0, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[4., 0., 0., 0., 1., 1., 1., 1.]

NullSpace M_0

{[0, 0, 0, 0, 1, 0, −1, 0], [1, 0, −1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, −1], [0, 1, 0, −1, 0, 0, 0, 0]}

NullSpace N_0

{[−1, 1, −1, 1, 0, 0, 0, 0], [−1, 0, −1, 0, 1, 0, 1, 0], [−1, 0, −1, 0, 0, 1, 0, 1]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[4., 0., 0., 0., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[−1, 1, −1, 1, 0, 0, 0, 0], [−1, 0, −1, 0, 1, 0, 1, 0], [−1, 0, −1, 0, 0, 1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 \end{pmatrix}$$

==

{2, 6}

R: [3, 8, 1, 1, 7, 4, 5, 5]
B: [6, 3, 8, 6, 2, 7, 4, 2]

TRACE TWO = 1

det AT = $\frac{1}{16}(1+t)^2(t)^2(-1+t)^2$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{1}{1073741824}(-6262-3387s-2864s^2-313s^3+627s^4+394s^5$

$+323s^6 - 8s^7 - 13s^8 - 15s^9 - 3s^{10} + s^{11})(5412 - 180s + 4s^2 - 75s^3 - 181 s^4 + 12s^5 - 4s^6 + 3s^7 + s^8)(-1 + s)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[1] v[3]) (1 + v[5] v[7])
"B CYCLES", (1 + v[2] v[3] v[8]) (1 + v[4] v[6] v[7])

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 0., 0.]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[0, 0, -1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, -1, 1]}

NullSpace of B^*

{[-1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, -1, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&0&0&4&0&0&0\\ 0&0&0&0&0&4&0&0\\ 0&0&0&0&0&0&4&0\\ 0&0&0&0&0&0&0&4\\ 4&0&0&0&0&0&0&0\\ 0&4&0&0&0&0&0&0\\ 0&0&4&0&0&0&0&0\\ 0&0&0&4&0&0&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&\frac{1}{2}&\frac{1}{2}&\frac{1}{4}&1&\frac{1}{2}&\frac{1}{2}&\frac{3}{4}\\ \frac{1}{2}&0&\frac{3}{4}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{4}&\frac{1}{2}\\ \frac{1}{2}&\frac{3}{4}&0&\frac{1}{4}&\frac{1}{2}&\frac{1}{4}&1&\frac{3}{4}\\ \frac{1}{4}&\frac{1}{2}&\frac{1}{4}&0&\frac{3}{4}&\frac{1}{2}&\frac{3}{4}&1\\ 1&\frac{1}{2}&\frac{1}{2}&\frac{3}{4}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{4}\\ \frac{1}{2}&1&\frac{1}{4}&\frac{1}{2}&\frac{1}{2}&0&\frac{3}{4}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{4}&1&\frac{3}{4}&\frac{1}{2}&\frac{3}{4}&0&\frac{1}{4}\\ \frac{3}{4}&\frac{1}{2}&\frac{3}{4}&1&\frac{1}{4}&\frac{1}{2}&\frac{1}{4}&0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[5] + v[2]v[6] + v[3]v[7] + v[4]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT2" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT3" = {{2, 5, 7, 8}, {1, 3, 4, 6}}

"PT4" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"PT5" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$\pi 2$ = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {4, 11, 17, 22}

$u2$ = [2, 2, 1, 4, 2, 2, 3, 3, 2, 2, 4, 1, 2, 1, 2, 1, 4, 3, 3, 2, 3, 4, 2, 2, 1, 3, 2, 1]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[4], [4], [1], [2]]
Action of B on ranges, [[3], [1], [2], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [2, 2, 3, 1, 3]

BPARTS [4, 3, 4, 5, 3]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{1}{4} & \frac{1}{8} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[9, 6, 8, 2, A, 7, 6, 8, 9, 7]

B–BLOCKS,

[8, 4, 9, 3, 1, 9, 8, 5, 4, 5]

with invariant measure, [1, 1, 1, 2, 2, 1, 1, 3, 3, 1]

N by blocks, N – check: true

b_1 = {1, 2, 3, 4}
b_2 = {3, 4, 5, 6}
b_3 = {5, 6, 7, 8}
b_4 = {1, 2, 4, 7}
b_5 = {3, 5, 6, 8}
b_6 = {1, 6, 7, 8}

$b_7 = \{2, 3, 4, 5\}$
$b_8 = \{2, 5, 7, 8\}$
$b_9 = \{1, 3, 4, 6\}$
$b_{10} = \{1, 2, 7, 8\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \\ h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: 23 ⊕ 4/3

$$\text{CLB} = \begin{pmatrix} 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 0 & 1 & 1 \\ -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 7}, {1, 3}}, true

Ω_B in Vec(K)? , {{4, 6, 7}, {2, 3, 8}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 4, 5, 6}, {1, 2, 7, 8}}

1, "range", [4, 8], [[8, 8, 4, 4, 4, 4, 8, 8], [4, 4, 8, 8, 8, 8, 4, 4]]

2, "range", [3, 7], [[7, 7, 3, 3, 3, 3, 7, 7], [3, 3, 7, 7, 7, 7, 3, 3]]

3, "range", [2, 6], [[6, 6, 2, 2, 2, 2, 6, 6], [2, 2, 6, 6, 6, 6, 2, 2]]

4, "range", [1, 5], [[5, 5, 1, 1, 1, 1, 5, 5], [1, 1, 5, 5, 5, 5, 1, 1]]

2, "partition", {{1, 6, 7, 8}, {2, 3, 4, 5}}

1, "range", [4, 8], [[8, 4, 4, 4, 4, 8, 8, 8], [4, 8, 8, 8, 8, 4, 4, 4]]

2, "range", [3, 7], [[7, 3, 3, 3, 3, 7, 7, 7], [3, 7, 7, 7, 7, 3, 3, 3]]

3, "range", [2, 6], [[6, 2, 2, 2, 2, 6, 6, 6], [2, 6, 6, 6, 6, 2, 2, 2]]

4, "range", [1, 5], [[5, 1, 1, 1, 1, 5, 5, 5], [1, 5, 5, 5, 5, 1, 1, 1]]

3, "partition", {{2, 5, 7, 8}, {1, 3, 4, 6}}

1, "range", [4, 8], [[8, 4, 8, 8, 4, 8, 4, 4], [4, 8, 4, 4, 8, 4, 8, 8]]

2, "range", [3, 7], [[7, 3, 7, 7, 3, 7, 3, 3], [3, 7, 3, 3, 7, 3, 7, 7]]

3, "range", [2, 6], [[6, 2, 6, 6, 2, 6, 2, 2], [2, 6, 2, 2, 6, 2, 6, 6]]

4, "range", [1, 5], [[5, 1, 5, 5, 1, 5, 1, 1], [1, 5, 1, 1, 5, 1, 5, 5]]

4, "partition", {{1, 2, 4, 7}, {3, 5, 6, 8}}

1, "range", [4, 8], [[8, 8, 4, 8, 4, 4, 8, 4], [4, 4, 8, 4, 8, 8, 4, 8]]

2, "range", [3, 7], [[7, 7, 3, 7, 3, 3, 7, 3], [3, 3, 7, 3, 7, 7, 3, 7]]

3, "range", [2, 6], [[6, 6, 2, 6, 2, 2, 6, 2], [2, 2, 6, 2, 6, 6, 2, 6]]

4, "range", [1, 5], [[5, 5, 1, 5, 1, 1, 5, 1], [1, 1, 5, 1, 5, 5, 1, 5]]

5, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [4, 8], [[8, 8, 8, 8, 4, 4, 4, 4], [4, 4, 4, 4, 8, 8, 8, 8]]

2, "range", [3, 7], [[7, 7, 7, 7, 3, 3, 3, 3], [3, 3, 3, 3, 7, 7, 7, 7]]

3, "range", [2, 6], [[6, 6, 6, 6, 2, 2, 2, 2], [2, 2, 2, 2, 6, 6, 6, 6]]

4, "range", [1, 5], [[5, 5, 5, 5, 1, 1, 1, 1], [1, 1, 1, 1, 5, 5, 5, 5]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2$ =
$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 11, 17, 22}

$u2$ =
$(2 \;\; 2 \;\; 1 \;\; 4 \;\; 2 \;\; 2 \;\; 3 \;\; 3 \;\; 2 \;\; 2 \;\; 4 \;\; 1 \;\; 2 \;\; 1 \;\; 2 \;\; 1 \;\; 4 \;\; 3 \;\; 3 \;\; 2 \;\; 3 \;\; 4 \;\; 2 \;\; 2 \;\; 1 \;\; 3 \;\; 2$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{16} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{3}{16} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{16} & \frac{1}{4} & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & 0 & \frac{1}{16} \\ \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & \frac{1}{4} & \frac{1}{16} & \frac{1}{8} & \frac{1}{16} & 0 \\ 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} \\ \frac{1}{8} & 0 & \frac{3}{16} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{16} & \frac{1}{8} \\ \frac{1}{8} & \frac{3}{16} & 0 & \frac{1}{16} & \frac{1}{8} & \frac{1}{16} & \frac{1}{4} & \frac{3}{16} \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{16} & 0 & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 2 & 2 & 3 & 0 & 2 & 2 & 1 \\ 2 & 4 & 1 & 2 & 2 & 0 & 3 & 2 \\ 2 & 1 & 4 & 3 & 2 & 3 & 0 & 1 \\ 3 & 2 & 3 & 4 & 1 & 2 & 1 & 0 \\ 0 & 2 & 2 & 1 & 4 & 2 & 2 & 3 \\ 2 & 0 & 3 & 2 & 2 & 4 & 1 & 2 \\ 2 & 3 & 0 & 1 & 2 & 1 & 4 & 3 \\ 1 & 2 & 1 & 0 & 3 & 2 & 3 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, 0, 0, 1, -1, 0, 0]$

$$\ker N_c = \begin{pmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & -t & 0 & t & 0 & -t & 0 & t \\ -s & -t & t & s & -s & -t & t & s \\ -s & 0 & s & 0 & -s & 0 & s & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & -1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & 0 & -s \\ 0 & 0 & s & -t \\ -s & 0 & t & 0 \\ -s & t & 0 & 0 \\ 0 & -t & 0 & s \\ 0 & 0 & -s & t \\ s & 0 & -t & 0 \\ s & -t & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} s & t & -s & s & 0 \\ t & 0 & -t & t+s & 0 \\ s & 0 & 0 & t+s & -s \\ s & t & 0 & s & -s \\ t & -t & s & t & 0 \\ s & 0 & t & 0 & 0 \\ t & 0 & 0 & 0 & s \\ t & -t & 0 & t & s \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 4 & 0 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & 1 & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{4} & 1 & 1 & \frac{1}{2} & \frac{3}{4} & 0 & 0 \\ \frac{1}{2} & \frac{1}{4} & 1 & 1 & \frac{1}{2} & \frac{3}{4} & 0 & 0 \\ 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & 0 & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} & 1 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 1 & 1 \\ \frac{1}{2} & \frac{3}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{3}{4} & \frac{1}{2} & 1 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{3}{4} & 1 & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{2} \\ 1 & \frac{3}{4} & \frac{1}{2} & 1 & 0 & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & 1 & \frac{3}{4} & \frac{1}{2} & 1 \\ \frac{1}{4} & 0 & \frac{3}{4} & \frac{1}{4} & \frac{3}{4} & 1 & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & 1 & \frac{3}{4} & \frac{1}{2} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew}$$

$$\mathrm{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&0&4&0&0&0\\ 0&4&0&0&0&4&0&0\\ 0&0&4&0&0&0&4&0\\ 0&0&0&4&0&0&0&4\\ 4&0&0&0&4&0&0&0\\ 0&4&0&0&0&4&0&0\\ 0&0&4&0&0&0&4&0\\ 0&0&0&4&0&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{1}{2}&\frac{1}{2}&\frac{3}{4}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{4}\\ \frac{1}{2}&1&\frac{1}{4}&\frac{1}{2}&\frac{1}{2}&0&\frac{3}{4}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{4}&1&\frac{3}{4}&\frac{1}{2}&\frac{3}{4}&0&\frac{1}{4}\\ \frac{3}{4}&\frac{1}{2}&\frac{3}{4}&1&\frac{1}{4}&\frac{1}{2}&\frac{1}{4}&0\\ 0&\frac{1}{2}&\frac{1}{2}&\frac{1}{4}&1&\frac{1}{2}&\frac{1}{2}&\frac{3}{4}\\ \frac{1}{2}&0&\frac{3}{4}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{4}&\frac{1}{2}\\ \frac{1}{2}&\frac{3}{4}&0&\frac{1}{4}&\frac{1}{2}&\frac{1}{4}&1&\frac{3}{4}\\ \frac{1}{4}&\frac{1}{2}&\frac{1}{4}&0&\frac{3}{4}&\frac{1}{2}&\frac{3}{4}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{3}{16}\ \frac{1}{8}\ \frac{3}{16}\ \frac{3}{16}\ \frac{1}{4}\ \frac{1}{16}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{16}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{16}\ \frac{1}{8}\ \frac{1}{8}\ 0\ \frac{3}{16}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

NM $(4\ 3\ 2\ 3\ 3\ 4\ 1\ 2\ 2\ 1\ 4\ 2\ 1\ 2\ 2\ 0\ 3\ 2\ 2\ 4)$

"IS MN in Vec(K)?", true

MN $(4\ 3\ 2\ 3\ 3\ 4\ 1\ 2\ 2\ 1\ 4\ 2\ 1\ 2\ 2\ 0\ 3\ 2\ 2\ 4)$

$\boldsymbol{\tau}$ = 32/1 , rank = 2 , ratio = 16/1 , n^2 / r = 32/1

$\boldsymbol{\tau}'$ = 32/1 , r' = 1/2 , $\boldsymbol{\tau} / n^2$ = 1/2

p^2 = 1/8 , min $\boldsymbol{\tau}$ = 8/1 , $\boldsymbol{\tau}$-check is positive? 24/1

max r = 8/1 , r-check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 5, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 40

dim span idems 16 vs no. of idems 20

"PT1" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT2" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT3" = {{2, 5, 7, 8}, {1, 3, 4, 6}}

"PT4" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"PT5" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \\ 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{5}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \\ \frac{3}{8} & \frac{7}{8} & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{5}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{1}{8} & \frac{7}{8} & \frac{5}{8} & \frac{3}{8} & \frac{5}{8} & \frac{-1}{8} & \frac{1}{8} \\ \frac{5}{8} & \frac{3}{8} & \frac{5}{8} & \frac{7}{8} & \frac{1}{8} & \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{5}{8} \\ \frac{3}{8} & \frac{-1}{8} & \frac{5}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{5}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{1}{8} & \frac{7}{8} & \frac{5}{8} \\ \frac{1}{8} & \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{5}{8} & \frac{3}{8} & \frac{5}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{3}{7} & \frac{3}{7} & \frac{5}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{1}{7} \\ \frac{3}{7} & 1 & \frac{1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{5}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{1}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{5}{7} & \frac{-1}{7} & \frac{1}{7} \\ \frac{5}{7} & \frac{3}{7} & \frac{5}{7} & 1 & \frac{1}{7} & \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{1}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{5}{7} \\ \frac{3}{7} & \frac{-1}{7} & \frac{5}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{1}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{5}{7} & \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} & \frac{1}{7} & 1 & \frac{5}{7} \\ \frac{1}{7} & \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} & \frac{5}{7} & \frac{3}{7} & \frac{5}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 1.309016994, 0.1909830058, 1.809016994, 0.6909830058]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 3.428571429, 1.496019422, 0.2182662923, 2.067447994, 0.7896948642]

NullSpace M_C

{[1, 0, 1, 1, 0, 1, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [1, 0, 0, 1, 0, 1, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, 1, 0, 0, 0, −1, 0, 0]}

NullSpace N_c

{[−1, 0, 0, 1, −1, 0, 0, 1], [−1, 1, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, −1, 0, 1, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 1.309016994, 0.1909830058, 1.809016994, 0.6909830058]

NullSpace M_0

{[0, 0, 0, 1, 0, 0, 0, −1], [0, 0, 1, 0, 0, 0, −1, 0], [0, 1, 0, 0, 0, −1, 0, 0], [1, 0, 0, 0, −1, 0, 0, 0]}

NullSpace N_0

{[0, 0, 1, −1, 0, 0, 1, −1], [0, 1, 0, −1, 0, 1, 0, −1], [1, 0, 0, −1, 1, 0, 0, −1]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −0.1909830058, −1.309016994, −0.6909830058, −1.809016994]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 1, −1, 0, 0, 1], [−1, 1, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, −1, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 2 & 1 & 4 & 2 & 2 & 3 \\ 2 & 0 & 3 & 2 & 2 & 4 & 1 & 2 \\ 2 & 3 & 0 & 1 & 2 & 1 & 4 & 3 \\ 1 & 2 & 1 & 0 & 3 & 2 & 3 & 4 \\ 4 & 2 & 2 & 3 & 0 & 2 & 2 & 1 \\ 2 & 4 & 1 & 2 & 2 & 0 & 3 & 2 \\ 2 & 1 & 4 & 3 & 2 & 3 & 0 & 1 \\ 3 & 2 & 3 & 4 & 1 & 2 & 1 & 0 \end{pmatrix}$$

==

{2, 8}

R: [3, 8, 1, 1, 7, 7, 5, 2]
B: [6, 3, 8, 6, 2, 4, 4, 5]

TRACE TWO = 1

det AT = $\frac{1}{16}(t)^2(1+t)^2(-1+t)^2$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{5}{1073741824}(2542 - 703s - 1255s^2 + 13s^3 + 347s^4 - 76s^5 - 71$

$s^6 + 34s^7 + 7s^8 - 5s^9 - 2s^{10} + s^{11}$) ($-1 + s$) ($-13332 + 7272s + 5180s^2 - 2019s^3 - 553s^4 + 216s^5 - 20s^6 - 13s^7 + 5s^8$)

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 2, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", (1 + v[1] v[3]) (1 + v[2] v[8]) (1 + v[5] v[7])
"B CYCLES", (1 + v[4] v[6]) (1 + v[2] v[3] v[5] v[8])

Eigenvalues

R: [0., 0., 1., −1., 1., −1., 1., −1.]

B: [1. I, −1. I, 0., 0., 1., −1., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of R^*

{[0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0]}

NullSpace of B^*

{[−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&0&0&0&0&4&0\\ 0&0&0&0&0&0&0&4\\ 0&0&0&0&4&0&0&0\\ 0&0&0&0&0&4&0&0\\ 0&0&4&0&0&0&0&0\\ 0&0&0&4&0&0&0&0\\ 4&0&0&0&0&0&0&0\\ 0&4&0&0&0&0&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&\frac{7}{11}&\frac{6}{11}&\frac{3}{11}&\frac{5}{11}&\frac{8}{11}&1&\frac{4}{11}\\ \frac{7}{11}&0&\frac{5}{11}&\frac{6}{11}&\frac{6}{11}&\frac{5}{11}&\frac{4}{11}&1\\ \frac{6}{11}&\frac{5}{11}&0&\frac{3}{11}&1&\frac{8}{11}&\frac{5}{11}&\frac{6}{11}\\ \frac{3}{11}&\frac{6}{11}&\frac{3}{11}&0&\frac{8}{11}&1&\frac{8}{11}&\frac{5}{11}\\ \frac{5}{11}&\frac{6}{11}&1&\frac{8}{11}&0&\frac{3}{11}&\frac{6}{11}&\frac{5}{11}\\ \frac{8}{11}&\frac{5}{11}&\frac{8}{11}&1&\frac{3}{11}&0&\frac{3}{11}&\frac{6}{11}\\ 1&\frac{4}{11}&\frac{5}{11}&\frac{8}{11}&\frac{6}{11}&\frac{3}{11}&0&\frac{7}{11}\\ \frac{4}{11}&1&\frac{6}{11}&\frac{5}{11}&\frac{5}{11}&\frac{6}{11}&\frac{7}{11}&0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 6, "for kernel rank", 2

degree 1: $\frac{1}{8}\,(\,v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]\,)$
degree 2: $\frac{1}{4}\,(\,v[1]v[7] + v[2]v[8] + v[3]v[5] + v[4]v[6]\,)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 5, 6, 8}, {2, 3, 4, 7}}

"PT4" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"PT5" = {{3, 6, 7, 8}, {1, 2, 4, 5}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {4, 6}

"RG3" = {3, 5}

"RG4" = {1, 7}

$\pi 2$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {6, 13, 15, 20}

$u2$ = [7, 6, 3, 5, 8, 11, 4, 5, 6, 6, 5, 4, 11, 3, 11, 8, 5, 6, 8, 11, 8, 5, 3, 6, 5, 3, 6, 7]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[1], [4], [4], [3]]
Action of B on ranges, [[3], [2], [1], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [6, 3, 3, 4, 4, 1]

BPARTS [2, 5, 1, 2, 6, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{3}{11} & \frac{2}{11} & \frac{2}{11} & \frac{1}{11} & \frac{1}{11} & \frac{2}{11} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[B, C, A, A, 6, 5, 3, 6, 5, 3, 1, 2]

B–BLOCKS,

[C, B, 9, 1, B, C, 2, 4, 7, 8, 9, 8]

with invariant measure, [2, 2, 1, 1, 2, 2, 1, 2, 2, 1, 3, 3]

N by blocks, N – check: true

b_1 = {1, 2, 3, 4}
b_2 = {5, 6, 7, 8}
b_3 = {3, 4, 7, 8}
b_4 = {3, 6, 7, 8}

$b_5 = \{1, 5, 6, 8\}$
$b_6 = \{2, 3, 4, 7\}$
$b_7 = \{1, 2, 4, 5\}$
$b_8 = \{1, 4, 5, 8\}$
$b_9 = \{2, 3, 6, 7\}$
$b_{10} = \{1, 2, 5, 6\}$
$b_{11} = \{1, 3, 4, 8\}$
$b_{12} = \{2, 5, 6, 7\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & 0 & 0 & h[1] & 0 \\ 0 & h[2] & 0 & 0 & 0 & 0 & 0 & h[1] \\ 0 & 0 & h[2] & 0 & h[1] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[2] & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[1] & 0 & h[2] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[1] & 0 & h[2] & 0 & 0 \\ h[1] & 0 & 0 & 0 & 0 & 0 & h[2] & 0 \\ 0 & h[1] & 0 & 0 & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: $23 \oplus 4/3$

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 7}, {1, 3}, {2, 8}}, false

Ω_B in Vec(K)? , {{4, 6}, {2, 3, 5, 8}}, false

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \left(\frac{3}{16} \;\; \frac{1}{8} \;\; \frac{3}{16} \;\; 0 \;\; \frac{3}{16} \;\; 0 \;\; \frac{3}{16} \;\; \frac{1}{8}\right)$ vs $\left(0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right)$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \left(0 \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{4} \;\; \frac{1}{8} \;\; \frac{1}{4} \;\; 0 \;\; \frac{1}{8}\right)$ vs $\left(0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right)$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 4, 8}, {2, 5, 6, 7}}

1, "range", [2, 8], [[8, 2, 8, 8, 2, 2, 2, 8], [2, 8, 2, 2, 8, 8, 8, 2]]

2, "range", [4, 6], [[6, 4, 6, 6, 4, 4, 4, 6], [4, 6, 4, 4, 6, 6, 6, 4]]

3, "range", [3, 5], [[5, 3, 5, 5, 3, 3, 3, 5], [3, 5, 3, 3, 5, 5, 5, 3]]

4, "range", [1, 7], [[7, 1, 7, 7, 1, 1, 1, 7], [1, 7, 1, 1, 7, 7, 7, 1]]

2, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [2, 8], [[8, 2, 2, 8, 8, 2, 2, 8], [2, 8, 8, 2, 2, 8, 8, 2]]

2, "range", [4, 6], [[6, 4, 4, 6, 6, 4, 4, 6], [4, 6, 6, 4, 4, 6, 6, 4]]

3, "range", [3, 5], [[5, 3, 3, 5, 5, 3, 3, 5], [3, 5, 5, 3, 3, 5, 5, 3]]

4, "range", [1, 7], [[7, 1, 1, 7, 7, 1, 1, 7], [1, 7, 7, 1, 1, 7, 7, 1]]

3, "partition", {{1, 5, 6, 8}, {2, 3, 4, 7}}

1, "range", [2, 8], [[8, 2, 2, 2, 8, 8, 2, 8], [2, 8, 8, 8, 2, 2, 8, 2]]

2, "range", [4, 6], [[6, 4, 4, 4, 6, 6, 4, 6], [4, 6, 6, 6, 4, 4, 6, 4]]

3, "range", [3, 5], [[5, 3, 3, 3, 5, 5, 3, 5], [3, 5, 5, 5, 3, 3, 5, 3]]

4, "range", [1, 7], [[7, 1, 1, 1, 7, 7, 1, 7], [1, 7, 7, 7, 1, 1, 7, 1]]

4, "partition", {{3, 4, 7, 8}, {1, 2, 5, 6}}

1, "range", [2, 8], [[8, 8, 2, 2, 8, 8, 2, 2], [2, 2, 8, 8, 2, 2, 8, 8]]

2, "range", [4, 6], [[6, 6, 4, 4, 6, 6, 4, 4], [4, 4, 6, 6, 4, 4, 6, 6]]

3, "range", [3, 5], [[5, 5, 3, 3, 5, 5, 3, 3], [3, 3, 5, 5, 3, 3, 5, 5]]

4, "range", [1, 7], [[7, 7, 1, 1, 7, 7, 1, 1], [1, 1, 7, 7, 1, 1, 7, 7]]

5, "partition", {{3, 6, 7, 8}, {1, 2, 4, 5}}

1, "range", [2, 8], [[8, 8, 2, 8, 8, 2, 2, 2], [2, 2, 8, 2, 2, 8, 8, 8]]

2, "range", [4, 6], [[6, 6, 4, 6, 6, 4, 4, 4], [4, 4, 6, 4, 4, 6, 6, 6]]

3, "range", [3, 5], [[5, 5, 3, 5, 5, 3, 3, 3], [3, 3, 5, 3, 3, 5, 5, 5]]

4, "range", [1, 7], [[7, 7, 1, 7, 7, 1, 1, 1], [1, 1, 7, 1, 1, 7, 7, 7]]

6, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [2, 8], [[8, 8, 8, 8, 2, 2, 2, 2], [2, 2, 2, 2, 8, 8, 8, 8]]

2, "range", [4, 6], [[6, 6, 6, 6, 4, 4, 4, 4], [4, 4, 4, 4, 6, 6, 6, 6]]

3, "range", [3, 5], [[5, 5, 5, 5, 3, 3, 3, 3], [3, 3, 3, 3, 5, 5, 5, 5]]

4, "range", [1, 7], [[7, 7, 7, 7, 1, 1, 1, 1], [1, 1, 1, 1, 7, 7, 7, 7]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2$ =
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

{6, 13, 15, 20}

$u2$ =
(7 6 3 5 8 11 4 5 6 6 5 4 11 3 11 8 5 6 8 11 8 5 3 6 5 3

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{11}{2} & \frac{11}{2} & \frac{11}{2} & \frac{11}{2} & \frac{11}{2} & \frac{11}{2} & \frac{11}{2} & \frac{11}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{4}{11} & 0 & 0 & 0 & 0 & 0 & \frac{7}{11} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6}{11} & 0 & 0 & 0 & 0 & 0 & \frac{5}{11} \\ 0 & \frac{5}{11} & 0 & 0 & 0 & 0 & 0 & \frac{6}{11} \\ 0 & \frac{5}{11} & 0 & 0 & 0 & 0 & 0 & \frac{6}{11} \\ 0 & \frac{6}{11} & 0 & 0 & 0 & 0 & 0 & \frac{5}{11} \\ 0 & \frac{7}{11} & 0 & 0 & 0 & 0 & 0 & \frac{4}{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{8}{11} & 0 & \frac{3}{11} & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{11} & 0 & \frac{6}{11} & 0 & 0 \\ 0 & 0 & 0 & \frac{8}{11} & 0 & \frac{3}{11} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{11} & 0 & \frac{8}{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{11} & 0 & \frac{8}{11} & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{11} & 0 & \frac{5}{11} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{5}{11} & 0 & \frac{6}{11} & 0 & 0 & 0 \\ 0 & 0 & \frac{6}{11} & 0 & \frac{5}{11} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{8}{11} & 0 & \frac{3}{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{11} & 0 & \frac{8}{11} & 0 & 0 & 0 \\ 0 & 0 & \frac{6}{11} & 0 & \frac{5}{11} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{11} & 0 & \frac{6}{11} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{4}{11} & 0 & 0 & 0 & 0 & 0 & \frac{7}{11} & 0 \\ \frac{5}{11} & 0 & 0 & 0 & 0 & 0 & \frac{6}{11} & 0 \\ \frac{8}{11} & 0 & 0 & 0 & 0 & 0 & \frac{3}{11} & 0 \\ \frac{6}{11} & 0 & 0 & 0 & 0 & 0 & \frac{5}{11} & 0 \\ \frac{3}{11} & 0 & 0 & 0 & 0 & 0 & \frac{8}{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{7}{11} & 0 & 0 & 0 & 0 & 0 & \frac{4}{11} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{11} & \frac{5}{44} & \frac{2}{11} & \frac{3}{22} & \frac{3}{44} & 0 & \frac{7}{44} \\ \frac{1}{11} & \frac{1}{4} & \frac{3}{22} & \frac{5}{44} & \frac{5}{44} & \frac{3}{22} & \frac{7}{44} & 0 \\ \frac{5}{44} & \frac{3}{22} & \frac{1}{4} & \frac{2}{11} & 0 & \frac{3}{44} & \frac{3}{22} & \frac{5}{44} \\ \frac{2}{11} & \frac{5}{44} & \frac{2}{11} & \frac{1}{4} & \frac{3}{44} & 0 & \frac{3}{44} & \frac{3}{22} \\ \frac{3}{22} & \frac{5}{44} & 0 & \frac{3}{44} & \frac{1}{4} & \frac{2}{11} & \frac{5}{44} & \frac{3}{22} \\ \frac{3}{44} & \frac{3}{22} & \frac{3}{44} & 0 & \frac{2}{11} & \frac{1}{4} & \frac{2}{11} & \frac{5}{44} \\ 0 & \frac{7}{44} & \frac{3}{22} & \frac{3}{44} & \frac{5}{44} & \frac{2}{11} & \frac{1}{4} & \frac{1}{11} \\ \frac{7}{44} & 0 & \frac{5}{44} & \frac{3}{22} & \frac{3}{22} & \frac{5}{44} & \frac{1}{11} & \frac{1}{4} \end{pmatrix} \quad NM =$$

$$\begin{pmatrix} 4 & \frac{16}{11} & \frac{20}{11} & \frac{32}{11} & \frac{24}{11} & \frac{12}{11} & 0 & \frac{28}{11} \\ \frac{16}{11} & 4 & \frac{24}{11} & \frac{20}{11} & \frac{20}{11} & \frac{24}{11} & \frac{28}{11} & 0 \\ \frac{20}{11} & \frac{24}{11} & 4 & \frac{32}{11} & 0 & \frac{12}{11} & \frac{24}{11} & \frac{20}{11} \\ \frac{32}{11} & \frac{20}{11} & \frac{32}{11} & 4 & \frac{12}{11} & 0 & \frac{12}{11} & \frac{24}{11} \\ \frac{24}{11} & \frac{20}{11} & 0 & \frac{12}{11} & 4 & \frac{32}{11} & \frac{20}{11} & \frac{24}{11} \\ \frac{12}{11} & \frac{24}{11} & \frac{12}{11} & 0 & \frac{32}{11} & 4 & \frac{32}{11} & \frac{20}{11} \\ 0 & \frac{28}{11} & \frac{24}{11} & \frac{12}{11} & \frac{20}{11} & \frac{32}{11} & 4 & \frac{16}{11} \\ \frac{28}{11} & 0 & \frac{20}{11} & \frac{24}{11} & \frac{24}{11} & \frac{20}{11} & \frac{16}{11} & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 0, 0, -1, 0, -1, 1, 0]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 & -1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & 0 & -t & 0 & -t & 0 & t \\ -s & s & t & -t & t & -t & -s & s \\ -s & 0 & s & 0 & s & 0 & -s & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & s & -t & 0 \\ 0 & t & 0 & s \\ s & 0 & 0 & t \\ s & 0 & -t & 0 \\ -s & 0 & 0 & -t \\ -s & 0 & t & 0 \\ 0 & -s & t & 0 \\ 0 & -t & 0 & -s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & s+t & 0 & s & -s \\ s & t & 0 & t & -t \\ t & s & -s & s & 0 \\ 0 & s+t & -s & s & 0 \\ -t & t & s & t & 0 \\ 0 & 0 & s & t & 0 \\ 0 & 0 & 0 & t & s \\ -s & s & 0 & s & t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 0 & 4 & 0 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & \frac{5}{11} & \frac{5}{11} & \frac{5}{11} & \frac{6}{11} & \frac{6}{11} & 0 & \frac{6}{11} \\ \frac{5}{11} & 1 & \frac{7}{11} & \frac{7}{11} & \frac{4}{11} & \frac{4}{11} & \frac{6}{11} & 0 \\ \frac{5}{11} & \frac{7}{11} & 1 & 1 & 0 & 0 & \frac{6}{11} & \frac{4}{11} \\ \frac{5}{11} & \frac{7}{11} & 1 & 1 & 0 & 0 & \frac{6}{11} & \frac{4}{11} \\ \frac{6}{11} & \frac{4}{11} & 0 & 0 & 1 & 1 & \frac{5}{11} & \frac{7}{11} \\ \frac{6}{11} & \frac{4}{11} & 0 & 0 & 1 & 1 & \frac{5}{11} & \frac{7}{11} \\ 0 & \frac{6}{11} & \frac{6}{11} & \frac{6}{11} & \frac{5}{11} & \frac{5}{11} & 1 & \frac{5}{11} \\ \frac{6}{11} & 0 & \frac{4}{11} & \frac{4}{11} & \frac{7}{11} & \frac{7}{11} & \frac{5}{11} & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{3}{11} & \frac{5}{11} & 1 & \frac{6}{11} & 0 & 0 & \frac{8}{11} \\ \frac{3}{11} & 1 & \frac{5}{11} & \frac{3}{11} & \frac{6}{11} & \frac{8}{11} & \frac{8}{11} & 0 \\ \frac{5}{11} & \frac{5}{11} & 1 & \frac{5}{11} & 0 & \frac{6}{11} & \frac{6}{11} & \frac{6}{11} \\ 1 & \frac{3}{11} & \frac{5}{11} & 1 & \frac{6}{11} & 0 & 0 & \frac{8}{11} \\ \frac{6}{11} & \frac{6}{11} & 0 & \frac{6}{11} & 1 & \frac{5}{11} & \frac{5}{11} & \frac{5}{11} \\ 0 & \frac{8}{11} & \frac{6}{11} & 0 & \frac{5}{11} & 1 & 1 & \frac{3}{11} \\ 0 & \frac{8}{11} & \frac{6}{11} & 0 & \frac{5}{11} & 1 & 1 & \frac{3}{11} \\ \frac{8}{11} & 0 & \frac{6}{11} & \frac{8}{11} & \frac{5}{11} & \frac{3}{11} & \frac{3}{11} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew}$$

$$\text{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&0&0&0&4&0\\ 0&4&0&0&0&0&0&4\\ 0&0&4&0&4&0&0&0\\ 0&0&0&4&0&4&0&0\\ 0&0&4&0&4&0&0&0\\ 0&0&0&4&0&4&0&0\\ 4&0&0&0&0&0&4&0\\ 0&4&0&0&0&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{4}{11}&\frac{5}{11}&\frac{8}{11}&\frac{6}{11}&\frac{3}{11}&0&\frac{7}{11}\\ \frac{4}{11}&1&\frac{6}{11}&\frac{5}{11}&\frac{5}{11}&\frac{6}{11}&\frac{7}{11}&0\\ \frac{5}{11}&\frac{6}{11}&1&\frac{8}{11}&0&\frac{3}{11}&\frac{6}{11}&\frac{5}{11}\\ \frac{8}{11}&\frac{5}{11}&\frac{8}{11}&1&\frac{3}{11}&0&\frac{3}{11}&\frac{6}{11}\\ \frac{6}{11}&\frac{5}{11}&0&\frac{3}{11}&1&\frac{8}{11}&\frac{5}{11}&\frac{6}{11}\\ \frac{3}{11}&\frac{6}{11}&\frac{3}{11}&0&\frac{8}{11}&1&\frac{8}{11}&\frac{5}{11}\\ 0&\frac{7}{11}&\frac{6}{11}&\frac{3}{11}&\frac{5}{11}&\frac{8}{11}&1&\frac{4}{11}\\ \frac{7}{11}&0&\frac{5}{11}&\frac{6}{11}&\frac{6}{11}&\frac{5}{11}&\frac{4}{11}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$$

T

$$\left(\frac{1}{4}\ \frac{2}{11}\ \frac{5}{44}\ \frac{2}{11}\ \frac{2}{11}\ \frac{1}{4}\ \frac{3}{22}\ \frac{5}{44}\ \frac{5}{44}\ \frac{3}{22}\ \frac{1}{4}\ \frac{1}{11}\ \frac{7}{44}\ 0\ \frac{3}{44}\ \frac{3}{22}\ \frac{2}{11}\ \frac{5}{44}\ \frac{1}{11}\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

NM

$$\left(4\ \frac{32}{11}\ \frac{20}{11}\ \frac{32}{11}\ \frac{32}{11}\ 4\ \frac{24}{11}\ \frac{20}{11}\ \frac{20}{11}\ \frac{24}{11}\ 4\ \frac{16}{11}\ \frac{28}{11}\ 0\ \frac{12}{11}\ \frac{24}{11}\ \frac{32}{11}\ \frac{20}{11}\ \frac{16}{11}\ 4\right)$$

"IS MN in Vec(K)?", true

MN

$$\left(4\ \frac{32}{11}\ \frac{20}{11}\ \frac{32}{11}\ \frac{32}{11}\ 4\ \frac{24}{11}\ \frac{20}{11}\ \frac{20}{11}\ \frac{24}{11}\ 4\ \frac{16}{11}\ \frac{28}{11}\ 0\ \frac{12}{11}\ \frac{24}{11}\ \frac{32}{11}\ \frac{20}{11}\ \frac{16}{11}\ 4\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 24/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 5, 6, 8}, {2, 3, 4, 7}}

"PT4" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"PT5" = {{3, 6, 7, 8}, {1, 2, 4, 5}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {4, 6}

"RG3" = {3, 5}

"RG4" = {1, 7}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{21}{88} & \frac{29}{88} & \frac{53}{88} & \frac{37}{88} & \frac{13}{88} & \frac{-1}{8} & \frac{45}{88} \\ \frac{21}{88} & \frac{7}{8} & \frac{37}{88} & \frac{29}{88} & \frac{29}{88} & \frac{37}{88} & \frac{45}{88} & \frac{-1}{8} \\ \frac{29}{88} & \frac{37}{88} & \frac{7}{8} & \frac{53}{88} & \frac{-1}{8} & \frac{13}{88} & \frac{37}{88} & \frac{29}{88} \\ \frac{53}{88} & \frac{29}{88} & \frac{53}{88} & \frac{7}{8} & \frac{13}{88} & \frac{-1}{8} & \frac{13}{88} & \frac{37}{88} \\ \frac{37}{88} & \frac{29}{88} & \frac{-1}{8} & \frac{13}{88} & \frac{7}{8} & \frac{53}{88} & \frac{29}{88} & \frac{37}{88} \\ \frac{13}{88} & \frac{37}{88} & \frac{13}{88} & \frac{-1}{8} & \frac{53}{88} & \frac{7}{8} & \frac{53}{88} & \frac{29}{88} \\ \frac{-1}{8} & \frac{45}{88} & \frac{37}{88} & \frac{13}{88} & \frac{29}{88} & \frac{53}{88} & \frac{7}{8} & \frac{21}{88} \\ \frac{45}{88} & \frac{-1}{8} & \frac{29}{88} & \frac{37}{88} & \frac{37}{88} & \frac{29}{88} & \frac{21}{88} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{3}{11} & \frac{29}{77} & \frac{53}{77} & \frac{37}{77} & \frac{13}{77} & \frac{-1}{7} & \frac{45}{77} \\ \frac{3}{11} & 1 & \frac{37}{77} & \frac{29}{77} & \frac{29}{77} & \frac{37}{77} & \frac{45}{77} & \frac{-1}{7} \\ \frac{29}{77} & \frac{37}{77} & 1 & \frac{53}{77} & \frac{-1}{7} & \frac{13}{77} & \frac{37}{77} & \frac{29}{77} \\ \frac{53}{77} & \frac{29}{77} & \frac{53}{77} & 1 & \frac{13}{77} & \frac{-1}{7} & \frac{13}{77} & \frac{37}{77} \\ \frac{37}{77} & \frac{29}{77} & \frac{-1}{7} & \frac{13}{77} & 1 & \frac{53}{77} & \frac{29}{77} & \frac{37}{77} \\ \frac{13}{77} & \frac{37}{77} & \frac{13}{77} & \frac{-1}{7} & \frac{53}{77} & 1 & \frac{53}{77} & \frac{29}{77} \\ \frac{-1}{7} & \frac{45}{77} & \frac{37}{77} & \frac{13}{77} & \frac{29}{77} & \frac{53}{77} & 1 & \frac{3}{11} \\ \frac{45}{77} & \frac{-1}{7} & \frac{29}{77} & \frac{37}{77} & \frac{37}{77} & \frac{29}{77} & \frac{3}{11} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 0.3083672838, 0.7685379357, 1.278775810, 1.644318969]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 3.428571429, 0.3524197529, 0.8783290695, 1.461458069, 1.879221680]

NullSpace M_C

{[1, 1, 1, 1, 0, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0], [0, 1, 1, 1, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0, 0, 0]}

NullSpace N_c

{[1, −1, 0, 0, 0, 0, 1, −1], [0, −1, 0, 1, 0, 1, 0, −1], [0, −1, 1, 0, 1, 0, 0, −1]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 0.3083672838, 0.7685379357, 1.278775810, 1.644318969]

NullSpace M_0

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0]}

NullSpace N_0

{[−1, 0, 0, 1, 0, 1, −1, 0], [−1, 1, 0, 0, 0, 0, −1, 1], [−1, 0, 1, 0, 1, 0, −1, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −0.3083672838, −0.7685379357, −1.278775810, −1.644318969]

NullSpace M

{}

NullSpace N

{[1, −1, 0, 0, 0, 0, 1, −1], [0, −1, 1, 0, 1, 0, 0, −1], [0, −1, 0, 1, 0, 1, 0, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 7 & 6 & 3 & 5 & 8 & 11 & 4 \\ 7 & 0 & 5 & 6 & 6 & 5 & 4 & 11 \\ 6 & 5 & 0 & 3 & 11 & 8 & 5 & 6 \\ 3 & 6 & 3 & 0 & 8 & 11 & 8 & 5 \\ 5 & 6 & 11 & 8 & 0 & 3 & 6 & 5 \\ 8 & 5 & 8 & 11 & 3 & 0 & 3 & 6 \\ 11 & 4 & 5 & 8 & 6 & 3 & 0 & 7 \\ 4 & 11 & 6 & 5 & 5 & 6 & 7 & 0 \end{pmatrix}$$

==

{3, 5}

R: [3, 3, 8, 1, 2, 7, 5, 5]
B: [6, 8, 1, 6, 7, 4, 4, 2]

TRACE TWO = 1

det AT = $\frac{1}{16}(-1+t)^2(t)^2(1+t)^2$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{-5}{536870912}(-1+s)(13332+7272s+1276s^2+893s^3+183s^4$

$+128s^5 + 68s^6 + 11s^7 + 5s^8)(2542 - 703s + 779s^2 - 664s^3 + 9s^4 - 28s^5 - 61s^6 + 56s^7 - 7s^8 - 5s^9 + 2s^{10})$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 1, "vs", 4

"R CYCLES", 1 + v[2] v[3] v[5] v[8]
"B CYCLES", (1 + v[2] v[8]) (1 + v[4] v[6])

Eigenvalues

R: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

NullSpace of B^*

{[0, 0, 0, 0, 0, −1, 1, 0], [−1, 0, 0, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&0&0&0&0&4&0\\ 0&0&0&0&0&0&0&4\\ 0&0&0&0&4&0&0&0\\ 0&0&0&0&0&4&0&0\\ 0&0&4&0&0&0&0&0\\ 0&0&0&4&0&0&0&0\\ 4&0&0&0&0&0&0&0\\ 0&4&0&0&0&0&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&\frac{2}{7}&\frac{4}{7}&\frac{2}{7}&\frac{3}{7}&\frac{5}{7}&1&\frac{5}{7}\\ \frac{2}{7}&0&\frac{4}{7}&\frac{4}{7}&\frac{3}{7}&\frac{3}{7}&\frac{5}{7}&1\\ \frac{4}{7}&\frac{4}{7}&0&\frac{5}{7}&1&\frac{2}{7}&\frac{3}{7}&\frac{3}{7}\\ \frac{2}{7}&\frac{4}{7}&\frac{5}{7}&0&\frac{2}{7}&1&\frac{5}{7}&\frac{3}{7}\\ \frac{3}{7}&\frac{3}{7}&1&\frac{2}{7}&0&\frac{5}{7}&\frac{4}{7}&\frac{4}{7}\\ \frac{5}{7}&\frac{3}{7}&\frac{2}{7}&1&\frac{5}{7}&0&\frac{2}{7}&\frac{4}{7}\\ 1&\frac{5}{7}&\frac{3}{7}&\frac{5}{7}&\frac{4}{7}&\frac{2}{7}&0&\frac{2}{7}\\ \frac{5}{7}&1&\frac{3}{7}&\frac{3}{7}&\frac{4}{7}&\frac{4}{7}&\frac{2}{7}&0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[7] + v[2]v[8] + v[3]v[5] + v[4]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"PT4" = {{3, 6, 7, 8}, {1, 2, 4, 5}}

"PT5" = {{1, 2, 3, 6}, {4, 5, 7, 8}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {4, 6}

"RG3" = {3, 5}

"RG4" = {1, 7}

$\pi 2$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {6, 13, 15, 20}

$u2$ = [2, 4, 2, 3, 5, 7, 5, 4, 4, 3, 3, 5, 7, 5, 7, 2, 3, 3, 2, 7, 5, 3, 5, 4, 4, 2, 4, 2]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [4], [1], [3]]
Action of B on ranges, [[1], [2], [4], [2]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

RPARTS [6, 3, 5, 5, 4, 4]

BPARTS [2, 2, 1, 4, 1, 4]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{7} & \frac{1}{7} & \frac{1}{14} & \frac{5}{14} & \frac{3}{14} & \frac{1}{14} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[6, 5, 6, 3, 3, C, 4, 9, C, 1, 2, 5]

B-BLOCKS,

[5, 6, A, B, 6, 5, 8, 7, A, 8, 7, B]

with invariant measure, [1, 1, 3, 1, 5, 5, 2, 2, 1, 2, 2, 3]

N by blocks, N - check: true

b_1 = {1, 2, 3, 4}
b_2 = {5, 6, 7, 8}
b_3 = {1, 2, 3, 6}
b_4 = {3, 4, 7, 8}
b_5 = {3, 6, 7, 8}

$b_6 = \{1, 2, 4, 5\}$
$b_7 = \{1, 4, 5, 8\}$
$b_8 = \{2, 3, 6, 7\}$
$b_9 = \{1, 2, 5, 6\}$
$b_{10} = \{1, 3, 4, 8\}$
$b_{11} = \{2, 5, 6, 7\}$
$b_{12} = \{4, 5, 7, 8\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & 0 & 0 & h[1] & 0 \\ 0 & h[2] & 0 & 0 & 0 & 0 & 0 & h[1] \\ 0 & 0 & h[2] & 0 & h[1] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[2] & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[1] & 0 & h[2] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[1] & 0 & h[2] & 0 & 0 \\ h[1] & 0 & 0 & 0 & 0 & 0 & h[2] & 0 \\ 0 & h[1] & 0 & 0 & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 28, Shape: $23 \oplus 5/3$

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3, 5, 8}}, true

Ω_B in Vec(K)? , {{2, 8}, {4, 6}}, false

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} \\ \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} \\ \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} \\ \frac{-3}{40} & \frac{29}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{-27}{40} & \frac{21}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{8} & 0 & \frac{3}{8} & 0 & \frac{3}{8} & 0 & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 4, 8}, {2, 5, 6, 7}}

1, "range", [2, 8], [[8, 2, 8, 8, 2, 2, 2, 8], [2, 8, 2, 2, 8, 8, 8, 2]]

2, "range", [4, 6], [[6, 4, 6, 6, 4, 4, 4, 6], [4, 6, 4, 4, 6, 6, 6, 4]]

3, "range", [3, 5], [[5, 3, 5, 5, 3, 3, 3, 5], [3, 5, 3, 3, 5, 5, 5, 3]]

4, "range", [1, 7], [[7, 1, 7, 7, 1, 1, 1, 7], [1, 7, 1, 1, 7, 7, 7, 1]]

2, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [2, 8], [[8, 2, 2, 8, 8, 2, 2, 8], [2, 8, 8, 2, 2, 8, 8, 2]]

2, "range", [4, 6], [[6, 4, 4, 6, 6, 4, 4, 6], [4, 6, 6, 4, 4, 6, 6, 4]]

3, "range", [3, 5], [[5, 3, 3, 5, 5, 3, 3, 5], [3, 5, 5, 3, 3, 5, 5, 3]]

4, "range", [1, 7], [[7, 1, 1, 7, 7, 1, 1, 7], [1, 7, 7, 1, 1, 7, 7, 1]]

3, "partition", {{3, 4, 7, 8}, {1, 2, 5, 6}}

1, "range", [2, 8], [[8, 8, 2, 2, 8, 8, 2, 2], [2, 2, 8, 8, 2, 2, 8, 8]]

2, "range", [4, 6], [[6, 6, 4, 4, 6, 6, 4, 4], [4, 4, 6, 6, 4, 4, 6, 6]]

3, "range", [3, 5], [[5, 5, 3, 3, 5, 5, 3, 3], [3, 3, 5, 5, 3, 3, 5, 5]]

4, "range", [1, 7], [[7, 7, 1, 1, 7, 7, 1, 1], [1, 1, 7, 7, 1, 1, 7, 7]]

4, "partition", {{3, 6, 7, 8}, {1, 2, 4, 5}}

1, "range", [2, 8], [[8, 8, 2, 8, 8, 2, 2, 2], [2, 2, 8, 2, 2, 8, 8, 8]]

2, "range", [4, 6], [[6, 6, 4, 6, 6, 4, 4, 4], [4, 4, 6, 4, 4, 6, 6, 6]]

3, "range", [3, 5], [[5, 5, 3, 5, 5, 3, 3, 3], [3, 3, 5, 3, 3, 5, 5, 5]]

4, "range", [1, 7], [[7, 7, 1, 7, 7, 1, 1, 1], [1, 1, 7, 1, 1, 7, 7, 7]]

5, "partition", {{1, 2, 3, 6}, {4, 5, 7, 8}}

1, "range", [2, 8], [[8, 8, 8, 2, 2, 8, 2, 2], [2, 2, 2, 8, 8, 2, 8, 8]]

2, "range", [4, 6], [[6, 6, 6, 4, 4, 6, 4, 4], [4, 4, 4, 6, 6, 4, 6, 6]]

3, "range", [3, 5], [[5, 5, 5, 3, 3, 5, 3, 3], [3, 3, 3, 5, 5, 3, 5, 5]]

4, "range", [1, 7], [[7, 7, 7, 1, 1, 7, 1, 1], [1, 1, 1, 7, 7, 1, 7, 7]]

6, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [2, 8], [[8, 8, 8, 8, 2, 2, 2, 2], [2, 2, 2, 2, 8, 8, 8, 8]]

2, "range", [4, 6], [[6, 6, 6, 6, 4, 4, 4, 4], [4, 4, 4, 4, 6, 6, 6, 6]]

3, "range", [3, 5], [[5, 5, 5, 5, 3, 3, 3, 3], [3, 3, 3, 3, 5, 5, 5, 5]]

4, "range", [1, 7], [[7, 7, 7, 7, 1, 1, 1, 1], [1, 1, 1, 1, 7, 7, 7, 7]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

{6, 13, 15, 20}

$u2 =$
$$\Big(2 \quad 4 \quad 2 \quad 3 \quad 5 \quad 7 \quad 5 \quad 4 \quad 4 \quad 3 \quad 3 \quad 5 \quad 7 \quad 5 \quad 7 \quad 2 \quad 3 \quad 3 \quad 2 \quad 7 \quad 5 \quad 3 \quad 5 \quad 4 \quad 4 \quad 2 \quad 4$$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{5}{7} & 0 & 0 & 0 & 0 & 0 & \frac{2}{7} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{7} & 0 & 0 & 0 & 0 & 0 & \frac{4}{7} \\ 0 & \frac{3}{7} & 0 & 0 & 0 & 0 & 0 & \frac{4}{7} \\ 0 & \frac{4}{7} & 0 & 0 & 0 & 0 & 0 & \frac{3}{7} \\ 0 & \frac{4}{7} & 0 & 0 & 0 & 0 & 0 & \frac{3}{7} \\ 0 & \frac{2}{7} & 0 & 0 & 0 & 0 & 0 & \frac{5}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{5}{7} & 0 & \frac{2}{7} & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{7} & 0 & \frac{4}{7} & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{7} & 0 & \frac{5}{7} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{7} & 0 & \frac{2}{7} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{7} & 0 & \frac{5}{7} & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{7} & 0 & \frac{3}{7} & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{3}{7} & 0 & \frac{4}{7} & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{7} & 0 & \frac{4}{7} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{7} & 0 & \frac{5}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{7} & 0 & \frac{2}{7} & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{7} & 0 & \frac{3}{7} & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{7} & 0 & \frac{3}{7} & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{5}{7} & 0 & 0 & 0 & 0 & 0 & \frac{2}{7} & 0 \\ \frac{3}{7} & 0 & 0 & 0 & 0 & 0 & \frac{4}{7} & 0 \\ \frac{5}{7} & 0 & 0 & 0 & 0 & 0 & \frac{2}{7} & 0 \\ \frac{4}{7} & 0 & 0 & 0 & 0 & 0 & \frac{3}{7} & 0 \\ \frac{2}{7} & 0 & 0 & 0 & 0 & 0 & \frac{5}{7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{2}{7} & 0 & 0 & 0 & 0 & 0 & \frac{5}{7} & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{5}{28} & \frac{3}{28} & \frac{5}{28} & \frac{1}{7} & \frac{1}{14} & 0 & \frac{1}{14} \\ \frac{5}{28} & \frac{1}{4} & \frac{3}{28} & \frac{3}{28} & \frac{1}{7} & \frac{1}{7} & \frac{1}{14} & 0 \\ \frac{3}{28} & \frac{3}{28} & \frac{1}{4} & \frac{1}{14} & 0 & \frac{5}{28} & \frac{1}{7} & \frac{1}{7} \\ \frac{5}{28} & \frac{3}{28} & \frac{1}{14} & \frac{1}{4} & \frac{5}{28} & 0 & \frac{1}{14} & \frac{1}{7} \\ \frac{1}{7} & \frac{1}{7} & 0 & \frac{5}{28} & \frac{1}{4} & \frac{1}{14} & \frac{3}{28} & \frac{3}{28} \\ \frac{1}{14} & \frac{1}{7} & \frac{5}{28} & 0 & \frac{1}{14} & \frac{1}{4} & \frac{5}{28} & \frac{3}{28} \\ 0 & \frac{1}{14} & \frac{1}{7} & \frac{1}{14} & \frac{3}{28} & \frac{5}{28} & \frac{1}{4} & \frac{5}{28} \\ \frac{1}{14} & 0 & \frac{1}{7} & \frac{1}{7} & \frac{3}{28} & \frac{3}{28} & \frac{5}{28} & \frac{1}{4} \end{pmatrix} \quad NM =$$

$$\begin{pmatrix} 4 & \frac{20}{7} & \frac{12}{7} & \frac{20}{7} & \frac{16}{7} & \frac{8}{7} & 0 & \frac{8}{7} \\ \frac{20}{7} & 4 & \frac{12}{7} & \frac{12}{7} & \frac{16}{7} & \frac{16}{7} & \frac{8}{7} & 0 \\ \frac{12}{7} & \frac{12}{7} & 4 & \frac{8}{7} & 0 & \frac{20}{7} & \frac{16}{7} & \frac{16}{7} \\ \frac{20}{7} & \frac{12}{7} & \frac{8}{7} & 4 & \frac{20}{7} & 0 & \frac{8}{7} & \frac{16}{7} \\ \frac{16}{7} & \frac{16}{7} & 0 & \frac{20}{7} & 4 & \frac{8}{7} & \frac{12}{7} & \frac{12}{7} \\ \frac{8}{7} & \frac{16}{7} & \frac{20}{7} & 0 & \frac{8}{7} & 4 & \frac{20}{7} & \frac{12}{7} \\ 0 & \frac{8}{7} & \frac{16}{7} & \frac{8}{7} & \frac{12}{7} & \frac{20}{7} & 4 & \frac{20}{7} \\ \frac{8}{7} & 0 & \frac{16}{7} & \frac{16}{7} & \frac{12}{7} & \frac{12}{7} & \frac{20}{7} & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 0, 1, -1, 1, -1, 0, 0]$

$$\ker N_C = \begin{pmatrix} 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & -1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -t & -s & s & t & s & t & -t & -s \\ -t & -s+t & s & 0 & s & 0 & -t & -s+t \\ -t+s & -s & 0 & t & 0 & t & -t+s & -s \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 0 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -s & t & 0 \\ 0 & -s & 0 & t \\ -t & 0 & 0 & s \\ -s & 0 & t & 0 \\ t & 0 & 0 & -s \\ s & 0 & -t & 0 \\ 0 & s & -t & 0 \\ 0 & s & 0 & -t \end{pmatrix}$$ RB checks

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & -t & t & 0 & s+t \\ 0 & 0 & t & -t & s+t \\ t & 0 & s & -s & s \\ s & -t & t & 0 & t \\ -t & 0 & t & s & t \\ -s & t & s & 0 & s \\ 0 & t & s & 0 & 0 \\ 0 & 0 & s & t & 0 \end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 4 & 0 & 4 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & 1 & \frac{4}{7} & \frac{3}{7} & \frac{3}{7} & \frac{4}{7} & 0 & 0 \\ 1 & 1 & \frac{4}{7} & \frac{3}{7} & \frac{3}{7} & \frac{4}{7} & 0 & 0 \\ \frac{4}{7} & \frac{4}{7} & 1 & \frac{2}{7} & 0 & \frac{5}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{2}{7} & 1 & \frac{5}{7} & 0 & \frac{4}{7} & \frac{4}{7} \\ \frac{3}{7} & \frac{3}{7} & 0 & \frac{5}{7} & 1 & \frac{2}{7} & \frac{4}{7} & \frac{4}{7} \\ \frac{4}{7} & \frac{4}{7} & \frac{5}{7} & 0 & \frac{2}{7} & 1 & \frac{3}{7} & \frac{3}{7} \\ 0 & 0 & \frac{3}{7} & \frac{4}{7} & \frac{4}{7} & \frac{3}{7} & 1 & 1 \\ 0 & 0 & \frac{3}{7} & \frac{4}{7} & \frac{4}{7} & \frac{3}{7} & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{3}{7} & \frac{2}{7} & 1 & \frac{5}{7} & 0 & 0 & \frac{4}{7} \\ \frac{3}{7} & 1 & \frac{2}{7} & \frac{3}{7} & \frac{5}{7} & \frac{4}{7} & \frac{4}{7} & 0 \\ \frac{2}{7} & \frac{2}{7} & 1 & \frac{2}{7} & 0 & \frac{5}{7} & \frac{5}{7} & \frac{5}{7} \\ 1 & \frac{3}{7} & \frac{2}{7} & 1 & \frac{5}{7} & 0 & 0 & \frac{4}{7} \\ \frac{5}{7} & \frac{5}{7} & 0 & \frac{5}{7} & 1 & \frac{2}{7} & \frac{2}{7} & \frac{2}{7} \\ 0 & \frac{4}{7} & \frac{5}{7} & 0 & \frac{2}{7} & 1 & 1 & \frac{3}{7} \\ 0 & \frac{4}{7} & \frac{5}{7} & 0 & \frac{2}{7} & 1 & 1 & \frac{3}{7} \\ \frac{4}{7} & 0 & \frac{5}{7} & \frac{4}{7} & \frac{2}{7} & \frac{3}{7} & \frac{3}{7} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew}$$

$$\mathrm{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&0&0&0&4&0\\ 0&4&0&0&0&0&0&4\\ 0&0&4&0&4&0&0&0\\ 0&0&0&4&0&4&0&0\\ 0&0&4&0&4&0&0&0\\ 0&0&0&4&0&4&0&0\\ 4&0&0&0&0&0&4&0\\ 0&4&0&0&0&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{5}{7}&\frac{3}{7}&\frac{5}{7}&\frac{4}{7}&\frac{2}{7}&0&\frac{2}{7}\\ \frac{5}{7}&1&\frac{3}{7}&\frac{3}{7}&\frac{4}{7}&\frac{4}{7}&\frac{2}{7}&0\\ \frac{3}{7}&\frac{3}{7}&1&\frac{2}{7}&0&\frac{5}{7}&\frac{4}{7}&\frac{4}{7}\\ \frac{5}{7}&\frac{3}{7}&\frac{2}{7}&1&\frac{5}{7}&0&\frac{2}{7}&\frac{4}{7}\\ \frac{4}{7}&\frac{4}{7}&0&\frac{5}{7}&1&\frac{2}{7}&\frac{3}{7}&\frac{3}{7}\\ \frac{2}{7}&\frac{4}{7}&\frac{5}{7}&0&\frac{2}{7}&1&\frac{5}{7}&\frac{3}{7}\\ 0&\frac{2}{7}&\frac{4}{7}&\frac{2}{7}&\frac{3}{7}&\frac{5}{7}&1&\frac{5}{7}\\ \frac{2}{7}&0&\frac{4}{7}&\frac{4}{7}&\frac{3}{7}&\frac{3}{7}&\frac{5}{7}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{1}{14}\ \frac{3}{28}\ \frac{5}{28}\ \frac{1}{14}\ \frac{1}{4}\ \frac{3}{28}\ \frac{3}{28}\ \frac{3}{28}\ \frac{3}{28}\ \frac{1}{4}\ \frac{5}{28}\ \frac{1}{14}\ 0\ \frac{1}{14}\ \frac{1}{7}\ \frac{5}{28}\ \frac{3}{28}\ \frac{5}{28}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM}\left(4\ \frac{8}{7}\ \frac{12}{7}\ \frac{20}{7}\ \frac{8}{7}\ 4\ \frac{12}{7}\ \frac{12}{7}\ \frac{12}{7}\ \frac{12}{7}\ 4\ \frac{20}{7}\ \frac{8}{7}\ 0\ \frac{8}{7}\ \frac{16}{7}\ \frac{20}{7}\ \frac{12}{7}\ \frac{20}{7}\ 4\right)$

"IS MN in Vec(K)?", true

$\mathrm{MN}\left(4\ \frac{8}{7}\ \frac{12}{7}\ \frac{20}{7}\ \frac{8}{7}\ 4\ \frac{12}{7}\ \frac{12}{7}\ \frac{12}{7}\ \frac{12}{7}\ 4\ \frac{20}{7}\ \frac{8}{7}\ 0\ \frac{8}{7}\ \frac{16}{7}\ \frac{20}{7}\ \frac{12}{7}\ \frac{20}{7}\ 4\right)$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 1/8$, min $\boldsymbol{\tau} = 8/1$, $\boldsymbol{\tau}$–check is positive? 24/1

max r = 8/1 , r–check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"PT4" = {{3, 6, 7, 8}, {1, 2, 4, 5}}

"PT5" = {{1, 2, 3, 6}, {4, 5, 7, 8}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {4, 6}

"RG3" = {3, 5}

"RG4" = {1, 7}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{33}{56} & \frac{17}{56} & \frac{33}{56} & \frac{25}{56} & \frac{9}{56} & \frac{-1}{8} & \frac{9}{56} \\ \frac{33}{56} & \frac{7}{8} & \frac{17}{56} & \frac{17}{56} & \frac{25}{56} & \frac{25}{56} & \frac{9}{56} & \frac{-1}{8} \\ \frac{17}{56} & \frac{17}{56} & \frac{7}{8} & \frac{9}{56} & \frac{-1}{8} & \frac{33}{56} & \frac{25}{56} & \frac{25}{56} \\ \frac{33}{56} & \frac{17}{56} & \frac{9}{56} & \frac{7}{8} & \frac{33}{56} & \frac{-1}{8} & \frac{9}{56} & \frac{25}{56} \\ \frac{25}{56} & \frac{25}{56} & \frac{-1}{8} & \frac{33}{56} & \frac{7}{8} & \frac{9}{56} & \frac{17}{56} & \frac{17}{56} \\ \frac{9}{56} & \frac{25}{56} & \frac{33}{56} & \frac{-1}{8} & \frac{9}{56} & \frac{7}{8} & \frac{33}{56} & \frac{17}{56} \\ \frac{-1}{8} & \frac{9}{56} & \frac{25}{56} & \frac{9}{56} & \frac{17}{56} & \frac{33}{56} & \frac{7}{8} & \frac{33}{56} \\ \frac{9}{56} & \frac{-1}{8} & \frac{25}{56} & \frac{25}{56} & \frac{17}{56} & \frac{17}{56} & \frac{33}{56} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{33}{49} & \frac{17}{49} & \frac{33}{49} & \frac{25}{49} & \frac{9}{49} & \frac{-1}{7} & \frac{9}{49} \\ \frac{33}{49} & 1 & \frac{17}{49} & \frac{17}{49} & \frac{25}{49} & \frac{25}{49} & \frac{9}{49} & \frac{-1}{7} \\ \frac{17}{49} & \frac{17}{49} & 1 & \frac{9}{49} & \frac{-1}{7} & \frac{33}{49} & \frac{25}{49} & \frac{25}{49} \\ \frac{33}{49} & \frac{17}{49} & \frac{9}{49} & 1 & \frac{33}{49} & \frac{-1}{7} & \frac{9}{49} & \frac{25}{49} \\ \frac{25}{49} & \frac{25}{49} & \frac{-1}{7} & \frac{33}{49} & 1 & \frac{9}{49} & \frac{17}{49} & \frac{17}{49} \\ \frac{9}{49} & \frac{25}{49} & \frac{33}{49} & \frac{-1}{7} & \frac{9}{49} & 1 & \frac{33}{49} & \frac{17}{49} \\ \frac{-1}{7} & \frac{9}{49} & \frac{25}{49} & \frac{9}{49} & \frac{17}{49} & \frac{33}{49} & 1 & \frac{33}{49} \\ \frac{9}{49} & \frac{-1}{7} & \frac{25}{49} & \frac{25}{49} & \frac{17}{49} & \frac{17}{49} & \frac{33}{49} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 0.2466909938, 1.753309006, 0.7884097369, 1.211590263]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 3.428571429, 0.2819325648, 2.003781721, 0.9010396996, 1.384674586]

NullSpace M_C

{[0, 0, −1, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [1, 1, 1, 1, 0, 0, 0, 0], [1, 1, 1, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0] }

NullSpace N_c

{[0, 1, 0, −1, 0, −1, 0, 1], [0, 0, 1, −1, 1, −1, 0, 0], [1, 0, 0, −1, 0, −1, 1, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 0.2466909938, 1.753309006, 0.7884097369, 1.211590263]

NullSpace M_0

{[0, 0, −1, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, 0, −1, 0, 1, 0, 0]}

NullSpace N_0

{[0, 1, 0, −1, 0, −1, 0, 1], [1, 0, 0, −1, 0, −1, 1, 0], [0, 0, 1, −1, 1, −1, 0, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −1.753309006, −0.2466909938, −1.211590263, −0.7884097369]

NullSpace M

{}

NullSpace N

{[−1, 1, 0, 0, 0, 0, −1, 1], [−1, 0, 0, 1, 0, 1, −1, 0], [−1, 0, 1, 0, 1, 0, −1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 4 & 2 & 3 & 5 & 7 & 5 \\ 2 & 0 & 4 & 4 & 3 & 3 & 5 & 7 \\ 4 & 4 & 0 & 5 & 7 & 2 & 3 & 3 \\ 2 & 4 & 5 & 0 & 2 & 7 & 5 & 3 \\ 3 & 3 & 7 & 2 & 0 & 5 & 4 & 4 \\ 5 & 3 & 2 & 7 & 5 & 0 & 2 & 4 \\ 7 & 5 & 3 & 5 & 4 & 2 & 0 & 2 \\ 5 & 7 & 3 & 3 & 4 & 4 & 2 & 0 \end{pmatrix}$$

==

{3, 7}

R: [3, 3, 8, 1, 7, 7, 4, 5]
B: [6, 8, 1, 6, 2, 4, 5, 2]

TRACE TWO = 1

det AT = $\frac{1}{16}(t)^2(-1+t)^2(1+t)^2$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{1}{1073741824}(-1+s)(-6262-3387s-2864s^2-313s^3+627$

$s^4 + 394s^5 + 323s^6 - 8s^7 - 13s^8 - 15s^9 - 3s^{10} + s^{11}$) ($5412 + 180s + 1452 s^2 + 101s^3 - 101s^4 + 20s^5 - 28s^6 + 3s^7 + s^8$)

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 3, "vs", 6

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 1, "vs", 4

"R CYCLES", 1 + v[1] v[3] v[4] v[5] v[8] v[7]
"B CYCLES", (1 + v[2] v[8]) (1 + v[4] v[6])

Eigenvalues

R: [0., 0., −1., 1., −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, −0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I]

B: [1., −1., 1., −1., 0., 0., 0., 0.]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&0&0&4&0&0&0\\ 0&0&0&0&0&4&0&0\\ 0&0&0&0&0&0&4&0\\ 0&0&0&0&0&0&0&4\\ 4&0&0&0&0&0&0&0\\ 0&4&0&0&0&0&0&0\\ 0&0&4&0&0&0&0&0\\ 0&0&0&4&0&0&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&\frac{1}{4}&\frac{1}{2}&\frac{1}{4}&1&\frac{3}{4}&\frac{1}{2}&\frac{3}{4}\\ \frac{1}{4}&0&\frac{1}{2}&\frac{1}{2}&\frac{3}{4}&1&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&0&\frac{3}{4}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{4}\\ \frac{1}{4}&\frac{1}{2}&\frac{3}{4}&0&\frac{3}{4}&\frac{1}{2}&\frac{1}{4}&1\\ 1&\frac{3}{4}&\frac{1}{2}&\frac{3}{4}&0&\frac{1}{4}&\frac{1}{2}&\frac{1}{4}\\ \frac{3}{4}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{4}&0&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&1&\frac{1}{4}&\frac{1}{2}&\frac{1}{2}&0&\frac{3}{4}\\ \frac{3}{4}&\frac{1}{2}&\frac{1}{4}&1&\frac{1}{4}&\frac{1}{2}&\frac{3}{4}&0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8] \right)$

degree 2: $\frac{1}{4}\left(v[1]v[5] + v[2]v[6] + v[3]v[7] + v[4]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 3, 8}, {4, 5, 6, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT3" = {{2, 5, 7, 8}, {1, 3, 4, 6}}

"PT4" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"PT5" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$\pi 2$ = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {4, 11, 17, 22}

$u2$ = [1, 2, 1, 4, 3, 2, 3, 2, 2, 3, 4, 2, 2, 3, 2, 2, 4, 1, 3, 2, 1, 4, 1, 2, 1, 2, 2, 3]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[4], [1], [2], [2]]
Action of B on ranges, [[3], [4], [1], [3]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

RPARTS [5, 1, 4, 1, 4]

BPARTS [2, 3, 3, 4, 4]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[8, 9, 9, 6, 1, 2, 5, 6, 5, 8]

B-BLOCKS,

[9, 8, 3, A, 7, 4, 3, 9, 8, A]

with invariant measure, [1, 1, 1, 1, 2, 2, 1, 3, 3, 1]

N by blocks, N - check: true

b_1 = {1, 2, 3, 4}
b_2 = {5, 6, 7, 8}
b_3 = {2, 5, 7, 8}
b_4 = {1, 4, 6, 7}
b_5 = {1, 2, 3, 8}
b_6 = {4, 5, 6, 7}

$b_7 = \{2, 3, 5, 8\}$
$b_8 = \{1, 2, 4, 7\}$
$b_9 = \{3, 5, 6, 8\}$
$b_{10} = \{1, 3, 4, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \\ h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: 23 ⊕ 4/3

$$\text{CLB} = \begin{pmatrix} 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3, 4, 5, 7, 8}}, true

Ω_B in Vec(K)? , {{2, 8}, {4, 6}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} \\ \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} \\ \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} \\ \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} & \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} \\ \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 3, 8}, {4, 5, 6, 7}}

1, "range", [4, 8], [[8, 8, 8, 4, 4, 4, 4, 8], [4, 4, 4, 8, 8, 8, 8, 4]]

2, "range", [3, 7], [[7, 7, 7, 3, 3, 3, 3, 7], [3, 3, 3, 7, 7, 7, 7, 3]]

3, "range", [2, 6], [[6, 6, 6, 2, 2, 2, 2, 6], [2, 2, 2, 6, 6, 6, 6, 2]]

4, "range", [1, 5], [[5, 5, 5, 1, 1, 1, 1, 5], [1, 1, 1, 5, 5, 5, 5, 1]]

2, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

2, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

3, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

3, "partition", {{2, 5, 7, 8}, {1, 3, 4, 6}}

1, "range", [4, 8], [[8, 4, 8, 8, 4, 8, 4, 4], [4, 8, 4, 4, 8, 4, 8, 8]]

2, "range", [3, 7], [[7, 3, 7, 7, 3, 7, 3, 3], [3, 7, 3, 3, 7, 3, 7, 7]]

3, "range", [2, 6], [[6, 2, 6, 6, 2, 6, 2, 2], [2, 6, 2, 2, 6, 2, 6, 6]]

4, "range", [1, 5], [[5, 1, 5, 5, 1, 5, 1, 1], [1, 5, 1, 1, 5, 1, 5, 5]]

4, "partition", {{1, 2, 4, 7}, {3, 5, 6, 8}}

1, "range", [4, 8], [[8, 8, 4, 8, 4, 4, 8, 4], [4, 4, 8, 4, 8, 8, 4, 8]]

2, "range", [3, 7], [[7, 7, 3, 7, 3, 3, 7, 3], [3, 3, 7, 3, 7, 7, 3, 7]]

3, "range", [2, 6], [[6, 6, 2, 6, 2, 2, 6, 2], [2, 2, 6, 2, 6, 6, 2, 6]]

4, "range", [1, 5], [[5, 5, 1, 5, 1, 1, 5, 1], [1, 1, 5, 1, 5, 5, 1, 5]]

5, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [4, 8], [[8, 8, 8, 8, 4, 4, 4, 4], [4, 4, 4, 4, 8, 8, 8, 8]]

2, "range", [3, 7], [[7, 7, 7, 7, 3, 3, 3, 3], [3, 3, 3, 3, 7, 7, 7, 7]]

3, "range", [2, 6], [[6, 6, 6, 6, 2, 2, 2, 2], [2, 2, 2, 2, 6, 6, 6, 6]]

4, "range", [1, 5], [[5, 5, 5, 5, 1, 1, 1, 1], [1, 1, 1, 1, 5, 5, 5, 5]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n–choose–rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2$ =
$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 11, 17, 22}

$u2$ =
$(1 \;\; 2 \;\; 1 \;\; 4 \;\; 3 \;\; 2 \;\; 3 \;\; 2 \;\; 2 \;\; 3 \;\; 4 \;\; 2 \;\; 2 \;\; 3 \;\; 2 \;\; 2 \;\; 4 \;\; 1 \;\; 3 \;\; 2 \;\; 1 \;\; 4 \;\; 1 \;\; 2 \;\; 1 \;\; 2 \;\; 2$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

idem-checks   N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \end{pmatrix}$$

idem-checks   N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & 0 & \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \\ \frac{3}{16} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} & 0 & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{16} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{3}{16} \\ \frac{3}{16} & \frac{1}{8} & \frac{1}{16} & \frac{1}{4} & \frac{1}{16} & \frac{1}{8} & \frac{3}{16} & 0 \\ 0 & \frac{1}{16} & \frac{1}{8} & \frac{1}{16} & \frac{1}{4} & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} \\ \frac{1}{16} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & 0 & \frac{3}{16} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{8} & \frac{3}{16} & 0 & \frac{3}{16} & \frac{1}{8} & \frac{1}{16} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 3 & 2 & 3 & 0 & 1 & 2 & 1 \\ 3 & 4 & 2 & 2 & 1 & 0 & 2 & 2 \\ 2 & 2 & 4 & 1 & 2 & 2 & 0 & 3 \\ 3 & 2 & 1 & 4 & 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 1 & 4 & 3 & 2 & 3 \\ 1 & 0 & 2 & 2 & 3 & 4 & 2 & 2 \\ 2 & 2 & 0 & 3 & 2 & 2 & 4 & 1 \\ 1 & 2 & 3 & 0 & 3 & 2 & 1 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, -1, 1, 0, 0, -1, 1, 0]$

$$\ker N_c = \begin{pmatrix} -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} t & -t & -s & s & t & -t & -s & s \\ s & 0 & -s & 0 & s & 0 & -s & 0 \\ 0 & -t & 0 & t & 0 & -t & 0 & t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix}1 & 0 & -1\end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & -t \\ 0 & -t & s & 0 \\ t & -s & 0 & 0 \\ s & 0 & 0 & -t \\ 0 & 0 & -s & t \\ 0 & t & -s & 0 \\ -t & s & 0 & 0 \\ -s & 0 & 0 & t \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & t & 0 & s \\ 0 & t & 0 & 0 & s \\ t & s & 0 & 0 & 0 \\ s & 0 & t & 0 & 0 \\ t+s & 0 & -t & t+s & -s \\ t+s & -t & 0 & t+s & -s \\ s & -s & 0 & t+s & 0 \\ t & 0 & -t & t+s & 0 \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix}4 & 0 & 0 & 4 & 0\end{pmatrix}$

$$
R N_0 R^* = \begin{pmatrix}
1 & 1 & \frac{3}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{4} & \frac{1}{2} \\
1 & 1 & \frac{3}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{4} & \frac{1}{2} \\
\frac{3}{4} & \frac{3}{4} & 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{3}{4} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & 0 \\
0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 1 & \frac{3}{4} & \frac{1}{2} \\
0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 1 & \frac{3}{4} & \frac{1}{2} \\
\frac{1}{4} & \frac{1}{4} & 0 & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & 1 & \frac{1}{4} \\
\frac{1}{2} & \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 1
\end{pmatrix}, R R^* = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
$$

$$
B N_0 B^* = \begin{pmatrix}
1 & \frac{1}{2} & \frac{1}{4} & 1 & 0 & \frac{1}{2} & \frac{3}{4} & 0 \\
\frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{3}{4} & \frac{1}{2} \\
\frac{1}{4} & \frac{1}{4} & 1 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & 0 & \frac{3}{4} \\
1 & \frac{1}{2} & \frac{1}{4} & 1 & 0 & \frac{1}{2} & \frac{3}{4} & 0 \\
0 & \frac{1}{2} & \frac{3}{4} & 0 & 1 & \frac{1}{2} & \frac{1}{4} & 1 \\
\frac{1}{2} & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{2} \\
\frac{3}{4} & \frac{3}{4} & 0 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & 1 & \frac{1}{4} \\
0 & \frac{1}{2} & \frac{3}{4} & 0 & 1 & \frac{1}{2} & \frac{1}{4} & 1
\end{pmatrix}, B B^* = \begin{pmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1
\end{pmatrix}
$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew}$$

$$\text{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&0&4&0&0&0\\ 0&4&0&0&0&4&0&0\\ 0&0&4&0&0&0&4&0\\ 0&0&0&4&0&0&0&4\\ 4&0&0&0&4&0&0&0\\ 0&4&0&0&0&4&0&0\\ 0&0&4&0&0&0&4&0\\ 0&0&0&4&0&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{3}{4}&\frac{1}{2}&\frac{3}{4}&0&\frac{1}{4}&\frac{1}{2}&\frac{1}{4}\\ \frac{3}{4}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{4}&0&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&1&\frac{1}{4}&\frac{1}{2}&\frac{1}{2}&0&\frac{3}{4}\\ \frac{3}{4}&\frac{1}{2}&\frac{1}{4}&1&\frac{1}{4}&\frac{1}{2}&\frac{3}{4}&0\\ 0&\frac{1}{4}&\frac{1}{2}&\frac{1}{4}&1&\frac{3}{4}&\frac{1}{2}&\frac{3}{4}\\ \frac{1}{4}&0&\frac{1}{2}&\frac{1}{2}&\frac{3}{4}&1&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&0&\frac{3}{4}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{4}\\ \frac{1}{4}&\frac{1}{2}&\frac{3}{4}&0&\frac{3}{4}&\frac{1}{2}&\frac{1}{4}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$$

$$T\left(\frac{1}{4}\ \frac{1}{16}\ \frac{1}{8}\ \frac{3}{16}\ \frac{1}{16}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{4}\ \frac{3}{16}\ \frac{1}{16}\ \frac{1}{8}\ \frac{1}{16}\ 0\ \frac{3}{16}\ \frac{1}{8}\ \frac{3}{16}\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

NM $(4\ \ 1\ \ 2\ \ 3\ \ 1\ \ 4\ \ 2\ \ 2\ \ 2\ \ 2\ \ 4\ \ 3\ \ 1\ \ 2\ \ 1\ \ 0\ \ 3\ \ 2\ \ 3\ \ 4)$

"IS MN in Vec(K)?", true

MN $(4\ \ 1\ \ 2\ \ 3\ \ 1\ \ 4\ \ 2\ \ 2\ \ 2\ \ 2\ \ 4\ \ 3\ \ 1\ \ 2\ \ 1\ \ 0\ \ 3\ \ 2\ \ 3\ \ 4)$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 24/1$$

$$\text{max r} = 8/1 \text{ , } r\text{–check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 5, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 40

dim span idems 16 vs no. of idems 20

"PT1" = {{1, 2, 3, 8}, {4, 5, 6, 7}}

"PT2" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT3" = {{2, 5, 7, 8}, {1, 3, 4, 6}}

"PT4" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"PT5" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$$M_C = \begin{pmatrix} 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \\ 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{5}{8} & \frac{3}{8} & \frac{5}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{1}{8} \\ \frac{5}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{5}{8} \\ \frac{5}{8} & \frac{3}{8} & \frac{1}{8} & \frac{7}{8} & \frac{1}{8} & \frac{3}{8} & \frac{5}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{1}{8} & \frac{7}{8} & \frac{5}{8} & \frac{3}{8} & \frac{5}{8} \\ \frac{1}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{5}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{5}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{3}{8} & \frac{5}{8} & \frac{-1}{8} & \frac{5}{8} & \frac{3}{8} & \frac{1}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{5}{7} & \frac{3}{7} & \frac{5}{7} & \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} & \frac{1}{7} \\ \frac{5}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & 1 & \frac{1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{5}{7} \\ \frac{5}{7} & \frac{3}{7} & \frac{1}{7} & 1 & \frac{1}{7} & \frac{3}{7} & \frac{5}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} & \frac{1}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{5}{7} \\ \frac{1}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{5}{7} & 1 & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{5}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} & \frac{5}{7} & \frac{-1}{7} & \frac{5}{7} & \frac{3}{7} & \frac{1}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 1.309016994, 0.1909830058, 1.809016994, 0.6909830058]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 3.428571429, 1.496019422, 0.2182662923, 2.067447994, 0.7896948642]

NullSpace M_C

{[1, 1, 1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [1, 1, 1, 0, 0, 0, 0, 1]}

NullSpace N_c

{[−1, 0, 0, 1, −1, 0, 0, 1], [−1, 1, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, −1, 0, 1, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 1.309016994, 0.1909830058, 1.809016994, 0.6909830058]

NullSpace M_0

{[0, 0, −1, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, −1]}

NullSpace N_0

{[−1, 0, 1, 0, −1, 0, 1, 0], [−1, 0, 0, 1, −1, 0, 0, 1], [−1, 1, 0, 0, −1, 1, 0, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −0.1909830058, −1.309016994, −0.6909830058, −1.809016994]

NullSpace M

{}

NullSpace N

{[0, 1, 0, −1, 0, 1, 0, −1], [0, 0, 1, −1, 0, 0, 1, −1], [1, 0, 0, −1, 1, 0, 0, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 4 & 3 & 2 & 3 \\ 1 & 0 & 2 & 2 & 3 & 4 & 2 & 2 \\ 2 & 2 & 0 & 3 & 2 & 2 & 4 & 1 \\ 1 & 2 & 3 & 0 & 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 3 & 0 & 1 & 2 & 1 \\ 3 & 4 & 2 & 2 & 1 & 0 & 2 & 2 \\ 2 & 2 & 4 & 1 & 2 & 2 & 0 & 3 \\ 3 & 2 & 1 & 4 & 1 & 2 & 3 & 0 \end{pmatrix}$$

==

20, [1, 1, 1, −1, −1, 1, 1, 1]

==

{4, 6}

R: [3, 3, 1, 6, 7, 4, 5, 5]
B: [6, 8, 8, 1, 2, 7, 4, 2]

TRACE TWO = 1

$$\det AT = \frac{1}{16}(1+t)^2 (t)^2 (-1+t)^2$$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{5}{1073741824}(-1+s)(-13332+7272s+5180s^2-2019s^3-553s^4+216s^5-20s^6-13s^7+5s^8)(2542-703s-1255s^2+13s^3+347s^4-76s^5-71s^6+34s^7+7s^8-5s^9-2s^{10}+s^{11})$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 2, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", (1 + v[1] v[3]) (1 + v[4] v[6]) (1 + v[5] v[7])
"B CYCLES", (1 + v[2] v[8]) (1 + v[1] v[4] v[6] v[7])

Eigenvalues

R: [0., 0., 1., -1., 1., -1., 1., -1.]

B: [1. I, -1. I, 0., 0., 1., -1., 1., -1.]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[-1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, -1, 1]}

NullSpace of B^*

{[0, -1, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, -1, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&0&0&0&0&4&0\\ 0&0&0&0&0&0&0&4\\ 0&0&0&0&4&0&0&0\\ 0&0&0&0&0&4&0&0\\ 0&0&4&0&0&0&0&0\\ 0&0&0&4&0&0&0&0\\ 4&0&0&0&0&0&0&0\\ 0&4&0&0&0&0&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&\frac{3}{11}&\frac{6}{11}&\frac{5}{11}&\frac{5}{11}&\frac{6}{11}&1&\frac{8}{11}\\ \frac{3}{11}&0&\frac{3}{11}&\frac{6}{11}&\frac{8}{11}&\frac{5}{11}&\frac{8}{11}&1\\ \frac{6}{11}&\frac{3}{11}&0&\frac{7}{11}&1&\frac{4}{11}&\frac{5}{11}&\frac{8}{11}\\ \frac{5}{11}&\frac{6}{11}&\frac{7}{11}&0&\frac{4}{11}&1&\frac{6}{11}&\frac{5}{11}\\ \frac{5}{11}&\frac{8}{11}&1&\frac{4}{11}&0&\frac{7}{11}&\frac{6}{11}&\frac{3}{11}\\ \frac{6}{11}&\frac{5}{11}&\frac{4}{11}&1&\frac{7}{11}&0&\frac{5}{11}&\frac{6}{11}\\ 1&\frac{8}{11}&\frac{5}{11}&\frac{6}{11}&\frac{6}{11}&\frac{5}{11}&0&\frac{3}{11}\\ \frac{8}{11}&1&\frac{8}{11}&\frac{5}{11}&\frac{3}{11}&\frac{6}{11}&\frac{3}{11}&0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 6, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[7] + v[2]v[8] + v[3]v[5] + v[4]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT2" = {{1, 5, 6, 8}, {2, 3, 4, 7}}

"PT3" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"PT4" = {{3, 6, 7, 8}, {1, 2, 4, 5}}

"PT5" = {{1, 2, 3, 6}, {4, 5, 7, 8}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {3, 5}

"RG3" = {4, 6}

"RG4" = {1, 7}

$\pi 2$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {6, 13, 15, 20}

$u2$ = [3, 6, 5, 5, 6, 11, 8, 3, 6, 8, 5, 8, 11, 7, 11, 4, 5, 8, 4, 11, 6, 5, 7, 6, 3, 5, 6, 3]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[2], [4], [3], [2]]
Action of B on ranges, [[1], [1], [4], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [4, 3, 3, 4, 6, 5]

BPARTS [2, 6, 1, 5, 1, 5]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{11} & \frac{1}{11} & \frac{1}{11} & \frac{2}{11} & \frac{3}{11} & \frac{2}{11} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[3, C, 1, B, 8, 5, 8, 5, 4, B, 4, 2]

B–BLOCKS,

[C, 3, 6, 7, 3, A, 9, C, 1, 2, 6, 7]

with invariant measure, [2, 2, 3, 1, 2, 2, 2, 2, 1, 1, 1, 3]

N by blocks, N – check: true

b_1 = {1, 2, 3, 4}
b_2 = {5, 6, 7, 8}
b_3 = {1, 2, 3, 6}
b_4 = {3, 4, 7, 8}

b_5 = {3, 6, 7, 8}
b_6 = {1, 4, 5, 8}
b_7 = {2, 3, 6, 7}
b_8 = {1, 2, 4, 5}
b_9 = {1, 5, 6, 8}
b_{10} = {2, 3, 4, 7}
b_{11} = {1, 2, 5, 6}
b_{12} = {4, 5, 7, 8}

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & 0 & 0 & 0 & h[2] & 0 \\ 0 & h[1] & 0 & 0 & 0 & 0 & 0 & h[2] \\ 0 & 0 & h[1] & 0 & h[2] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[1] & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[2] & 0 & h[1] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[2] & 0 & h[1] & 0 & 0 \\ h[2] & 0 & 0 & 0 & 0 & 0 & h[1] & 0 \\ 0 & h[2] & 0 & 0 & 0 & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: 23 ⊕ 4/3

$$\text{CLB} = \begin{pmatrix} 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 7}, {1, 3}, {4, 6}}, false

Ω_B in Vec(K)? , {{1, 4, 6, 7}, {2, 8}}, false

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} \end{pmatrix}$$

Omega−checks true, true

"I−V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{3}{16} & 0 & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{4} & 0 & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [2, 8], [[8, 2, 2, 8, 8, 2, 2, 8], [2, 8, 8, 2, 2, 8, 8, 2]]

2, "range", [3, 5], [[5, 3, 3, 5, 5, 3, 3, 5], [3, 5, 5, 3, 3, 5, 5, 3]]

3, "range", [4, 6], [[6, 4, 4, 6, 6, 4, 4, 6], [4, 6, 6, 4, 4, 6, 6, 4]]

4, "range", [1, 7], [[7, 1, 1, 7, 7, 1, 1, 7], [1, 7, 7, 1, 1, 7, 7, 1]]

2, "partition", {{1, 5, 6, 8}, {2, 3, 4, 7}}

1, "range", [2, 8], [[8, 2, 2, 2, 8, 8, 2, 8], [2, 8, 8, 8, 2, 2, 8, 2]]

2, "range", [3, 5], [[5, 3, 3, 3, 5, 5, 3, 5], [3, 5, 5, 5, 3, 3, 5, 3]]

3, "range", [4, 6], [[6, 4, 4, 4, 6, 6, 4, 6], [4, 6, 6, 6, 4, 4, 6, 4]]

4, "range", [1, 7], [[7, 1, 1, 1, 7, 7, 1, 7], [1, 7, 7, 7, 1, 1, 7, 1]]

3, "partition", {{3, 4, 7, 8}, {1, 2, 5, 6}}

1, "range", [2, 8], [[8, 8, 2, 2, 8, 8, 2, 2], [2, 2, 8, 8, 2, 2, 8, 8]]

2, "range", [3, 5], [[5, 5, 3, 3, 5, 5, 3, 3], [3, 3, 5, 5, 3, 3, 5, 5]]

3, "range", [4, 6], [[6, 6, 4, 4, 6, 6, 4, 4], [4, 4, 6, 6, 4, 4, 6, 6]]

4, "range", [1, 7], [[7, 7, 1, 1, 7, 7, 1, 1], [1, 1, 7, 7, 1, 1, 7, 7]]

4, "partition", {{3, 6, 7, 8}, {1, 2, 4, 5}}

1, "range", [2, 8], [[8, 8, 2, 8, 8, 2, 2, 2], [2, 2, 8, 2, 2, 8, 8, 8]]

2, "range", [3, 5], [[5, 5, 3, 5, 5, 3, 3, 3], [3, 3, 5, 3, 3, 5, 5, 5]]

3, "range", [4, 6], [[6, 6, 4, 6, 6, 4, 4, 4], [4, 4, 6, 4, 4, 6, 6, 6]]

4, "range", [1, 7], [[7, 7, 1, 7, 7, 1, 1, 1], [1, 1, 7, 1, 1, 7, 7, 7]]

5, "partition", {{1, 2, 3, 6}, {4, 5, 7, 8}}

1, "range", [2, 8], [[8, 8, 8, 2, 2, 8, 2, 2], [2, 2, 2, 8, 8, 2, 8, 8]]

2, "range", [3, 5], [[5, 5, 5, 3, 3, 5, 3, 3], [3, 3, 3, 5, 5, 3, 5, 5]]

3, "range", [4, 6], [[6, 6, 6, 4, 4, 6, 4, 4], [4, 4, 4, 6, 6, 4, 6, 6]]

4, "range", [1, 7], [[7, 7, 7, 1, 1, 7, 1, 1], [1, 1, 1, 7, 7, 1, 7, 7]]

6, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [2, 8], [[8, 8, 8, 8, 2, 2, 2, 2], [2, 2, 2, 2, 8, 8, 8, 8]]

2, "range", [3, 5], [[5, 5, 5, 5, 3, 3, 3, 3], [3, 3, 3, 3, 5, 5, 5, 5]]

3, "range", [4, 6], [[6, 6, 6, 6, 4, 4, 4, 4], [4, 4, 4, 4, 6, 6, 6, 6]]

4, "range", [1, 7], [[7, 7, 7, 7, 1, 1, 1, 1], [1, 1, 1, 1, 7, 7, 7, 7]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2$ =
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

{6, 13, 15, 20}

$u2$ =
$$\begin{pmatrix} 3 & 6 & 5 & 5 & 6 & 11 & 8 & 3 & 6 & 8 & 5 & 8 & 11 & 7 & 11 & 4 & 5 & 8 & 4 & 11 & 6 & 5 & 7 & 6 & 3 & 5 \end{pmatrix}$$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{11}{2} & \frac{11}{2} & \frac{11}{2} & \frac{11}{2} & \frac{11}{2} & \frac{11}{2} & \frac{11}{2} & \frac{11}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{8}{11} & 0 & 0 & 0 & 0 & 0 & \frac{3}{11} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{8}{11} & 0 & 0 & 0 & 0 & 0 & \frac{3}{11} \\ 0 & \frac{5}{11} & 0 & 0 & 0 & 0 & 0 & \frac{6}{11} \\ 0 & \frac{3}{11} & 0 & 0 & 0 & 0 & 0 & \frac{8}{11} \\ 0 & \frac{6}{11} & 0 & 0 & 0 & 0 & 0 & \frac{5}{11} \\ 0 & \frac{3}{11} & 0 & 0 & 0 & 0 & 0 & \frac{8}{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{5}{11} & 0 & \frac{6}{11} & 0 & 0 & 0 \\ 0 & 0 & \frac{8}{11} & 0 & \frac{3}{11} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{11} & 0 & \frac{7}{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{7}{11} & 0 & \frac{4}{11} & 0 & 0 & 0 \\ 0 & 0 & \frac{6}{11} & 0 & \frac{5}{11} & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{11} & 0 & \frac{8}{11} & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & \frac{6}{11} & 0 & \frac{5}{11} & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{11} & 0 & \frac{6}{11} & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{11} & 0 & \frac{7}{11} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{7}{11} & 0 & \frac{4}{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{11} & 0 & \frac{6}{11} & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{11} & 0 & \frac{5}{11} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{8}{11} & 0 & 0 & 0 & 0 & 0 & \frac{3}{11} & 0 \\ \frac{5}{11} & 0 & 0 & 0 & 0 & 0 & \frac{6}{11} & 0 \\ \frac{6}{11} & 0 & 0 & 0 & 0 & 0 & \frac{5}{11} & 0 \\ \frac{6}{11} & 0 & 0 & 0 & 0 & 0 & \frac{5}{11} & 0 \\ \frac{5}{11} & 0 & 0 & 0 & 0 & 0 & \frac{6}{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{3}{11} & 0 & 0 & 0 & 0 & 0 & \frac{8}{11} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{2}{11} & \frac{5}{44} & \frac{3}{22} & \frac{3}{22} & \frac{5}{44} & 0 & \frac{3}{44} \\ \frac{2}{11} & \frac{1}{4} & \frac{2}{11} & \frac{5}{44} & \frac{3}{44} & \frac{3}{22} & \frac{3}{44} & 0 \\ \frac{5}{44} & \frac{2}{11} & \frac{1}{4} & \frac{1}{11} & 0 & \frac{7}{44} & \frac{3}{22} & \frac{3}{44} \\ \frac{3}{22} & \frac{5}{44} & \frac{1}{11} & \frac{1}{4} & \frac{7}{44} & 0 & \frac{5}{44} & \frac{3}{22} \\ \frac{3}{22} & \frac{3}{44} & 0 & \frac{7}{44} & \frac{1}{4} & \frac{1}{11} & \frac{5}{44} & \frac{2}{11} \\ \frac{5}{44} & \frac{3}{22} & \frac{7}{44} & 0 & \frac{1}{11} & \frac{1}{4} & \frac{3}{22} & \frac{5}{44} \\ 0 & \frac{3}{44} & \frac{3}{22} & \frac{5}{44} & \frac{5}{44} & \frac{3}{22} & \frac{1}{4} & \frac{2}{11} \\ \frac{3}{44} & 0 & \frac{3}{44} & \frac{3}{22} & \frac{2}{11} & \frac{5}{44} & \frac{2}{11} & \frac{1}{4} \end{pmatrix} \quad NM =$$

$$\begin{pmatrix} 4 & \frac{32}{11} & \frac{20}{11} & \frac{24}{11} & \frac{24}{11} & \frac{20}{11} & 0 & \frac{12}{11} \\ \frac{32}{11} & 4 & \frac{32}{11} & \frac{20}{11} & \frac{12}{11} & \frac{24}{11} & \frac{12}{11} & 0 \\ \frac{20}{11} & \frac{32}{11} & 4 & \frac{16}{11} & 0 & \frac{28}{11} & \frac{24}{11} & \frac{12}{11} \\ \frac{24}{11} & \frac{20}{11} & \frac{16}{11} & 4 & \frac{28}{11} & 0 & \frac{20}{11} & \frac{24}{11} \\ \frac{24}{11} & \frac{12}{11} & 0 & \frac{28}{11} & 4 & \frac{16}{11} & \frac{20}{11} & \frac{32}{11} \\ \frac{20}{11} & \frac{24}{11} & \frac{28}{11} & 0 & \frac{16}{11} & 4 & \frac{24}{11} & \frac{20}{11} \\ 0 & \frac{12}{11} & \frac{24}{11} & \frac{20}{11} & \frac{20}{11} & \frac{24}{11} & 4 & \frac{32}{11} \\ \frac{12}{11} & 0 & \frac{12}{11} & \frac{24}{11} & \frac{32}{11} & \frac{20}{11} & \frac{32}{11} & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, -1, 1, 0, 1, 0, 0, -1]$

$$\ker N_C = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & -1 & 1 & 0 & 1 & 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & -t & 0 & t & 0 & t & 0 & -t \\ t & -t & -s & s & -s & s & t & -t \\ s & 0 & -s & 0 & -s & 0 & s & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -t & s \\ -t & 0 & 0 & s \\ -t & -s & 0 & 0 \\ 0 & -t & -s & 0 \\ t & s & 0 & 0 \\ 0 & t & s & 0 \\ 0 & 0 & t & -s \\ t & 0 & 0 & -s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & s & 0 & 0 & t \\ t & s & -t & 0 & t \\ t & 0 & -t & s & t \\ 0 & 0 & 0 & t & s \\ s & 0 & t & -s & s \\ s+t & 0 & 0 & -t & t \\ s+t & -s & 0 & 0 & s \\ s & -s & t & 0 & s \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 0 & 4 \end{pmatrix}$

$$R N_0 R^* = \begin{pmatrix} 1 & 1 & \frac{5}{11} & \frac{7}{11} & \frac{6}{11} & \frac{4}{11} & 0 & 0 \\ 1 & 1 & \frac{5}{11} & \frac{7}{11} & \frac{6}{11} & \frac{4}{11} & 0 & 0 \\ \frac{5}{11} & \frac{5}{11} & 1 & \frac{5}{11} & 0 & \frac{6}{11} & \frac{6}{11} & \frac{6}{11} \\ \frac{7}{11} & \frac{7}{11} & \frac{5}{11} & 1 & \frac{6}{11} & 0 & \frac{4}{11} & \frac{4}{11} \\ \frac{6}{11} & \frac{6}{11} & 0 & \frac{6}{11} & 1 & \frac{5}{11} & \frac{5}{11} & \frac{5}{11} \\ \frac{4}{11} & \frac{4}{11} & \frac{6}{11} & 0 & \frac{5}{11} & 1 & \frac{7}{11} & \frac{7}{11} \\ 0 & 0 & \frac{6}{11} & \frac{4}{11} & \frac{5}{11} & \frac{7}{11} & 1 & 1 \\ 0 & 0 & \frac{6}{11} & \frac{4}{11} & \frac{5}{11} & \frac{7}{11} & 1 & 1 \end{pmatrix}, R R^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & \frac{5}{11} & \frac{5}{11} & \frac{5}{11} & \frac{6}{11} & \frac{6}{11} & 0 & \frac{6}{11} \\ \frac{5}{11} & 1 & 1 & \frac{3}{11} & 0 & \frac{8}{11} & \frac{6}{11} & 0 \\ \frac{5}{11} & 1 & 1 & \frac{3}{11} & 0 & \frac{8}{11} & \frac{6}{11} & 0 \\ \frac{5}{11} & \frac{3}{11} & \frac{3}{11} & 1 & \frac{8}{11} & 0 & \frac{6}{11} & \frac{8}{11} \\ \frac{6}{11} & 0 & 0 & \frac{8}{11} & 1 & \frac{3}{11} & \frac{5}{11} & 1 \\ \frac{6}{11} & \frac{8}{11} & \frac{8}{11} & 0 & \frac{3}{11} & 1 & \frac{5}{11} & \frac{3}{11} \\ 0 & \frac{6}{11} & \frac{6}{11} & \frac{6}{11} & \frac{5}{11} & \frac{5}{11} & 1 & \frac{5}{11} \\ \frac{6}{11} & 0 & 0 & \frac{8}{11} & 1 & \frac{3}{11} & \frac{5}{11} & 1 \end{pmatrix}, B B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\,T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew}$$

$$\mathrm{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&0&0&0&4&0\\ 0&4&0&0&0&0&0&4\\ 0&0&4&0&4&0&0&0\\ 0&0&0&4&0&4&0&0\\ 0&0&4&0&4&0&0&0\\ 0&0&0&4&0&4&0&0\\ 4&0&0&0&0&0&4&0\\ 0&4&0&0&0&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{8}{11}&\frac{5}{11}&\frac{6}{11}&\frac{6}{11}&\frac{5}{11}&0&\frac{3}{11}\\ \frac{8}{11}&1&\frac{8}{11}&\frac{5}{11}&\frac{3}{11}&\frac{6}{11}&\frac{3}{11}&0\\ \frac{5}{11}&\frac{8}{11}&1&\frac{4}{11}&0&\frac{7}{11}&\frac{6}{11}&\frac{3}{11}\\ \frac{6}{11}&\frac{5}{11}&\frac{4}{11}&1&\frac{7}{11}&0&\frac{5}{11}&\frac{6}{11}\\ \frac{6}{11}&\frac{3}{11}&0&\frac{7}{11}&1&\frac{4}{11}&\frac{5}{11}&\frac{8}{11}\\ \frac{5}{11}&\frac{6}{11}&\frac{7}{11}&0&\frac{4}{11}&1&\frac{6}{11}&\frac{5}{11}\\ 0&\frac{3}{11}&\frac{6}{11}&\frac{5}{11}&\frac{5}{11}&\frac{6}{11}&1&\frac{8}{11}\\ \frac{3}{11}&0&\frac{3}{11}&\frac{6}{11}&\frac{8}{11}&\frac{5}{11}&\frac{8}{11}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$$

T

$$\left(\frac{1}{4}\ \frac{1}{11}\ \frac{5}{44}\ \frac{3}{22}\ \frac{1}{11}\ \frac{1}{4}\ \frac{2}{11}\ \frac{5}{44}\ \frac{5}{44}\ \frac{2}{11}\ \frac{1}{4}\ \frac{2}{11}\ \frac{3}{44}\ 0\ \frac{5}{44}\ \frac{3}{22}\ \frac{3}{22}\ \frac{5}{44}\ \frac{2}{11}\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

NM

$$\left(4\ \frac{16}{11}\ \frac{20}{11}\ \frac{24}{11}\ \frac{16}{11}\ 4\ \frac{32}{11}\ \frac{20}{11}\ \frac{20}{11}\ \frac{32}{11}\ 4\ \frac{32}{11}\ \frac{12}{11}\ 0\ \frac{20}{11}\ \frac{24}{11}\ \frac{24}{11}\ \frac{20}{11}\ \frac{32}{11}\ 4\right)$$

"IS MN in Vec(K)?", true

MN

$$\left(4\ \frac{16}{11}\ \frac{20}{11}\ \frac{24}{11}\ \frac{16}{11}\ 4\ \frac{32}{11}\ \frac{20}{11}\ \frac{20}{11}\ \frac{32}{11}\ 4\ \frac{32}{11}\ \frac{12}{11}\ 0\ \frac{20}{11}\ \frac{24}{11}\ \frac{24}{11}\ \frac{20}{11}\ \frac{32}{11}\ 4\right)$$

$$\boldsymbol{\tau} = 32/1\text{ , rank} = 2\text{ , ratio} = 16/1\text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1\text{ , } r' = 1/2\text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/8\text{ , min } \boldsymbol{\tau} = 8/1\text{ , } \boldsymbol{\tau}\text{-check is positive? } 24/1$$

$$\text{max r} = 8/1\text{ , } r\text{-check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT2" = {{1, 5, 6, 8}, {2, 3, 4, 7}}

"PT3" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"PT4" = {{3, 6, 7, 8}, {1, 2, 4, 5}}

"PT5" = {{1, 2, 3, 6}, {4, 5, 7, 8}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {3, 5}

"RG3" = {4, 6}

"RG4" = {1, 7}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{53}{88} & \frac{29}{88} & \frac{37}{88} & \frac{37}{88} & \frac{29}{88} & \frac{-1}{8} & \frac{13}{88} \\ \frac{53}{88} & \frac{7}{8} & \frac{53}{88} & \frac{29}{88} & \frac{13}{88} & \frac{37}{88} & \frac{13}{88} & \frac{-1}{8} \\ \frac{29}{88} & \frac{53}{88} & \frac{7}{8} & \frac{21}{88} & \frac{-1}{8} & \frac{45}{88} & \frac{37}{88} & \frac{13}{88} \\ \frac{37}{88} & \frac{29}{88} & \frac{21}{88} & \frac{7}{8} & \frac{45}{88} & \frac{-1}{8} & \frac{29}{88} & \frac{37}{88} \\ \frac{37}{88} & \frac{13}{88} & \frac{-1}{8} & \frac{45}{88} & \frac{7}{8} & \frac{21}{88} & \frac{29}{88} & \frac{53}{88} \\ \frac{29}{88} & \frac{37}{88} & \frac{45}{88} & \frac{-1}{8} & \frac{21}{88} & \frac{7}{8} & \frac{37}{88} & \frac{29}{88} \\ \frac{-1}{8} & \frac{13}{88} & \frac{37}{88} & \frac{29}{88} & \frac{29}{88} & \frac{37}{88} & \frac{7}{8} & \frac{53}{88} \\ \frac{13}{88} & \frac{-1}{8} & \frac{13}{88} & \frac{37}{88} & \frac{53}{88} & \frac{29}{88} & \frac{53}{88} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{53}{77} & \frac{29}{77} & \frac{37}{77} & \frac{37}{77} & \frac{29}{77} & \frac{-1}{7} & \frac{13}{77} \\ \frac{53}{77} & 1 & \frac{53}{77} & \frac{29}{77} & \frac{13}{77} & \frac{37}{77} & \frac{13}{77} & \frac{-1}{7} \\ \frac{29}{77} & \frac{53}{77} & 1 & \frac{3}{11} & \frac{-1}{7} & \frac{45}{77} & \frac{37}{77} & \frac{13}{77} \\ \frac{37}{77} & \frac{29}{77} & \frac{3}{11} & 1 & \frac{45}{77} & \frac{-1}{7} & \frac{29}{77} & \frac{37}{77} \\ \frac{37}{77} & \frac{13}{77} & \frac{-1}{7} & \frac{45}{77} & 1 & \frac{3}{11} & \frac{29}{77} & \frac{53}{77} \\ \frac{29}{77} & \frac{37}{77} & \frac{45}{77} & \frac{-1}{7} & \frac{3}{11} & 1 & \frac{37}{77} & \frac{29}{77} \\ \frac{-1}{7} & \frac{13}{77} & \frac{37}{77} & \frac{29}{77} & \frac{29}{77} & \frac{37}{77} & 1 & \frac{53}{77} \\ \frac{13}{77} & \frac{-1}{7} & \frac{13}{77} & \frac{37}{77} & \frac{53}{77} & \frac{29}{77} & \frac{53}{77} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 0.3083672838, 0.7685379357, 1.278775810, 1.644318969]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 3.428571429, 0.3524197529, 0.8783290695, 1.461458069, 1.879221680]

NullSpace M_C

{[0, 0, 0, −1, 0, 1, 0, 0], [0, 0, 1, 0, −1, 0, 0, 0], [1, 0, 0, 1, 1, 0, 0, 1], [0, 1, 0, 0, 0, 0, 0, −1], [0, 0, 0, 1, 1, 0, 1, 1] }

NullSpace N_c

{[−1, 0, 0, 1, 0, 1, −1, 0], [−1, 1, 0, 0, 0, 0, −1, 1], [−1, 0, 1, 0, 1, 0, −1, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 0.3083672838, 0.7685379357, 1.278775810, 1.644318969]

NullSpace M_0

{[0, 1, 0, 0, 0, 0, 0, −1], [0, 0, −1, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0, −1, 0], [0, 0, 0, 1, 0, −1, 0, 0]}

NullSpace N_0

{[0, 0, 1, −1, 1, −1, 0, 0], [1, 0, 0, −1, 0, −1, 1, 0], [0, 1, 0, −1, 0, −1, 0, 1]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −0.3083672838, −0.7685379357, −1.278775810, −1.644318969]

NullSpace M

{}

NullSpace N

{[0, 1, 0, −1, 0, −1, 0, 1], [0, 0, 1, −1, 1, −1, 0, 0], [1, 0, 0, −1, 0, −1, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 6 & 5 & 5 & 6 & 11 & 8 \\ 3 & 0 & 3 & 6 & 8 & 5 & 8 & 11 \\ 6 & 3 & 0 & 7 & 11 & 4 & 5 & 8 \\ 5 & 6 & 7 & 0 & 4 & 11 & 6 & 5 \\ 5 & 8 & 11 & 4 & 0 & 7 & 6 & 3 \\ 6 & 5 & 4 & 11 & 7 & 0 & 5 & 6 \\ 11 & 8 & 5 & 6 & 6 & 5 & 0 & 3 \\ 8 & 11 & 8 & 5 & 3 & 6 & 3 & 0 \end{pmatrix}$$

==

{4, 8}

R: [3, 3, 1, 6, 7, 7, 5, 2]
B: [6, 8, 8, 1, 2, 4, 4, 5]

TRACE TWO = 1

det AT = $\frac{1}{16}(t)^2(1+t)^2(-1+t)^2$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{1}{1073741824}(-6262 - 3387s - 2864s^2 - 313s^3 + 627s^4 + 394s^5$

$+323s^6 - 8s^7 - 13s^8 - 15s^9 - 3s^{10} + s^{11}$) ($5412 - 180s + 4s^2 - 75s^3 - 181 s^4 + 12s^5 - 4s^6 + 3s^7 + s^8$) ($-1 + s$)

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[5] v[7]) (1 + v[1] v[3])
"B CYCLES", (1 + v[2] v[5] v[8]) (1 + v[1] v[4] v[6])

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 0., 0.]

NullSpace of R

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of R^*

{[-1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, -1, 1, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, 0, -1, 1, 0], [0, 1, -1, 0, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&0&0&4&0&0&0\\ 0&0&0&0&0&4&0&0\\ 0&0&0&0&0&0&4&0\\ 0&0&0&0&0&0&0&4\\ 4&0&0&0&0&0&0&0\\ 0&4&0&0&0&0&0&0\\ 0&0&4&0&0&0&0&0\\ 0&0&0&4&0&0&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&\frac{1}{4}&\frac{1}{2}&\frac{3}{4}&1&\frac{3}{4}&\frac{1}{2}&\frac{1}{4}\\ \frac{1}{4}&0&\frac{1}{4}&\frac{1}{2}&\frac{3}{4}&1&\frac{3}{4}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{4}&0&\frac{1}{2}&\frac{1}{2}&\frac{3}{4}&1&\frac{1}{2}\\ \frac{3}{4}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{4}&\frac{1}{2}&\frac{1}{2}&1\\ 1&\frac{3}{4}&\frac{1}{2}&\frac{1}{4}&0&\frac{1}{4}&\frac{1}{2}&\frac{3}{4}\\ \frac{3}{4}&1&\frac{3}{4}&\frac{1}{2}&\frac{1}{4}&0&\frac{1}{4}&\frac{1}{2}\\ \frac{1}{2}&\frac{3}{4}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{4}&0&\frac{1}{2}\\ \frac{1}{4}&\frac{1}{2}&\frac{1}{2}&1&\frac{3}{4}&\frac{1}{2}&\frac{1}{2}&0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[5] + v[2]v[6] + v[3]v[7] + v[4]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 3, 8}, {4, 5, 6, 7}}

"PT2" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT3" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT4" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"PT5" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$\pi 2$ = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {4, 11, 17, 22}

$u2$ = [1, 2, 3, 4, 3, 2, 1, 1, 2, 3, 4, 3, 2, 2, 2, 3, 4, 2, 1, 2, 2, 4, 1, 2, 3, 1, 2, 2]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [4], [2], [2]]
Action of B on ranges, [[4], [1], [1], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [1, 4, 2, 4, 1]

BPARTS [3, 3, 5, 1, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{3}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[9, 6, A, 7, 2, 7, 6, 4, 9, A]

B-BLOCKS,

[A, 5, 9, 8, 1, A, 9, 3, 8, 5]

with invariant measure, [1, 1, 1, 1, 2, 1, 1, 2, 3, 3]

N by blocks, N - check: true

b_1 = {1, 2, 3, 4}
b_2 = {3, 4, 5, 6}
b_3 = {5, 6, 7, 8}
b_4 = {1, 2, 7, 8}
b_5 = {1, 6, 7, 8}
b_6 = {1, 2, 4, 7}

$b_7 = \{3, 5, 6, 8\}$
$b_8 = \{2, 3, 4, 5\}$
$b_9 = \{1, 2, 3, 8\}$
$b_{10} = \{4, 5, 6, 7\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \\ h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: $23 \oplus 4/3$

$$\text{CLB} = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 7}, {1, 3}}, true

Ω_B in Vec(K)? , {{2, 5, 8}, {1, 4, 6}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \left(\frac{1}{4} \;\; 0 \;\; \frac{1}{4} \;\; 0 \;\; \frac{1}{4} \;\; 0 \;\; \frac{1}{4} \;\; 0\right)$ vs $\left(0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right)$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \left(\frac{1}{6} \;\; \frac{1}{6} \;\; 0 \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; 0 \;\; \frac{1}{6}\right)$ vs $\left(0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right)$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 3, 8}, {4, 5, 6, 7}}

1, "range", [4, 8], [[8, 8, 8, 4, 4, 4, 4, 8], [4, 4, 4, 8, 8, 8, 8, 4]]

2, "range", [3, 7], [[7, 7, 7, 3, 3, 3, 3, 7], [3, 3, 3, 7, 7, 7, 7, 3]]

3, "range", [2, 6], [[6, 6, 6, 2, 2, 2, 2, 6], [2, 2, 2, 6, 6, 6, 6, 2]]

4, "range", [1, 5], [[5, 5, 5, 1, 1, 1, 1, 5], [1, 1, 1, 5, 5, 5, 5, 1]]

2, "partition", {{3, 4, 5, 6}, {1, 2, 7, 8}}

1, "range", [4, 8], [[8, 8, 4, 4, 4, 4, 8, 8], [4, 4, 8, 8, 8, 8, 4, 4]]

2, "range", [3, 7], [[7, 7, 3, 3, 3, 3, 7, 7], [3, 3, 7, 7, 7, 7, 3, 3]]

3, "range", [2, 6], [[6, 6, 2, 2, 2, 2, 6, 6], [2, 2, 6, 6, 6, 6, 2, 2]]

4, "range", [1, 5], [[5, 5, 1, 1, 1, 1, 5, 5], [1, 1, 5, 5, 5, 5, 1, 1]]

3, "partition", {{1, 6, 7, 8}, {2, 3, 4, 5}}

1, "range", [4, 8], [[8, 4, 4, 4, 4, 8, 8, 8], [4, 8, 8, 8, 8, 4, 4, 4]]

2, "range", [3, 7], [[7, 3, 3, 3, 3, 7, 7, 7], [3, 7, 7, 7, 7, 3, 3, 3]]

3, "range", [2, 6], [[6, 2, 2, 2, 2, 6, 6, 6], [2, 6, 6, 6, 6, 2, 2, 2]]

4, "range", [1, 5], [[5, 1, 1, 1, 1, 5, 5, 5], [1, 5, 5, 5, 5, 1, 1, 1]]

4, "partition", {{1, 2, 4, 7}, {3, 5, 6, 8}}

1, "range", [4, 8], [[8, 8, 4, 8, 4, 4, 8, 4], [4, 4, 8, 4, 8, 8, 4, 8]]

2, "range", [3, 7], [[7, 7, 3, 7, 3, 3, 7, 3], [3, 3, 7, 3, 7, 7, 3, 7]]

3, "range", [2, 6], [[6, 6, 2, 6, 2, 2, 6, 2], [2, 2, 6, 2, 6, 6, 2, 6]]

4, "range", [1, 5], [[5, 5, 1, 5, 1, 1, 5, 1], [1, 1, 5, 1, 5, 5, 1, 5]]

5, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [4, 8], [[8, 8, 8, 8, 4, 4, 4, 4], [4, 4, 4, 4, 8, 8, 8, 8]]

2, "range", [3, 7], [[7, 7, 7, 7, 3, 3, 3, 3], [3, 3, 3, 3, 7, 7, 7, 7]]

3, "range", [2, 6], [[6, 6, 6, 6, 2, 2, 2, 2], [2, 2, 2, 2, 6, 6, 6, 6]]

4, "range", [1, 5], [[5, 5, 5, 5, 1, 1, 1, 1], [1, 1, 1, 1, 5, 5, 5, 5]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 11, 17, 22}

$u2 =$
$(1 \;\; 2 \;\; 3 \;\; 4 \;\; 3 \;\; 2 \;\; 1 \;\; 1 \;\; 2 \;\; 3 \;\; 4 \;\; 3 \;\; 2 \;\; 2 \;\; 2 \;\; 3 \;\; 4 \;\; 2 \;\; 1 \;\; 2 \;\; 2 \;\; 4 \;\; 1 \;\; 2 \;\; 3 \;\; 1 \;\; 2$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem−checks} \quad \text{N0−checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \text{idem−checks} \quad \text{N0−checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{3}{16} & \frac{1}{8} & \frac{1}{16} & 0 & \frac{1}{16} & \frac{1}{8} & \frac{3}{16} \\ \frac{3}{16} & \frac{1}{4} & \frac{3}{16} & \frac{1}{8} & \frac{1}{16} & 0 & \frac{1}{16} & \frac{1}{8} \\ \frac{1}{8} & \frac{3}{16} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} & 0 & \frac{1}{8} \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{3}{16} & \frac{1}{8} & \frac{1}{8} & 0 \\ 0 & \frac{1}{16} & \frac{1}{8} & \frac{3}{16} & \frac{1}{4} & \frac{3}{16} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{16} & 0 & \frac{1}{16} & \frac{1}{8} & \frac{3}{16} & \frac{1}{4} & \frac{3}{16} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{16} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} & \frac{1}{4} & \frac{1}{8} \\ \frac{3}{16} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{16} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 2 & 3 & 4 & 2 & 2 & 1 & 0 & 2 \\ 1 & 2 & 2 & 4 & 3 & 2 & 2 & 0 \\ 0 & 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ 1 & 0 & 1 & 2 & 3 & 4 & 3 & 2 \\ 2 & 1 & 0 & 2 & 2 & 3 & 4 & 2 \\ 3 & 2 & 2 & 0 & 1 & 2 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 0, 1, -1, 0, 0, 1, -1]$

$$\ker N_c = \begin{pmatrix} 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & -1 & 1 \end{pmatrix} \quad \begin{pmatrix} -s & t & s & -t & -s & t & s & -t \\ -s & 0 & s & 0 & -s & 0 & s & 0 \\ -s+t & s & 0 & -t & -s+t & s & 0 & -t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 0 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & t & -s & 0 \\ 0 & 0 & -s & t \\ -s & 0 & 0 & t \\ -t & s & 0 & 0 \\ 0 & -t & s & 0 \\ 0 & 0 & s & -t \\ s & 0 & 0 & -t \\ t & -s & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & s & 0 & t & -s \\ s+t & s & 0 & 0 & -s \\ s+t & s & -s & 0 & 0 \\ t & t & -t & s & 0 \\ t & t & 0 & -t & s \\ 0 & t & 0 & 0 & s \\ 0 & t & s & 0 & 0 \\ s & s & t & -s & 0 \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 & 0 & 0 & 0 \end{pmatrix}$

$$R N_0 R^* = \begin{pmatrix} 1 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{3}{4} \\ 1 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{3}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 1 & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{3}{4} & 1 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{3}{4} & 1 & 1 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{4} \\ \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix}, R R^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & 1 & 1 & \frac{3}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{4} \\ \frac{1}{2} & 1 & 1 & \frac{3}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 1 & \frac{3}{4} \\ \frac{1}{2} & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 1 & \frac{3}{4} \\ \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & 1 \end{pmatrix}, B B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathrm{Skew}$$

$$\mathrm{Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 \\ 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \\ \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{16}\ \frac{1}{8}\ \frac{1}{4}\ \frac{3}{16}\ \frac{1}{8}\ \frac{1}{8}\ \frac{3}{16}\ \frac{1}{4}\ \frac{3}{16}\ \frac{3}{16}\ \frac{1}{8}\ \frac{1}{16}\ 0\ \frac{1}{16}\ \frac{1}{8}\ \frac{3}{16}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

NM $(4\ 2\ 2\ 1\ 2\ 4\ 3\ 2\ 2\ 3\ 4\ 3\ 3\ 2\ 1\ 0\ 1\ 2\ 3\ 4)$

"IS MN in Vec(K)?", true

MN $(4\ 2\ 2\ 1\ 2\ 4\ 3\ 2\ 2\ 3\ 4\ 3\ 3\ 2\ 1\ 0\ 1\ 2\ 3\ 4)$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 1/8$, min $\boldsymbol{\tau} = 8/1$, $\boldsymbol{\tau}$–check is positive? 24/1

max r = 8/1 , r–check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 5, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 40

dim span idems 16 vs no. of idems 20

"PT1" = {{1, 2, 3, 8}, {4, 5, 6, 7}}

"PT2" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT3" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT4" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"PT5" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \\ 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{5}{8} & \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{5}{8} \\ \frac{5}{8} & \frac{7}{8} & \frac{5}{8} & \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{5}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{3}{8} \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{5}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{5}{8} & \frac{7}{8} & \frac{5}{8} & \frac{3}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{5}{8} & \frac{7}{8} & \frac{5}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{5}{8} & \frac{7}{8} & \frac{3}{8} \\ \frac{5}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{5}{7} & \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} & \frac{5}{7} \\ \frac{5}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{5}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} & \frac{3}{7} \\ \frac{1}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} & \frac{5}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} & \frac{5}{7} & 1 & \frac{5}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{5}{7} & 1 & \frac{3}{7} \\ \frac{5}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} & \frac{3}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 1.309016994, 0.1909830058, 1.809016994, 0.6909830058]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 3.428571429, 1.496019422, 0.2182662923, 2.067447994, 0.7896948642]

NullSpace M_C

{[0, −1, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [1, 1, 1, 0, 0, 0, 0, 1], [1, 1, 1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0] }

NullSpace N_c

{[0, 0, −1, 1, 0, 0, −1, 1], [0, 1, −1, 0, 0, 1, −1, 0], [1, 0, −1, 0, 1, 0, −1, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 1.309016994, 0.1909830058, 1.809016994, 0.6909830058]

NullSpace M_0

{[0, 0, 1, 0, 0, 0, −1, 0], [0, 0, 0, 1, 0, 0, 0, −1], [−1, 0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, −1, 0, 0]}

NullSpace N_0

{[−1, 0, 0, 1, −1, 0, 0, 1], [−1, 1, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, −1, 0, 1, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −0.1909830058, −1.309016994, −0.6909830058, −1.809016994]

NullSpace M

{}

NullSpace N

{[1, −1, 0, 0, 1, −1, 0, 0], [0, −1, 1, 0, 0, −1, 1, 0], [0, −1, 0, 1, 0, −1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ 1 & 0 & 1 & 2 & 3 & 4 & 3 & 2 \\ 2 & 1 & 0 & 2 & 2 & 3 & 4 & 2 \\ 3 & 2 & 2 & 0 & 1 & 2 & 2 & 4 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 2 & 3 & 4 & 2 & 2 & 1 & 0 & 2 \\ 1 & 2 & 2 & 4 & 3 & 2 & 2 & 0 \end{pmatrix}$$

==

{5, 7}

R: [3, 3, 1, 1, 2, 7, 4, 5]
B: [6, 8, 8, 6, 7, 4, 5, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 7

Level 2 det = $\frac{1}{67108864} (-1 + s) (4141 - 2121s - 557s^2 + 327s^3 + 269s^4 - 139s^5 - 15s^6 + 13s^7 + 2s^8) (-1023 - 33s + 78s^2 - 30s^3 - 32s^4 + 18s^6 - 2$

$s^7 - s^8 + s^9$)

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", 1 + v[1] v[3]
"B CYCLES", (1 + v[2] v[8]) (1 + v[4] v[6]) (1 + v[5] v[7])

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [0., 0., 1., −1., 1., −1., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, −1, 1, 0, 0, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B^*

{[0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&4&0&0&0&0&0\\ 0&0&0&4&0&0&0&0\\ 4&0&0&0&0&0&0&0\\ 0&4&0&0&0&0&0&0\\ 0&0&0&0&0&0&4&0\\ 0&0&0&0&0&0&0&4\\ 0&0&0&0&4&0&0&0\\ 0&0&0&0&0&4&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&\frac{1}{2}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&0&\frac{1}{2}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 1&\frac{1}{2}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&1&\frac{1}{2}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{2}&1&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{2}&1\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{2}&0&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{2}&0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 1 "Trace mark", 0, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[3] + v[2]v[4] + v[5]v[7] + v[6]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 4, 6, 7}, {1, 2, 5, 8}}

"PT2" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT3" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT4" = {{1, 4, 7, 8}, {2, 3, 5, 6}}

"PT5" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT6" = {{2, 3, 7, 8}, {1, 4, 5, 6}}

"PT7" = {{3, 4, 5, 8}, {1, 2, 6, 7}}

"PT8" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"RG1" = {6, 8}

"RG2" = {5, 7}

"RG3" = {2, 4}

"RG4" = {1, 3}

$\pi 2 = [0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]$

supp $\pi 2$ = {2, 9, 24, 27}

$u2 = [1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1]$

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[2], [3], [4], [4]]
Action of B on ranges, [[3], [2], [1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [7, 2, 8, 1, 1, 8, 2, 7]

BPARTS [6, 5, 3, 4, 6, 5, 3, 4]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[10, 3, 2, C, 1, 3, 10, B, 7, C, 6, 6, 7, 1, B, 2]

B-BLOCKS,

[D, E, F, F, 8, A, 8, 5, A, 9, 4, 5, E, D, 4, 9]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N - check: true

b_1 = {3, 4, 6, 7}
b_2 = {3, 4, 5, 6}

b_3 = {1, 2, 7, 8}
b_4 = {2, 3, 7, 8}
b_5 = {1, 4, 7, 8}
b_6 = {3, 4, 5, 8}
b_7 = {3, 4, 7, 8}
b_8 = {2, 3, 5, 6}
b_9 = {1, 4, 5, 8}
b_{10} = {2, 3, 6, 7}
b_{11} = {1, 2, 5, 8}
b_{12} = {1, 2, 5, 6}
b_{13} = {1, 4, 5, 6}
b_{14} = {1, 4, 6, 7}
b_{15} = {2, 3, 5, 8}
b_{16} = {1, 2, 6, 7}

dim(span of partition vectors), rank(N_0), rank(N):   5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & h[1] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[2] & 0 & h[1] & 0 & 0 & 0 & 0 \\ h[1] & 0 & h[2] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[1] & 0 & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & 0 & h[1] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[2] & 0 & h[1] \\ 0 & 0 & 0 & 0 & h[1] & 0 & h[2] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[1] & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 24, Shape: $18 \oplus 6/3$

$$CLB = \begin{pmatrix} -1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3}}, true

Ω_B in Vec(K)? , {{5, 7}, {2, 8}, {4, 6}}, false

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} \\ \frac{-3}{40} & \frac{29}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{-27}{40} & \frac{21}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} \\ \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} & \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} \\ \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \left(\frac{1}{2} \ \ 0 \ \ \frac{1}{2} \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0\right)$ vs $\left(\frac{1}{2} \ \ 0 \ \ \frac{1}{2} \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0\right)$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \left(0 \ \ \frac{3}{16} \ \ 0 \ \ \frac{3}{16} \ \ \frac{1}{8} \ \ \frac{3}{16} \ \ \frac{1}{8} \ \ \frac{3}{16}\right)$ vs $(0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0)$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 4, 6, 7}, {1, 2, 5, 8}}

1, "range", [6, 8], [[8, 8, 6, 6, 8, 6, 6, 8], [6, 6, 8, 8, 6, 8, 8, 6]]

2, "range", [5, 7], [[7, 7, 5, 5, 7, 5, 5, 7], [5, 5, 7, 7, 5, 7, 7, 5]]

3, "range", [2, 4], [[4, 4, 2, 2, 4, 2, 2, 4], [2, 2, 4, 4, 2, 4, 4, 2]]

4, "range", [1, 3], [[3, 3, 1, 1, 3, 1, 1, 3], [1, 1, 3, 3, 1, 3, 3, 1]]

2, "partition", {{3, 4, 5, 6}, {1, 2, 7, 8}}

1, "range", [6, 8], [[8, 8, 6, 6, 6, 6, 8, 8], [6, 6, 8, 8, 8, 8, 6, 6]]

2, "range", [5, 7], [[7, 7, 5, 5, 5, 5, 7, 7], [5, 5, 7, 7, 7, 7, 5, 5]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 2, 4, 4], [2, 2, 4, 4, 4, 4, 2, 2]]

4, "range", [1, 3], [[3, 3, 1, 1, 1, 1, 3, 3], [1, 1, 3, 3, 3, 3, 1, 1]]

3, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [6, 8], [[8, 6, 6, 8, 8, 6, 6, 8], [6, 8, 8, 6, 6, 8, 8, 6]]

2, "range", [5, 7], [[7, 5, 5, 7, 7, 5, 5, 7], [5, 7, 7, 5, 5, 7, 7, 5]]

3, "range", [2, 4], [[4, 2, 2, 4, 4, 2, 2, 4], [2, 4, 4, 2, 2, 4, 4, 2]]

4, "range", [1, 3], [[3, 1, 1, 3, 3, 1, 1, 3], [1, 3, 3, 1, 1, 3, 3, 1]]

4, "partition", {{1, 4, 7, 8}, {2, 3, 5, 6}}

1, "range", [6, 8], [[8, 6, 6, 8, 6, 6, 8, 8], [6, 8, 8, 6, 8, 8, 6, 6]]

2, "range", [5, 7], [[7, 5, 5, 7, 5, 5, 7, 7], [5, 7, 7, 5, 7, 7, 5, 5]]

3, "range", [2, 4], [[4, 2, 2, 4, 2, 2, 4, 4], [2, 4, 4, 2, 4, 4, 2, 2]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 1, 3, 3], [1, 3, 3, 1, 3, 3, 1, 1]]

5, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [6, 8], [[8, 6, 6, 8, 6, 8, 8, 6], [6, 8, 8, 6, 8, 6, 6, 8]]

2, "range", [5, 7], [[7, 5, 5, 7, 5, 7, 7, 5], [5, 7, 7, 5, 7, 5, 5, 7]]

3, "range", [2, 4], [[4, 2, 2, 4, 2, 4, 4, 2], [2, 4, 4, 2, 4, 2, 2, 4]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 3, 3, 1], [1, 3, 3, 1, 3, 1, 1, 3]]

6, "partition", {{2, 3, 7, 8}, {1, 4, 5, 6}}

1, "range", [6, 8], [[8, 6, 6, 8, 8, 8, 6, 6], [6, 8, 8, 6, 6, 6, 8, 8]]

2, "range", [5, 7], [[7, 5, 5, 7, 7, 7, 5, 5], [5, 7, 7, 5, 5, 5, 7, 7]]

3, "range", [2, 4], [[4, 2, 2, 4, 4, 4, 2, 2], [2, 4, 4, 2, 2, 2, 4, 4]]

4, "range", [1, 3], [[3, 1, 1, 3, 3, 3, 1, 1], [1, 3, 3, 1, 1, 1, 3, 3]]

7, "partition", {{3, 4, 5, 8}, {1, 2, 6, 7}}

1, "range", [6, 8], [[8, 8, 6, 6, 6, 8, 8, 6], [6, 6, 8, 8, 8, 6, 6, 8]]

2, "range", [5, 7], [[7, 7, 5, 5, 5, 7, 7, 5], [5, 5, 7, 7, 7, 5, 5, 7]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 4, 4, 2], [2, 2, 4, 4, 4, 2, 2, 4]]

4, "range", [1, 3], [[3, 3, 1, 1, 1, 3, 3, 1], [1, 1, 3, 3, 3, 1, 1, 3]]

8, "partition", {{3, 4, 7, 8}, {1, 2, 5, 6}}

1, "range", [6, 8], [[8, 8, 6, 6, 8, 8, 6, 6], [6, 6, 8, 8, 6, 6, 8, 8]]

2, "range", [5, 7], [[7, 7, 5, 5, 7, 7, 5, 5], [5, 5, 7, 7, 5, 5, 7, 7]]

3, "range", [2, 4], [[4, 4, 2, 2, 4, 4, 2, 2], [2, 2, 4, 4, 2, 2, 4, 4]]

4, "range", [1, 3], [[3, 3, 1, 1, 3, 3, 1, 1], [1, 1, 3, 3, 1, 1, 3, 3]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2$ =
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

{2, 9, 24, 27}

$u2$ =
$$\begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \end{pmatrix}$$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$u1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_7 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_8 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 2 & 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 0 & 2 & 2 & 2 & 2 \\ 0 & 2 & 4 & 2 & 2 & 2 & 2 & 2 \\ 2 & 0 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 2 & 4 & 2 & 0 \\ 2 & 2 & 2 & 2 & 0 & 2 & 4 & 2 \\ 2 & 2 & 2 & 2 & 2 & 0 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 0, 1, 0, 0, -1, 0, -1]$

$$\ker N_c = \begin{pmatrix} 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & -1 & 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s & t & -s & t & s & -t & s & -t \\ -s & s & -s & s & t & -t & t & -t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s & -t & 0 \\ 0 & s & t & 0 \\ 0 & -s & t & 0 \\ 0 & -s & -t & 0 \\ -s & 0 & 0 & t \\ t & 0 & 0 & s \\ s & 0 & 0 & -t \\ -t & 0 & 0 & -s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & -1 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & s & 0 & t & 0 \\ 0 & s & t & -t & t \\ 0 & -s & s+t & -t & s+t \\ 0 & -s & s & t & s \\ -s & 0 & s & 0 & s+t \\ t & 0 & 0 & 0 & s \\ s & 0 & t & 0 & 0 \\ -t & 0 & s+t & 0 & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 4 & 0 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad Skew\ T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad Skew$$

$$Omega = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&4&0&0&0&0&0\\ 0&4&0&4&0&0&0&0\\ 4&0&4&0&0&0&0&0\\ 0&4&0&4&0&0&0&0\\ 0&0&0&0&4&0&4&0\\ 0&0&0&0&0&4&0&4\\ 0&0&0&0&4&0&4&0\\ 0&0&0&0&0&4&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{1}{2}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&1&\frac{1}{2}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 0&\frac{1}{2}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&0&\frac{1}{2}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{2}&0&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{2}&0\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{2}&1&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{2}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$$

$$T \left(\frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ 0\ \frac{1}{8}\ \frac{1}{4}\right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \left(4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 2\ 2\ 0\ 2\ 4\right)$$

"IS MN in Vec(K)?", true

$$\mathrm{MN} \left(4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 2\ 2\ 0\ 2\ 4\right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 24/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 8, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 64

dim span idems 16 vs no. of idems 32

"PT1" = {{3, 4, 6, 7}, {1, 2, 5, 8}}

"PT2" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT3" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT4" = {{1, 4, 7, 8}, {2, 3, 5, 6}}

"PT5" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT6" = {{2, 3, 7, 8}, {1, 4, 5, 6}}

"PT7" = {{3, 4, 5, 8}, {1, 2, 6, 7}}

"PT8" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"RG1" = {6, 8}

"RG2" = {5, 7}

"RG3" = {2, 4}

"RG4" = {1, 3}

$$M_c = \begin{pmatrix} 3 & -1 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 3 & -1 & -1 & -1 & -1 \\ 3 & -1 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & -1 & 3 \\ -1 & -1 & -1 & -1 & 3 & -1 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[3., 0., 0., 0., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[3.428571429, 0., 0., 0., 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[1, 1, 0, 0, 1, 0, 0, 1], [0, −1, 0, 1, 0, 0, 0, 0], [1, 1, 0, 0, 1, 1, 0, 0], [0, 0, 0, 0, −1, 0, 1, 0], [−1, 0, 1, 0, 0, 0, 0, 0]}

NullSpace N_C

{[0, −1, 0, −1, 1, 0, 1, 0], [0, −1, 0, −1, 0, 1, 0, 1], [1, −1, 1, −1, 0, 0, 0, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[4., 0., 0., 0., 1., 1., 1., 1.]

NullSpace M_0

{[0, 0, 0, 0, 1, 0, −1, 0], [0, 1, 0, −1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, −1], [1, 0, −1, 0, 0, 0, 0, 0]}

NullSpace N_0

{[−1, 0, −1, 0, 0, 1, 0, 1], [−1, 1, −1, 1, 0, 0, 0, 0], [−1, 0, −1, 0, 1, 0, 1, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[4., 0., 0., 0., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[0, 0, 0, 0, 1, −1, 1, −1], [0, 1, 0, 1, 0, −1, 0, −1], [1, 0, 1, 0, 0, −1, 0, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 \end{pmatrix}$$

==

{6, 8}

R: [3, 3, 1, 1, 7, 4, 5, 2]
B: [6, 8, 8, 6, 2, 7, 4, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 7

Level 2 det = $\frac{1}{67108864}(-1+s)(4141+2121s+811s^2+65s^3-67s^4+43s^5+41s^6+11s^7+2s^8)(-1023-33s-82s^2+98s^3+32s^4-14s^6-2s^7-s^8+s^9)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[5] v[7]) (1 + v[1] v[3])
"B CYCLES", (1 + v[4] v[6] v[7]) (1 + v[2] v[5] v[8])

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0]}

NullSpace of B^*

{[−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 1, 0, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 1   "Trace mark", 2, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{4}$ ($v[1]v[3] + v[2]v[4] + v[5]v[7] + v[6]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 4, 6, 7}, {1, 2, 5, 8}}

"PT2" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT3" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT4" = {{1, 4, 7, 8}, {2, 3, 5, 6}}

"PT5" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT6" = {{2, 3, 7, 8}, {1, 4, 5, 6}}

"PT7" = {{3, 4, 5, 8}, {1, 2, 6, 7}}

"PT8" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"RG1" = {6, 8}

"RG2" = {5, 7}

"RG3" = {2, 4}

"RG4" = {1, 3}

$\pi 2$ = [0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]

supp $\pi 2$ = {2, 9, 24, 27}

$u2$ = [1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [2], [4], [4]]
Action of B on ranges, [[2], [3], [1], [1]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

RPARTS [8, 7, 1, 2, 2, 1, 7, 8]

BPARTS [5, 4, 6, 3, 5, 4, 6, 3]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[C, 10, 6, B, 2, 10, C, 3, 1, B, 7, 7, 1, 2, 3, 6]

B–BLOCKS,

[E, 5, 8, 8, A, 4, A, 9, 4, D, F, 9, 5, E, F, D]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

$b_1 = \{3, 4, 6, 7\}$
$b_2 = \{3, 4, 5, 6\}$
$b_3 = \{1, 2, 7, 8\}$
$b_4 = \{2, 3, 7, 8\}$
$b_5 = \{1, 4, 7, 8\}$
$b_6 = \{3, 4, 5, 8\}$
$b_7 = \{3, 4, 7, 8\}$
$b_8 = \{2, 3, 5, 6\}$
$b_9 = \{1, 4, 5, 8\}$
$b_{10} = \{2, 3, 6, 7\}$
$b_{11} = \{1, 2, 5, 8\}$
$b_{12} = \{1, 2, 5, 6\}$
$b_{13} = \{1, 4, 5, 6\}$
$b_{14} = \{1, 4, 6, 7\}$
$b_{15} = \{2, 3, 5, 8\}$
$b_{16} = \{1, 2, 6, 7\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & h[2] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[1] & 0 & h[2] & 0 & 0 & 0 & 0 \\ h[2] & 0 & h[1] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[2] & 0 & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & 0 & h[2] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[1] & 0 & h[2] \\ 0 & 0 & 0 & 0 & h[2] & 0 & h[1] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[2] & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 25, Shape: $18 \oplus 7/5$

$$\text{CLB} = \begin{pmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 7}, {1, 3}}, false

Ω_B in Vec(K)? , {{4, 6, 7}, {2, 5, 8}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{3}{8} & 0 & \frac{3}{8} & 0 & \frac{1}{8} & 0 & \frac{1}{8} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 4, 6, 7}, {1, 2, 5, 8}}

1, "range", [6, 8], [[8, 8, 6, 6, 8, 6, 6, 8], [6, 6, 8, 8, 6, 8, 8, 6]]

2, "range", [5, 7], [[7, 7, 5, 5, 7, 5, 5, 7], [5, 5, 7, 7, 5, 7, 7, 5]]

3, "range", [2, 4], [[4, 4, 2, 2, 4, 2, 2, 4], [2, 2, 4, 4, 2, 4, 4, 2]]

4, "range", [1, 3], [[3, 3, 1, 1, 3, 1, 1, 3], [1, 1, 3, 3, 1, 3, 3, 1]]

2, "partition", {{3, 4, 5, 6}, {1, 2, 7, 8}}

1, "range", [6, 8], [[8, 8, 6, 6, 6, 6, 8, 8], [6, 6, 8, 8, 8, 8, 6, 6]]

2, "range", [5, 7], [[7, 7, 5, 5, 5, 5, 7, 7], [5, 5, 7, 7, 7, 7, 5, 5]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 2, 4, 4], [2, 2, 4, 4, 4, 4, 2, 2]]

4, "range", [1, 3], [[3, 3, 1, 1, 1, 1, 3, 3], [1, 1, 3, 3, 3, 3, 1, 1]]

3, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [6, 8], [[8, 6, 6, 8, 8, 6, 6, 8], [6, 8, 8, 6, 6, 8, 8, 6]]

2, "range", [5, 7], [[7, 5, 5, 7, 7, 5, 5, 7], [5, 7, 7, 5, 5, 7, 7, 5]]

3, "range", [2, 4], [[4, 2, 2, 4, 4, 2, 2, 4], [2, 4, 4, 2, 2, 4, 4, 2]]

4, "range", [1, 3], [[3, 1, 1, 3, 3, 1, 1, 3], [1, 3, 3, 1, 1, 3, 3, 1]]

4, "partition", {{1, 4, 7, 8}, {2, 3, 5, 6}}

1, "range", [6, 8], [[8, 6, 6, 8, 6, 6, 8, 8], [6, 8, 8, 6, 8, 8, 6, 6]]

2, "range", [5, 7], [[7, 5, 5, 7, 5, 5, 7, 7], [5, 7, 7, 5, 7, 7, 5, 5]]

3, "range", [2, 4], [[4, 2, 2, 4, 2, 2, 4, 4], [2, 4, 4, 2, 4, 4, 2, 2]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 1, 3, 3], [1, 3, 3, 1, 3, 3, 1, 1]]

5, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [6, 8], [[8, 6, 6, 8, 6, 8, 8, 6], [6, 8, 8, 6, 8, 6, 6, 8]]

2, "range", [5, 7], [[7, 5, 5, 7, 5, 7, 7, 5], [5, 7, 7, 5, 7, 5, 5, 7]]

3, "range", [2, 4], [[4, 2, 2, 4, 2, 4, 4, 2], [2, 4, 4, 2, 4, 2, 2, 4]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 3, 3, 1], [1, 3, 3, 1, 3, 1, 1, 3]]

6, "partition", {{2, 3, 7, 8}, {1, 4, 5, 6}}

1, "range", [6, 8], [[8, 6, 6, 8, 8, 8, 6, 6], [6, 8, 8, 6, 6, 6, 8, 8]]

2, "range", [5, 7], [[7, 5, 5, 7, 7, 7, 5, 5], [5, 7, 7, 5, 5, 5, 7, 7]]

3, "range", [2, 4], [[4, 2, 2, 4, 4, 4, 2, 2], [2, 4, 4, 2, 2, 2, 4, 4]]

4, "range", [1, 3], [[3, 1, 1, 3, 3, 3, 1, 1], [1, 3, 3, 1, 1, 1, 3, 3]]

7, "partition", {{3, 4, 5, 8}, {1, 2, 6, 7}}

1, "range", [6, 8], [[8, 8, 6, 6, 6, 8, 8, 6], [6, 6, 8, 8, 8, 6, 6, 8]]

2, "range", [5, 7], [[7, 7, 5, 5, 5, 7, 7, 5], [5, 5, 7, 7, 7, 5, 5, 7]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 4, 4, 2], [2, 2, 4, 4, 4, 2, 2, 4]]

4, "range", [1, 3], [[3, 3, 1, 1, 1, 3, 3, 1], [1, 1, 3, 3, 3, 1, 1, 3]]

8, "partition", {{3, 4, 7, 8}, {1, 2, 5, 6}}

1, "range", [6, 8], [[8, 8, 6, 6, 8, 8, 6, 6], [6, 6, 8, 8, 6, 6, 8, 8]]

2, "range", [5, 7], [[7, 7, 5, 5, 7, 7, 5, 5], [5, 5, 7, 7, 5, 5, 7, 7]]

3, "range", [2, 4], [[4, 4, 2, 2, 4, 4, 2, 2], [2, 2, 4, 4, 2, 2, 4, 4]]

4, "range", [1, 3], [[3, 3, 1, 1, 3, 3, 1, 1], [1, 1, 3, 3, 1, 1, 3, 3]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

{2, 9, 24, 27}

$u2 =$
$$\begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \end{pmatrix}$$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$u1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_7 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_8 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 2 & 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 0 & 2 & 2 & 2 & 2 \\ 0 & 2 & 4 & 2 & 2 & 2 & 2 & 2 \\ 2 & 0 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 2 & 4 & 2 & 0 \\ 2 & 2 & 2 & 2 & 0 & 2 & 4 & 2 \\ 2 & 2 & 2 & 2 & 2 & 0 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 0, 1, 0, 0, -1, 0, -1]$

$$\ker N_c = \begin{pmatrix} -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s & t & -s & t & s & -t & s & -t \\ -s & s & -s & s & t & -t & t & -t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 0 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -t & 0 & 0 & s \\ t & 0 & 0 & s \\ t & 0 & 0 & -s \\ -t & 0 & 0 & -s \\ 0 & -t & s & 0 \\ 0 & s & t & 0 \\ 0 & t & -s & 0 \\ 0 & -s & -t & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & t & s & -s & 0 \\ s+t & -t & s+t & -s & 0 \\ t & -t & t & s & 0 \\ 0 & t & 0 & s & 0 \\ 0 & 0 & s & 0 & t \\ s & 0 & s+t & 0 & -s \\ s+t & 0 & t & 0 & -t \\ t & 0 & 0 & 0 & s \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 4 & 0 & 0 \end{pmatrix}$

$$R N_0 R^* = \begin{pmatrix} 1 & 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \end{pmatrix}, R R^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \end{pmatrix}, B B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\,T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew}$$

$$\mathrm{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&4&0&0&0&0&0\\ 0&4&0&4&0&0&0&0\\ 4&0&4&0&0&0&0&0\\ 0&4&0&4&0&0&0&0\\ 0&0&0&0&4&0&4&0\\ 0&0&0&0&0&4&0&4\\ 0&0&0&0&4&0&4&0\\ 0&0&0&0&0&4&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{1}{2}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&1&\frac{1}{2}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 0&\frac{1}{2}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&0&\frac{1}{2}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{2}&0&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{2}&0\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{2}&1&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{2}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ 0\ \frac{1}{8}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

NM $(4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 2\ 2\ 0\ 2\ 4)$

"IS MN in Vec(K)?", true

MN $(4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 2\ 2\ 0\ 2\ 4)$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 1/8$, min $\boldsymbol{\tau} = 8/1$, $\boldsymbol{\tau}$–check is positive? 24/1

max r = 8/1 , r–check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 8, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 64

dim span idems 16 vs no. of idems 32

"PT1" = {{3, 4, 6, 7}, {1, 2, 5, 8}}

"PT2" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT3" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT4" = {{1, 4, 7, 8}, {2, 3, 5, 6}}

"PT5" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT6" = {{2, 3, 7, 8}, {1, 4, 5, 6}}

"PT7" = {{3, 4, 5, 8}, {1, 2, 6, 7}}

"PT8" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"RG1" = {6, 8}

"RG2" = {5, 7}

"RG3" = {2, 4}

"RG4" = {1, 3}

$$M_c = \begin{pmatrix} 3 & -1 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 3 & -1 & -1 & -1 & -1 \\ 3 & -1 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & -1 & 3 \\ -1 & -1 & -1 & -1 & 3 & -1 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[3., 0., 0., 0., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[3.428571429, 0., 0., 0., 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 1, 0, −1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 0, 1, 0], [0, 0, 1, 1, 1, 0, 0, 1], [0, 0, 1, 1, 1, 1, 0, 0], [1, 0, −1, 0, 0, 0, 0, 0] }

NullSpace N_c

{[0, −1, 0, −1, 0, 1, 0, 1], [1, −1, 1, −1, 0, 0, 0, 0], [0, −1, 0, −1, 1, 0, 1, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[4., 0., 0., 0., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 0, 1, 0], [0, 0, 0, 0, 0, −1, 0, 1]}

NullSpace N_0

{[−1, 0, −1, 0, 0, 1, 0, 1], [−1, 1, −1, 1, 0, 0, 0, 0], [−1, 0, −1, 0, 1, 0, 1, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[4., 0., 0., 0., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[−1, 1, −1, 1, 0, 0, 0, 0], [−1, 0, −1, 0, 1, 0, 1, 0], [−1, 0, −1, 0, 0, 1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 \end{pmatrix}$$

===

40, [1, -1, 1, 1, -1, 1, -1, 1]
===

60, [1, 1, 1, -1, 1, 1, -1, -1]

===

{2, 3, 5, 8}

R: [3, 8, 8, 1, 2, 7, 5, 2]
B: [6, 3, 1, 6, 7, 4, 4, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{-7}{33554432}(31 - s + 9s^2 + s^3)(1 + s)^2(-1 + s)(101 - 53s + 37s^2 - 7s^3 + 2s^4)(33 + 7s^2)(41 - 11s - 7s^2 - s^3 + 2s^4)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[2] v[8]
"B CYCLES", 1 + v[4] v[6]

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

NullSpace of B^*

{[−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{5}{9} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & \frac{4}{9} \\ \frac{5}{9} & 0 & \frac{1}{3} & \frac{4}{9} & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} & 1 \\ \frac{2}{3} & \frac{1}{3} & 0 & \frac{5}{9} & 1 & \frac{4}{9} & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{4}{9} & \frac{5}{9} & 0 & \frac{4}{9} & 1 & \frac{2}{3} & \frac{5}{9} \\ \frac{1}{3} & \frac{2}{3} & 1 & \frac{4}{9} & 0 & \frac{5}{9} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{5}{9} & \frac{4}{9} & 1 & \frac{5}{9} & 0 & \frac{1}{3} & \frac{4}{9} \\ 1 & \frac{4}{9} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & \frac{5}{9} \\ \frac{4}{9} & 1 & \frac{2}{3} & \frac{5}{9} & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[7] + v[2]v[8] + v[3]v[5] + v[4]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 5, 6, 8}, {2, 3, 4, 7}}

"PT4" = {{3, 6, 7, 8}, {1, 2, 4, 5}}

"PT5" = {{4, 5, 7, 8}, {1, 2, 3, 6}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {4, 6}

"RG3" = {3, 5}

"RG4" = {1, 7}

$$\pi 2 = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]$$

supp $\pi 2$ = {6, 13, 15, 20}

$$u2 = [5, 6, 3, 3, 6, 9, 4, 3, 4, 6, 5, 4, 9, 5, 9, 4, 3, 6, 4, 9, 6, 5, 5, 6, 3, 3, 4, 5]$$

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[1], [4], [1], [3]]
Action of B on ranges, [[3], [2], [4], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [6, 3, 3, 5, 2, 2]

BPARTS [2, 4, 1, 4, 6, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[8, 7, 8, 7, 4, 9, 9, 6, 6, 1, 2, 3]

B−BLOCKS,

[3, 8, 7, 2, A, B, 5, A, C, 5, 8, 7]

with invariant measure, [1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 1, 1]

N by blocks, N − check: true

$b_1 = \{4, 5, 7, 8\}$
$b_2 = \{1, 2, 3, 4\}$
$b_3 = \{5, 6, 7, 8\}$
$b_4 = \{1, 2, 3, 6\}$
$b_5 = \{3, 6, 7, 8\}$
$b_6 = \{1, 5, 6, 8\}$
$b_7 = \{1, 4, 5, 8\}$
$b_8 = \{2, 3, 6, 7\}$
$b_9 = \{2, 3, 4, 7\}$
$b_{10} = \{1, 2, 4, 5\}$
$b_{11} = \{1, 3, 4, 8\}$
$b_{12} = \{2, 5, 6, 7\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & 0 & 0 & 0 & h[2] & 0 \\ 0 & h[1] & 0 & 0 & 0 & 0 & 0 & h[2] \\ 0 & 0 & h[1] & 0 & h[2] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[1] & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[2] & 0 & h[1] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[2] & 0 & h[1] & 0 & 0 \\ h[2] & 0 & 0 & 0 & 0 & 0 & h[1] & 0 \\ 0 & h[2] & 0 & 0 & 0 & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: $18 \oplus 4/3$

$$CLB = \begin{pmatrix} 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 8}}, true

Ω_B in Vec(K)? , {{4, 6}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} \\ \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} \\ \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} \\ \frac{-3}{40} & \frac{29}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{-27}{40} & \frac{21}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} \end{pmatrix}$$

Omega-checks true, true

$$\pi_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \text{ vs } \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \text{ vs } \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{1, 3, 4, 8}, {2, 5, 6, 7}}

1, "range", [2, 8], [[8, 2, 8, 8, 2, 2, 2, 8], [2, 8, 2, 2, 8, 8, 8, 2]]

2, "range", [4, 6], [[6, 4, 6, 6, 4, 4, 4, 6], [4, 6, 4, 4, 6, 6, 6, 4]]

3, "range", [3, 5], [[5, 3, 5, 5, 3, 3, 3, 5], [3, 5, 3, 3, 5, 5, 5, 3]]

4, "range", [1, 7], [[7, 1, 7, 7, 1, 1, 1, 7], [1, 7, 1, 1, 7, 7, 7, 1]]

2, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [2, 8], [[8, 2, 2, 8, 8, 2, 2, 8], [2, 8, 8, 2, 2, 8, 8, 2]]

2, "range", [4, 6], [[6, 4, 4, 6, 6, 4, 4, 6], [4, 6, 6, 4, 4, 6, 6, 4]]

3, "range", [3, 5], [[5, 3, 3, 5, 5, 3, 3, 5], [3, 5, 5, 3, 3, 5, 5, 3]]

4, "range", [1, 7], [[7, 1, 1, 7, 7, 1, 1, 7], [1, 7, 7, 1, 1, 7, 7, 1]]

3, "partition", {{1, 5, 6, 8}, {2, 3, 4, 7}}

1, "range", [2, 8], [[8, 2, 2, 2, 8, 8, 2, 8], [2, 8, 8, 8, 2, 2, 8, 2]]

2, "range", [4, 6], [[6, 4, 4, 4, 6, 6, 4, 6], [4, 6, 6, 6, 4, 4, 6, 4]]

3, "range", [3, 5], [[5, 3, 3, 3, 5, 5, 3, 5], [3, 5, 5, 5, 3, 3, 5, 3]]

4, "range", [1, 7], [[7, 1, 1, 1, 7, 7, 1, 7], [1, 7, 7, 7, 1, 1, 7, 1]]

4, "partition", {{3, 6, 7, 8}, {1, 2, 4, 5}}

1, "range", [2, 8], [[8, 8, 2, 8, 8, 2, 2, 2], [2, 2, 8, 2, 2, 8, 8, 8]]

2, "range", [4, 6], [[6, 6, 4, 6, 6, 4, 4, 4], [4, 4, 6, 4, 4, 6, 6, 6]]

3, "range", [3, 5], [[5, 5, 3, 5, 5, 3, 3, 3], [3, 3, 5, 3, 3, 5, 5, 5]]

4, "range", [1, 7], [[7, 7, 1, 7, 7, 1, 1, 1], [1, 1, 7, 1, 1, 7, 7, 7]]

5, "partition", {{4, 5, 7, 8}, {1, 2, 3, 6}}

1, "range", [2, 8], [[8, 8, 8, 2, 2, 8, 2, 2], [2, 2, 2, 8, 8, 2, 8, 8]]

2, "range", [4, 6], [[6, 6, 6, 4, 4, 6, 4, 4], [4, 4, 4, 6, 6, 4, 6, 6]]

3, "range", [3, 5], [[5, 5, 5, 3, 3, 5, 3, 3], [3, 3, 3, 5, 5, 3, 5, 5]]

4, "range", [1, 7], [[7, 7, 7, 1, 1, 7, 1, 1], [1, 1, 1, 7, 7, 1, 7, 7]]

6, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [2, 8], [[8, 8, 8, 8, 2, 2, 2, 2], [2, 2, 2, 2, 8, 8, 8, 8]]

2, "range", [4, 6], [[6, 6, 6, 6, 4, 4, 4, 4], [4, 4, 4, 4, 6, 6, 6, 6]]

3, "range", [3, 5], [[5, 5, 5, 5, 3, 3, 3, 3], [3, 3, 3, 3, 5, 5, 5, 5]]

4, "range", [1, 7], [[7, 7, 7, 7, 1, 1, 1, 1], [1, 1, 1, 1, 7, 7, 7, 7]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{6, 13, 15, 20}

$u2 =$
$(5 \;\; 6 \;\; 3 \;\; 3 \;\; 6 \;\; 9 \;\; 4 \;\; 3 \;\; 4 \;\; 6 \;\; 5 \;\; 4 \;\; 9 \;\; 5 \;\; 9 \;\; 4 \;\; 3 \;\; 6 \;\; 4 \;\; 9 \;\; 6 \;\; 5 \;\; 5 \;\; 6 \;\; 3 \;\; 3 \;\; 4 \;\; 5$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{4}{9} & 0 & 0 & 0 & 0 & 0 & \frac{5}{9} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{5}{9} & 0 & 0 & 0 & 0 & 0 & \frac{4}{9} \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{4}{9} & 0 & 0 & 0 & 0 & 0 & \frac{5}{9} \\ 0 & \frac{5}{9} & 0 & 0 & 0 & 0 & 0 & \frac{4}{9} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{9} & 0 & \frac{5}{9} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{9} & 0 & \frac{5}{9} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{9} & 0 & \frac{5}{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{4}{9} & 0 & 0 & 0 & 0 & 0 & \frac{5}{9} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{5}{9} & 0 & 0 & 0 & 0 & 0 & \frac{4}{9} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{9} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & 0 & \frac{5}{36} \\ \frac{1}{9} & \frac{1}{4} & \frac{1}{6} & \frac{5}{36} & \frac{1}{12} & \frac{1}{9} & \frac{5}{36} & 0 \\ \frac{1}{12} & \frac{1}{6} & \frac{1}{4} & \frac{1}{9} & 0 & \frac{5}{36} & \frac{1}{6} & \frac{1}{12} \\ \frac{1}{6} & \frac{5}{36} & \frac{1}{9} & \frac{1}{4} & \frac{5}{36} & 0 & \frac{1}{12} & \frac{1}{9} \\ \frac{1}{6} & \frac{1}{12} & 0 & \frac{5}{36} & \frac{1}{4} & \frac{1}{9} & \frac{1}{12} & \frac{1}{6} \\ \frac{1}{12} & \frac{1}{9} & \frac{5}{36} & 0 & \frac{1}{9} & \frac{1}{4} & \frac{1}{6} & \frac{5}{36} \\ 0 & \frac{5}{36} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{4} & \frac{1}{9} \\ \frac{5}{36} & 0 & \frac{1}{12} & \frac{1}{9} & \frac{1}{6} & \frac{5}{36} & \frac{1}{9} & \frac{1}{4} \end{pmatrix} \quad NM =$$

$$\begin{pmatrix} 4 & \frac{16}{9} & \frac{4}{3} & \frac{8}{3} & \frac{8}{3} & \frac{4}{3} & 0 & \frac{20}{9} \\ \frac{16}{9} & 4 & \frac{8}{3} & \frac{20}{9} & \frac{4}{3} & \frac{16}{9} & \frac{20}{9} & 0 \\ \frac{4}{3} & \frac{8}{3} & 4 & \frac{16}{9} & 0 & \frac{20}{9} & \frac{8}{3} & \frac{4}{3} \\ \frac{8}{3} & \frac{20}{9} & \frac{16}{9} & 4 & \frac{20}{9} & 0 & \frac{4}{3} & \frac{16}{9} \\ \frac{8}{3} & \frac{4}{3} & 0 & \frac{20}{9} & 4 & \frac{16}{9} & \frac{4}{3} & \frac{8}{3} \\ \frac{4}{3} & \frac{16}{9} & \frac{20}{9} & 0 & \frac{16}{9} & 4 & \frac{8}{3} & \frac{20}{9} \\ 0 & \frac{20}{9} & \frac{8}{3} & \frac{4}{3} & \frac{4}{3} & \frac{8}{3} & 4 & \frac{16}{9} \\ \frac{20}{9} & 0 & \frac{4}{3} & \frac{16}{9} & \frac{8}{3} & \frac{20}{9} & \frac{16}{9} & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 1, 0, -1, 0, -1, 0, 1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & -1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -t+s & -s & 0 & t & 0 & t & -t+s & -s \\ -t & 0 & t & 0 & t & 0 & -t & 0 \\ -t & -s & s & t & s & t & -t & -s \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -s & 0 & t \\ 0 & -t & -s & 0 \\ t & 0 & -s & 0 \\ s & 0 & 0 & t \\ -t & 0 & s & 0 \\ -s & 0 & 0 & -t \\ 0 & s & 0 & -t \\ 0 & t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s+t & 0 & 0 & s & -s \\ t & 0 & s & t & -t \\ t & -t & s & t & 0 \\ s+t & -s & 0 & s & 0 \\ s & t & -s & s & 0 \\ 0 & s & 0 & t & 0 \\ 0 & 0 & 0 & t & s \\ s & 0 & -s & s & t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 4 & 0 \end{pmatrix}$

$$R N_0 R^* = \begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & 1 & \frac{5}{9} & 0 & \frac{4}{9} & \frac{2}{3} & 0 \\ \frac{1}{3} & 1 & 1 & \frac{5}{9} & 0 & \frac{4}{9} & \frac{2}{3} & 0 \\ \frac{1}{3} & \frac{5}{9} & \frac{5}{9} & 1 & \frac{4}{9} & 0 & \frac{2}{3} & \frac{4}{9} \\ \frac{2}{3} & 0 & 0 & \frac{4}{9} & 1 & \frac{5}{9} & \frac{1}{3} & 1 \\ \frac{2}{3} & \frac{4}{9} & \frac{4}{9} & 0 & \frac{5}{9} & 1 & \frac{1}{3} & \frac{5}{9} \\ 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & 0 & \frac{4}{9} & 1 & \frac{5}{9} & \frac{1}{3} & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & \frac{5}{9} & \frac{1}{3} & 1 & \frac{2}{3} & 0 & 0 & \frac{4}{9} \\ \frac{5}{9} & 1 & \frac{1}{3} & \frac{5}{9} & \frac{2}{3} & \frac{4}{9} & \frac{4}{9} & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & \frac{1}{3} & 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ 1 & \frac{5}{9} & \frac{1}{3} & 1 & \frac{2}{3} & 0 & 0 & \frac{4}{9} \\ \frac{2}{3} & \frac{2}{3} & 0 & \frac{2}{3} & 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{4}{9} & \frac{2}{3} & 0 & \frac{1}{3} & 1 & 1 & \frac{5}{9} \\ 0 & \frac{4}{9} & \frac{2}{3} & 0 & \frac{1}{3} & 1 & 1 & \frac{5}{9} \\ \frac{4}{9} & 0 & \frac{2}{3} & \frac{4}{9} & \frac{1}{3} & \frac{5}{9} & \frac{5}{9} & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\,T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew}$$

$$\mathrm{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&0&0&0&4&0\\ 0&4&0&0&0&0&0&4\\ 0&0&4&0&4&0&0&0\\ 0&0&0&4&0&4&0&0\\ 0&0&4&0&4&0&0&0\\ 0&0&0&4&0&4&0&0\\ 4&0&0&0&0&0&4&0\\ 0&4&0&0&0&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{4}{9}&\frac{1}{3}&\frac{2}{3}&\frac{2}{3}&\frac{1}{3}&0&\frac{5}{9}\\ \frac{4}{9}&1&\frac{2}{3}&\frac{5}{9}&\frac{1}{3}&\frac{4}{9}&\frac{5}{9}&0\\ \frac{1}{3}&\frac{2}{3}&1&\frac{4}{9}&0&\frac{5}{9}&\frac{2}{3}&\frac{1}{3}\\ \frac{2}{3}&\frac{5}{9}&\frac{4}{9}&1&\frac{5}{9}&0&\frac{1}{3}&\frac{4}{9}\\ \frac{2}{3}&\frac{1}{3}&0&\frac{5}{9}&1&\frac{4}{9}&\frac{1}{3}&\frac{2}{3}\\ \frac{1}{3}&\frac{4}{9}&\frac{5}{9}&0&\frac{4}{9}&1&\frac{2}{3}&\frac{5}{9}\\ 0&\frac{5}{9}&\frac{2}{3}&\frac{1}{3}&\frac{1}{3}&\frac{2}{3}&1&\frac{4}{9}\\ \frac{5}{9}&0&\frac{1}{3}&\frac{4}{9}&\frac{2}{3}&\frac{5}{9}&\frac{4}{9}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{1}{9}\ \frac{5}{36}\ \frac{1}{6}\ \frac{1}{9}\ \frac{1}{4}\ \frac{1}{6}\ \frac{1}{12}\ \frac{5}{36}\ \frac{1}{6}\ \frac{1}{4}\ \frac{1}{9}\ \frac{5}{36}\ 0\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{9}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM}\left(4\ \frac{16}{9}\ \frac{20}{9}\ \frac{8}{3}\ \frac{16}{9}\ 4\ \frac{8}{3}\ \frac{4}{3}\ \frac{20}{9}\ \frac{8}{3}\ 4\ \frac{16}{9}\ \frac{20}{9}\ 0\ \frac{4}{3}\ \frac{8}{3}\ \frac{8}{3}\ \frac{4}{3}\ \frac{16}{9}\ 4\right)$

"IS MN in Vec(K)?", true

$\mathrm{MN}\left(4\ \frac{16}{9}\ \frac{20}{9}\ \frac{8}{3}\ \frac{16}{9}\ 4\ \frac{8}{3}\ \frac{4}{3}\ \frac{20}{9}\ \frac{8}{3}\ 4\ \frac{16}{9}\ \frac{20}{9}\ 0\ \frac{4}{3}\ \frac{8}{3}\ \frac{8}{3}\ \frac{4}{3}\ \frac{16}{9}\ 4\right)$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 1/8$, min $\boldsymbol{\tau} = 8/1$, $\boldsymbol{\tau}$–check is positive? 24/1

max r = 8/1 , r–check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 5, 6, 8}, {2, 3, 4, 7}}

"PT4" = {{3, 6, 7, 8}, {1, 2, 4, 5}}

"PT5" = {{4, 5, 7, 8}, {1, 2, 3, 6}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {4, 6}

"RG3" = {3, 5}

"RG4" = {1, 7}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{23}{72} & \frac{5}{24} & \frac{13}{24} & \frac{13}{24} & \frac{5}{24} & \frac{-1}{8} & \frac{31}{72} \\ \frac{23}{72} & \frac{7}{8} & \frac{13}{24} & \frac{31}{72} & \frac{5}{24} & \frac{23}{72} & \frac{31}{72} & \frac{-1}{8} \\ \frac{5}{24} & \frac{13}{24} & \frac{7}{8} & \frac{23}{72} & \frac{-1}{8} & \frac{31}{72} & \frac{13}{24} & \frac{5}{24} \\ \frac{13}{24} & \frac{31}{72} & \frac{23}{72} & \frac{7}{8} & \frac{31}{72} & \frac{-1}{8} & \frac{5}{24} & \frac{23}{72} \\ \frac{13}{24} & \frac{5}{24} & \frac{-1}{8} & \frac{31}{72} & \frac{7}{8} & \frac{23}{72} & \frac{5}{24} & \frac{13}{24} \\ \frac{5}{24} & \frac{23}{72} & \frac{31}{72} & \frac{-1}{8} & \frac{23}{72} & \frac{7}{8} & \frac{13}{24} & \frac{31}{72} \\ \frac{-1}{8} & \frac{31}{72} & \frac{13}{24} & \frac{5}{24} & \frac{5}{24} & \frac{13}{24} & \frac{7}{8} & \frac{23}{72} \\ \frac{31}{72} & \frac{-1}{8} & \frac{5}{24} & \frac{23}{72} & \frac{13}{24} & \frac{31}{72} & \frac{23}{72} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{23}{63} & \frac{5}{21} & \frac{13}{21} & \frac{13}{21} & \frac{5}{21} & \frac{-1}{7} & \frac{31}{63} \\ \frac{23}{63} & 1 & \frac{13}{21} & \frac{31}{63} & \frac{5}{21} & \frac{23}{63} & \frac{31}{63} & \frac{-1}{7} \\ \frac{5}{21} & \frac{13}{21} & 1 & \frac{23}{63} & \frac{-1}{7} & \frac{31}{63} & \frac{13}{21} & \frac{5}{21} \\ \frac{13}{21} & \frac{31}{63} & \frac{23}{63} & 1 & \frac{31}{63} & \frac{-1}{7} & \frac{5}{21} & \frac{23}{63} \\ \frac{13}{21} & \frac{5}{21} & \frac{-1}{7} & \frac{31}{63} & 1 & \frac{23}{63} & \frac{5}{21} & \frac{13}{21} \\ \frac{5}{21} & \frac{23}{63} & \frac{31}{63} & \frac{-1}{7} & \frac{23}{63} & 1 & \frac{13}{21} & \frac{31}{63} \\ \frac{-1}{7} & \frac{31}{63} & \frac{13}{21} & \frac{5}{21} & \frac{5}{21} & \frac{13}{21} & 1 & \frac{23}{63} \\ \frac{31}{63} & \frac{-1}{7} & \frac{5}{21} & \frac{23}{63} & \frac{13}{21} & \frac{31}{63} & \frac{23}{63} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 1.203158569, 0.5746192085, 1.608015106, 0.6142071162]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 3.428571429, 1.375038365, 0.6567076669, 1.837731550, 0.7019509901]

NullSpace M_C

{[0, −1, 0, 0, 0, 0, 0, 1], [1, 1, 1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 1, 0, 0, 0], [1, 1, 1, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace N_c

{[0, −1, 1, 0, 1, 0, 0, −1], [1, −1, 0, 0, 0, 0, 1, −1], [0, −1, 0, 1, 0, 1, 0, −1]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 1.203158569, 0.5746192085, 1.608015106, 0.6142071162]

NullSpace M_0

{[0, 0, −1, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, 0, −1, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

NullSpace N_0

{[1, 0, −1, 0, −1, 0, 1, 0], [0, 0, −1, 1, −1, 1, 0, 0], [0, 1, −1, 0, −1, 0, 0, 1]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −0.5746192085, −1.203158569, −0.6142071162, −1.608015106]

NullSpace M

{}

NullSpace N

{[1, 0, −1, 0, −1, 0, 1, 0], [0, 1, −1, 0, −1, 0, 0, 1], [0, 0, −1, 1, −1, 1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 5 & 6 & 3 & 3 & 6 & 9 & 4 \\ 5 & 0 & 3 & 4 & 6 & 5 & 4 & 9 \\ 6 & 3 & 0 & 5 & 9 & 4 & 3 & 6 \\ 3 & 4 & 5 & 0 & 4 & 9 & 6 & 5 \\ 3 & 6 & 9 & 4 & 0 & 5 & 6 & 3 \\ 6 & 5 & 4 & 9 & 5 & 0 & 3 & 4 \\ 9 & 4 & 3 & 6 & 6 & 3 & 0 & 5 \\ 4 & 9 & 6 & 5 & 3 & 4 & 5 & 0 \end{pmatrix}$$

==

{2, 3, 6, 7}

R: [3, 8, 8, 1, 7, 4, 4, 5]
B: [6, 3, 1, 6, 2, 7, 5, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{5}{33554432}(31 - s - 7s^2 + s^3)(11 + 5s)(41 + 19s + s^2 + s^3 + 2s^4)(1 + s)^2(-3 + s)(-1 + s)(101 + 53s - 27s^2 - 9s^3 + 2s^4)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 3, "vs", 6

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", 1 + v[1] v[3] v[4] v[5] v[8] v[7]
"B CYCLES", 1 + v[1] v[2] v[3] v[5] v[6] v[7]

Eigenvalues

R: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

B: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

NullSpace of R

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[0, -1, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, -1, 1, 0]}

NullSpace of B^*

{[-1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, -1, 0, 0, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&0&0&4&0&0&0\\ 0&0&0&0&0&4&0&0\\ 0&0&0&0&0&0&4&0\\ 0&0&0&0&0&0&0&4\\ 4&0&0&0&0&0&0&0\\ 0&4&0&0&0&0&0&0\\ 0&0&4&0&0&0&0&0\\ 0&0&0&4&0&0&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&\frac{3}{5}&\frac{2}{5}&\frac{1}{5}&1&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\\ \frac{3}{5}&0&\frac{1}{5}&\frac{4}{5}&\frac{2}{5}&1&\frac{4}{5}&\frac{1}{5}\\ \frac{2}{5}&\frac{1}{5}&0&\frac{3}{5}&\frac{3}{5}&\frac{4}{5}&1&\frac{2}{5}\\ \frac{1}{5}&\frac{4}{5}&\frac{3}{5}&0&\frac{4}{5}&\frac{1}{5}&\frac{2}{5}&1\\ 1&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}&0&\frac{3}{5}&\frac{2}{5}&\frac{1}{5}\\ \frac{2}{5}&1&\frac{4}{5}&\frac{1}{5}&\frac{3}{5}&0&\frac{1}{5}&\frac{4}{5}\\ \frac{3}{5}&\frac{4}{5}&1&\frac{2}{5}&\frac{2}{5}&\frac{1}{5}&0&\frac{3}{5}\\ \frac{4}{5}&\frac{1}{5}&\frac{2}{5}&1&\frac{1}{5}&\frac{4}{5}&\frac{3}{5}&0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3   "Trace mark", 4, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8] \right)$
degree 2: $\frac{1}{4}\left(v[1]v[5] + v[2]v[6] + v[3]v[7] + v[4]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{4, 5, 6, 7}, {1, 2, 3, 8}}

"PT2" = {{2, 3, 5, 8}, {1, 4, 6, 7}}

"PT3" = {{2, 5, 7, 8}, {1, 3, 4, 6}}

"PT4" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$\pi 2$ = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {4, 11, 17, 22}

$u2$ = [3, 2, 1, 5, 2, 3, 4, 1, 4, 2, 5, 4, 1, 3, 3, 4, 5, 2, 4, 1, 2, 5, 3, 2, 1, 1, 4, 3]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[4], [1], [1], [2]]
Action of B on ranges, [[3], [4], [2], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [4, 1, 2, 2]

BPARTS [2, 3, 4, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[4, 5, 4, 7, 6, 3, 2, 5]

B-BLOCKS,

[3, 4, 5, 1, 8, 5, 4, 2]

with invariant measure, [1, 1, 1, 2, 2, 1, 1, 1]

N by blocks, N - check: true

b_1 = {2, 5, 7, 8}
b_2 = {1, 2, 3, 4}
b_3 = {5, 6, 7, 8}
b_4 = {2, 3, 5, 8}
b_5 = {1, 4, 6, 7}
b_6 = {4, 5, 6, 7}
b_7 = {1, 2, 3, 8}
b_8 = {1, 3, 4, 6}

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \\ h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: $23 \oplus 4/3$

$$\text{CLB} = \begin{pmatrix} -1 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3, 4, 5, 7, 8}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 5, 6, 7}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} \\ \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} \\ \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} \\ \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} & \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} \\ \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} \end{pmatrix}$$

Omega–checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4, 5, 6, 7}, {1, 2, 3, 8}}

1, "range", [4, 8], [[8, 8, 8, 4, 4, 4, 4, 8], [4, 4, 4, 8, 8, 8, 8, 4]]

2, "range", [3, 7], [[7, 7, 7, 3, 3, 3, 3, 7], [3, 3, 3, 7, 7, 7, 7, 3]]

3, "range", [2, 6], [[6, 6, 6, 2, 2, 2, 2, 6], [2, 2, 2, 6, 6, 6, 6, 2]]

4, "range", [1, 5], [[5, 5, 5, 1, 1, 1, 1, 5], [1, 1, 1, 5, 5, 5, 5, 1]]

2, "partition", {{2, 3, 5, 8}, {1, 4, 6, 7}}

1, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

2, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

3, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

3, "partition", {{2, 5, 7, 8}, {1, 3, 4, 6}}

1, "range", [4, 8], [[8, 4, 8, 8, 4, 8, 4, 4], [4, 8, 4, 4, 8, 4, 8, 8]]

2, "range", [3, 7], [[7, 3, 7, 7, 3, 7, 3, 3], [3, 7, 3, 3, 7, 3, 7, 7]]

3, "range", [2, 6], [[6, 2, 6, 6, 2, 6, 2, 2], [2, 6, 2, 2, 6, 2, 6, 6]]

4, "range", [1, 5], [[5, 1, 5, 5, 1, 5, 1, 1], [1, 5, 1, 1, 5, 1, 5, 5]]

4, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [4, 8], [[8, 8, 8, 8, 4, 4, 4, 4], [4, 4, 4, 4, 8, 8, 8, 8]]

2, "range", [3, 7], [[7, 7, 7, 7, 3, 3, 3, 3], [3, 3, 3, 3, 7, 7, 7, 7]]

3, "range", [2, 6], [[6, 6, 6, 6, 2, 2, 2, 2], [2, 2, 2, 2, 6, 6, 6, 6]]

4, "range", [1, 5], [[5, 5, 5, 5, 1, 1, 1, 1], [1, 1, 1, 1, 5, 5, 5, 5]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 = []

g_2 = [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 11, 17, 22}

$u2 =$
$(3 \;\; 2 \;\; 1 \;\; 5 \;\; 2 \;\; 3 \;\; 4 \;\; 1 \;\; 4 \;\; 2 \;\; 5 \;\; 4 \;\; 1 \;\; 3 \;\; 3 \;\; 4 \;\; 5 \;\; 2 \;\; 4 \;\; 1 \;\; 2 \;\; 5 \;\; 3 \;\; 2 \;\; 1 \;\; 1 \;\; 4$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{5}{2} & \frac{5}{2} & \frac{5}{2} & \frac{5}{2} & \frac{5}{2} & \frac{5}{2} & \frac{5}{2} & \frac{5}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} \\ 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} \\ 0 & 0 & 0 & \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 \\ 0 & 0 & \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 \\ 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 \\ 0 & 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} & 0 & 0 \\ 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} & 0 & 0 \\ 0 & \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 \\ \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 \\ \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 \\ \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 \\ \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{10} & \frac{3}{20} & \frac{1}{5} & 0 & \frac{3}{20} & \frac{1}{10} & \frac{1}{20} \\ \frac{1}{10} & \frac{1}{4} & \frac{1}{5} & \frac{1}{20} & \frac{3}{20} & 0 & \frac{1}{20} & \frac{1}{5} \\ \frac{3}{20} & \frac{1}{5} & \frac{1}{4} & \frac{1}{10} & \frac{1}{10} & \frac{1}{20} & 0 & \frac{3}{20} \\ \frac{1}{5} & \frac{1}{20} & \frac{1}{10} & \frac{1}{4} & \frac{1}{20} & \frac{1}{5} & \frac{3}{20} & 0 \\ 0 & \frac{3}{20} & \frac{1}{10} & \frac{1}{20} & \frac{1}{4} & \frac{1}{10} & \frac{3}{20} & \frac{1}{5} \\ \frac{3}{20} & 0 & \frac{1}{20} & \frac{1}{5} & \frac{1}{10} & \frac{1}{4} & \frac{1}{5} & \frac{1}{20} \\ \frac{1}{10} & \frac{1}{20} & 0 & \frac{3}{20} & \frac{3}{20} & \frac{1}{5} & \frac{1}{4} & \frac{1}{10} \\ \frac{1}{20} & \frac{1}{5} & \frac{3}{20} & 0 & \frac{1}{5} & \frac{1}{20} & \frac{1}{10} & \frac{1}{4} \end{pmatrix} \quad NM =$$

$$\begin{pmatrix} 4 & \frac{8}{5} & \frac{12}{5} & \frac{16}{5} & 0 & \frac{12}{5} & \frac{8}{5} & \frac{4}{5} \\ \frac{8}{5} & 4 & \frac{16}{5} & \frac{4}{5} & \frac{12}{5} & 0 & \frac{4}{5} & \frac{16}{5} \\ \frac{12}{5} & \frac{16}{5} & 4 & \frac{8}{5} & \frac{8}{5} & \frac{4}{5} & 0 & \frac{12}{5} \\ \frac{16}{5} & \frac{4}{5} & \frac{8}{5} & 4 & \frac{4}{5} & \frac{16}{5} & \frac{12}{5} & 0 \\ 0 & \frac{12}{5} & \frac{8}{5} & \frac{4}{5} & 4 & \frac{8}{5} & \frac{12}{5} & \frac{16}{5} \\ \frac{12}{5} & 0 & \frac{4}{5} & \frac{16}{5} & \frac{8}{5} & 4 & \frac{16}{5} & \frac{4}{5} \\ \frac{8}{5} & \frac{4}{5} & 0 & \frac{12}{5} & \frac{12}{5} & \frac{16}{5} & 4 & \frac{8}{5} \\ \frac{4}{5} & \frac{16}{5} & \frac{12}{5} & 0 & \frac{16}{5} & \frac{4}{5} & \frac{8}{5} & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, -1, 0, 1, 0, -1, 0, 1]$

$$\ker N_c = \begin{pmatrix} -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & -t & -s+t & s & 0 & -t & -s+t & s \\ t & -t & -s & s & t & -t & -s & s \\ s & 0 & -s & 0 & s & 0 & -s & 0 \end{pmatrix} \quad RB$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 0 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & t & -s \\ -s & 0 & 0 & -t \\ -s & -t & 0 & 0 \\ 0 & -s & t & 0 \\ 0 & 0 & -t & s \\ s & 0 & 0 & t \\ s & t & 0 & 0 \\ 0 & s & -t & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} t & s & 0 & -t & t \\ s & t & -s & 0 & s \\ s & 0 & -s & 0 & t+s \\ t & 0 & 0 & -t & t+s \\ s & -s & 0 & t & s \\ t & -t & s & 0 & t \\ t & 0 & s & 0 & 0 \\ s & 0 & 0 & t & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 0 & 4 \end{pmatrix}$$

$$R N_0 R^* = \begin{pmatrix} 1 & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & 0 & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{3}{5} & 1 & 1 & \frac{1}{5} & \frac{2}{5} & 0 & 0 & \frac{4}{5} \\ \frac{3}{5} & 1 & 1 & \frac{1}{5} & \frac{2}{5} & 0 & 0 & \frac{4}{5} \\ \frac{3}{5} & \frac{1}{5} & \frac{1}{5} & 1 & \frac{2}{5} & \frac{4}{5} & \frac{4}{5} & 0 \\ 0 & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & 1 & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} \\ \frac{2}{5} & 0 & 0 & \frac{4}{5} & \frac{3}{5} & 1 & 1 & \frac{1}{5} \\ \frac{2}{5} & 0 & 0 & \frac{4}{5} & \frac{3}{5} & 1 & 1 & \frac{1}{5} \\ \frac{2}{5} & \frac{4}{5} & \frac{4}{5} & 0 & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} & 1 \end{pmatrix}, R R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & \frac{1}{5} & \frac{3}{5} & 1 & 0 & \frac{4}{5} & \frac{2}{5} & 0 \\ \frac{1}{5} & 1 & \frac{3}{5} & \frac{1}{5} & \frac{4}{5} & 0 & \frac{2}{5} & \frac{4}{5} \\ \frac{3}{5} & \frac{3}{5} & 1 & \frac{3}{5} & \frac{2}{5} & \frac{2}{5} & 0 & \frac{2}{5} \\ 1 & \frac{1}{5} & \frac{3}{5} & 1 & 0 & \frac{4}{5} & \frac{2}{5} & 0 \\ 0 & \frac{4}{5} & \frac{2}{5} & 0 & 1 & \frac{1}{5} & \frac{3}{5} & 1 \\ \frac{4}{5} & 0 & \frac{2}{5} & \frac{4}{5} & \frac{1}{5} & 1 & \frac{3}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{2}{5} & 0 & \frac{2}{5} & \frac{3}{5} & \frac{3}{5} & 1 & \frac{3}{5} \\ 0 & \frac{4}{5} & \frac{2}{5} & 0 & 1 & \frac{1}{5} & \frac{3}{5} & 1 \end{pmatrix}, B B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew}$$

$$\mathrm{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&0&4&0&0&0\\0&4&0&0&0&4&0&0\\0&0&4&0&0&0&4&0\\0&0&0&4&0&0&0&4\\4&0&0&0&4&0&0&0\\0&4&0&0&0&4&0&0\\0&0&4&0&0&0&4&0\\0&0&0&4&0&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}&0&\frac{3}{5}&\frac{2}{5}&\frac{1}{5}\\ \frac{2}{5}&1&\frac{4}{5}&\frac{1}{5}&\frac{3}{5}&0&\frac{1}{5}&\frac{4}{5}\\ \frac{3}{5}&\frac{4}{5}&1&\frac{2}{5}&\frac{2}{5}&\frac{1}{5}&0&\frac{3}{5}\\ \frac{4}{5}&\frac{1}{5}&\frac{2}{5}&1&\frac{1}{5}&\frac{4}{5}&\frac{3}{5}&0\\ 0&\frac{3}{5}&\frac{2}{5}&\frac{1}{5}&1&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\\ \frac{3}{5}&0&\frac{1}{5}&\frac{4}{5}&\frac{2}{5}&1&\frac{4}{5}&\frac{1}{5}\\ \frac{2}{5}&\frac{1}{5}&0&\frac{3}{5}&\frac{3}{5}&\frac{4}{5}&1&\frac{2}{5}\\ \frac{1}{5}&\frac{4}{5}&\frac{3}{5}&0&\frac{4}{5}&\frac{1}{5}&\frac{2}{5}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{1}{10}\ \frac{1}{20}\ \frac{1}{5}\ \frac{1}{10}\ \frac{1}{4}\ \frac{1}{5}\ \frac{3}{20}\ \frac{1}{20}\ \frac{1}{5}\ \frac{1}{4}\ \frac{1}{10}\ \frac{1}{20}\ \frac{1}{10}\ \frac{3}{20}\ 0\ \frac{1}{5}\ \frac{3}{20}\ \frac{1}{10}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM}\left(4\ \frac{8}{5}\ \frac{4}{5}\ \frac{16}{5}\ \frac{8}{5}\ 4\ \frac{16}{5}\ \frac{12}{5}\ \frac{4}{5}\ \frac{16}{5}\ 4\ \frac{8}{5}\ \frac{4}{5}\ \frac{8}{5}\ \frac{12}{5}\ 0\ \frac{16}{5}\ \frac{12}{5}\ \frac{8}{5}\ 4\right)$

"IS MN in Vec(K)?", true

$\mathrm{MN}\left(4\ \frac{8}{5}\ \frac{4}{5}\ \frac{16}{5}\ \frac{8}{5}\ 4\ \frac{16}{5}\ \frac{12}{5}\ \frac{4}{5}\ \frac{16}{5}\ 4\ \frac{8}{5}\ \frac{4}{5}\ \frac{8}{5}\ \frac{12}{5}\ 0\ \frac{16}{5}\ \frac{12}{5}\ \frac{8}{5}\ 4\right)$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 1/8$, min $\boldsymbol{\tau} = 8/1$, $\boldsymbol{\tau}$–check is positive? 24/1

max r = 8/1 , r–check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 4, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 32

dim span idems 16 vs no. of idems 16

"PT1" = {{4, 5, 6, 7}, {1, 2, 3, 8}}

"PT2" = {{2, 3, 5, 8}, {1, 4, 6, 7}}

"PT3" = {{2, 5, 7, 8}, {1, 3, 4, 6}}

"PT4" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \\ 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{11}{40} & \frac{19}{40} & \frac{27}{40} & \frac{-1}{8} & \frac{19}{40} & \frac{11}{40} & \frac{3}{40} \\ \frac{11}{40} & \frac{7}{8} & \frac{27}{40} & \frac{3}{40} & \frac{19}{40} & \frac{-1}{8} & \frac{3}{40} & \frac{27}{40} \\ \frac{19}{40} & \frac{27}{40} & \frac{7}{8} & \frac{11}{40} & \frac{11}{40} & \frac{3}{40} & \frac{-1}{8} & \frac{19}{40} \\ \frac{27}{40} & \frac{3}{40} & \frac{11}{40} & \frac{7}{8} & \frac{3}{40} & \frac{27}{40} & \frac{19}{40} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{19}{40} & \frac{11}{40} & \frac{3}{40} & \frac{7}{8} & \frac{11}{40} & \frac{19}{40} & \frac{27}{40} \\ \frac{19}{40} & \frac{-1}{8} & \frac{3}{40} & \frac{27}{40} & \frac{11}{40} & \frac{7}{8} & \frac{27}{40} & \frac{3}{40} \\ \frac{11}{40} & \frac{3}{40} & \frac{-1}{8} & \frac{19}{40} & \frac{19}{40} & \frac{27}{40} & \frac{7}{8} & \frac{11}{40} \\ \frac{3}{40} & \frac{27}{40} & \frac{19}{40} & \frac{-1}{8} & \frac{27}{40} & \frac{3}{40} & \frac{11}{40} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix}$$

N_C–scaled =

$$\begin{pmatrix} 1 & \frac{11}{35} & \frac{19}{35} & \frac{27}{35} & \frac{-1}{7} & \frac{19}{35} & \frac{11}{35} & \frac{3}{35} \\ \frac{11}{35} & 1 & \frac{27}{35} & \frac{3}{35} & \frac{19}{35} & \frac{-1}{7} & \frac{3}{35} & \frac{27}{35} \\ \frac{19}{35} & \frac{27}{35} & 1 & \frac{11}{35} & \frac{11}{35} & \frac{3}{35} & \frac{-1}{7} & \frac{19}{35} \\ \frac{27}{35} & \frac{3}{35} & \frac{11}{35} & 1 & \frac{3}{35} & \frac{27}{35} & \frac{19}{35} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{19}{35} & \frac{11}{35} & \frac{3}{35} & 1 & \frac{11}{35} & \frac{19}{35} & \frac{27}{35} \\ \frac{19}{35} & \frac{-1}{7} & \frac{3}{35} & \frac{27}{35} & \frac{11}{35} & 1 & \frac{27}{35} & \frac{3}{35} \\ \frac{11}{35} & \frac{3}{35} & \frac{-1}{7} & \frac{19}{35} & \frac{19}{35} & \frac{27}{35} & 1 & \frac{11}{35} \\ \frac{3}{35} & \frac{27}{35} & \frac{19}{35} & \frac{-1}{7} & \frac{27}{35} & \frac{3}{35} & \frac{11}{35} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 1.365685425, 0.2343145752, 2.094427191, 0.3055728092]

Eigenvalues M_C−scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 0., 0., 3.428571429, 1.560783343, 0.2677880860, 2.393631075, 0.349226067]

NullSpace M_C

{[1, 0, 0, 0, 0, 1, 1, 1], [−1, 0, 0, 0, 1, 0, 0, 0], [0, 0, 1, 0, 0, 0, −1, 0], [1, 1, 0, 0, 0, 0, 1, 1], [0, 0, 0, 1, 0, 0, 0, −1]}

NullSpace N_c

{[0, 0, 1, −1, 0, 0, 1, −1], [0, 1, 0, −1, 0, 1, 0, −1], [1, 0, 0, −1, 1, 0, 0, −1]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 1.365685425, 0.2343145752, 2.094427191, 0.3055728092]

NullSpace M_0

{[0, −1, 0, 0, 0, 1, 0, 0], [0, 0, 0, −1, 0, 0, 0, 1], [1, 0, 0, 0, −1, 0, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0]}

NullSpace N_0

{[−1, 0, 0, 1, −1, 0, 0, 1], [−1, 1, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, −1, 0, 1, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −0.2343145752, −1.365685425, −0.3055728092, −2.094427191]

NullSpace M

{}

NullSpace N

{[0, −1, 1, 0, 0, −1, 1, 0], [1, −1, 0, 0, 1, −1, 0, 0], [0, −1, 0, 1, 0, −1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 1 & 5 & 2 & 3 & 4 \\ 3 & 0 & 1 & 4 & 2 & 5 & 4 & 1 \\ 2 & 1 & 0 & 3 & 3 & 4 & 5 & 2 \\ 1 & 4 & 3 & 0 & 4 & 1 & 2 & 5 \\ 5 & 2 & 3 & 4 & 0 & 3 & 2 & 1 \\ 2 & 5 & 4 & 1 & 3 & 0 & 1 & 4 \\ 3 & 4 & 5 & 2 & 2 & 1 & 0 & 3 \\ 4 & 1 & 2 & 5 & 1 & 4 & 3 & 0 \end{pmatrix}$$

==

{2, 4, 5, 7}

R: [3, 8, 1, 6, 2, 7, 4, 5]
B: [6, 3, 8, 1, 7, 4, 5, 2]

TRACE TWO = 2

det AT = $\frac{-1}{4} (t)^2 (1 + t^2)^2$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 8

Level 2 det = $\frac{-5}{134217728} (11 + 3 s + s^2 + s^3) (-1 + s)^2 (-3 + s) (101 - 62 s$

$-6s^2 + 2s^3 + 5s^4)\,(1+s)^2\,(-31 - 2s^2 + s^4)\,(-41 - 8s - 4s^2 - 4s^3 + s^4)$

RANK of R is 8

R ranking is 1, "vs", 8

RBAR ranking 1, "vs", 8

RANK of B is 8

B ranking is 1, "vs", 8

BBAR ranking 1, "vs", 8

"R CYCLES", (1 + v[1] v[3]) (1 + v[4] v[6] v[7]) (1 + v[2] v[5] v[8])
"B CYCLES", (1 + v[2] v[3] v[8]) (1 + v[1] v[4] v[6]) (1 + v[5] v[7])

Eigenvalues

R: [-1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., 1., 1.]

B: [-1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., 1., 1.]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 8

"RANK of the KERNEL is ", 8

"IdemSolvability Check", 3 "Trace mark", 8, "Rank mark", 8, "for kernel rank", 8

degree 1: $\frac{1}{8}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8] \right)$

degree 2: $\frac{1}{24}\left(v[2]v[3] + v[3]v[7] + v[5]v[8] + v[1]v[6] + v[6]v[7] + v[2]v[5] + v[1]v[7] + v[7]v[8] + v[4]v[5] + 6v[6]v[8] + v[4]v[8] + v[4]v[7] + v[2]v[6] + v[5]v[6] + 6v[2]v[4] + v[3]v[5] + v[1]v[2] + 6v[5]v[7] + 6v[1]v[3] + v[2]v[8] + v[4]v[6] + v[3]v[6] + v[3]v[4] + v[1]v[4] + v[2]v[7] + v[3]v[8] + v[1]v[5] + v[1]v[8] \right)$

degree 3 : $\frac{1}{24}\left(4v[2]v[5]v[7] + 3v[1]v[5]v[8] + 3v[1]v[5]v[6] + 3v[1]v[4]v[7] + 4v[5]v[6]v[7] + 3v[2]v[3]v[7] + 3v[3]v[4]v[5] + 3v[3]v[6]v[7] + 3v[4]v[5]v[8] + 3v[3]v[5]v[6] + 4v[1]v[3]v[4] + 4v[5]v[7]v[8] + 3v[2]v[3]v[6] + 4v[3]v[5]v[7] + 4v[3]v[6]v[8] + 4v[1]v[3]v[7] + 3v[3]v[4]v[7] + 3v[1]v[2]v[8] + 3v[1]v[4]v[8] + 3v[1]v[2]v[5] + 4v[2]v[4]v[7] + 3v[3]v[5]v[8] + 4v[1]v[2]v[3] + 3v[2]v[3]v[8] + 3v[1]v[7]v[8] + 3v[2]v[5]v[8] + 4v[1]v[5]v[7] + 3v[1]v[4]v[6] + 3v[4]v[6]v[7] + 4v[6]v[7]v[8] + 3v[3]v[4]v[6] + 4v[5]v[6]v[8] + 3v[3]v[4]v[8] + 3v[4]v[7]v[8] + 3v[2]v[3]v[5] + 4v[1]v[2]v[4] + 4v[1]v[3]v[5] + 3v[2]v[5]v[6] + 3v[1]v[2]v[6] + 3v[1]v[4]v[5] + 3v[3]v[7]v[8] + 3v[2]v[6]v[7] + 4v[2]v[4]v[6] + 3v[2]v[7]v[8] + 3v[4]v[5]v[6] + 4v[1]v[3]v[8] + 4v[2]v[4]v[5] + 4v[4]v[5]v[7] + 4v[2]v[3]v[4] + 4v[2]v[4]v[8] + 4v[4]v[6]v[8] + 3v[1]v[2]v[7] + 4v[1]v[6]v[8] + 3v[1]v[6]v[7] + 4v[1]v[3]v[6] + 4v[2]v[6]v[8] \right)$

degree 4 : $\frac{1}{6}(v[1]v[3]v[7]v[8] + 3v[2]v[3]v[5]v[8] + v[1]v[5]v[6]v[7] + v[2]v[3]v[4]v[8] + v[1]v[5]v[7]v[8] + v[1]v[5]v[6]v[8] + 3v[1]v[4]v[5]v[8] + v[1]v[3]v[5]v[8] + v[1]v[4]v[5]v[7] + v[1]v[2]v[3]v[7] + 3v[2]v[3]v[5]v[6] + v[4]v[6]v[7]v[8] + v[3]v[5]v[6]v[8] + v[3]v[4]v[5]v[7] + v[2]v[4]v[7]v[8] + 3v[2]v[3]v[7]v[8] + v[1]v[2]v[5]v[7] + v[1]v[2]v[4]v[8] + v[1]v[2]v[4]v[5] + v[1]v[2]v[3]v[6] + v[1]v[2]v[3]v[5] + v$

[2]v[6]v[7]v[8] + v[3]v[5]v[6]v[7] + 3v[1]v[4]v[7]v[8] + 8v[1]v[2]v[3]v[4] + 8v[2]v[
4]v[5]v[7] + v[3]v[4]v[6]v[8] + v[1]v[2]v[3]v[8] + 3v[3]v[4]v[7]v[8] + v[4]v[5]v[7]
v[8] + v[2]v[5]v[8]v[7] + 8v[2]v[4]v[6]v[8] + v[2]v[3]v[6]v[8] + 8v[1]v[3]v[5]v[7]
+ v[1]v[3]v[4]v[6] + 3v[1]v[4]v[6]v[7] + v[1]v[2]v[4]v[7] + v[4]v[5]v[6]v[7] + v[2]v
[4]v[5]v[8] + v[1]v[3]v[4]v[7] + v[2]v[4]v[5]v[6] + 3v[1]v[2]v[5]v[8] + v[1]v[3]v[5]
v[6] + 3v[3]v[4]v[5]v[6] + v[1]v[3]v[4]v[5] + v[2]v[3]v[4]v[6] + 3v[1]v[2]v[5]v[6]
+ v[4]v[5]v[6]v[8] + v[3]v[5]v[7]v[8] + v[2]v[4]v[6]v[7] + 3v[1]v[2]v[7]v[8] + v[2]v
[3]v[5]v[7] + 3v[3]v[4]v[5]v[8] + v[1]v[3]v[4]v[8] + v[2]v[5]v[6]v[7] + 3v[2]v[3]v[
6]v[7] + v[3]v[6]v[7]v[8] + 3v[1]v[4]v[5]v[6] + v[1]v[4]v[6]v[8] + 3v[1]v[2]v[6]v[7]
+ v[2]v[3]v[4]v[7] + v[2]v[5]v[6]v[8] + 3v[3]v[4]v[6]v[7] + v[1]v[6]v[7]v[8] + v[1]v
[3]v[6]v[7] + v[2]v[3]v[4]v[5] + v[1]v[2]v[6]v[8] + 8v[1]v[3]v[6]v[8] + 8v[5]v[6]v[7
]v[8] + v[1]v[2]v[4]v[6])

degree 5 : $\frac{1}{24}$ (4v[2]v[4]v[5]v[6]v[7] + 3v[3]v[4]v[5]v[7]v[8] + 3v[1]v[2]v[3]v[6]v[
7] + 3v[1]v[2]v[4]v[6]v[7] + 3v[3]v[4]v[6]v[7]v[8] + 4v[1]v[2]v[4]v[5]v[7] + 4v[2]
v[4]v[5]v[7]v[8] + 4v[1]v[2]v[3]v[4]v[8] + 3v[1]v[2]v[4]v[5]v[6] + 3v[1]v[2]v[5]v[
6]v[7] + 4v[1]v[3]v[6]v[7]v[8] + 4v[2]v[3]v[4]v[5]v[7] + 3v[3]v[4]v[5]v[6]v[7] + 4
v[2]v[5]v[6]v[7]v[8] + 3v[1]v[4]v[5]v[6]v[8] + 4v[1]v[3]v[5]v[7]v[8] + 4v[1]v[3]v[
4]v[5]v[7] + 3v[1]v[3]v[4]v[7]v[8] + 4v[1]v[3]v[5]v[6]v[7] + 3v[2]v[3]v[5]v[6]v[8]
+ 4v[3]v[5]v[6]v[7]v[8] + 3v[2]v[3]v[4]v[5]v[6] + 4v[1]v[3]v[4]v[6]v[8] + 4v[1]v[2
]v[3]v[4]v[6] + 3v[1]v[2]v[4]v[7]v[8] + 4v[1]v[2]v[3]v[5]v[7] + 3v[2]v[3]v[4]v[5]v[
8] + 3v[3]v[4]v[5]v[6]v[8] + 4v[1]v[2]v[3]v[4]v[7] + 3v[1]v[3]v[4]v[5]v[8] + 4v[1]
v[2]v[3]v[6]v[8] + 3v[2]v[3]v[5]v[8]v[7] + 4v[2]v[3]v[4]v[6]v[8] + 4v[2]v[4]v[6]v[
8]v[7] + 4v[2]v[4]v[5]v[6]v[8] + 3v[1]v[3]v[4]v[6]v[7] + 3v[1]v[2]v[3]v[5]v[8] + 3
v[2]v[3]v[6]v[7]v[8] + 3v[1]v[4]v[5]v[6]v[7] + 4v[1]v[2]v[4]v[6]v[8] + 3v[1]v[2]v[
3]v[5]v[6] + 3v[1]v[2]v[4]v[5]v[8] + 3v[1]v[2]v[6]v[7]v[8] + 4v[1]v[2]v[3]v[4]v[5]
+ 4v[4]v[5]v[6]v[7]v[8] + 3v[1]v[2]v[5]v[6]v[8] + 3v[2]v[3]v[4]v[7]v[8] + 4v[1]v[5
]v[6]v[7]v[8] + 3v[1]v[2]v[3]v[7]v[8] + 3v[1]v[2]v[5]v[8]v[7] + 3v[2]v[3]v[4]v[6]v[
7] + 4v[1]v[3]v[5]v[6]v[8] + 3v[1]v[3]v[4]v[5]v[6] + 3v[1]v[4]v[6]v[7]v[8] + 3v[1]v
[4]v[5]v[7]v[8] + 3v[2]v[3]v[5]v[6]v[7])

degree 6 : $\frac{1}{24}$ (6v[1]v[2]v[3]v[4]v[5]v[7] + v[1]v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]
v[5]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[4]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[
4]v[5]v[8] + v[1]v[2]v[3]v[4]v[7]v[8] + v[3]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]
v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[7] + v[2]v[3]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[8
]v[7] + 6v[2]v[4]v[5]v[6]v[8]v[7] + v[1]v[2]v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[5]v[6]v
[7] + 6v[1]v[2]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[5]v[6]v[8] + v[1]v[3]v[4]v[5]v[8]v[7
] + v[1]v[2]v[3]v[5]v[8]v[7] + v[1]v[2]v[4]v[6]v[8]v[7] + v[1]v[2]v[3]v[4]v[5]v[6] +
v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[5]v[6]v[8] + v[1]v[2]v[5]v[6]v[7]v[8] + v[1
]v[2]v[3]v[4]v[6]v[7] + v[1]v[2]v[4]v[5]v[7]v[8] + 6v[1]v[3]v[5]v[6]v[7]v[8] + v[1]v

[2]v[3]v[6]v[7]v[8])
degree 7 : $\frac{1}{8}$ (v[1]v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[8]v[7] + v[1]v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[8]v[7] + v[1]v[3]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[8]v[7])
degree 8 : 1 (v[1]) (v[2]) (v[3]) (v[8]) (v[7]) (v[5]) (v[4]) (v[6])

Group spectrum $1 + t + 2t^2 + 2t^3 + 3t^4 + 2t^5 + 2t^6 + t^7 + t^8$

KERNEL STRUCTURE

"PT1" = {{8}, {4}, {1}, {5}, {6}, {3}, {7}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$\pi 8 = [1]$$

supp π8 = {1}

$$u8 = [1]$$

supp u8 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[8, 7, 6, 1, 2, 3, 5, 4]

B–BLOCKS,

[6, 5, 2, 7, 3, 8, 4, 1]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

b_1 = {8}
b_2 = {4}
b_3 = {1}
b_4 = {5}
b_5 = {6}
b_6 = {3}
b_7 = {7}
b_8 = {2}

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[3] & h[2] & h[1] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[3] & h[2] & h[1] & h[2] & h[2] & h[2] & h[2] \\ h[1] & h[2] & h[3] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[1] & h[2] & h[3] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[3] & h[2] & h[1] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[3] & h[2] & h[1] \\ h[2] & h[2] & h[2] & h[2] & h[1] & h[2] & h[3] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[1] & h[2] & h[3] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 24, Shape: 23 ⊕ 1/0

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3}, {2, 5, 8}, {4, 6, 7}}, true

Ω_B in Vec(K)? , {{2, 3, 8}, {1, 4, 6}, {5, 7}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} \\ \frac{-3}{40} & \frac{29}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{-27}{40} & \frac{21}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} \\ \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} & \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} \\ \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} \end{pmatrix}$$

Omega−checks true, true

"I−V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{8}, {4}, {1}, {5}, {6}, {3}, {7}, {2}}

1, "range", [1, 2, 3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 4, 3, 2, 1], [8, 7, 6, 5, 4, 1, 2, 3], [8, 7, 6, 5, 2, 3, 4, 1], [8, 7, 6, 5, 2, 1, 4, 3], [8, 5, 6, 7, 4, 3, 2, 1], [8, 5, 6, 7, 4, 1, 2, 3], [8, 5, 6, 7, 2, 3, 4, 1], [8, 5, 6, 7, 2, 1, 4, 3], [8, 4, 6, 2, 3, 7, 1, 5], [8, 4, 6, 2, 3, 5, 1, 7], [8, 4, 6, 2, 1, 7, 3, 5], [8, 4, 6, 2, 1, 5, 3, 7], [8, 3, 6, 1, 7, 4, 5, 2], [8, 3, 6, 1, 7, 2, 5, 4], [8, 3, 6, 1, 5, 4, 7, 2], [8, 3, 6, 1, 5, 2, 7, 4], [8, 2, 6, 4, 3, 7, 1, 5], [8, 2, 6, 4, 3, 5, 1, 7], [8, 2, 6, 4, 1, 7, 3, 5], [8, 2, 6, 4, 1, 5, 3, 7], [8, 1, 6, 3, 7, 4, 5, 2], [8, 1, 6, 3, 7, 2, 5, 4], [8, 1, 6, 3, 5, 4, 7, 2], [8, 1, 6, 3, 5, 2, 7, 4], [7, 8, 5, 6, 3, 4, 1, 2], [7, 8, 5, 6, 3, 2, 1, 4], [7, 8, 5, 6, 1, 4, 3, 2], [7, 8, 5, 6, 1, 2, 3, 4], [7, 6, 5, 8, 3, 4, 1, 2], [7, 6, 5, 8, 3, 2, 1, 4], [7, 6, 5, 8, 1, 4, 3, 2], [7, 6, 5, 8, 1, 2, 3, 4], [7, 4, 5, 2, 8, 3, 6, 1], [7, 4, 5, 2, 8, 1, 6, 3], [7, 4, 5, 2, 6, 3, 8, 1], [7, 4, 5, 2, 6, 1, 8, 3], [7, 3, 5, 1, 4, 8, 2, 6], [7, 3, 5, 1, 4, 6, 2, 8], [7, 3, 5, 1, 2, 8, 4, 6], [7, 3, 5, 1, 2, 6, 4, 8], [7, 2, 5, 4, 8, 3, 6, 1], [7, 2, 5, 4, 8, 1, 6, 3], [7, 2, 5, 4, 6, 3, 8, 1], [7, 2, 5, 4, 6, 1, 8, 3], [7, 1, 5, 3, 4, 8, 2, 6], [7, 1, 5, 3, 4, 6, 2, 8], [7, 1, 5, 3, 2, 8, 4, 6], [7, 1, 5, 3, 2, 6, 4, 8], [6, 7, 8, 5, 4, 3, 2, 1], [6, 7, 8, 5, 4, 1, 2, 3], [6, 7, 8, 5, 2, 3, 4, 1], [6, 7, 8, 5, 2, 1, 4, 3], [6, 5, 8, 7, 4, 3, 2, 1], [6, 5, 8, 7, 4, 1, 2, 3], [6, 5, 8, 7, 2, 3, 4, 1], [6, 5, 8, 7, 2, 1, 4, 3], [6, 4, 8, 2, 3, 7, 1, 5], [6, 4, 8, 2, 3, 5, 1, 7], [6, 4, 8, 2, 1, 7, 3, 5], [6, 4, 8, 2, 1, 5, 3, 7], [6, 3, 8, 1, 7, 4, 5, 2], [6, 3, 8, 1, 7, 2, 5, 4], [6, 3, 8, 1, 5, 4, 7, 2], [6, 3, 8, 1, 5, 2, 7, 4], [6, 2, 8, 4, 3, 7, 1, 5], [6, 2, 8, 4, 3, 5, 1, 7], [6, 2, 8, 4, 1, 7, 3, 5], [6, 2, 8, 4, 1, 5, 3, 7], [6, 1, 8, 3, 7, 4, 5, 2], [6, 1, 8, 3, 7, 2, 5, 4], [6, 1, 8, 3, 5, 4, 7, 2], [6, 1, 8, 3, 5, 2, 7, 4], [5, 8, 7, 6, 3, 4, 1, 2], [5, 8, 7, 6, 3, 2, 1, 4], [5, 8, 7, 6, 1, 4, 3, 2], [5, 8, 7, 6, 1, 2, 3, 4], [5, 6, 7, 8, 3, 4, 1, 2], [5, 6, 7, 8, 3, 2, 1, 4], [5, 6, 7, 8, 1, 4, 3, 2], [5, 6, 7, 8, 1, 2, 3, 4], [5, 4, 7, 2, 8, 3, 6, 1], [5, 4, 7, 2, 8, 1, 6, 3], [5, 4, 7, 2, 6, 3, 8, 1], [5, 4, 7, 2, 6, 1, 8, 3], [5, 3, 7, 1, 4, 8, 2, 6], [5, 3, 7, 1, 4, 6, 2, 8], [5, 3, 7, 1, 2, 8, 4, 6], [5, 3, 7, 1, 2, 6, 4, 8], [5, 2, 7, 4, 8, 3, 6, 1], [5, 2, 7, 4, 8, 1, 6, 3], [5, 2, 7, 4, 6, 3, 8, 1], [5, 2, 7, 4, 6, 1, 8, 3], [5, 1, 7, 3, 4, 8, 2, 6], [5, 1, 7, 3, 4, 6, 2, 8], [5, 1, 7, 3, 2, 8, 4, 6], [5, 1, 7, 3, 2, 6, 4, 8], [4, 8, 2, 6, 7, 3, 5, 1], [4, 8, 2, 6, 7, 1, 5, 3], [4, 8, 2, 6, 5, 3, 7, 1], [4, 8, 2, 6, 5, 1, 7, 3], [4, 7, 2, 5, 3, 8, 1, 6], [4, 7, 2, 5, 3, 6, 1, 8], [4, 7, 2, 5, 1, 8, 3, 6], [4, 7, 2, 5, 1, 6, 3, 8], [4, 6, 2, 8, 7, 3, 5, 1], [4, 6, 2, 8, 7, 1, 5, 3], [4, 6, 2, 8, 5, 3, 7, 1], [4, 6, 2, 8, 5, 1, 7, 3], [4, 5, 2, 7, 3, 8, 1, 6], [4, 5, 2, 7, 3, 6, 1, 8], [4, 5, 2, 7, 1, 8, 3, 6], [4, 5, 2, 7, 1, 6, 3, 8], [4, 3, 2, 1, 8, 7, 6, 5], [4, 3, 2, 1, 8, 5, 6, 7], [4, 3, 2, 1, 6, 7, 8, 5], [4, 3, 2, 1, 6, 5, 8, 7], [4, 1, 2, 3, 8, 7, 6, 5], [4, 1, 2, 3, 8, 5, 6, 7], [4, 1, 2, 3, 6, 7, 8, 5], [4, 1, 2, 3, 6, 5, 8, 7], [3, 8, 1, 6, 4, 7, 2, 5], [3, 8, 1, 6, 4, 5, 2, 7], [3, 8, 1, 6, 2, 7, 4, 5], [3, 8, 1, 6, 2, 5, 4, 7], [3, 7, 1, 5, 8, 4, 6, 2], [3, 7, 1, 5, 8, 2, 6, 4], [3, 7, 1, 5, 6, 4, 8, 2], [3, 7, 1, 5, 6, 2, 8, 4], [3, 6, 1, 8, 4, 7, 2, 5], [3, 6, 1, 8, 4, 5, 2, 7], [3, 6, 1, 8, 2, 7, 4, 5], [3, 6, 1, 8, 2, 5, 4, 7], [3, 5, 1, 7, 8, 4, 6, 2], [3, 5, 1, 7, 8, 2, 6, 4], [3, 5, 1, 7, 6, 4, 8, 2], [3, 5, 1, 7, 6, 2, 8, 4], [3, 4, 1, 2, 7, 8, 5, 6], [3, 4, 1, 2, 7, 6, 5, 8], [3, 4, 1, 2, 5, 8, 7, 6], [3, 4, 1, 2, 5, 6, 7, 8], [3, 2, 1, 4, 7, 8, 5, 6], [3, 2, 1, 4, 7, 6, 5, 8], [3, 2, 1, 4, 5, 8, 7, 6], [3, 2, 1, 4, 5, 6, 7, 8], [2, 8, 4, 6, 7, 3, 5, 1], [2, 8, 4, 6, 7, 1, 5, 3], [2, 8, 4, 6, 5, 3, 7, 1], [2, 8, 4, 6, 5, 1, 7, 3], [2, 7, 4, 5, 3, 8, 1, 6], [2, 7, 4, 5, 3, 6, 1, 8], [2, 7, 4, 5, 1, 8, 3, 6], [2, 7, 4, 5, 1, 6, 3, 8], [2, 6, 4, 8, 7, 3, 5, 1], [2, 6, 4, 8, 7, 1, 5, 3], [2, 6, 4, 8, 5, 3, 7, 1], [2, 6, 4, 8, 5, 1, 7, 3], [2, 5, 4, 7, 3, 8, 1, 6], [2, 5, 4, 7, 3, 6, 1, 8], [2, 5, 4, 7, 1, 8, 3, 6], [2, 5, 4, 7, 1, 6, 3, 8], [2, 3, 4, 1, 8, 7, 6, 5], [2, 3, 4, 1, 8, 5, 6, 7], [2, 3, 4, 1, 6, 7, 8, 5], [2, 3, 4, 1, 6, 5, 8, 7], [2, 1, 4, 3, 8, 7, 6, 5], [2, 1, 4, 3, 8, 5, 6, 7], [2, 1, 4, 3, 6, 7, 8, 5], [2, 1, 4, 3, 6, 5, 8, 7], [1, 8, 3, 6, 4, 7, 2, 5], [1, 8, 3, 6, 4, 5, 2, 7], [1, 8, 3, 6, 2, 7, 4, 5], [1, 8, 3, 6, 2, 5, 4, 7], [1, 7, 3, 5, 8, 4, 6, 2], [1, 7, 3, 5, 8, 2, 6, 4], [1, 7, 3, 5, 6, 4, 8, 2], [1, 7, 3, 5, 6, 2, 8, 4], [1, 6, 3, 8, 4, 7, 2, 5], [1, 6, 3, 8, 4, 5, 2, 7], [1, 6, 3, 8, 2, 7, 4, 5], [1, 6, 3, 8, 2, 5, 4, 7], [1, 5, 3, 7, 8, 4, 6, 2], [1, 5, 3, 7, 8, 2, 6, 4], [1, 5, 3, 7, 6, 4, 8, 2], [1, 5, 3, 7, 6, 2, 8, 4], [1, 4, 3, 2, 7, 8, 5, 6], [1, 4, 3, 2, 7, 6, 5, 8], [1, 4, 3, 2, 5, 8, 7, 6], [1, 4, 3, 2, 5, 6, 7, 8], [1, 2, 3, 4, 7, 8, 5, 6], [1, 2, 3, 4, 7, 6, 5, 8], [1, 2, 3, 4, 5, 8, 7, 6], [1, 2, 3, 4, 5, 6, 7, 8]]

"group has", 192, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

g_1 =   [[1, 8], [2, 7], [3, 6], [4, 5]]

g_2 =   [[1, 8, 3, 6], [2, 7], [4, 5]]

g_3 =   [[1, 8], [2, 7, 4, 5], [3, 6]]

g_4 =   [[1, 8, 3, 6], [2, 7, 4, 5]]

g_5 =   [[1, 8], [2, 5, 4, 7], [3, 6]]

linear dimension, 26

"Symmetric?", true

Is Z in Vec(K)? true
$(-24h[3]-24h[1]-8h[2]\ \ 4h[2]\ \ 4h[2]\ \ 4h[2]\ \ -24h[1]-8h[2]\ \ 4h[2]\ \ 4h[2]\ \ 24$

"Basis for Z(G)"

1, "coeff", 24

$$Z[\,1\,]=\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}$$

2, "coeff", 4

$$Z[\,2\,]=\begin{pmatrix}0&1&0&1&1&1&1&1\\1&0&1&0&1&1&1&1\\0&1&0&1&1&1&1&1\\1&0&1&0&1&1&1&1\\1&1&1&1&0&1&0&1\\1&1&1&1&1&0&1&0\\1&1&1&1&0&1&0&1\\1&1&1&1&1&0&1&0\end{pmatrix}$$

3, "coeff", 24

$$Z[3] = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 6. & -2. & -2. & -2. & 0 & 0 & 0 & 0 \\ 1. & -1. & 1. & -1. & 1. & -1. & 1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 2t^3 + 3t^4 + 2t^5 + 2t^6 + t^7 + t^8$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 11t^4 + 18t^5 + 36t^6 + 58t^7 + 102t^8 + 160t^9 + 258t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 8 = (1)$

{1}

$u8 = (1)$

{1}

picheck $(1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1)$

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$\pi 7 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$u7 = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$\pi 6 =$

$$(2 \quad 2$$

$u6 =$

$$\left(\frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32} \quad \frac{1}{32}\right.$$

picheck $\begin{pmatrix} 42 & 42 & 42 & 42 & 42 & 42 & 42 & 42 \end{pmatrix}$

$\pi 5 =$

$$(6 \quad 6$$

$u5 =$

$$\left(\frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{25}\right.$$

picheck $\begin{pmatrix} 210 & 210 & 210 & 210 & 210 & 210 & 210 & 210 \end{pmatrix}$

$\pi 4 =$

$$(24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 2$$

$u4 =$

$$\left(\frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{51}\right.$$

picheck $\begin{pmatrix} 840 & 840 & 840 & 840 & 840 & 840 & 840 & 840 \end{pmatrix}$

$\pi 3 =$

$$(120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120$$

$u3 =$

$$\left(\frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096}\right.$$

picheck $\begin{pmatrix}2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520\end{pmatrix}$

$\pi 2 =$
$\begin{pmatrix}720 & 720 & 720 & 720 & 720 & 720 & 720 & 720 & 720 & 720 & 720 & 720 & 720 & 720 & 720\end{pmatrix}$

$u2 =$
$\begin{pmatrix}\frac{45}{16384} & \frac{45}{16384} & \frac{45}{16384} & \frac{45}{16384} & \frac{45}{16384} & \frac{45}{16384} & \frac{45}{16384} & \frac{45}{16384} & \frac{45}{16384} & \frac{45}{16384} & \frac{45}{1638}\end{pmatrix}$

picheck $\begin{pmatrix}5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040\end{pmatrix}$

$\pi 1 = \begin{pmatrix}5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072}\end{pmatrix}$

picheck $\begin{pmatrix}5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0, 0, 0]$

$\ker N_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} s+t & s+t & s+t & s+t & s+t & s+t & s+t & s+t \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -t & -t & -t & -t & s-t & -t & -t \\ 0 & 0 & s & 0 & t & 0 & 0 \\ 0 & 0 & t & 0 & 0 & 0 & s \\ -s & -s & -s & -s & -s & -s & t-s \\ 0 & t & 0 & s & 0 & 0 & 0 \\ 0 & s & 0 & 0 & 0 & t & 0 \\ t & 0 & 0 & 0 & 0 & s & 0 \\ s & 0 & 0 & t & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_C = \begin{pmatrix} 0&0&0&0&0&0&1&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&1&0&0&0&0&0 \end{pmatrix} \begin{pmatrix} t&0&0&s&0&0&0&0 \\ 0&0&s&t&0&0&0&0 \\ 0&0&t&0&0&0&s&0 \\ s&0&0&0&0&t&0&0 \\ 0&0&0&0&s&0&0&t \\ 0&t&0&0&0&0&0&s \\ 0&s&0&0&0&t&0&0 \\ 0&0&0&0&t&s&0&0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, BB^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 8, "vs", 8

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad Skew\ T = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad Skew$$

$$Omega = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 1T + 48\Omega$$

Ω

$$\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{-3}{4}\ \frac{9}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{5}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right.$$

$$T\left(1\ 0\ 0\ 0\ \frac{3}{2}\ 0\ 1\ -1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\right)$$

"IS NM in Vec(K)?", true

NM

$$\left(7\ 6\ 6\ 6\ \frac{27}{2}\ 6\ 7\ -37\ 55\ 6\ 6\ 31\ 6\ 6\ 6\ 6\ 7\ 6\ 6\ 6\ 6\ 6\ 6\ 6\ 6\ 7\right.$$

"IS MN in Vec(K)?", true

MN

$$\left(7\ 6\ 6\ 6\ \frac{27}{2}\ 6\ 7\ -37\ 55\ 6\ 6\ 31\ 6\ 6\ 6\ 6\ 7\ 6\ 6\ 6\ 6\ 6\ 6\ 6\ 6\ 7\right.$$

$$\boldsymbol{\tau} = 8/1\ ,\ \mathrm{rank} = 8\ ,\ \mathrm{ratio} = 1/1\ ,\ n^2 / r = 8/1$$

$$\boldsymbol{\tau}' = 56/1\ ,\ r' = 7/8\ ,\ \boldsymbol{\tau} / n^2 = 1/8$$

$$p^2 = 1/8\ ,\ \min\ \boldsymbol{\tau} = 8/1\ ,\ \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\max\ \mathrm{r} = 8/1\ ,\ r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 192

KERNEL HAS LINEAR DIMENSION 26
out of total no. of elements equal to 192

dim span idems 1 vs no. of idems 1

"PT1" = {{8}, {4}, {1}, {5}, {6}, {3}, {7}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0] }

NullSpace N_C

{[1, 1, 1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 0, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1]}

NullSpace N_0

{}

Eigenvalues M

[7., −1., −1., −1., −1., −1., −1., −1.]

Eigenvalues N

[7., −1., −1., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0&1&0&1&1&1&1&1 \\ 1&0&1&0&1&1&1&1 \\ 0&1&0&1&1&1&1&1 \\ 1&0&1&0&1&1&1&1 \\ 1&1&1&1&0&1&0&1 \\ 1&1&1&1&1&0&1&0 \\ 1&1&1&1&0&1&0&1 \\ 1&1&1&1&1&0&1&0 \end{pmatrix} \quad \begin{pmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

==

{2, 4, 6, 8}

R: [3, 8, 1, 6, 7, 4, 5, 2]
B: [6, 3, 8, 1, 2, 7, 4, 5]

TRACE TWO = 4

$\det AT = \frac{1}{4} (t)^2 (1 + t^2)^2$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 8

Level 2 det $= \frac{5}{134217728} (-41 + 36s + 4s^2 - 8s^3 + s^4) (-31 - 4s + 6s^2 - 4s^3 + s^4) (33 - 2s^2 + s^4) (-1 + s)^4 (101 + 58s + 26s^2 + 10s^3 + 5s^4)$

RANK of R is 8

R ranking is 1, "vs", 8

RBAR ranking 1, "vs", 8

RANK of B is 8

B ranking is 1, "vs", 8

BBAR ranking 1, "vs", 8

"R CYCLES", (1 + v[1] v[3]) (1 + v[2] v[8]) (v[4] v[6] + 1) (1 + v[5] v[7])
"B CYCLES", (1 + v[1] v[4] v[6] v[7]) (1 + v[2] v[3] v[5] v[8])

Eigenvalues

R: [1., −1., 1., −1., 1., −1., 1., −1.]

B: [−1., 1., 1. I, −1. I, −1., 1., 1. I, −1. I]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 4

$$\text{PROTO-M} = \begin{pmatrix} 0&1&1&1&1&1&1&1\\ 1&0&1&1&1&1&1&1\\ 1&1&0&1&1&1&1&1\\ 1&1&1&0&1&1&1&1\\ 1&1&1&1&0&1&1&1\\ 1&1&1&1&1&0&1&1\\ 1&1&1&1&1&1&0&1\\ 1&1&1&1&1&1&1&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&1&1&1&1&1&1&1\\ 1&0&1&1&1&1&1&1\\ 1&1&0&1&1&1&1&1\\ 1&1&1&0&1&1&1&1\\ 1&1&1&1&0&1&1&1\\ 1&1&1&1&1&0&1&1\\ 1&1&1&1&1&1&0&1\\ 1&1&1&1&1&1&1&0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 8

"RANK of the KERNEL is ", 8

"IdemSolvability Check", 3 "Trace mark", 8, "Rank mark", 8, "for kernel rank", 8

degree 1: $\frac{1}{8}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8] \right)$

degree 2: $\frac{1}{16}\left(v[2]v[3] + 4v[3]v[7] + v[5]v[8] + v[1]v[6] + v[6]v[7] + v[2]v[5] + 4v[1]v[7] + v[7]v[8] + v[4]v[5] + 4v[6]v[8] + 4v[4]v[8] + v[4]v[7] + 4v[2]v[6] + v[5]v[6] + 4v[2]v[4] + 4v[3]v[5] + v[1]v[2] + 4v[5]v[7] + 4v[1]v[3] + 4v[2]v[8] + 4v[4]v[6] + v[3]v[6] + v[3]v[4] + v[1]v[4] + v[2]v[7] + v[3]v[8] + 4v[1]v[5] + v[1]v[8] \right)$

degree 3 : $\frac{1}{16}$ ($v[2]v[5]v[7] + 2v[1]v[5]v[8] + 2v[1]v[5]v[6] + v[1]v[4]v[7] + v[5]v[6]v[7] + 2v[2]v[3]v[7] + v[3]v[4]v[5] + 2v[3]v[6]v[7] + 2v[4]v[5]v[8] + v[3]v[5]v[6] + v[1]v[3]v[4] + v[5]v[7]v[8] + 2v[2]v[3]v[6] + 2v[3]v[5]v[7] + v[3]v[6]v[8] + 2v[1]v[3]v[7] + 2v[3]v[4]v[7] + v[1]v[2]v[8] + 2v[1]v[4]v[8] + 2v[1]v[2]v[5] + v[2]v[4$

]v[7] + v[3]v[5]v[8] + v[1]v[2]v[3] + v[2]v[3]v[8] + v[1]v[7]v[8] + v[2]v[5]v[8] + 2
v[1]v[5]v[7] + v[1]v[4]v[6] + v[4]v[6]v[7] + v[6]v[7]v[8] + v[3]v[4]v[6] + v[5]v[6]v[
8] + 2 v[3]v[4]v[8] + 2 v[4]v[7]v[8] + v[2]v[3]v[5] + v[1]v[2]v[4] + 2 v[1]v[3]v[5] +
2 v[2]v[5]v[6] + 2 v[1]v[2]v[6] + 2 v[1]v[4]v[5] + 2 v[3]v[7]v[8] + 2 v[2]v[6]v[7] + 2 v
[2]v[4]v[6] + v[2]v[7]v[8] + v[4]v[5]v[6] + v[1]v[3]v[8] + v[2]v[4]v[5] + v[4]v[5]v[
7] + v[2]v[3]v[4] + 2 v[2]v[4]v[8] + 2 v[4]v[6]v[8] + v[1]v[2]v[7] + v[1]v[6]v[8] + v[
1]v[6]v[7] + v[1]v[3]v[6] + 2 v[2]v[6]v[8])

degree 4 : $\frac{1}{4}$ (v[1]v[3]v[7]v[8] + 4 v[2]v[3]v[5]v[8] + v[1]v[5]v[6]v[7] + v[2]v[3]v[4
]v[8] + v[1]v[5]v[7]v[8] + 2 v[1]v[5]v[6]v[8] + 4 v[1]v[4]v[5]v[8] + v[1]v[3]v[5]v[8]
+ v[1]v[4]v[5]v[7] + v[1]v[2]v[3]v[7] + 2 v[2]v[3]v[5]v[6] + v[4]v[6]v[7]v[8] + 2 v[3
]v[5]v[6]v[8] + v[3]v[4]v[5]v[7] + v[2]v[4]v[7]v[8] + 2 v[2]v[3]v[7]v[8] + v[1]v[2]v[
5]v[7] + v[1]v[2]v[4]v[8] + 2 v[1]v[2]v[4]v[5] + 2 v[1]v[2]v[3]v[6] + v[1]v[2]v[3]v[5
] + v[2]v[6]v[7]v[8] + v[3]v[5]v[6]v[7] + 2 v[1]v[4]v[7]v[8] + 4 v[1]v[2]v[3]v[4] + 4
v[2]v[4]v[5]v[7] + v[3]v[4]v[6]v[8] + 2 v[1]v[2]v[3]v[8] + 4 v[3]v[4]v[7]v[8] + 2 v[4]
v[5]v[7]v[8] + 2 v[2]v[5]v[8]v[7] + 8v[2]v[4]v[6]v[8] + v[2]v[3]v[6]v[8] + 8v[1]v[3]
v[5]v[7] + 2 v[1]v[3]v[4]v[6] + 4 v[1]v[4]v[6]v[7] + 2 v[1]v[2]v[4]v[7] + 2 v[4]v[5]v[
6]v[7] + v[2]v[4]v[5]v[8] + v[1]v[3]v[4]v[7] + v[2]v[4]v[5]v[6] + 2 v[1]v[2]v[5]v[8]
+ v[1]v[3]v[5]v[6] + 4 v[3]v[4]v[5]v[6] + v[1]v[3]v[4]v[5] + v[2]v[3]v[4]v[6] + 4 v[1
]v[2]v[5]v[6] + v[4]v[5]v[6]v[8] + v[3]v[5]v[7]v[8] + v[2]v[4]v[6]v[7] + 4 v[1]v[2]v[
7]v[8] + v[2]v[3]v[5]v[7] + 2 v[3]v[4]v[5]v[8] + 2 v[1]v[3]v[4]v[8] + 2 v[2]v[5]v[6]v[
7] + 4 v[2]v[3]v[6]v[7] + 2 v[3]v[6]v[7]v[8] + 2 v[1]v[4]v[5]v[6] + v[1]v[4]v[6]v[8] +
2 v[1]v[2]v[6]v[7] + 2 v[2]v[3]v[4]v[7] + v[2]v[5]v[6]v[8] + 2 v[3]v[4]v[6]v[7] + 2 v[
1]v[6]v[7]v[8] + v[1]v[3]v[6]v[7] + 2 v[2]v[3]v[4]v[5] + v[1]v[2]v[6]v[8] + 4 v[1]v[3]
v[6]v[8] + 4 v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[6])

degree 5 : $\frac{1}{16}$ (v[2]v[4]v[5]v[6]v[7] + 2 v[3]v[4]v[5]v[7]v[8] + 2 v[1]v[2]v[3]v[6]v[7
] + v[1]v[2]v[4]v[6]v[7] + 2 v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[7] + v[2]v[4]v[
5]v[7]v[8] + v[1]v[2]v[3]v[4]v[8] + 2 v[1]v[2]v[4]v[5]v[6] + 2 v[1]v[2]v[5]v[6]v[7]
+ v[1]v[3]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[7] + v[3]v[4]v[5]v[6]v[7] + v[2]v[5]v[6]
v[7]v[8] + 2 v[1]v[4]v[5]v[6]v[8] + 2 v[1]v[3]v[5]v[7]v[8] + 2 v[1]v[3]v[4]v[5]v[7] +
2 v[1]v[3]v[4]v[7]v[8] + 2 v[1]v[3]v[5]v[6]v[7] + v[2]v[3]v[5]v[6]v[8] + v[3]v[5]v[6]
v[7]v[8] + v[2]v[3]v[4]v[5]v[6] + v[1]v[3]v[4]v[6]v[8] + v[1]v[2]v[3]v[4]v[6] + v[1]
v[2]v[4]v[7]v[8] + 2 v[1]v[2]v[3]v[5]v[7] + v[2]v[3]v[4]v[5]v[8] + v[3]v[4]v[5]v[6]v
[8] + v[1]v[2]v[3]v[4]v[7] + 2 v[1]v[3]v[4]v[5]v[8] + v[1]v[2]v[3]v[6]v[8] + v[2]v[3]
v[5]v[8]v[7] + 2 v[2]v[3]v[4]v[6]v[8] + 2 v[2]v[4]v[6]v[8]v[7] + 2 v[2]v[4]v[5]v[6]v[
8] + v[1]v[3]v[4]v[6]v[7] + v[1]v[2]v[3]v[5]v[8] + 2 v[2]v[3]v[6]v[7]v[8] + v[1]v[4]v
[5]v[6]v[7] + 2 v[1]v[2]v[4]v[6]v[8] + 2 v[1]v[2]v[3]v[5]v[6] + 2 v[1]v[2]v[4]v[5]v[8]
+ v[1]v[2]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[5] + v[4]v[5]v[6]v[7]v[8] + 2 v[1]v[2]v[5]
v[6]v[8] + 2 v[2]v[3]v[4]v[7]v[8] + v[1]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[7]v[8] + v[1

]$v[2]v[5]v[8]v[7] + 2v[2]v[3]v[4]v[6]v[7] + v[1]v[3]v[5]v[6]v[8] + v[1]v[3]v[4]v[5]v[6] + v[1]v[4]v[6]v[7]v[8] + 2v[1]v[4]v[5]v[7]v[8] + 2v[2]v[3]v[5]v[6]v[7]$)

degree 6 : $\frac{1}{16}$ ($4v[1]v[2]v[3]v[4]v[5]v[7] + v[1]v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[7]v[8] + 4v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[4]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[8] + v[1]v[2]v[3]v[4]v[7]v[8] + v[3]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[7] + v[2]v[3]v[5]v[6]v[7]v[8] + 4v[2]v[3]v[4]v[6]v[8]v[7] + 4v[2]v[4]v[5]v[6]v[8]v[7] + 4v[1]v[2]v[3]v[5]v[6]v[7] + 4v[1]v[3]v[4]v[5]v[6]v[7] + 4v[1]v[2]v[3]v[4]v[6]v[8] + 4v[1]v[2]v[4]v[5]v[6]v[8] + 4v[1]v[3]v[4]v[5]v[8]v[7] + 4v[1]v[2]v[3]v[5]v[8]v[7] + 4v[1]v[2]v[4]v[6]v[8]v[7] + v[1]v[2]v[3]v[4]v[5]v[6] + v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[5]v[6]v[8] + v[1]v[2]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7] + v[1]v[2]v[4]v[5]v[7]v[8] + 4v[1]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[6]v[7]v[8]$)

degree 7 : $\frac{1}{8}$ ($v[1]v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[8]v[7] + v[1]v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[8]v[7] + v[1]v[3]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[8]v[7]$)

degree 8 : 1 ($v[1]$) ($v[2]$) ($v[3]$) ($v[8]$) ($v[7]$) ($v[5]$) ($v[4]$) ($v[6]$)

Group spectrum $1 + t + 4t^2 + 5t^3 + 9t^4 + 5t^5 + 4t^6 + t^7 + t^8$

KERNEL STRUCTURE

"PT1" = {{8}, {4}, {1}, {5}, {6}, {3}, {7}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$\pi 8 = [1]$$

supp $\pi 8$ = {1}

$$u8 = [1]$$

supp u8 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[8, 5, 6, 7, 2, 3, 4, 1]

B–BLOCKS,

[6, 7, 2, 1, 3, 8, 5, 4]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

$b_1 = \{8\}$
$b_2 = \{4\}$
$b_3 = \{1\}$
$b_4 = \{5\}$
$b_5 = \{6\}$
$b_6 = \{3\}$
$b_7 = \{7\}$
$b_8 = \{2\}$

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[3] & h[4] & h[2] & h[5] & h[3] & h[6] & h[2] \\ h[2] & h[1] & h[3] & h[4] & h[2] & h[5] & h[3] & h[6] \\ h[4] & h[2] & h[1] & h[3] & h[6] & h[2] & h[5] & h[3] \\ h[3] & h[4] & h[2] & h[1] & h[3] & h[6] & h[2] & h[5] \\ h[5] & h[3] & h[6] & h[2] & h[1] & h[3] & h[4] & h[2] \\ h[2] & h[5] & h[3] & h[6] & h[2] & h[1] & h[3] & h[4] \\ h[6] & h[2] & h[5] & h[3] & h[4] & h[2] & h[1] & h[3] \\ h[3] & h[6] & h[2] & h[5] & h[3] & h[4] & h[2] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 8, Shape: $6 \oplus 2/0$

$$\mathrm{CLB} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3}, {2, 8}, {5, 7}, {4, 6}}, true

Ω_B in Vec(K)? , {{2, 3, 5, 8}, {1, 4, 6, 7}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{8}, {4}, {1}, {5}, {6}, {3}, {7}, {2}}

1, "range", [1, 2, 3, 4, 5, 6, 7, 8], [[8, 5, 6, 7, 4, 1, 2, 3], [8, 1, 6, 3, 4, 5, 2, 7], [7, 8, 5, 6, 3, 4, 1, 2], [7, 4, 5, 2, 3, 8, 1, 6], [6, 7, 8, 5, 2, 3, 4, 1], [6, 3, 8, 1, 2, 7, 4, 5], [5, 6, 7, 8, 1, 2, 3, 4], [5, 2, 7, 4, 1, 6, 3, 8], [4, 5, 2, 7, 8, 1, 6, 3], [4, 1, 2, 3, 8, 5, 6, 7], [3, 8, 1, 6, 7, 4, 5, 2], [3, 4, 1, 2, 7, 8, 5, 6], [2, 7, 4, 5, 6, 3, 8, 1], [2, 3, 4, 1, 6, 7, 8, 5], [1, 6, 3, 8, 5, 2, 7, 4], [1, 2, 3, 4, 5, 6, 7, 8]] "group has", 16, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

g_1 =   [[1, 8, 3, 6], [2, 5, 4, 7]]

g_2 =   [[1, 8, 7, 2], [3, 6, 5, 4]]

g_3 =   [[1, 7], [2, 8], [3, 5], [4, 6]]

g_4 =   [[1, 7], [2, 4], [3, 5], [6, 8]]

g_5 =   [[1, 6, 3, 8], [2, 7, 4, 5]]

linear dimension, 12

"Symmetric?", false

Is Z in Vec(K)? true

$\big(0 \quad h[4] \quad -2h[3]+2h[6] \quad 2h[3] \quad 0 \quad h[2] \quad 2h[5]-2h[1] \quad 2h[1] \quad h[4] \quad 2h[3] \quad h[2] \quad 2$

"Basis for Z(G)"

1, "coeff", 2

Z[1] = $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

3, "coeff", 2

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

4, "coeff", 1

$$Z[\ 4\ ] = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

5, "coeff", 2

$$Z[\ 5\ ] = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

6, "coeff", 2

$$Z[\ 6\ ] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
1, 5, true
1, 6, true
2, 3, true
2, 4, true
2, 5, true
2, 6, true
3, 4, true
3, 5, true
3, 6, true
4, 5, true
4, 6, true
5, 6, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0 & 0 & 0 & 0 & -2. & 2. & 2.I & -2.I \\ 1. & -1. & 1. & -1. & 1. & -1. & 1. & -1. \\ 0 & 0 & 0 & 0 & -2. & 2. & 2.I & -2.I \\ 1. & -1. & 1. & -1. & 1. & -1. & 1. & -1. \\ 1. & -1. & 1. & -1. & 1. & -1. & 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 8 \\ 0 & 0 & 4 & 4 & 0 & 0 & 4 & 8 & 0 & 0 & 4 & 4 & 0 & 0 & 8 & 28 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 56 \\ 2 & 2 & 6 & 6 & 2 & 2 & 6 & 14 & 2 & 2 & 6 & 6 & 2 & 2 & 14 & 70 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 56 \\ 0 & 0 & 4 & 4 & 0 & 0 & 4 & 8 & 0 & 0 & 4 & 4 & 0 & 0 & 8 & 28 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 8 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 4t^2 + 5t^3 + 9t^4 + 5t^5 + 4t^6 + t^7 + t^8$

Molien Series to order 10: $1 + t + 5t^2 + 11t^3 + 32t^4 + 63t^5 + 137t^6 + 253t^7 + 475t^8 + 806t^9 + 1366t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 8 = (1)$

{1}

$u8 = (1)$

{1}

picheck $(1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1)$

$\pi = \left(\frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8}\right)$

$\pi 7 = (1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1)$

$u7 = \left(\frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8}\right)$

picheck $(7 \;\; 7 \;\; 7 \;\; 7 \;\; 7 \;\; 7 \;\; 7 \;\; 7)$

$\pi 6 =$
$(2 \;\; 2$

$u6 =$
$\left(\frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32}\right.$

picheck $(42 \;\; 42 \;\; 42 \;\; 42 \;\; 42 \;\; 42 \;\; 42 \;\; 42)$

$\pi 5 =$
$(6 \;\; 6$

$u5 =$
$\left(\frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{25}\right.$

picheck $(210 \;\; 210 \;\; 210 \;\; 210 \;\; 210 \;\; 210 \;\; 210 \;\; 210)$

$\pi 4 =$
$(24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 2$

$u4 =$
$\left(\frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{51}\right.$

picheck $(840 \;\; 840 \;\; 840 \;\; 840 \;\; 840 \;\; 840 \;\; 840 \;\; 840)$

$\pi 3 =$
$(120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120$

$u3 =$
$\left(\frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096}\right.$

picheck $(2520 \;\; 2520 \;\; 2520 \;\; 2520 \;\; 2520 \;\; 2520 \;\; 2520 \;\; 2520)$

$\pi 2 =$
$(720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720$

$u2 =$
$\left(\frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{1638}\right.$

picheck $(5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040)$

$\pi 1 = (5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040)$

$u1 = \left(\frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072}\right)$

picheck $(5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040)$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0, 0, 0]$

$\ker N_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} s+t & s+t & s+t & s+t & s+t & s+t & s+t & s+t \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} t & 0 & 0 & 0 & s & 0 & 0 \\ -s & -s & -s & -s & t-s & -s & -s \\ -t & -t & -t & -t & -t & -t & s-t \\ s & 0 & 0 & 0 & 0 & 0 & t \\ 0 & s & 0 & 0 & 0 & t & 0 \\ 0 & t & s & 0 & 0 & 0 & 0 \\ 0 & 0 & t & s & 0 & 0 & 0 \\ 0 & 0 & 0 & t & 0 & s & 0 \end{pmatrix}$$ RB checks

$$\ker M_C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & t & s \\ 0 & s & 0 & 0 & 0 & 0 & 0 & t \\ s & t & 0 & 0 & 0 & 0 & 0 & 0 \\ t & 0 & 0 & 0 & 0 & 0 & s & 0 \\ 0 & 0 & s & 0 & t & 0 & 0 & 0 \\ 0 & 0 & t & s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t & 0 & s & 0 & 0 \\ 0 & 0 & 0 & 0 & s & t & 0 & 0 \end{pmatrix}$$ RB checks

$$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, BB^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 8, "vs", 8

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad Skew\ T = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad Skew$$

$$Omega = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 1T + 48\Omega$$

$\Omega \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$T \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

"IS NM in Vec(K)?", true

$\text{NM} \begin{pmatrix} 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$

"IS MN in Vec(K)?", true

$\text{MN} \begin{pmatrix} 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$

$$\boldsymbol{\tau} = 8/1 \text{ , rank} = 8 \text{ , ratio} = 1/1 \text{ , } n^2 / r = 8/1$$

$$\boldsymbol{\tau}' = 56/1 \text{ , } r' = 7/8 \text{ , } \boldsymbol{\tau} / n^2 = 1/8$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 16

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 1 vs no. of idems 1

"PT1" = {{8}, {4}, {1}, {5}, {6}, {3}, {7}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1., 1., 1.]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0] }

NullSpace N_C

{[1, 1, 1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, −1, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [1, −1, 0, 0, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 1, 0], [0, −1, 0, 1, 0, 0, 0, 0]}

NullSpace N_0

{}

Eigenvalues M

[7., −1., −1., −1., −1., −1., −1., −1.]

Eigenvalues N

[7., −1., −1., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&1&0&1&0&1\\ 1&0&1&0&1&0&1&0\\ 0&1&0&1&0&1&0&1\\ 1&0&1&0&1&0&1&0\\ 0&1&0&1&0&1&0&1\\ 1&0&1&0&1&0&1&0\\ 0&1&0&1&0&1&0&1\\ 1&0&1&0&1&0&1&0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad commutator = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$1, 3: \quad commutator = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$1, 4: \quad commutator = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$2, 3 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$2, 4 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$3, 4 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

===

80, [1, −1, 1, −1, 1, 1, −1, −1]

===

{3, 4, 5, 6}

R: [3, 3, 8, 6, 2, 4, 5, 5]

B: [6, 8, 1, 1, 7, 7, 4, 2]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det $= \frac{-7}{67108864} (1+s)^2 (31 - s + 9s^2 + s^3)(33 + 7s^2)(-1+s)(41 - 13s - 5s^2 + s^3)(101 - 53s + 37s^2 - 7s^3 + 2s^4)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 2, "vs", 6

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", (1 + v[2] v[3] v[5] v[8]) (1 + v[4] v[6])
"B CYCLES", (1 + v[2] v[8]) (1 + v[1] v[4] v[6] v[7])

Eigenvalues

R: [1. I, −1. I, 0., 0., 1., −1., 1., −1.]

B: [1. I, −1. I, 0., 0., 1., −1., 1., −1.]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of R^*

{[1, −1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

NullSpace of B^*

{[0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} & \frac{5}{9} & \frac{4}{9} & 1 & \frac{2}{3} \\ \frac{1}{3} & 0 & \frac{5}{9} & \frac{2}{3} & \frac{4}{9} & \frac{1}{3} & \frac{2}{3} & 1 \\ \frac{4}{9} & \frac{5}{9} & 0 & \frac{1}{3} & 1 & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} \\ \frac{5}{9} & \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} & 1 & \frac{4}{9} & \frac{1}{3} \\ \frac{5}{9} & \frac{4}{9} & 1 & \frac{2}{3} & 0 & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} \\ \frac{4}{9} & \frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 & \frac{5}{9} & \frac{2}{3} \\ 1 & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} & \frac{4}{9} & \frac{5}{9} & 0 & \frac{1}{3} \\ \frac{2}{3} & 1 & \frac{4}{9} & \frac{1}{3} & \frac{5}{9} & \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 2 degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{4}$ ($v[1]v[7] + v[2]v[8] + v[3]v[5] + v[4]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7}}

"PT2" = {{1, 5, 6, 8}, {2, 3, 4, 7}}

"PT3" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"PT4" = {{3, 6, 7, 8}, {1, 2, 4, 5}}

"PT5" = {{4, 5, 7, 8}, {1, 2, 3, 6}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {4, 6}

"RG3" = {3, 5}

"RG4" = {1, 7}

$\pi 2$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {6, 13, 15, 20}

$u2$ = [3, 4, 5, 5, 4, 9, 6, 5, 6, 4, 3, 6, 9, 3, 9, 6, 5, 4, 6, 9, 4, 3, 3, 4, 5, 5, 6, 3]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [2], [1], [3]]
Action of B on ranges, [[1], [4], [4], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [5, 3, 5, 6, 4, 3]

BPARTS [2, 6, 1, 3, 1, 3]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{9} & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[6, C, 5, 7, 4, 2, 3, 4, 1, 5, C, 1]

B–BLOCKS,

[9, 5, C, 8, 9, C, 5, B, A, 2, 3, 8]

with invariant measure, [2, 1, 1, 2, 2, 1, 1, 2, 2, 1, 1, 2]

N by blocks, N – check: true

$b_1 = \{4, 5, 7, 8\}$
$b_2 = \{1, 2, 3, 4\}$
$b_3 = \{5, 6, 7, 8\}$
$b_4 = \{1, 2, 3, 6\}$
$b_5 = \{3, 4, 7, 8\}$
$b_6 = \{3, 6, 7, 8\}$
$b_7 = \{1, 2, 4, 5\}$
$b_8 = \{1, 3, 4, 8\}$
$b_9 = \{2, 5, 6, 7\}$
$b_{10} = \{1, 5, 6, 8\}$
$b_{11} = \{2, 3, 4, 7\}$
$b_{12} = \{1, 2, 5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N):   5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & 0 & 0 & h[1] & 0 \\ 0 & h[2] & 0 & 0 & 0 & 0 & 0 & h[1] \\ 0 & 0 & h[2] & 0 & h[1] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[2] & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[1] & 0 & h[2] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[1] & 0 & h[2] & 0 & 0 \\ h[1] & 0 & 0 & 0 & 0 & 0 & h[2] & 0 \\ 0 & h[1] & 0 & 0 & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: $18 \oplus 4/3$

$$\text{CLB} = \begin{pmatrix} 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3, 5, 8}, {4, 6}}, false

Ω_B in Vec(K)? , {{1, 4, 6, 7}, {2, 8}}, false

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} \\ \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} \\ \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} \\ \frac{-3}{40} & \frac{29}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{-27}{40} & \frac{21}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{3}{16} & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & \frac{1}{8} & 0 & \frac{3}{16} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{3}{16} & \frac{1}{8} & 0 & \frac{3}{16} & 0 & \frac{3}{16} & \frac{3}{16} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 4, 8}, {2, 5, 6, 7}}

1, "range", [2, 8], [[8, 2, 8, 8, 2, 2, 2, 8], [2, 8, 2, 2, 8, 8, 8, 2]]

2, "range", [4, 6], [[6, 4, 6, 6, 4, 4, 4, 6], [4, 6, 4, 4, 6, 6, 6, 4]]

3, "range", [3, 5], [[5, 3, 5, 5, 3, 3, 3, 5], [3, 5, 3, 3, 5, 5, 5, 3]]

4, "range", [1, 7], [[7, 1, 7, 7, 1, 1, 1, 7], [1, 7, 1, 1, 7, 7, 7, 1]]

2, "partition", {{1, 5, 6, 8}, {2, 3, 4, 7}}

1, "range", [2, 8], [[8, 2, 2, 2, 8, 8, 2, 8], [2, 8, 8, 8, 2, 2, 8, 2]]

2, "range", [4, 6], [[6, 4, 4, 4, 6, 6, 4, 6], [4, 6, 6, 6, 4, 4, 6, 4]]

3, "range", [3, 5], [[5, 3, 3, 3, 5, 5, 3, 5], [3, 5, 5, 5, 3, 3, 5, 3]]

4, "range", [1, 7], [[7, 1, 1, 1, 7, 7, 1, 7], [1, 7, 7, 7, 1, 1, 7, 1]]

3, "partition", {{3, 4, 7, 8}, {1, 2, 5, 6}}

1, "range", [2, 8], [[8, 8, 2, 2, 8, 8, 2, 2], [2, 2, 8, 8, 2, 2, 8, 8]]

2, "range", [4, 6], [[6, 6, 4, 4, 6, 6, 4, 4], [4, 4, 6, 6, 4, 4, 6, 6]]

3, "range", [3, 5], [[5, 5, 3, 3, 5, 5, 3, 3], [3, 3, 5, 5, 3, 3, 5, 5]]

4, "range", [1, 7], [[7, 7, 1, 1, 7, 7, 1, 1], [1, 1, 7, 7, 1, 1, 7, 7]]

4, "partition", {{3, 6, 7, 8}, {1, 2, 4, 5}}

1, "range", [2, 8], [[8, 8, 2, 8, 8, 2, 2, 2], [2, 2, 8, 2, 2, 8, 8, 8]]

2, "range", [4, 6], [[6, 6, 4, 6, 6, 4, 4, 4], [4, 4, 6, 4, 4, 6, 6, 6]]

3, "range", [3, 5], [[5, 5, 3, 5, 5, 3, 3, 3], [3, 3, 5, 3, 3, 5, 5, 5]]

4, "range", [1, 7], [[7, 7, 1, 7, 7, 1, 1, 1], [1, 1, 7, 1, 1, 7, 7, 7]]

5, "partition", {{4, 5, 7, 8}, {1, 2, 3, 6}}

1, "range", [2, 8], [[8, 8, 8, 2, 2, 8, 2, 2], [2, 2, 2, 8, 8, 2, 8, 8]]

2, "range", [4, 6], [[6, 6, 6, 4, 4, 6, 4, 4], [4, 4, 4, 6, 6, 4, 6, 6]]

3, "range", [3, 5], [[5, 5, 5, 3, 3, 5, 3, 3], [3, 3, 3, 5, 5, 3, 5, 5]]

4, "range", [1, 7], [[7, 7, 7, 1, 1, 7, 1, 1], [1, 1, 1, 7, 7, 1, 7, 7]]

6, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [2, 8], [[8, 8, 8, 8, 2, 2, 2, 2], [2, 2, 2, 2, 8, 8, 8, 8]]

2, "range", [4, 6], [[6, 6, 6, 6, 4, 4, 4, 4], [4, 4, 4, 4, 6, 6, 6, 6]]

3, "range", [3, 5], [[5, 5, 5, 5, 3, 3, 3, 3], [3, 3, 3, 3, 5, 5, 5, 5]]

4, "range", [1, 7], [[7, 7, 7, 7, 1, 1, 1, 1], [1, 1, 1, 1, 7, 7, 7, 7]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2$ =
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

{6, 13, 15, 20}

$u2$ =
$$\big(3 \;\; 4 \;\; 5 \;\; 5 \;\; 4 \;\; 9 \;\; 6 \;\; 5 \;\; 6 \;\; 4 \;\; 3 \;\; 6 \;\; 9 \;\; 3 \;\; 9 \;\; 6 \;\; 5 \;\; 4 \;\; 6 \;\; 9 \;\; 4 \;\; 3 \;\; 3 \;\; 4 \;\; 5 \;\; 5 \;\; 6 \;\; 3$$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

$\pi = \begin{pmatrix}\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8}\end{pmatrix}$

$\pi 1 = \begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2}\end{pmatrix}$

picheck $\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{4}{9} & 0 & 0 & 0 & 0 & 0 & \frac{5}{9} \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{5}{9} & 0 & 0 & 0 & 0 & 0 & \frac{4}{9} \\ 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{4}{9} & 0 & \frac{5}{9} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{9} & 0 & \frac{5}{9} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{9} & 0 & \frac{5}{9} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{9} & 0 & \frac{4}{9} & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{5}{9} & 0 & 0 & 0 & 0 & 0 & \frac{4}{9} & 0 \\ \frac{4}{9} & 0 & 0 & 0 & 0 & 0 & \frac{5}{9} & 0 \\ \frac{4}{9} & 0 & 0 & 0 & 0 & 0 & \frac{5}{9} & 0 \\ \frac{5}{9} & 0 & 0 & 0 & 0 & 0 & \frac{4}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{6} & \frac{5}{36} & \frac{1}{9} & \frac{1}{9} & \frac{5}{36} & 0 & \frac{1}{12} \\ \frac{1}{6} & \frac{1}{4} & \frac{1}{9} & \frac{1}{12} & \frac{5}{36} & \frac{1}{6} & \frac{1}{12} & 0 \\ \frac{5}{36} & \frac{1}{9} & \frac{1}{4} & \frac{1}{6} & 0 & \frac{1}{12} & \frac{1}{9} & \frac{5}{36} \\ \frac{1}{9} & \frac{1}{12} & \frac{1}{6} & \frac{1}{4} & \frac{1}{12} & 0 & \frac{5}{36} & \frac{1}{6} \\ \frac{1}{9} & \frac{5}{36} & 0 & \frac{1}{12} & \frac{1}{4} & \frac{1}{6} & \frac{5}{36} & \frac{1}{9} \\ \frac{5}{36} & \frac{1}{6} & \frac{1}{12} & 0 & \frac{1}{6} & \frac{1}{4} & \frac{1}{9} & \frac{1}{12} \\ 0 & \frac{1}{12} & \frac{1}{9} & \frac{5}{36} & \frac{5}{36} & \frac{1}{9} & \frac{1}{4} & \frac{1}{6} \\ \frac{1}{12} & 0 & \frac{5}{36} & \frac{1}{6} & \frac{1}{9} & \frac{1}{12} & \frac{1}{6} & \frac{1}{4} \end{pmatrix} \quad NM =$$

$$\begin{pmatrix} 4 & \frac{8}{3} & \frac{20}{9} & \frac{16}{9} & \frac{16}{9} & \frac{20}{9} & 0 & \frac{4}{3} \\ \frac{8}{3} & 4 & \frac{16}{9} & \frac{4}{3} & \frac{20}{9} & \frac{8}{3} & \frac{4}{3} & 0 \\ \frac{20}{9} & \frac{16}{9} & 4 & \frac{8}{3} & 0 & \frac{4}{3} & \frac{16}{9} & \frac{20}{9} \\ \frac{16}{9} & \frac{4}{3} & \frac{8}{3} & 4 & \frac{4}{3} & 0 & \frac{20}{9} & \frac{8}{3} \\ \frac{16}{9} & \frac{20}{9} & 0 & \frac{4}{3} & 4 & \frac{8}{3} & \frac{20}{9} & \frac{16}{9} \\ \frac{20}{9} & \frac{8}{3} & \frac{4}{3} & 0 & \frac{8}{3} & 4 & \frac{16}{9} & \frac{4}{3} \\ 0 & \frac{4}{3} & \frac{16}{9} & \frac{20}{9} & \frac{20}{9} & \frac{16}{9} & 4 & \frac{8}{3} \\ \frac{4}{3} & 0 & \frac{20}{9} & \frac{8}{3} & \frac{16}{9} & \frac{4}{3} & \frac{8}{3} & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 0, 1, 0, 1, 0, -1, 0]$

$$\ker N_C = \begin{pmatrix} 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -t & t & s & -s & s & -s & -t & t \\ -t & 0 & s & -s+t & s & -s+t & -t & 0 \\ 0 & s & 0 & -s & 0 & -s & 0 & s \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & -1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s & t & 0 \\ -t & s & 0 & 0 \\ -s & 0 & 0 & t \\ 0 & 0 & s & t \\ s & 0 & 0 & -t \\ 0 & 0 & -s & -t \\ 0 & -s & -t & 0 \\ t & -s & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 & 0 & 1 & -1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & s & -s & s+t & 0 \\ t & s & -s & s & 0 \\ s & t & 0 & t & -t \\ 0 & t & 0 & s+t & -t \\ -s & s & 0 & s & t \\ 0 & s & 0 & 0 & t \\ 0 & t & s & 0 & 0 \\ -t & t & s & t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 0 & 4 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & \frac{5}{9} & \frac{1}{3} & \frac{4}{9} & \frac{2}{3} & 0 & 0 \\ 1 & 1 & \frac{5}{9} & \frac{1}{3} & \frac{4}{9} & \frac{2}{3} & 0 & 0 \\ \frac{5}{9} & \frac{5}{9} & 1 & \frac{1}{3} & 0 & \frac{2}{3} & \frac{4}{9} & \frac{4}{9} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 1 & \frac{2}{3} & 0 & \frac{2}{3} & \frac{2}{3} \\ \frac{4}{9} & \frac{4}{9} & 0 & \frac{2}{3} & 1 & \frac{1}{3} & \frac{5}{9} & \frac{5}{9} \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 & \frac{1}{3} & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{4}{9} & \frac{2}{3} & \frac{5}{9} & \frac{1}{3} & 1 & 1 \\ 0 & 0 & \frac{4}{9} & \frac{2}{3} & \frac{5}{9} & \frac{1}{3} & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & \frac{1}{3} & \frac{5}{9} & \frac{5}{9} & \frac{4}{9} & \frac{4}{9} & 0 & \frac{2}{3} \\ \frac{1}{3} & 1 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ \frac{5}{9} & \frac{1}{3} & 1 & 1 & 0 & 0 & \frac{4}{9} & \frac{2}{3} \\ \frac{5}{9} & \frac{1}{3} & 1 & 1 & 0 & 0 & \frac{4}{9} & \frac{2}{3} \\ \frac{4}{9} & \frac{2}{3} & 0 & 0 & 1 & 1 & \frac{5}{9} & \frac{1}{3} \\ \frac{4}{9} & \frac{2}{3} & 0 & 0 & 1 & 1 & \frac{5}{9} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{4}{9} & \frac{4}{9} & \frac{5}{9} & \frac{5}{9} & 1 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 1 \end{pmatrix}, B B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew}$$

$$\text{Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} & \frac{4}{9} & \frac{5}{9} & 0 & \frac{1}{3} \\ \frac{2}{3} & 1 & \frac{4}{9} & \frac{1}{3} & \frac{5}{9} & \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{5}{9} & \frac{4}{9} & 1 & \frac{2}{3} & 0 & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} \\ \frac{4}{9} & \frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 & \frac{5}{9} & \frac{2}{3} \\ \frac{4}{9} & \frac{5}{9} & 0 & \frac{1}{3} & 1 & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} \\ \frac{5}{9} & \frac{2}{3} & \frac{1}{3} & 0 & \frac{2}{3} & 1 & \frac{4}{9} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} & \frac{5}{9} & \frac{4}{9} & 1 & \frac{2}{3} \\ \frac{1}{3} & 0 & \frac{5}{9} & \frac{2}{3} & \frac{4}{9} & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \right)$$

$$T \left(\frac{1}{4} \; \frac{1}{6} \; \frac{1}{12} \; \frac{1}{9} \; \frac{1}{6} \; \frac{1}{4} \; \frac{1}{9} \; \frac{5}{36} \; \frac{1}{12} \; \frac{1}{9} \; \frac{1}{4} \; \frac{1}{6} \; \frac{1}{12} \; 0 \; \frac{5}{36} \; \frac{1}{9} \; \frac{1}{9} \; \frac{5}{36} \; \frac{1}{6} \; \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \left(4 \; \frac{8}{3} \; \frac{4}{3} \; \frac{16}{9} \; \frac{8}{3} \; 4 \; \frac{16}{9} \; \frac{20}{9} \; \frac{4}{3} \; \frac{16}{9} \; 4 \; \frac{8}{3} \; \frac{4}{3} \; 0 \; \frac{20}{9} \; \frac{16}{9} \; \frac{16}{9} \; \frac{20}{9} \; \frac{8}{3} \; 4 \right)$$

"IS MN in Vec(K)?", true

$$\mathrm{MN} \left(4 \; \frac{8}{3} \; \frac{4}{3} \; \frac{16}{9} \; \frac{8}{3} \; 4 \; \frac{16}{9} \; \frac{20}{9} \; \frac{4}{3} \; \frac{16}{9} \; 4 \; \frac{8}{3} \; \frac{4}{3} \; 0 \; \frac{20}{9} \; \frac{16}{9} \; \frac{16}{9} \; \frac{20}{9} \; \frac{8}{3} \; 4 \right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 24/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7}}

"PT2" = {{1, 5, 6, 8}, {2, 3, 4, 7}}

"PT3" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"PT4" = {{3, 6, 7, 8}, {1, 2, 4, 5}}

"PT5" = {{4, 5, 7, 8}, {1, 2, 3, 6}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {4, 6}

"RG3" = {3, 5}

"RG4" = {1, 7}

$$M_C = \begin{pmatrix} 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{13}{24} & \frac{31}{72} & \frac{23}{72} & \frac{23}{72} & \frac{31}{72} & \frac{-1}{8} & \frac{5}{24} \\ \frac{13}{24} & \frac{7}{8} & \frac{23}{72} & \frac{5}{24} & \frac{31}{72} & \frac{13}{24} & \frac{5}{24} & \frac{-1}{8} \\ \frac{31}{72} & \frac{23}{72} & \frac{7}{8} & \frac{13}{24} & \frac{-1}{8} & \frac{5}{24} & \frac{23}{72} & \frac{31}{72} \\ \frac{23}{72} & \frac{5}{24} & \frac{13}{24} & \frac{7}{8} & \frac{5}{24} & \frac{-1}{8} & \frac{31}{72} & \frac{13}{24} \\ \frac{23}{72} & \frac{31}{72} & \frac{-1}{8} & \frac{5}{24} & \frac{7}{8} & \frac{13}{24} & \frac{31}{72} & \frac{23}{72} \\ \frac{31}{72} & \frac{13}{24} & \frac{5}{24} & \frac{-1}{8} & \frac{13}{24} & \frac{7}{8} & \frac{23}{72} & \frac{5}{24} \\ \frac{-1}{8} & \frac{5}{24} & \frac{23}{72} & \frac{31}{72} & \frac{31}{72} & \frac{23}{72} & \frac{7}{8} & \frac{13}{24} \\ \frac{5}{24} & \frac{-1}{8} & \frac{31}{72} & \frac{13}{24} & \frac{23}{72} & \frac{5}{24} & \frac{13}{24} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{13}{21} & \frac{31}{63} & \frac{23}{63} & \frac{23}{63} & \frac{31}{63} & \frac{-1}{7} & \frac{5}{21} \\ \frac{13}{21} & 1 & \frac{23}{63} & \frac{5}{21} & \frac{31}{63} & \frac{13}{21} & \frac{5}{21} & \frac{-1}{7} \\ \frac{31}{63} & \frac{23}{63} & 1 & \frac{13}{21} & \frac{-1}{7} & \frac{5}{21} & \frac{23}{63} & \frac{31}{63} \\ \frac{23}{63} & \frac{5}{21} & \frac{13}{21} & 1 & \frac{5}{21} & \frac{-1}{7} & \frac{31}{63} & \frac{13}{21} \\ \frac{23}{63} & \frac{31}{63} & \frac{-1}{7} & \frac{5}{21} & 1 & \frac{13}{21} & \frac{31}{63} & \frac{23}{63} \\ \frac{31}{63} & \frac{13}{21} & \frac{5}{21} & \frac{-1}{7} & \frac{13}{21} & 1 & \frac{23}{63} & \frac{5}{21} \\ \frac{-1}{7} & \frac{5}{21} & \frac{23}{63} & \frac{31}{63} & \frac{31}{63} & \frac{23}{63} & 1 & \frac{13}{21} \\ \frac{5}{21} & \frac{-1}{7} & \frac{31}{63} & \frac{13}{21} & \frac{23}{63} & \frac{5}{21} & \frac{13}{21} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 1.203158569, 0.5746192085, 1.608015106, 0.6142071162]

Eigenvalues M_C−scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 0., 0., 3.428571429, 1.375038365, 0.6567076669, 1.837731550, 0.7019509901]

NullSpace M_C

{[0, 0, −1, 0, 1, 0, 0, 0], [1, 1, 1, 1, 0, 0, 0, 0], [1, 1, 1, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

NullSpace N_C

{[−1, 1, 0, 0, 0, 0, −1, 1], [−1, 0, 1, 0, 1, 0, −1, 0], [−1, 0, 0, 1, 0, 1, −1, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 1.203158569, 0.5746192085, 1.608015106, 0.6142071162]

NullSpace M_0

{[−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0]}

NullSpace N_0

{[−1, 1, 0, 0, 0, 0, −1, 1], [−1, 0, 1, 0, 1, 0, −1, 0], [−1, 0, 0, 1, 0, 1, −1, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −0.5746192085, −1.203158569, −0.6142071162, −1.608015106]

NullSpace M

{}

NullSpace N

{[0, −1, 1, 0, 1, 0, 0, −1], [0, −1, 0, 1, 0, 1, 0, −1], [1, −1, 0, 0, 0, 0, 1, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 4 & 5 & 5 & 4 & 9 & 6 \\ 3 & 0 & 5 & 6 & 4 & 3 & 6 & 9 \\ 4 & 5 & 0 & 3 & 9 & 6 & 5 & 4 \\ 5 & 6 & 3 & 0 & 6 & 9 & 4 & 3 \\ 5 & 4 & 9 & 6 & 0 & 3 & 4 & 5 \\ 4 & 3 & 6 & 9 & 3 & 0 & 5 & 6 \\ 9 & 6 & 5 & 4 & 4 & 5 & 0 & 3 \\ 6 & 9 & 4 & 3 & 5 & 6 & 3 & 0 \end{pmatrix}$$

==

{3, 4, 7, 8}

R: [3, 3, 8, 6, 7, 7, 4, 2]
B: [6, 8, 1, 1, 2, 4, 5, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{-13}{67108864}(-41 - 5s + 13s^2 + s^3)(31 - s - 7s^2 + s^3)(101 + 53 s - 27s^2 - 9s^3 + 2s^4)(-33 + 4s + 13s^2)(-1 + s)(1 + s)^2$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 3, "vs", 6

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[2] v[3] v[8]) (1 + v[4] v[6] v[7])
"B CYCLES", (1 + v[1] v[4] v[6]) (1 + v[2] v[5] v[8])

Eigenvalues

R: [1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 0., 0.]

B: [1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of R^*

{[0, 0, 0, 0, 1, −1, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of B^*

{[0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{5} & \frac{4}{5} & \frac{3}{5} & 1 & \frac{4}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{1}{5} & 0 & \frac{3}{5} & \frac{2}{5} & \frac{4}{5} & 1 & \frac{2}{5} & \frac{3}{5} \\ \frac{4}{5} & \frac{3}{5} & 0 & \frac{1}{5} & \frac{1}{5} & \frac{2}{5} & 1 & \frac{4}{5} \\ \frac{3}{5} & \frac{2}{5} & \frac{1}{5} & 0 & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & 1 \\ 1 & \frac{4}{5} & \frac{1}{5} & \frac{2}{5} & 0 & \frac{1}{5} & \frac{4}{5} & \frac{3}{5} \\ \frac{4}{5} & 1 & \frac{2}{5} & \frac{3}{5} & \frac{1}{5} & 0 & \frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{2}{5} & 1 & \frac{4}{5} & \frac{4}{5} & \frac{3}{5} & 0 & \frac{1}{5} \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & 1 & \frac{3}{5} & \frac{2}{5} & \frac{1}{5} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{4}$ ($v[1]v[5] + v[2]v[6] + v[3]v[7] + v[4]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT2" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT3" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"PT4" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$\pi 2 = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]$

supp $\pi 2$ = {4, 11, 17, 22}

$u2 = [1, 4, 3, 5, 4, 1, 2, 3, 2, 4, 5, 2, 3, 1, 1, 2, 5, 4, 2, 3, 4, 5, 1, 4, 3, 3, 2, 1]$

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [1], [2], [2]]
Action of B on ranges, [[4], [4], [1], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [3, 1, 4, 1]

BPARTS [2, 4, 1, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{2}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[8, 3, 6, 3, 8, 2, 1, 7]

B–BLOCKS,

[3, 8, 4, 1, 2, 3, 8, 5]

with invariant measure, [1, 1, 2, 1, 1, 1, 1, 2]

N by blocks, N – check: true

b_1 = {1, 2, 3, 4}
b_2 = {5, 6, 7, 8}
b_3 = {3, 4, 5, 6}
b_4 = {1, 6, 7, 8}
b_5 = {2, 3, 4, 5}
b_6 = {1, 2, 4, 7}
b_7 = {3, 5, 6, 8}
b_8 = {1, 2, 7, 8}

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \\ h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: 23 ⊕ 4/3

$$CLB = \begin{pmatrix} 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 0 & 1 & 1 \\ -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3, 8}, {4, 6, 7}}, true

Ω_B in Vec(K)? , {{2, 5, 8}, {1, 4, 6}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} \\ \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} \\ \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} \\ \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} & \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} \\ \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 4, 5, 6}, {1, 2, 7, 8}}

1, "range", [4, 8], [[8, 8, 4, 4, 4, 4, 8, 8], [4, 4, 8, 8, 8, 8, 4, 4]]

2, "range", [3, 7], [[7, 7, 3, 3, 3, 3, 7, 7], [3, 3, 7, 7, 7, 7, 3, 3]]

3, "range", [2, 6], [[6, 6, 2, 2, 2, 2, 6, 6], [2, 2, 6, 6, 6, 6, 2, 2]]

4, "range", [1, 5], [[5, 5, 1, 1, 1, 1, 5, 5], [1, 1, 5, 5, 5, 5, 1, 1]]

2, "partition", {{1, 6, 7, 8}, {2, 3, 4, 5}}

1, "range", [4, 8], [[8, 4, 4, 4, 4, 8, 8, 8], [4, 8, 8, 8, 8, 4, 4, 4]]

2, "range", [3, 7], [[7, 3, 3, 3, 3, 7, 7, 7], [3, 7, 7, 7, 7, 3, 3, 3]]

3, "range", [2, 6], [[6, 2, 2, 2, 2, 6, 6, 6], [2, 6, 6, 6, 6, 2, 2, 2]]

4, "range", [1, 5], [[5, 1, 1, 1, 1, 5, 5, 5], [1, 5, 5, 5, 5, 1, 1, 1]]

3, "partition", {{1, 2, 4, 7}, {3, 5, 6, 8}}

1, "range", [4, 8], [[8, 8, 4, 8, 4, 4, 8, 4], [4, 4, 8, 4, 8, 8, 4, 8]]

2, "range", [3, 7], [[7, 7, 3, 7, 3, 3, 7, 3], [3, 3, 7, 3, 7, 7, 3, 7]]

3, "range", [2, 6], [[6, 6, 2, 6, 2, 2, 6, 2], [2, 2, 6, 2, 6, 6, 2, 6]]

4, "range", [1, 5], [[5, 5, 1, 5, 1, 1, 5, 1], [1, 1, 5, 1, 5, 5, 1, 5]]

4, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [4, 8], [[8, 8, 8, 8, 4, 4, 4, 4], [4, 4, 4, 4, 8, 8, 8, 8]]

2, "range", [3, 7], [[7, 7, 7, 7, 3, 3, 3, 3], [3, 3, 3, 3, 7, 7, 7, 7]]

3, "range", [2, 6], [[6, 6, 6, 6, 2, 2, 2, 2], [2, 2, 2, 2, 6, 6, 6, 6]]

4, "range", [1, 5], [[5, 5, 5, 5, 1, 1, 1, 1], [1, 1, 1, 1, 5, 5, 5, 5]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

g_1 =   []

g_2 =   [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\,2\,] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 11, 17, 22}

$u2 =$
$(1 \;\; 4 \;\; 3 \;\; 5 \;\; 4 \;\; 1 \;\; 2 \;\; 3 \;\; 2 \;\; 4 \;\; 5 \;\; 2 \;\; 3 \;\; 1 \;\; 1 \;\; 2 \;\; 5 \;\; 4 \;\; 2 \;\; 3 \;\; 4 \;\; 5 \;\; 1 \;\; 4 \;\; 3 \;\; 3 \;\; 2$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

$\pi = \begin{pmatrix}\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8}\end{pmatrix}$

$\pi 1 = \begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{5}{2} & \frac{5}{2} & \frac{5}{2} & \frac{5}{2} & \frac{5}{2} & \frac{5}{2} & \frac{5}{2} & \frac{5}{2}\end{pmatrix}$

picheck $\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} \\ 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} \\ 0 & 0 & 0 & \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} \\ 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} \\ 0 & 0 & 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} & 0 \\ 0 & 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 \\ 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 \\ 0 & \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} & 0 & 0 & 0 \\ \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} & 0 & 0 & 0 \\ \frac{2}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{5} & 0 & 0 & 0 & \frac{4}{5} & 0 & 0 & 0 \\ \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} & 0 & 0 & 0 \\ \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{5} & \frac{1}{20} & \frac{1}{10} & 0 & \frac{1}{20} & \frac{1}{5} & \frac{3}{20} \\ \frac{1}{5} & \frac{1}{4} & \frac{1}{10} & \frac{3}{20} & \frac{1}{20} & 0 & \frac{3}{20} & \frac{1}{10} \\ \frac{1}{20} & \frac{1}{10} & \frac{1}{4} & \frac{1}{5} & \frac{1}{5} & \frac{3}{20} & 0 & \frac{1}{20} \\ \frac{1}{10} & \frac{3}{20} & \frac{1}{5} & \frac{1}{4} & \frac{3}{20} & \frac{1}{10} & \frac{1}{20} & 0 \\ 0 & \frac{1}{20} & \frac{1}{5} & \frac{3}{20} & \frac{1}{4} & \frac{1}{5} & \frac{1}{20} & \frac{1}{10} \\ \frac{1}{20} & 0 & \frac{3}{20} & \frac{1}{10} & \frac{1}{5} & \frac{1}{4} & \frac{1}{10} & \frac{3}{20} \\ \frac{1}{5} & \frac{3}{20} & 0 & \frac{1}{20} & \frac{1}{20} & \frac{1}{10} & \frac{1}{4} & \frac{1}{5} \\ \frac{3}{20} & \frac{1}{10} & \frac{1}{20} & 0 & \frac{1}{10} & \frac{3}{20} & \frac{1}{5} & \frac{1}{4} \end{pmatrix} \quad NM =$$

$$\begin{pmatrix} 4 & \frac{16}{5} & \frac{4}{5} & \frac{8}{5} & 0 & \frac{4}{5} & \frac{16}{5} & \frac{12}{5} \\ \frac{16}{5} & 4 & \frac{8}{5} & \frac{12}{5} & \frac{4}{5} & 0 & \frac{12}{5} & \frac{8}{5} \\ \frac{4}{5} & \frac{8}{5} & 4 & \frac{16}{5} & \frac{16}{5} & \frac{12}{5} & 0 & \frac{4}{5} \\ \frac{8}{5} & \frac{12}{5} & \frac{16}{5} & 4 & \frac{12}{5} & \frac{8}{5} & \frac{4}{5} & 0 \\ 0 & \frac{4}{5} & \frac{16}{5} & \frac{12}{5} & 4 & \frac{16}{5} & \frac{4}{5} & \frac{8}{5} \\ \frac{4}{5} & 0 & \frac{12}{5} & \frac{8}{5} & \frac{16}{5} & 4 & \frac{8}{5} & \frac{12}{5} \\ \frac{16}{5} & \frac{12}{5} & 0 & \frac{4}{5} & \frac{4}{5} & \frac{8}{5} & 4 & \frac{16}{5} \\ \frac{12}{5} & \frac{8}{5} & \frac{4}{5} & 0 & \frac{8}{5} & \frac{12}{5} & \frac{16}{5} & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 0, 1, 0, -1, 0, 1, 0]$

$$\ker N_c = \begin{pmatrix} -1 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} t & -t+s & -s & 0 & t & -t+s & -s & 0 \\ 0 & -t & 0 & t & 0 & -t & 0 & t \\ t & -t & -s & s & t & -t & -s & s \end{pmatrix}$$

RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & t & -s & 0 \\ 0 & 0 & -s & t \\ -t & 0 & 0 & s \\ -t & s & 0 & 0 \\ 0 & -t & s & 0 \\ 0 & 0 & s & -t \\ t & 0 & 0 & -s \\ t & -s & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} t & 0 & 0 & s & 0 \\ t & 0 & -t & s & t \\ t+s & -t & -s & 0 & t+s \\ t+s & -t & 0 & 0 & t \\ s & 0 & 0 & -s & t+s \\ s & 0 & t & -s & s \\ 0 & t & s & 0 & 0 \\ 0 & t & 0 & 0 & s \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 0 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & \frac{1}{5} & \frac{3}{5} & 0 & 0 & \frac{4}{5} & \frac{2}{5} \\ 1 & 1 & \frac{1}{5} & \frac{3}{5} & 0 & 0 & \frac{4}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} & 1 & \frac{3}{5} & \frac{4}{5} & \frac{4}{5} & 0 & \frac{2}{5} \\ \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & 1 & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & 0 \\ 0 & 0 & \frac{4}{5} & \frac{2}{5} & 1 & 1 & \frac{1}{5} & \frac{3}{5} \\ 0 & 0 & \frac{4}{5} & \frac{2}{5} & 1 & 1 & \frac{1}{5} & \frac{3}{5} \\ \frac{4}{5} & \frac{4}{5} & 0 & \frac{2}{5} & \frac{1}{5} & \frac{1}{5} & 1 & \frac{3}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & 0 & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & 1 \end{pmatrix}, \ \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{2}{5} & \frac{4}{5} & \frac{4}{5} \\ \frac{3}{5} & 1 & \frac{3}{5} & \frac{3}{5} & \frac{2}{5} & 0 & \frac{2}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{3}{5} & 1 & 1 & \frac{4}{5} & \frac{2}{5} & 0 & 0 \\ \frac{1}{5} & \frac{3}{5} & 1 & 1 & \frac{4}{5} & \frac{2}{5} & 0 & 0 \\ 0 & \frac{2}{5} & \frac{4}{5} & \frac{4}{5} & 1 & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{2}{5} & 0 & \frac{2}{5} & \frac{2}{5} & \frac{3}{5} & 1 & \frac{3}{5} & \frac{3}{5} \\ \frac{4}{5} & \frac{2}{5} & 0 & 0 & \frac{1}{5} & \frac{3}{5} & 1 & 1 \\ \frac{4}{5} & \frac{2}{5} & 0 & 0 & \frac{1}{5} & \frac{3}{5} & 1 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathrm{Skew}$$

$$\mathrm{Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 \\ 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{4}{5} & \frac{1}{5} & \frac{2}{5} & 0 & \frac{1}{5} & \frac{4}{5} & \frac{3}{5} \\ \frac{4}{5} & 1 & \frac{2}{5} & \frac{3}{5} & \frac{1}{5} & 0 & \frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{2}{5} & 1 & \frac{4}{5} & \frac{4}{5} & \frac{3}{5} & 0 & \frac{1}{5} \\ \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & 1 & \frac{3}{5} & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & \frac{1}{5} & \frac{4}{5} & \frac{3}{5} & 1 & \frac{4}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{1}{5} & 0 & \frac{3}{5} & \frac{2}{5} & \frac{4}{5} & 1 & \frac{2}{5} & \frac{3}{5} \\ \frac{4}{5} & \frac{3}{5} & 0 & \frac{1}{5} & \frac{1}{5} & \frac{2}{5} & 1 & \frac{4}{5} \\ \frac{3}{5} & \frac{2}{5} & \frac{1}{5} & 0 & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$\Omega \left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T \left(\frac{1}{4}\ \frac{1}{5}\ \frac{3}{20}\ \frac{1}{10}\ \frac{1}{5}\ \frac{1}{4}\ \frac{1}{10}\ \frac{1}{20}\ \frac{3}{20}\ \frac{1}{10}\ \frac{1}{4}\ \frac{1}{5}\ \frac{3}{20}\ \frac{1}{5}\ \frac{1}{20}\ 0\ \frac{1}{10}\ \frac{1}{20}\ \frac{1}{5}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

$\text{NM} \left(4\ \frac{16}{5}\ \frac{12}{5}\ \frac{8}{5}\ \frac{16}{5}\ 4\ \frac{8}{5}\ \frac{4}{5}\ \frac{12}{5}\ \frac{8}{5}\ 4\ \frac{16}{5}\ \frac{12}{5}\ \frac{16}{5}\ \frac{4}{5}\ 0\ \frac{8}{5}\ \frac{4}{5}\ \frac{16}{5}\ 4\right)$

"IS MN in Vec(K)?", true

$\text{MN} \left(4\ \frac{16}{5}\ \frac{12}{5}\ \frac{8}{5}\ \frac{16}{5}\ 4\ \frac{8}{5}\ \frac{4}{5}\ \frac{12}{5}\ \frac{8}{5}\ 4\ \frac{16}{5}\ \frac{12}{5}\ \frac{16}{5}\ \frac{4}{5}\ 0\ \frac{8}{5}\ \frac{4}{5}\ \frac{16}{5}\ 4\right)$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 24/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 4, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 32

dim span idems 16 vs no. of idems 16

"PT1" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT2" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT3" = {{1, 2, 4, 7}, {3, 5, 6, 8}}

"PT4" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \\ 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{27}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-1}{8} & \frac{3}{40} & \frac{27}{40} & \frac{19}{40} \\ \frac{27}{40} & \frac{7}{8} & \frac{11}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-1}{8} & \frac{19}{40} & \frac{11}{40} \\ \frac{3}{40} & \frac{11}{40} & \frac{7}{8} & \frac{27}{40} & \frac{27}{40} & \frac{19}{40} & \frac{-1}{8} & \frac{3}{40} \\ \frac{11}{40} & \frac{19}{40} & \frac{27}{40} & \frac{7}{8} & \frac{19}{40} & \frac{11}{40} & \frac{3}{40} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{3}{40} & \frac{27}{40} & \frac{19}{40} & \frac{7}{8} & \frac{27}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{3}{40} & \frac{-1}{8} & \frac{19}{40} & \frac{11}{40} & \frac{27}{40} & \frac{7}{8} & \frac{11}{40} & \frac{19}{40} \\ \frac{27}{40} & \frac{19}{40} & \frac{-1}{8} & \frac{3}{40} & \frac{3}{40} & \frac{11}{40} & \frac{7}{8} & \frac{27}{40} \\ \frac{19}{40} & \frac{11}{40} & \frac{3}{40} & \frac{-1}{8} & \frac{11}{40} & \frac{19}{40} & \frac{27}{40} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{27}{35} & \frac{3}{35} & \frac{11}{35} & \frac{-1}{7} & \frac{3}{35} & \frac{27}{35} & \frac{19}{35} \\ \frac{27}{35} & 1 & \frac{11}{35} & \frac{19}{35} & \frac{3}{35} & \frac{-1}{7} & \frac{19}{35} & \frac{11}{35} \\ \frac{3}{35} & \frac{11}{35} & 1 & \frac{27}{35} & \frac{27}{35} & \frac{19}{35} & \frac{-1}{7} & \frac{3}{35} \\ \frac{11}{35} & \frac{19}{35} & \frac{27}{35} & 1 & \frac{19}{35} & \frac{11}{35} & \frac{3}{35} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{3}{35} & \frac{27}{35} & \frac{19}{35} & 1 & \frac{27}{35} & \frac{3}{35} & \frac{11}{35} \\ \frac{3}{35} & \frac{-1}{7} & \frac{19}{35} & \frac{11}{35} & \frac{27}{35} & 1 & \frac{11}{35} & \frac{19}{35} \\ \frac{27}{35} & \frac{19}{35} & \frac{-1}{7} & \frac{3}{35} & \frac{3}{35} & \frac{11}{35} & 1 & \frac{27}{35} \\ \frac{19}{35} & \frac{11}{35} & \frac{3}{35} & \frac{-1}{7} & \frac{11}{35} & \frac{19}{35} & \frac{27}{35} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 1.365685425, 0.2343145752, 2.094427191, 0.3055728092]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 3.428571429, 1.560783343, 0.2677880860, 2.393631075, 0.349226067]

NullSpace M_C

{[0, 0, 0, 1, 0, 0, 0, −1], [0, 0, 0, 0, 1, 1, 1, 1], [0, 0, 1, 0, 0, 0, −1, 0], [1, 0, 0, 0, 0, 1, 1, 1], [0, 1, 0, 0, 0, −1, 0, 0]}

NullSpace N_C

{[0, 0, 1, −1, 0, 0, 1, −1], [0, 1, 0, −1, 0, 1, 0, −1], [1, 0, 0, −1, 1, 0, 0, −1]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 1.365685425, 0.2343145752, 2.094427191, 0.3055728092]

NullSpace M_0

{[0, 0, −1, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, −1, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, −1]}

NullSpace N_0

{[−1, 0, 1, 0, −1, 0, 1, 0], [−1, 0, 0, 1, −1, 0, 0, 1], [−1, 1, 0, 0, −1, 1, 0, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −0.2343145752, −1.365685425, −0.3055728092, −2.094427191]

NullSpace M

{}

NullSpace N

{[0, 0, 1, −1, 0, 0, 1, −1], [1, 0, 0, −1, 1, 0, 0, −1], [0, 1, 0, −1, 0, 1, 0, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 4 & 3 & 5 & 4 & 1 & 2 \\ 1 & 0 & 3 & 2 & 4 & 5 & 2 & 3 \\ 4 & 3 & 0 & 1 & 1 & 2 & 5 & 4 \\ 3 & 2 & 1 & 0 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 0 & 1 & 4 & 3 \\ 4 & 5 & 2 & 3 & 1 & 0 & 3 & 2 \\ 1 & 2 & 5 & 4 & 4 & 3 & 0 & 1 \\ 2 & 3 & 4 & 5 & 3 & 2 & 1 & 0 \end{pmatrix}$$

==

{5, 6, 7, 8}

R: [3, 3, 1, 1, 2, 4, 4, 2]
B: [6, 8, 8, 6, 7, 7, 5, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 6

Level 2 det = $\frac{-1}{134217728}(1+s)^3(-11+3s-s^2+s^3)(3+s)(-1+s)(-31-2s^2+s^4)(101-44s+10s^2-4s^3+s^4)(41+16s+6s^2+s^4)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[1] v[3]
"B CYCLES", 1 + v[5] v[7]

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, 0, 1, −1, 0], [0, 0, 0, 0, 1, 0, 0, −1], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, −1, 0, 0, 0, 0]}

NullSpace of B^*

{[0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8] \right)$

degree 2: $\frac{1}{4}$ ($v[1]v[3] + v[2]v[4] + v[5]v[7] + v[6]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 5, 8}, {3, 4, 6, 7}}

"PT2" = {{2, 3, 5, 6}, {1, 4, 7, 8}}

"PT3" = {{2, 3, 7, 8}, {1, 4, 5, 6}}

"PT4" = {{3, 4, 5, 8}, {1, 2, 6, 7}}

"RG1" = {6, 8}

"RG2" = {5, 7}

"RG3" = {2, 4}

"RG4" = {1, 3}

$\pi 2$ = [0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]

supp $\pi 2$ = {2, 9, 24, 27}

$u2$ = [1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [3], [4], [4]]
Action of B on ranges, [[2], [2], [1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [4, 1, 1, 4]

BPARTS [3, 2, 2, 3]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[8, 3, 1, 8, 3, 4, 4, 1]

B−BLOCKS,

[5, 7, 5, 6, 2, 7, 2, 6]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{3, 4, 5, 8\}$
$b_2 = \{2, 3, 5, 6\}$
$b_3 = \{1, 2, 5, 8\}$
$b_4 = \{3, 4, 6, 7\}$
$b_5 = \{2, 3, 7, 8\}$
$b_6 = \{1, 4, 5, 6\}$
$b_7 = \{1, 4, 7, 8\}$
$b_8 = \{1, 2, 6, 7\}$

dim(span of partition vectors), rank(N_0), rank(N):   5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & h[2] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[1] & 0 & h[2] & 0 & 0 & 0 & 0 \\ h[2] & 0 & h[1] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[2] & 0 & h[1] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[1] & 0 & h[2] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[1] & 0 & h[2] \\ 0 & 0 & 0 & 0 & h[2] & 0 & h[1] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[2] & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 20, Shape: 15 ⊕ 5/3

$$\mathrm{CLB} = \begin{pmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3}}, true

Ω_B in Vec(K)? , {{5, 7}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{-9}{40} & \frac{7}{40} \\ \frac{-3}{40} & \frac{29}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{-27}{40} & \frac{21}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} \\ \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} & \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} \\ \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 5, 8}, {3, 4, 6, 7}}

1, "range", [6, 8], [[8, 8, 6, 6, 8, 6, 6, 8], [6, 6, 8, 8, 6, 8, 8, 6]]

2, "range", [5, 7], [[7, 7, 5, 5, 7, 5, 5, 7], [5, 5, 7, 7, 5, 7, 7, 5]]

3, "range", [2, 4], [[4, 4, 2, 2, 4, 2, 2, 4], [2, 2, 4, 4, 2, 4, 4, 2]]

4, "range", [1, 3], [[3, 3, 1, 1, 3, 1, 1, 3], [1, 1, 3, 3, 1, 3, 3, 1]]

2, "partition", {{2, 3, 5, 6}, {1, 4, 7, 8}}

1, "range", [6, 8], [[8, 6, 6, 8, 6, 6, 8, 8], [6, 8, 8, 6, 8, 8, 6, 6]]

2, "range", [5, 7], [[7, 5, 5, 7, 5, 5, 7, 7], [5, 7, 7, 5, 7, 7, 5, 5]]

3, "range", [2, 4], [[4, 2, 2, 4, 2, 2, 4, 4], [2, 4, 4, 2, 4, 4, 2, 2]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 1, 3, 3], [1, 3, 3, 1, 3, 3, 1, 1]]

3, "partition", {{2, 3, 7, 8}, {1, 4, 5, 6}}

1, "range", [6, 8], [[8, 6, 6, 8, 8, 8, 6, 6], [6, 8, 8, 6, 6, 6, 8, 8]]

2, "range", [5, 7], [[7, 5, 5, 7, 7, 7, 5, 5], [5, 7, 7, 5, 5, 5, 7, 7]]

3, "range", [2, 4], [[4, 2, 2, 4, 4, 4, 2, 2], [2, 4, 4, 2, 2, 2, 4, 4]]

4, "range", [1, 3], [[3, 1, 1, 3, 3, 3, 1, 1], [1, 3, 3, 1, 1, 1, 3, 3]]

4, "partition", {{3, 4, 5, 8}, {1, 2, 6, 7}}

1, "range", [6, 8], [[8, 8, 6, 6, 6, 8, 8, 6], [6, 6, 8, 8, 8, 6, 6, 8]]

2, "range", [5, 7], [[7, 7, 5, 5, 5, 7, 7, 5], [5, 5, 7, 7, 7, 5, 5, 7]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 4, 4, 2], [2, 2, 4, 4, 4, 2, 2, 4]]

4, "range", [1, 3], [[3, 3, 1, 1, 1, 3, 3, 1], [1, 1, 3, 3, 3, 1, 1, 3]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$

{2, 9, 24, 27}

$u2 =$
$\begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \end{pmatrix}$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 2 & 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 0 & 2 & 2 & 2 & 2 \\ 0 & 2 & 4 & 2 & 2 & 2 & 2 & 2 \\ 2 & 0 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 2 & 4 & 2 & 0 \\ 2 & 2 & 2 & 2 & 0 & 2 & 4 & 2 \\ 2 & 2 & 2 & 2 & 2 & 0 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 1, 1, -1, -1, -1, -1]$

$$\ker N_c = \begin{pmatrix} -1 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -s & s & -s & s & t & -t & t & -t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s & s & -s & s & t & -t & t & -t \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} t & 0 & 0 & -s \\ -t & 0 & 0 & -s \\ -t & 0 & 0 & s \\ t & 0 & 0 & s \\ 0 & -t & s & 0 \\ 0 & -t & -s & 0 \\ 0 & t & -s & 0 \\ 0 & t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & s & t & 0 \\ t & 0 & s & 0 & 0 \\ s+t & 0 & -s & s & 0 \\ s & 0 & -s & s+t & 0 \\ s & t & 0 & s & -s \\ 0 & t & 0 & 0 & s \\ t & -t & 0 & t & s \\ s+t & -t & 0 & s+t & -s \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 4 & 0 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew}$$

$$\text{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&4&0&0&0&0&0\\ 0&4&0&4&0&0&0&0\\ 4&0&4&0&0&0&0&0\\ 0&4&0&4&0&0&0&0\\ 0&0&0&0&4&0&4&0\\ 0&0&0&0&0&4&0&4\\ 0&0&0&0&4&0&4&0\\ 0&0&0&0&0&4&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{1}{2}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&1&\frac{1}{2}&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 0&\frac{1}{2}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&0&\frac{1}{2}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{2}&0&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{2}&0\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{2}&1&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&\frac{1}{2}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{3}{8}\ \frac{3}{8}\ \frac{3}{8}\ \frac{3}{8}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ 0\ \frac{1}{8}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

NM $(6\ 6\ 6\ 6\ 2\ 4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 2\ 2\ 0\ 2\ 4)$

"IS MN in Vec(K)?", true

MN $(6\ 6\ 6\ 6\ 2\ 4\ 2\ 2\ 2\ 2\ 4\ 2\ 2\ 2\ 2\ 2\ 2\ 0\ 2\ 4)$

$\boldsymbol{\tau}$ = 32/1 , rank = 2 , ratio = 16/1 , n^2 / r = 32/1

$\boldsymbol{\tau}'$ = 32/1 , r' = 1/2 , $\boldsymbol{\tau} / n^2$ = 1/2

p^2 = 1/8 , min $\boldsymbol{\tau}$ = 8/1 , $\boldsymbol{\tau}$-check is positive? 24/1

max r = 8/1 , r-check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 4, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 32

dim span idems 16 vs no. of idems 16

"PT1" = {{1, 2, 5, 8}, {3, 4, 6, 7}}

"PT2" = {{2, 3, 5, 6}, {1, 4, 7, 8}}

"PT3" = {{2, 3, 7, 8}, {1, 4, 5, 6}}

"PT4" = {{3, 4, 5, 8}, {1, 2, 6, 7}}

"RG1" = {6, 8}

"RG2" = {5, 7}

"RG3" = {2, 4}

"RG4" = {1, 3}

$$M_C = \begin{pmatrix} 3 & -1 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 3 & -1 & -1 & -1 & -1 \\ 3 & -1 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & -1 & 3 \\ -1 & -1 & -1 & -1 & 3 & -1 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & -1 & 3 \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[3., 0., 0., 0., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[3.428571429, 0., 0., 0., 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 0, 0, 0, 1, 0, −1], [0, 0, 0, 0, 1, 0, −1, 0], [0, 0, 1, 1, 0, 0, 1, 1], [0, 1, 0, −1, 0, 0, 0, 0], [1, 0, 0, 1, 0, 0, 1, 1] }

NullSpace N_c

{[0, 1, 0, 1, 0, −1, 0, −1], [1, 0, 1, 0, 0, −1, 0, −1], [0, 0, 0, 0, 1, −1, 1, −1]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[4., 0., 0., 0., 1., 1., 1., 1.]

NullSpace M_0

{[0, −1, 0, 1, 0, 0, 0, 0], [1, 0, −1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, −1, 0], [0, 0, 0, 0, 0, 1, 0, −1]}

NullSpace N_0

{[−1, 0, −1, 0, 0, 1, 0, 1], [−1, 1, −1, 1, 0, 0, 0, 0], [−1, 0, −1, 0, 1, 0, 1, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[4., 0., 0., 0., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[−1, 1, −1, 1, 0, 0, 0, 0], [−1, 0, −1, 0, 1, 0, 1, 0], [−1, 0, −1, 0, 0, 1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 \end{pmatrix}$$

==

100, [1, −1, −1, −1, −1, −1, 1, 1]
==

120, [1, 1, 1, −1, −1, −1, −1, −1]

==

{2, 3, 4, 5, 6, 8}

R: [3, 8, 8, 6, 2, 4, 5, 2]
B: [6, 3, 1, 1, 7, 7, 4, 5]

TRACE TWO = 1

det AT = $\frac{1}{16} (t)^2 (1 + t)^2 (-1 + t)^2$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{-5}{536870912}\left(13332 + 7272 s + 1276 s^2 + 893 s^3 + 183 s^4 + 128 s^5 + 68 s^6 + 11 s^7 + 5 s^8\right)\left(2542 - 703 s + 779 s^2 - 664 s^3 + 9 s^4 - 28 s^5 - 61 s^6 + 56 s^7 - 7 s^8 - 5 s^9 + 2 s^{10}\right)(-1 + s)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 1, "vs", 4

"R CYCLES", (1 + v[2] v[8]) (1 + v[4] v[6])
"B CYCLES", 1 + v[1] v[4] v[6] v[7]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of R^*

{[0, −1, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

NullSpace of B^*

{[0, 0, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO–M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{5}{7} & \frac{4}{7} & \frac{4}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{2}{7} \\ \frac{5}{7} & 0 & \frac{2}{7} & \frac{4}{7} & \frac{5}{7} & \frac{3}{7} & \frac{2}{7} & 1 \\ \frac{4}{7} & \frac{2}{7} & 0 & \frac{2}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{5}{7} \\ \frac{4}{7} & \frac{4}{7} & \frac{2}{7} & 0 & \frac{5}{7} & 1 & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{5}{7} & 1 & \frac{5}{7} & 0 & \frac{2}{7} & \frac{4}{7} & \frac{2}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{5}{7} & 1 & \frac{2}{7} & 0 & \frac{4}{7} & \frac{4}{7} \\ 1 & \frac{2}{7} & \frac{3}{7} & \frac{3}{7} & \frac{4}{7} & \frac{4}{7} & 0 & \frac{5}{7} \\ \frac{2}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{2}{7} & \frac{4}{7} & \frac{5}{7} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{4}$ ($v[1]v[7] + v[2]v[8] + v[3]v[5] + v[4]v[6]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 5, 6, 8}, {2, 3, 4, 7}}

"PT4" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"PT5" = {{4, 5, 7, 8}, {1, 2, 3, 6}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {4, 6}

"RG3" = {3, 5}

"RG4" = {1, 7}

$\pi 2$ = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {6, 13, 15, 20}

$u2$ = [5, 4, 4, 3, 3, 7, 2, 2, 4, 5, 3, 2, 7, 2, 7, 5, 3, 5, 5, 7, 3, 3, 2, 4, 2, 4, 4, 5]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[1], [2], [1], [3]]
Action of B on ranges, [[3], [4], [4], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [5, 2, 3, 5, 2, 3]

BPARTS [3, 4, 1, 1, 6, 3]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{3}{14} & \frac{1}{7} & \frac{5}{14} & \frac{1}{14} & \frac{1}{7} & \frac{1}{14} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[9, B, C, 6, 1, 8, 6, 9, 8, 1, C, B]

B−BLOCKS,

[3, C, B, C, B, 2, 5, 7, A, 4, 4, 5]

with invariant measure, [2, 1, 1, 3, 3, 2, 1, 2, 2, 1, 5, 5]

N by blocks, N − check: true

$b_1 = \{4, 5, 7, 8\}$
$b_2 = \{1, 2, 3, 4\}$
$b_3 = \{5, 6, 7, 8\}$
$b_4 = \{1, 3, 4, 8\}$
$b_5 = \{2, 5, 6, 7\}$
$b_6 = \{1, 2, 3, 6\}$
$b_7 = \{3, 4, 7, 8\}$
$b_8 = \{1, 4, 5, 8\}$
$b_9 = \{2, 3, 6, 7\}$
$b_{10} = \{1, 2, 5, 6\}$
$b_{11} = \{1, 5, 6, 8\}$
$b_{12} = \{2, 3, 4, 7\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & 0 & 0 & h[1] & 0 \\ 0 & h[2] & 0 & 0 & 0 & 0 & 0 & h[1] \\ 0 & 0 & h[2] & 0 & h[1] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[2] & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[1] & 0 & h[2] & 0 & 0 & 0 \\ 0 & 0 & 0 & h[1] & 0 & h[2] & 0 & 0 \\ h[1] & 0 & 0 & 0 & 0 & 0 & h[2] & 0 \\ 0 & h[1] & 0 & 0 & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 28, Shape: 23 ⊕ 5/3

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 8}, {4, 6}}, false

Ω_B in Vec(K)? , {{1, 4, 6, 7}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} \\ \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} \\ \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} \\ \frac{-3}{40} & \frac{29}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{-27}{40} & \frac{21}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} \\ \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} \\ \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{3}{8} & 0 & \frac{1}{8} & 0 & \frac{1}{8} & 0 & \frac{3}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 4, 8}, {2, 5, 6, 7}}

1, "range", [2, 8], [[8, 2, 8, 8, 2, 2, 2, 8], [2, 8, 2, 2, 8, 8, 8, 2]]

2, "range", [4, 6], [[6, 4, 6, 6, 4, 4, 4, 6], [4, 6, 4, 4, 6, 6, 6, 4]]

3, "range", [3, 5], [[5, 3, 5, 5, 3, 3, 3, 5], [3, 5, 3, 3, 5, 5, 5, 3]]

4, "range", [1, 7], [[7, 1, 7, 7, 1, 1, 1, 7], [1, 7, 1, 1, 7, 7, 7, 1]]

2, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [2, 8], [[8, 2, 2, 8, 8, 2, 2, 8], [2, 8, 8, 2, 2, 8, 8, 2]]

2, "range", [4, 6], [[6, 4, 4, 6, 6, 4, 4, 6], [4, 6, 6, 4, 4, 6, 6, 4]]

3, "range", [3, 5], [[5, 3, 3, 5, 5, 3, 3, 5], [3, 5, 5, 3, 3, 5, 5, 3]]

4, "range", [1, 7], [[7, 1, 1, 7, 7, 1, 1, 7], [1, 7, 7, 1, 1, 7, 7, 1]]

3, "partition", {{1, 5, 6, 8}, {2, 3, 4, 7}}

1, "range", [2, 8], [[8, 2, 2, 2, 8, 8, 2, 8], [2, 8, 8, 8, 2, 2, 8, 2]]

2, "range", [4, 6], [[6, 4, 4, 4, 6, 6, 4, 6], [4, 6, 6, 6, 4, 4, 6, 4]]

3, "range", [3, 5], [[5, 3, 3, 3, 5, 5, 3, 5], [3, 5, 5, 5, 3, 3, 5, 3]]

4, "range", [1, 7], [[7, 1, 1, 1, 7, 7, 1, 7], [1, 7, 7, 7, 1, 1, 7, 1]]

4, "partition", {{3, 4, 7, 8}, {1, 2, 5, 6}}

1, "range", [2, 8], [[8, 8, 2, 2, 8, 8, 2, 2], [2, 2, 8, 8, 2, 2, 8, 8]]

2, "range", [4, 6], [[6, 6, 4, 4, 6, 6, 4, 4], [4, 4, 6, 6, 4, 4, 6, 6]]

3, "range", [3, 5], [[5, 5, 3, 3, 5, 5, 3, 3], [3, 3, 5, 5, 3, 3, 5, 5]]

4, "range", [1, 7], [[7, 7, 1, 1, 7, 7, 1, 1], [1, 1, 7, 7, 1, 1, 7, 7]]

5, "partition", {{4, 5, 7, 8}, {1, 2, 3, 6}}

1, "range", [2, 8], [[8, 8, 8, 2, 2, 8, 2, 2], [2, 2, 2, 8, 8, 2, 8, 8]]

2, "range", [4, 6], [[6, 6, 6, 4, 4, 6, 4, 4], [4, 4, 4, 6, 6, 4, 6, 6]]

3, "range", [3, 5], [[5, 5, 5, 3, 3, 5, 3, 3], [3, 3, 3, 5, 5, 3, 5, 5]]

4, "range", [1, 7], [[7, 7, 7, 1, 1, 7, 1, 1], [1, 1, 1, 7, 7, 1, 7, 7]]

6, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [2, 8], [[8, 8, 8, 8, 2, 2, 2, 2], [2, 2, 2, 2, 8, 8, 8, 8]]

2, "range", [4, 6], [[6, 6, 6, 6, 4, 4, 4, 4], [4, 4, 4, 4, 6, 6, 6, 6]]

3, "range", [3, 5], [[5, 5, 5, 5, 3, 3, 3, 3], [3, 3, 3, 3, 5, 5, 5, 5]]

4, "range", [1, 7], [[7, 7, 7, 7, 1, 1, 1, 1], [1, 1, 1, 1, 7, 7, 7, 7]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\left(h[1] \quad h[2]\right)$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {[3, 7], 17}, {[3, 8], 18}, {19, [4, 5]}, {20, [4, 6]}, {[4, 7], 21}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {[5, 8], 25}, {[6, 7], 26}, {27, [6, 8]}, {[7, 8], 28}

KERNEL HIERARCHY

$\pi 2 =$
$\left(0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right)$

{6, 13, 15, 20}

$u2 =$
$(5\ 4\ 4\ 3\ 3\ 7\ 2\ 2\ 4\ 5\ 3\ 2\ 7\ 2\ 7\ 5\ 3\ 5\ 5\ 7\ 3\ 3\ 2\ 4\ 2\ 4\ 4$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\left(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\right)$

$\pi = \left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$\pi 1 = \left(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\right)$

$u1 = \left(\frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\ \frac{7}{2}\right)$

picheck $\left(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\right)$

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{2}{7} & 0 & 0 & 0 & 0 & 0 & \frac{5}{7} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{5}{7} & 0 & 0 & 0 & 0 & 0 & \frac{2}{7} \\ 0 & \frac{3}{7} & 0 & 0 & 0 & 0 & 0 & \frac{4}{7} \\ 0 & \frac{2}{7} & 0 & 0 & 0 & 0 & 0 & \frac{5}{7} \\ 0 & \frac{4}{7} & 0 & 0 & 0 & 0 & 0 & \frac{3}{7} \\ 0 & \frac{5}{7} & 0 & 0 & 0 & 0 & 0 & \frac{2}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & \frac{3}{7} & 0 & \frac{4}{7} & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{7} & 0 & \frac{4}{7} & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{7} & 0 & \frac{2}{7} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{7} & 0 & \frac{5}{7} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{7} & 0 & \frac{3}{7} & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{7} & 0 & \frac{3}{7} & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & 0 & \frac{3}{7} & 0 & \frac{4}{7} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{7} & 0 & \frac{2}{7} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{7} & 0 & \frac{2}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{7} & 0 & \frac{5}{7} & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{7} & 0 & \frac{3}{7} & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{7} & 0 & \frac{5}{7} & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{7} & 0 & 0 & 0 & 0 & 0 & \frac{5}{7} & 0 \\ \frac{3}{7} & 0 & 0 & 0 & 0 & 0 & \frac{4}{7} & 0 \\ \frac{3}{7} & 0 & 0 & 0 & 0 & 0 & \frac{4}{7} & 0 \\ \frac{4}{7} & 0 & 0 & 0 & 0 & 0 & \frac{3}{7} & 0 \\ \frac{4}{7} & 0 & 0 & 0 & 0 & 0 & \frac{3}{7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{5}{7} & 0 & 0 & 0 & 0 & 0 & \frac{2}{7} & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_6 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{14} & \frac{3}{28} & \frac{3}{28} & \frac{1}{7} & \frac{1}{7} & 0 & \frac{5}{28} \\ \frac{1}{14} & \frac{1}{4} & \frac{5}{28} & \frac{3}{28} & \frac{1}{14} & \frac{1}{7} & \frac{5}{28} & 0 \\ \frac{3}{28} & \frac{5}{28} & \frac{1}{4} & \frac{5}{28} & 0 & \frac{1}{14} & \frac{1}{7} & \frac{1}{14} \\ \frac{3}{28} & \frac{3}{28} & \frac{5}{28} & \frac{1}{4} & \frac{1}{14} & 0 & \frac{1}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{1}{14} & 0 & \frac{1}{14} & \frac{1}{4} & \frac{5}{28} & \frac{3}{28} & \frac{5}{28} \\ \frac{1}{7} & \frac{1}{7} & \frac{1}{14} & 0 & \frac{5}{28} & \frac{1}{4} & \frac{3}{28} & \frac{3}{28} \\ 0 & \frac{5}{28} & \frac{1}{7} & \frac{1}{7} & \frac{3}{28} & \frac{3}{28} & \frac{1}{4} & \frac{1}{14} \\ \frac{5}{28} & 0 & \frac{1}{14} & \frac{1}{7} & \frac{5}{28} & \frac{3}{28} & \frac{1}{14} & \frac{1}{4} \end{pmatrix} \quad NM =$$

$$\begin{pmatrix} 4 & \frac{8}{7} & \frac{12}{7} & \frac{12}{7} & \frac{16}{7} & \frac{16}{7} & 0 & \frac{20}{7} \\ \frac{8}{7} & 4 & \frac{20}{7} & \frac{12}{7} & \frac{8}{7} & \frac{16}{7} & \frac{20}{7} & 0 \\ \frac{12}{7} & \frac{20}{7} & 4 & \frac{20}{7} & 0 & \frac{8}{7} & \frac{16}{7} & \frac{8}{7} \\ \frac{12}{7} & \frac{12}{7} & \frac{20}{7} & 4 & \frac{8}{7} & 0 & \frac{16}{7} & \frac{16}{7} \\ \frac{16}{7} & \frac{8}{7} & 0 & \frac{8}{7} & 4 & \frac{20}{7} & \frac{12}{7} & \frac{20}{7} \\ \frac{16}{7} & \frac{16}{7} & \frac{8}{7} & 0 & \frac{20}{7} & 4 & \frac{12}{7} & \frac{12}{7} \\ 0 & \frac{20}{7} & \frac{16}{7} & \frac{16}{7} & \frac{12}{7} & \frac{12}{7} & 4 & \frac{8}{7} \\ \frac{20}{7} & 0 & \frac{8}{7} & \frac{16}{7} & \frac{20}{7} & \frac{12}{7} & \frac{8}{7} & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, 0, 0, 0, 0, -1, 1]$

$$\ker N_c = \begin{pmatrix} 0 & 0 & -1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -s & 0 & s & 0 & s & 0 & -s \\ -t & 0 & t & 0 & t & 0 & -t & 0 \\ -t & -s & s & t & s & t & -t & -s \end{pmatrix} \quad RB$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 1 & -1 \end{pmatrix}$

$$\text{ker } M_0 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -s & t & 0 & 0 \\ -t & 0 & 0 & s \\ 0 & 0 & -t & s \\ 0 & s & -t & 0 \\ 0 & 0 & t & -s \\ 0 & -s & t & 0 \\ s & -t & 0 & 0 \\ t & 0 & 0 & -s \end{pmatrix} \text{ RB checks}$$

$$\text{ker } M_c = \begin{pmatrix} 1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & -s & s+t & 0 & 0 \\ t & -t & t & 0 & s \\ t & 0 & t & -t & s \\ t & 0 & s+t & -t & 0 \\ s & 0 & s & t & -s \\ s & 0 & 0 & t & 0 \\ t & s & 0 & 0 & 0 \\ s & t & s & 0 & -s \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 4 & 0 & 0 \end{pmatrix}$

$$R N_0 R^* = \begin{pmatrix} 1 & \frac{2}{7} & \frac{2}{7} & \frac{2}{7} & \frac{5}{7} & \frac{5}{7} & 0 & \frac{5}{7} \\ \frac{2}{7} & 1 & 1 & \frac{3}{7} & 0 & \frac{4}{7} & \frac{5}{7} & 0 \\ \frac{2}{7} & 1 & 1 & \frac{3}{7} & 0 & \frac{4}{7} & \frac{5}{7} & 0 \\ \frac{2}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{4}{7} & 0 & \frac{5}{7} & \frac{4}{7} \\ \frac{5}{7} & 0 & 0 & \frac{4}{7} & 1 & \frac{3}{7} & \frac{2}{7} & 1 \\ \frac{5}{7} & \frac{4}{7} & \frac{4}{7} & 0 & \frac{3}{7} & 1 & \frac{2}{7} & \frac{3}{7} \\ 0 & \frac{5}{7} & \frac{5}{7} & \frac{5}{7} & \frac{2}{7} & \frac{2}{7} & 1 & \frac{2}{7} \\ \frac{5}{7} & 0 & 0 & \frac{4}{7} & 1 & \frac{3}{7} & \frac{2}{7} & 1 \end{pmatrix}, R R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & \frac{2}{7} & \frac{4}{7} & \frac{4}{7} & \frac{3}{7} & \frac{3}{7} & 0 & \frac{5}{7} \\ \frac{2}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{4}{7} & \frac{4}{7} & \frac{5}{7} & 0 \\ \frac{4}{7} & \frac{3}{7} & 1 & 1 & 0 & 0 & \frac{3}{7} & \frac{4}{7} \\ \frac{4}{7} & \frac{3}{7} & 1 & 1 & 0 & 0 & \frac{3}{7} & \frac{4}{7} \\ \frac{3}{7} & \frac{4}{7} & 0 & 0 & 1 & 1 & \frac{4}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{4}{7} & 0 & 0 & 1 & 1 & \frac{4}{7} & \frac{3}{7} \\ 0 & \frac{5}{7} & \frac{3}{7} & \frac{3}{7} & \frac{4}{7} & \frac{4}{7} & 1 & \frac{2}{7} \\ \frac{5}{7} & 0 & \frac{4}{7} & \frac{4}{7} & \frac{3}{7} & \frac{3}{7} & \frac{2}{7} & 1 \end{pmatrix}, B B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew}$$

$$\text{Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{2}{7} & \frac{3}{7} & \frac{3}{7} & \frac{4}{7} & \frac{4}{7} & 0 & \frac{5}{7} \\ \frac{2}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{2}{7} & \frac{4}{7} & \frac{5}{7} & 0 \\ \frac{3}{7} & \frac{5}{7} & 1 & \frac{5}{7} & 0 & \frac{2}{7} & \frac{4}{7} & \frac{2}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{5}{7} & 1 & \frac{2}{7} & 0 & \frac{4}{7} & \frac{4}{7} \\ \frac{4}{7} & \frac{2}{7} & 0 & \frac{2}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{5}{7} \\ \frac{4}{7} & \frac{4}{7} & \frac{2}{7} & 0 & \frac{5}{7} & 1 & \frac{3}{7} & \frac{3}{7} \\ 0 & \frac{5}{7} & \frac{4}{7} & \frac{4}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{2}{7} \\ \frac{5}{7} & 0 & \frac{2}{7} & \frac{4}{7} & \frac{5}{7} & \frac{3}{7} & \frac{2}{7} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{5}{28}\ \frac{3}{28}\ \frac{3}{28}\ \frac{5}{28}\ \frac{1}{4}\ \frac{5}{28}\ \frac{3}{28}\ \frac{3}{28}\ \frac{5}{28}\ \frac{1}{4}\ \frac{1}{14}\ \frac{5}{28}\ 0\ \frac{1}{7}\ \frac{1}{7}\ \frac{3}{28}\ \frac{3}{28}\ \frac{1}{14}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM}\left(4\ \frac{20}{7}\ \frac{12}{7}\ \frac{12}{7}\ \frac{20}{7}\ 4\ \frac{20}{7}\ \frac{12}{7}\ \frac{12}{7}\ \frac{20}{7}\ 4\ \frac{8}{7}\ \frac{20}{7}\ 0\ \frac{16}{7}\ \frac{16}{7}\ \frac{12}{7}\ \frac{12}{7}\ \frac{8}{7}\ 4\right)$

"IS MN in Vec(K)?", true

$\mathrm{MN}\left(4\ \frac{20}{7}\ \frac{12}{7}\ \frac{12}{7}\ \frac{20}{7}\ 4\ \frac{20}{7}\ \frac{12}{7}\ \frac{12}{7}\ \frac{20}{7}\ 4\ \frac{8}{7}\ \frac{20}{7}\ 0\ \frac{16}{7}\ \frac{16}{7}\ \frac{12}{7}\ \frac{12}{7}\ \frac{8}{7}\ 4\right)$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 1/8$, min $\boldsymbol{\tau} = 8/1$, $\boldsymbol{\tau}$–check is positive? 24/1

max r = 8/1 , r–check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 6, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 48

dim span idems 16 vs no. of idems 24

"PT1" = {{1, 3, 4, 8}, {2, 5, 6, 7}}

"PT2" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT3" = {{1, 5, 6, 8}, {2, 3, 4, 7}}

"PT4" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"PT5" = {{4, 5, 7, 8}, {1, 2, 3, 6}}

"PT6" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {2, 8}

"RG2" = {4, 6}

"RG3" = {3, 5}

"RG4" = {1, 7}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & -1 & 3 & -1 & -1 \\ 3 & -1 & -1 & -1 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{9}{56} & \frac{17}{56} & \frac{17}{56} & \frac{25}{56} & \frac{25}{56} & \frac{-1}{8} & \frac{33}{56} \\ \frac{9}{56} & \frac{7}{8} & \frac{33}{56} & \frac{17}{56} & \frac{9}{56} & \frac{25}{56} & \frac{33}{56} & \frac{-1}{8} \\ \frac{17}{56} & \frac{33}{56} & \frac{7}{8} & \frac{33}{56} & \frac{-1}{8} & \frac{9}{56} & \frac{25}{56} & \frac{9}{56} \\ \frac{17}{56} & \frac{17}{56} & \frac{33}{56} & \frac{7}{8} & \frac{9}{56} & \frac{-1}{8} & \frac{25}{56} & \frac{25}{56} \\ \frac{25}{56} & \frac{9}{56} & \frac{-1}{8} & \frac{9}{56} & \frac{7}{8} & \frac{33}{56} & \frac{17}{56} & \frac{33}{56} \\ \frac{25}{56} & \frac{25}{56} & \frac{9}{56} & \frac{-1}{8} & \frac{33}{56} & \frac{7}{8} & \frac{17}{56} & \frac{17}{56} \\ \frac{-1}{8} & \frac{33}{56} & \frac{25}{56} & \frac{25}{56} & \frac{17}{56} & \frac{17}{56} & \frac{7}{8} & \frac{9}{56} \\ \frac{33}{56} & \frac{-1}{8} & \frac{9}{56} & \frac{25}{56} & \frac{33}{56} & \frac{17}{56} & \frac{9}{56} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{9}{49} & \frac{17}{49} & \frac{17}{49} & \frac{25}{49} & \frac{25}{49} & \frac{-1}{7} & \frac{33}{49} \\ \frac{9}{49} & 1 & \frac{33}{49} & \frac{17}{49} & \frac{9}{49} & \frac{25}{49} & \frac{33}{49} & \frac{-1}{7} \\ \frac{17}{49} & \frac{33}{49} & 1 & \frac{33}{49} & \frac{-1}{7} & \frac{9}{49} & \frac{25}{49} & \frac{9}{49} \\ \frac{17}{49} & \frac{17}{49} & \frac{33}{49} & 1 & \frac{9}{49} & \frac{-1}{7} & \frac{25}{49} & \frac{25}{49} \\ \frac{25}{49} & \frac{9}{49} & \frac{-1}{7} & \frac{9}{49} & 1 & \frac{33}{49} & \frac{17}{49} & \frac{33}{49} \\ \frac{25}{49} & \frac{25}{49} & \frac{9}{49} & \frac{-1}{7} & \frac{33}{49} & 1 & \frac{17}{49} & \frac{17}{49} \\ \frac{-1}{7} & \frac{33}{49} & \frac{25}{49} & \frac{25}{49} & \frac{17}{49} & \frac{17}{49} & 1 & \frac{9}{49} \\ \frac{33}{49} & \frac{-1}{7} & \frac{9}{49} & \frac{25}{49} & \frac{33}{49} & \frac{17}{49} & \frac{9}{49} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 0.2466909938, 1.753309006, 0.7884097369, 1.211590263]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 3.428571429, 0.2819325648, 2.003781721, 0.9010396996, 1.384674586]

NullSpace M_C

{[0, 0, 0, 0, 1, 1, 1, 1], [0, 0, 0, 1, 1, 0, 1, 1], [1, 0, 0, 0, 0, 0, −1, 0], [0, 1, 0, 0, 0, 0, 0, −1], [0, 0, 1, 0, −1, 0, 0, 0]}

NullSpace N_c

{[1, −1, 0, 0, 0, 0, 1, −1], [0, −1, 1, 0, 1, 0, 0, −1], [0, −1, 0, 1, 0, 1, 0, −1]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 0.2466909938, 1.753309006, 0.7884097369, 1.211590263]

NullSpace M_0

{[0, 0, −1, 0, 1, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

NullSpace N_0

{[0, 0, 1, −1, 1, −1, 0, 0], [1, 0, 0, −1, 0, −1, 1, 0], [0, 1, 0, −1, 0, −1, 0, 1]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −1.753309006, −0.2466909938, −1.211590263, −0.7884097369]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 1, 0, 1, −1, 0], [−1, 1, 0, 0, 0, 0, −1, 1], [−1, 0, 1, 0, 1, 0, −1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 5 & 4 & 4 & 3 & 3 & 7 & 2 \\ 5 & 0 & 2 & 4 & 5 & 3 & 2 & 7 \\ 4 & 2 & 0 & 2 & 7 & 5 & 3 & 5 \\ 4 & 4 & 2 & 0 & 5 & 7 & 3 & 3 \\ 3 & 5 & 7 & 5 & 0 & 2 & 4 & 2 \\ 3 & 3 & 5 & 7 & 2 & 0 & 4 & 4 \\ 7 & 2 & 3 & 3 & 4 & 4 & 0 & 5 \\ 2 & 7 & 5 & 3 & 2 & 4 & 5 & 0 \end{pmatrix}$$

==

{2, 3, 4, 6, 7, 8}

R: [3, 8, 8, 6, 7, 4, 4, 2]
B: [6, 3, 1, 1, 2, 7, 5, 5]

TRACE TWO = 1

det AT = $\frac{1}{16}(-1+t)^2(1+t)^2(t)^2$

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 8

Level 2 det = $\frac{1}{1073741824}(-1+s)(5412+180s+1452s^2+101s^3-101s^4+$

$20s^5 - 28s^6 + 3s^7 + s^8$) ($-6262 - 3387s - 2864s^2 - 313s^3 + 627s^4 + 394s^5 + 323s^6 - 8s^7 - 13s^8 - 15s^9 - 3s^{10} + s^{11}$)

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[2] v[8]) (1 + v[4] v[6])
"B CYCLES", 1 + v[1] v[2] v[3] v[5] v[6] v[7]

Eigenvalues

R: [1., -1., 1., -1., 0., 0., 0., 0.]

B: [0., 0., -1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, 0, 1, -1, 0], [0, 1, -1, 0, 0, 0, 0, 0]}

NullSpace of B^*

{[0, 0, 1, -1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, -1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 1 & \frac{3}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & 1 \\ 1 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & 1 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3   "Trace mark", 2, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[5] + v[2]v[6] + v[3]v[7] + v[4]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 3, 8}, {4, 5, 6, 7}}

"PT2" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT3" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT4" = {{2, 5, 7, 8}, {1, 3, 4, 6}}

"PT5" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$\pi 2$ = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]

supp $\pi 2$ = {4, 11, 17, 22}

$u2$ = [3, 2, 2, 4, 1, 2, 2, 1, 2, 1, 4, 3, 2, 1, 2, 3, 4, 3, 2, 2, 3, 4, 3, 2, 2, 1, 2, 1]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [1], [1], [2]]
Action of B on ranges, [[4], [4], [2], [3]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [1, 2, 1, 3, 2]

BPARTS [2, 4, 4, 5, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{8} & \frac{3}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[4, 7, 6, 7, 9, 8, 4, 8, 9, 5]

B-BLOCKS,

[7, 4, 2, A, A, 3, 3, 7, 4, 1]

with invariant measure, [1, 1, 2, 3, 1, 1, 3, 1, 1, 2]

N by blocks, N - check: true

b_1 = {1, 2, 3, 4}
b_2 = {5, 6, 7, 8}
b_3 = {2, 5, 7, 8}
b_4 = {1, 6, 7, 8}
b_5 = {1, 4, 6, 7}
b_6 = {2, 3, 5, 8}

$b_7 = \{2, 3, 4, 5\}$
$b_8 = \{1, 2, 3, 8\}$
$b_9 = \{4, 5, 6, 7\}$
$b_{10} = \{1, 3, 4, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & 0 & h[1] & 0 & 0 & 0 \\ 0 & h[2] & 0 & 0 & 0 & h[1] & 0 & 0 \\ 0 & 0 & h[2] & 0 & 0 & 0 & h[1] & 0 \\ 0 & 0 & 0 & h[2] & 0 & 0 & 0 & h[1] \\ h[1] & 0 & 0 & 0 & h[2] & 0 & 0 & 0 \\ 0 & h[1] & 0 & 0 & 0 & h[2] & 0 & 0 \\ 0 & 0 & h[1] & 0 & 0 & 0 & h[2] & 0 \\ 0 & 0 & 0 & h[1] & 0 & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 27, Shape: 23 ⊕ 4/3

$$\text{CLB} = \begin{pmatrix} -1 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 8}, {4, 6}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 5, 6, 7}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} \\ \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} \\ \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} \\ \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} \\ \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} & \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} \\ \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} \end{pmatrix}$$

Omega–checks true, true

"I–V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 3, 8}, {4, 5, 6, 7}}

1, "range", [4, 8], [[8, 8, 8, 4, 4, 4, 4, 8], [4, 4, 4, 8, 8, 8, 8, 4]]

2, "range", [3, 7], [[7, 7, 7, 3, 3, 3, 3, 7], [3, 3, 3, 7, 7, 7, 7, 3]]

3, "range", [2, 6], [[6, 6, 6, 2, 2, 2, 2, 6], [2, 2, 2, 6, 6, 6, 6, 2]]

4, "range", [1, 5], [[5, 5, 5, 1, 1, 1, 1, 5], [1, 1, 1, 5, 5, 5, 5, 1]]

2, "partition", {{1, 6, 7, 8}, {2, 3, 4, 5}}

1, "range", [4, 8], [[8, 4, 4, 4, 4, 8, 8, 8], [4, 8, 8, 8, 8, 4, 4, 4]]

2, "range", [3, 7], [[7, 3, 3, 3, 3, 7, 7, 7], [3, 7, 7, 7, 7, 3, 3, 3]]

3, "range", [2, 6], [[6, 2, 2, 2, 2, 6, 6, 6], [2, 6, 6, 6, 6, 2, 2, 2]]

4, "range", [1, 5], [[5, 1, 1, 1, 1, 5, 5, 5], [1, 5, 5, 5, 5, 1, 1, 1]]

3, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [4, 8], [[8, 4, 4, 8, 4, 8, 8, 4], [4, 8, 8, 4, 8, 4, 4, 8]]

2, "range", [3, 7], [[7, 3, 3, 7, 3, 7, 7, 3], [3, 7, 7, 3, 7, 3, 3, 7]]

3, "range", [2, 6], [[6, 2, 2, 6, 2, 6, 6, 2], [2, 6, 6, 2, 6, 2, 2, 6]]

4, "range", [1, 5], [[5, 1, 1, 5, 1, 5, 5, 1], [1, 5, 5, 1, 5, 1, 1, 5]]

4, "partition", {{2, 5, 7, 8}, {1, 3, 4, 6}}

1, "range", [4, 8], [[8, 4, 8, 8, 4, 8, 4, 4], [4, 8, 4, 4, 8, 4, 8, 8]]

2, "range", [3, 7], [[7, 3, 7, 7, 3, 7, 3, 3], [3, 7, 3, 3, 7, 3, 7, 7]]

3, "range", [2, 6], [[6, 2, 6, 6, 2, 6, 2, 2], [2, 6, 2, 2, 6, 2, 6, 6]]

4, "range", [1, 5], [[5, 1, 5, 5, 1, 5, 1, 1], [1, 5, 1, 1, 5, 1, 5, 5]]

5, "partition", {{1, 2, 3, 4}, {5, 6, 7, 8}}

1, "range", [4, 8], [[8, 8, 8, 8, 4, 4, 4, 4], [4, 4, 4, 4, 8, 8, 8, 8]]

2, "range", [3, 7], [[7, 7, 7, 7, 3, 3, 3, 3], [3, 3, 3, 3, 7, 7, 7, 7]]

3, "range", [2, 6], [[6, 6, 6, 6, 2, 2, 2, 2], [2, 2, 2, 2, 6, 6, 6, 6]]

4, "range", [1, 5], [[5, 5, 5, 5, 1, 1, 1, 1], [1, 1, 1, 1, 5, 5, 5, 5]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2$ =
$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{4, 11, 17, 22}

$u2$ =
$(3 \quad 2 \quad 2 \quad 4 \quad 1 \quad 2 \quad 2 \quad 1 \quad 2 \quad 1 \quad 4 \quad 3 \quad 2 \quad 1 \quad 2 \quad 3 \quad 4 \quad 3 \quad 2 \quad 2 \quad 3 \quad 4 \quad 3 \quad 2 \quad 2 \quad 1 \quad 2$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

idem−checks

$$PP_4 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

idem−checks

$$PP_5 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{16} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{3}{16} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{16} & \frac{1}{4} & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & 0 & \frac{1}{16} & \frac{1}{8} \\ \frac{1}{8} & \frac{3}{16} & \frac{1}{4} & \frac{3}{16} & \frac{1}{8} & \frac{1}{16} & 0 & \frac{1}{16} \\ \frac{1}{8} & \frac{1}{8} & \frac{3}{16} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} & 0 \\ 0 & \frac{3}{16} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{16} & \frac{1}{8} & \frac{1}{8} \\ \frac{3}{16} & 0 & \frac{1}{16} & \frac{1}{8} & \frac{1}{16} & \frac{1}{4} & \frac{3}{16} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{16} & 0 & \frac{1}{16} & \frac{1}{8} & \frac{3}{16} & \frac{1}{4} & \frac{3}{16} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{16} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 1 & 2 & 2 & 0 & 3 & 2 & 2 \\ 1 & 4 & 3 & 2 & 3 & 0 & 1 & 2 \\ 2 & 3 & 4 & 3 & 2 & 1 & 0 & 1 \\ 2 & 2 & 3 & 4 & 2 & 2 & 1 & 0 \\ 0 & 3 & 2 & 2 & 4 & 1 & 2 & 2 \\ 3 & 0 & 1 & 2 & 1 & 4 & 3 & 2 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 3 \\ 2 & 2 & 1 & 0 & 2 & 2 & 3 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 0, 0, 1, -1, 0, 0, 1]$

$$\ker N_c = \begin{pmatrix} 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} t & s & -t & -s & t & s & -t & -s \\ 0 & t & s-t & -s & 0 & t & s-t & -s \\ t & 0 & -t & 0 & t & 0 & -t & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix}1 & -1 & 0\end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s & -t & 0 \\ -s & t & 0 & 0 \\ -s & 0 & 0 & t \\ 0 & 0 & -s & t \\ 0 & -s & t & 0 \\ s & -t & 0 & 0 \\ s & 0 & 0 & -t \\ 0 & 0 & s & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} s & 0 & t & -s & s \\ t & 0 & 0 & -t & t+s \\ t & -t & 0 & 0 & t+s \\ t & -t & s & 0 & t \\ t & 0 & -t & s & t \\ s & 0 & 0 & t & 0 \\ s & t & 0 & 0 & 0 \\ s & t & -s & 0 & s \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix}4 & 0 & 0 & 0 & 4\end{pmatrix}$

$$R N_0 R^* = \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} \\ \frac{1}{4} & 1 & 1 & \frac{1}{2} & \frac{3}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 1 & 1 & \frac{1}{2} & \frac{3}{4} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 1 & 1 & \frac{1}{2} \\ \frac{3}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 1 & 1 & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} & 0 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} & 1 & 1 & \frac{1}{4} & \frac{1}{2} & 0 & 0 \\ \frac{3}{4} & \frac{1}{2} & 1 & 1 & \frac{1}{4} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{3}{4} & \frac{1}{4} & \frac{1}{4} & 1 & \frac{1}{4} & \frac{3}{4} & \frac{3}{4} \\ \frac{3}{4} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 & \frac{3}{4} & \frac{1}{2} & 1 & 1 \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 & \frac{3}{4} & \frac{1}{2} & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew}$$

$$\text{Omega} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&0&4&0&0&0\\ 0&4&0&0&0&4&0&0\\ 0&0&4&0&0&0&4&0\\ 0&0&0&4&0&0&0&4\\ 4&0&0&0&4&0&0&0\\ 0&4&0&0&0&4&0&0\\ 0&0&4&0&0&0&4&0\\ 0&0&0&4&0&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{1}{4}&\frac{1}{2}&\frac{1}{2}&0&\frac{3}{4}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{4}&1&\frac{3}{4}&\frac{1}{2}&\frac{3}{4}&0&\frac{1}{4}&\frac{1}{2}\\ \frac{1}{2}&\frac{3}{4}&1&\frac{3}{4}&\frac{1}{2}&\frac{1}{4}&0&\frac{1}{4}\\ \frac{1}{2}&\frac{1}{2}&\frac{3}{4}&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{4}&0\\ 0&\frac{3}{4}&\frac{1}{2}&\frac{1}{2}&1&\frac{1}{4}&\frac{1}{2}&\frac{1}{2}\\ \frac{3}{4}&0&\frac{1}{4}&\frac{1}{2}&\frac{1}{4}&1&\frac{3}{4}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{4}&0&\frac{1}{4}&\frac{1}{2}&\frac{3}{4}&1&\frac{3}{4}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{4}&0&\frac{1}{2}&\frac{1}{2}&\frac{3}{4}&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{3}{16}\ \frac{1}{8}\ \frac{1}{8}\ \frac{3}{16}\ \frac{1}{4}\ \frac{3}{16}\ \frac{1}{8}\ \frac{1}{8}\ \frac{3}{16}\ \frac{1}{4}\ \frac{1}{16}\ \frac{1}{8}\ \frac{1}{8}\ \frac{3}{16}\ 0\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{16}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

NM $(4\ 3\ 2\ 2\ 3\ 4\ 3\ 2\ 2\ 3\ 4\ 1\ 2\ 2\ 3\ 0\ 2\ 2\ 1\ 4)$

"IS MN in Vec(K)?", true

MN $(4\ 3\ 2\ 2\ 3\ 4\ 3\ 2\ 2\ 3\ 4\ 1\ 2\ 2\ 3\ 0\ 2\ 2\ 1\ 4)$

$\boldsymbol{\tau}$ = 32/1 , rank = 2 , ratio = 16/1 , n^2 / r = 32/1

$\boldsymbol{\tau}'$ = 32/1 , r' = 1/2 , $\boldsymbol{\tau} / n^2$ = 1/2

p^2 = 1/8 , min $\boldsymbol{\tau}$ = 8/1 , $\boldsymbol{\tau}$–check is positive? 24/1

max r = 8/1 , r–check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 5, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 40

dim span idems 16 vs no. of idems 20

"PT1" = {{1, 2, 3, 8}, {4, 5, 6, 7}}

"PT2" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"PT3" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT4" = {{2, 5, 7, 8}, {1, 3, 4, 6}}

"PT5" = {{1, 2, 3, 4}, {5, 6, 7, 8}}

"RG1" = {4, 8}

"RG2" = {3, 7}

"RG3" = {2, 6}

"RG4" = {1, 5}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \\ 3 & -1 & -1 & -1 & 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 & -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 & -1 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{5}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{1}{8} & \frac{7}{8} & \frac{5}{8} & \frac{3}{8} & \frac{5}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{5}{8} & \frac{7}{8} & \frac{5}{8} & \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{1}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{5}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{5}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{5}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{1}{8} & \frac{7}{8} & \frac{5}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{5}{8} & \frac{7}{8} & \frac{5}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{1}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{5}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \\ 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{5}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{1}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{5}{7} & \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{5}{7} & 1 & \frac{5}{7} & \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} & \frac{1}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{5}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{5}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{1}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{5}{7} & \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} & \frac{1}{7} & 1 & \frac{5}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} & \frac{1}{7} & \frac{3}{7} & \frac{5}{7} & 1 & \frac{5}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{1}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{5}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 3., 1.309016994, 0.1909830058, 1.809016994, 0.6909830058]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 3.428571429, 1.496019422, 0.2182662923, 2.067447994, 0.7896948642]

NullSpace M_C

{[0, 0, −1, 0, 0, 0, 1, 0], [0, 0, 0, −1, 0, 0, 0, 1], [0, 0, 1, 1, 1, 1, 0, 0], [1, 0, 0, 0, −1, 0, 0, 0], [0, 1, 1, 1, 1, 0, 0, 0] }

NullSpace N_c

{[−1, 1, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, −1, 0, 1, 0], [−1, 0, 0, 1, −1, 0, 0, 1]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 4., 1.309016994, 0.1909830058, 1.809016994, 0.6909830058]

NullSpace M_0

{[0, 0, 0, −1, 0, 0, 0, 1], [−1, 0, 0, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 1, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0]}

NullSpace N_0

{[0, −1, 0, 1, 0, −1, 0, 1], [1, −1, 0, 0, 1, −1, 0, 0], [0, −1, 1, 0, 0, −1, 1, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 4., −0.1909830058, −1.309016994, −0.6909830058, −1.809016994]

NullSpace M

{}

NullSpace N

{[0, 0, −1, 1, 0, 0, −1, 1], [0, 1, −1, 0, 0, 1, −1, 0], [1, 0, −1, 0, 1, 0, −1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 3 & 2 & 2 & 4 & 1 & 2 & 2 \\ 3 & 0 & 1 & 2 & 1 & 4 & 3 & 2 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 3 \\ 2 & 2 & 1 & 0 & 2 & 2 & 3 & 4 \\ 4 & 1 & 2 & 2 & 0 & 3 & 2 & 2 \\ 1 & 4 & 3 & 2 & 3 & 0 & 1 & 2 \\ 2 & 3 & 4 & 3 & 2 & 1 & 0 & 1 \\ 2 & 2 & 3 & 4 & 2 & 2 & 1 & 0 \end{pmatrix}$$

==

{2, 4, 5, 6, 7, 8}

R: [3, 8, 1, 6, 2, 4, 4, 2]
B: [6, 3, 8, 1, 7, 7, 5, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 7

Level 2 det $= \frac{1}{67108864} (-1023 - 33s + 78s^2 - 30s^3 - 32s^4 + 18s^6 - 2s^7 - s^8 + s^9)(4141 - 2121s - 557s^2 + 327s^3 + 269s^4 - 139s^5 - 15s^6 + 13s^7 + 2s^8)$

$(-1 + s)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 2, "vs", 6

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 1, "vs", 2

"R CYCLES", (1 + v[1] v[3]) (1 + v[2] v[8]) (1 + v[4] v[6])
"B CYCLES", 1 + v[5] v[7]

Eigenvalues

R: [0., 0., 1., −1., 1., −1., 1., −1.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, 0, −1, 1, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 1 "Trace mark", 0, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8] \right)$

degree 2: $\frac{1}{4}\left(v[1]v[3] + v[2]v[4] + v[5]v[7] + v[6]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 4, 6, 7}, {1, 2, 5, 8}}

"PT2" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT3" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT4" = {{2, 3, 5, 6}, {1, 4, 7, 8}}

"PT5" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT6" = {{1, 4, 5, 6}, {2, 3, 7, 8}}

"PT7" = {{3, 4, 5, 8}, {1, 2, 6, 7}}

"PT8" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"RG1" = {6, 8}

"RG2" = {5, 7}

"RG3" = {2, 4}

"RG4" = {1, 3}

$\pi 2$ = [0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]

supp $\pi 2$ = {2, 9, 24, 27}

$u2$ = [1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1]

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[3], [3], [1], [4]]
Action of B on ranges, [[2], [2], [4], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [5, 5, 3, 3, 1, 1, 7, 7]

BPARTS [8, 2, 8, 2, 6, 4, 6, 4]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[A, B, A, 8, 10, 10, 4, 4, B, 1, 2, 5, 1, 8, 2, 5]

B-BLOCKS,

[C, 6, 9, 6, F, 7, 9, C, 3, D, F, E, E, 3, 7, D]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N - check: true

b_1 = {3, 4, 6, 7}
b_2 = {1, 2, 5, 8}

$b_3 = \{3, 4, 5, 6\}$
$b_4 = \{1, 4, 5, 8\}$
$b_5 = \{3, 4, 5, 8\}$
$b_6 = \{3, 4, 7, 8\}$
$b_7 = \{2, 3, 5, 6\}$
$b_8 = \{2, 3, 6, 7\}$
$b_9 = \{1, 2, 7, 8\}$
$b_{10} = \{1, 4, 6, 7\}$
$b_{11} = \{2, 3, 5, 8\}$
$b_{12} = \{1, 2, 5, 6\}$
$b_{13} = \{1, 4, 5, 6\}$
$b_{14} = \{1, 4, 7, 8\}$
$b_{15} = \{2, 3, 7, 8\}$
$b_{16} = \{1, 2, 6, 7\}$

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & h[1] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[2] & 0 & h[1] & 0 & 0 & 0 & 0 \\ h[1] & 0 & h[2] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[1] & 0 & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & 0 & h[1] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[2] & 0 & h[1] \\ 0 & 0 & 0 & 0 & h[1] & 0 & h[2] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[1] & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 24, Shape: $18 \oplus 6/3$

$$\mathrm{CLB} = \begin{pmatrix} 1 & 1 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 3}, {2, 8}, {4, 6}}, false

Ω_B in Vec(K)? , {{5, 7}}, true

$$V = \begin{pmatrix} \frac{3}{40} & \frac{11}{40} & \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{-29}{40} & \frac{-13}{40} & \frac{19}{40} \\ \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} \\ \frac{27}{40} & \frac{-21}{40} & \frac{3}{40} & \frac{11}{40} & \frac{-13}{40} & \frac{19}{40} & \frac{3}{40} & \frac{-29}{40} \\ \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} \\ \frac{-3}{40} & \frac{29}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{-27}{40} & \frac{21}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-11}{40} & \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} \\ \frac{13}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{29}{40} & \frac{-27}{40} & \frac{21}{40} & \frac{-3}{40} & \frac{-11}{40} \\ \frac{1}{40} & \frac{17}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{-19}{40} & \frac{-3}{40} & \frac{-11}{40} & \frac{13}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & 0 & \frac{3}{16} & 0 & \frac{3}{16} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 4, 6, 7}, {1, 2, 5, 8}}

1, "range", [6, 8], [[8, 8, 6, 6, 8, 6, 6, 8], [6, 6, 8, 8, 6, 8, 8, 6]]

2, "range", [5, 7], [[7, 7, 5, 5, 7, 5, 5, 7], [5, 5, 7, 7, 5, 7, 7, 5]]

3, "range", [2, 4], [[4, 4, 2, 2, 4, 2, 2, 4], [2, 2, 4, 4, 2, 4, 4, 2]]

4, "range", [1, 3], [[3, 3, 1, 1, 3, 1, 1, 3], [1, 1, 3, 3, 1, 3, 3, 1]]

2, "partition", {{3, 4, 5, 6}, {1, 2, 7, 8}}

1, "range", [6, 8], [[8, 8, 6, 6, 6, 6, 8, 8], [6, 6, 8, 8, 8, 8, 6, 6]]

2, "range", [5, 7], [[7, 7, 5, 5, 5, 5, 7, 7], [5, 5, 7, 7, 7, 7, 5, 5]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 2, 4, 4], [2, 2, 4, 4, 4, 4, 2, 2]]

4, "range", [1, 3], [[3, 3, 1, 1, 1, 1, 3, 3], [1, 1, 3, 3, 3, 3, 1, 1]]

3, "partition", {{1, 4, 5, 8}, {2, 3, 6, 7}}

1, "range", [6, 8], [[8, 6, 6, 8, 8, 6, 6, 8], [6, 8, 8, 6, 6, 8, 8, 6]]

2, "range", [5, 7], [[7, 5, 5, 7, 7, 5, 5, 7], [5, 7, 7, 5, 5, 7, 7, 5]]

3, "range", [2, 4], [[4, 2, 2, 4, 4, 2, 2, 4], [2, 4, 4, 2, 2, 4, 4, 2]]

4, "range", [1, 3], [[3, 1, 1, 3, 3, 1, 1, 3], [1, 3, 3, 1, 1, 3, 3, 1]]

4, "partition", {{2, 3, 5, 6}, {1, 4, 7, 8}}

1, "range", [6, 8], [[8, 6, 6, 8, 6, 6, 8, 8], [6, 8, 8, 6, 8, 8, 6, 6]]

2, "range", [5, 7], [[7, 5, 5, 7, 5, 5, 7, 7], [5, 7, 7, 5, 7, 7, 5, 5]]

3, "range", [2, 4], [[4, 2, 2, 4, 2, 2, 4, 4], [2, 4, 4, 2, 4, 4, 2, 2]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 1, 3, 3], [1, 3, 3, 1, 3, 3, 1, 1]]

5, "partition", {{1, 4, 6, 7}, {2, 3, 5, 8}}

1, "range", [6, 8], [[8, 6, 6, 8, 6, 8, 8, 6], [6, 8, 8, 6, 8, 6, 6, 8]]

2, "range", [5, 7], [[7, 5, 5, 7, 5, 7, 7, 5], [5, 7, 7, 5, 7, 5, 5, 7]]

3, "range", [2, 4], [[4, 2, 2, 4, 2, 4, 4, 2], [2, 4, 4, 2, 4, 2, 2, 4]]

4, "range", [1, 3], [[3, 1, 1, 3, 1, 3, 3, 1], [1, 3, 3, 1, 3, 1, 1, 3]]

6, "partition", {{1, 4, 5, 6}, {2, 3, 7, 8}}

1, "range", [6, 8], [[8, 6, 6, 8, 8, 8, 6, 6], [6, 8, 8, 6, 6, 6, 8, 8]]

2, "range", [5, 7], [[7, 5, 5, 7, 7, 7, 5, 5], [5, 7, 7, 5, 5, 5, 7, 7]]

3, "range", [2, 4], [[4, 2, 2, 4, 4, 4, 2, 2], [2, 4, 4, 2, 2, 2, 4, 4]]

4, "range", [1, 3], [[3, 1, 1, 3, 3, 3, 1, 1], [1, 3, 3, 1, 1, 1, 3, 3]]

7, "partition", {{3, 4, 5, 8}, {1, 2, 6, 7}}

1, "range", [6, 8], [[8, 8, 6, 6, 6, 8, 8, 6], [6, 6, 8, 8, 8, 6, 6, 8]]

2, "range", [5, 7], [[7, 7, 5, 5, 5, 7, 7, 5], [5, 5, 7, 7, 7, 5, 5, 7]]

3, "range", [2, 4], [[4, 4, 2, 2, 2, 4, 4, 2], [2, 2, 4, 4, 4, 2, 2, 4]]

4, "range", [1, 3], [[3, 3, 1, 1, 1, 3, 3, 1], [1, 1, 3, 3, 3, 1, 1, 3]]

8, "partition", {{3, 4, 7, 8}, {1, 2, 5, 6}}

1, "range", [6, 8], [[8, 8, 6, 6, 8, 8, 6, 6], [6, 6, 8, 8, 6, 6, 8, 8]]

2, "range", [5, 7], [[7, 7, 5, 5, 7, 7, 5, 5], [5, 5, 7, 7, 5, 5, 7, 7]]

3, "range", [2, 4], [[4, 4, 2, 2, 4, 4, 2, 2], [2, 2, 4, 4, 2, 2, 4, 4]]

4, "range", [1, 3], [[3, 3, 1, 1, 3, 3, 1, 1], [1, 1, 3, 3, 1, 1, 3, 3]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {[2, 6], 11}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

{2, 9, 24, 27}

$u2 =$
$$\begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \end{pmatrix}$$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$u1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_3 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_5 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$ idem-checks

$$PP_6 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$ idem-checks

$$PP_7 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_8 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 2 & 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 0 & 2 & 2 & 2 & 2 \\ 0 & 2 & 4 & 2 & 2 & 2 & 2 & 2 \\ 2 & 0 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 2 & 4 & 2 & 0 \\ 2 & 2 & 2 & 2 & 0 & 2 & 4 & 2 \\ 2 & 2 & 2 & 2 & 2 & 0 & 2 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 1, 0, 1, -1, 0, -1, 0]$

$$\ker N_c = \begin{pmatrix} -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -s+t & 0 & -s+t & 0 & 0 & -t+s & 0 & -t+s \\ -s & s & -s & s & t & -t & t & -t \\ -s & s & -s & s & t & -t & t & -t \end{pmatrix}$$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} t & 0 & 0 & -s \\ -s & 0 & 0 & -t \\ -t & 0 & 0 & s \\ s & 0 & 0 & t \\ 0 & -t & s & 0 \\ 0 & -t & -s & 0 \\ 0 & t & -s & 0 \\ 0 & t & s & 0 \end{pmatrix}$$ RB checks

$$\ker M_c = \begin{pmatrix} 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} t & 0 & 0 & s & 0 \\ -s & 0 & s & t & s \\ -t & 0 & s+t & -s & s+t \\ s & 0 & t & -t & t \\ 0 & -t & t & 0 & s+t \\ 0 & -t & s+t & 0 & t \\ 0 & t & s & 0 & 0 \\ 0 & t & 0 & 0 & s \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 0 & 4 & 0 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad Skew\ T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad Skew$$

$$Omega = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 0 & 4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \left(\frac{1}{8} \right)$$

$$T \left(\frac{1}{4} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{4} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{4} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ 0 \ \frac{1}{8} \ \frac{1}{4} \right)$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \left(4 \ 2 \ 2 \ 2 \ 2 \ 4 \ 2 \ 2 \ 2 \ 2 \ 4 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 0 \ 2 \ 4 \right)$$

"IS MN in Vec(K)?", true

$$\mathrm{MN} \left(4 \ 2 \ 2 \ 2 \ 2 \ 4 \ 2 \ 2 \ 2 \ 2 \ 4 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 0 \ 2 \ 4 \right)$$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 24/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 8, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 20
out of total no. of elements equal to 64

dim span idems 16 vs no. of idems 32

"PT1" = {{3, 4, 6, 7}, {1, 2, 5, 8}}

"PT2" = {{3, 4, 5, 6}, {1, 2, 7, 8}}

"PT3" = {{1, 4, 5, 8}, {2, 3, 6, 7}}

"PT4" = {{2, 3, 5, 6}, {1, 4, 7, 8}}

"PT5" = {{1, 4, 6, 7}, {2, 3, 5, 8}}

"PT6" = {{1, 4, 5, 6}, {2, 3, 7, 8}}

"PT7" = {{3, 4, 5, 8}, {1, 2, 6, 7}}

"PT8" = {{3, 4, 7, 8}, {1, 2, 5, 6}}

"RG1" = {6, 8}

"RG2" = {5, 7}

"RG3" = {2, 4}

"RG4" = {1, 3}

$$M_C = \begin{pmatrix} 3 & -1 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 3 & -1 & -1 & -1 & -1 \\ 3 & -1 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & -1 & 3 \\ -1 & -1 & -1 & -1 & 3 & -1 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & -1 & 3 \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} & \frac{-1}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & \frac{3}{7} & \frac{-1}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{3}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[3., 0., 0., 0., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[3.428571429, 0., 0., 0., 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[1, 0, 0, 1, 1, 0, 0, 1], [0, 0, 0, 0, −1, 0, 1, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [1, 0, 0, 1, 1, 1, 0, 0], [0, 1, 0, −1, 0, 0, 0, 0]}

NullSpace N_C

{[0, −1, 0, −1, 0, 1, 0, 1], [0, −1, 0, −1, 1, 0, 1, 0], [1, −1, 1, −1, 0, 0, 0, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[4., 0., 0., 0., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 0, 1, 0], [0, 0, 0, 0, 0, −1, 0, 1], [0, 1, 0, −1, 0, 0, 0, 0]}

NullSpace N_0

{[−1, 0, −1, 0, 1, 0, 1, 0], [−1, 0, −1, 0, 0, 1, 0, 1], [−1, 1, −1, 1, 0, 0, 0, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[4., 0., 0., 0., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{[−1, 1, −1, 1, 0, 0, 0, 0], [−1, 0, −1, 0, 1, 0, 1, 0], [−1, 0, −1, 0, 0, 1, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 \end{pmatrix}$$