T - Run

[2, 3, 5, 5, 1, 1], [3, 4, 6, 6, 4, 2]

$\tilde{\pi}$ = [1, 1, 1, 1, 1, 1]
$\boldsymbol{\delta}$ = [2, 2, 2, 2, 2, 2]

POSSIBLE RANKS

1 x 6
2 x 3

BASE DETERMINANT 91/512, .1777343750

NullSpace of Δ

{5, 6}, {1, 2, 3, 4}

Nullspace of A

[{6},{5}] `,` [{2, 4},{1, 3}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[4] v[5]

Degree 3
v[2] v[3] v[6] + v[1] v[3] v[5] + v[2] v[4] v[6] + v[1] v[3] v[6]

Degree 4
v[1] v[2] v[3] v[5] + v[1] v[2] v[4] v[5] + v[1] v[2] v[3] v[6] + v[1] v[2] v[4] v[6]

Degree 5
2 v[1] v[3] v[4] v[5] v[6] + 2 v[2] v[3] v[4] v[5] v[6]

Degree 6
4 v[1] v[2] v[3] v[4] v[5] v[6]

==

{3, 5}

R: [2, 3, 6, 5, 4, 1]
B: [3, 4, 5, 6, 1, 2]

TRACE TWO = 2

det AT = $\frac{1}{2}(t)^2(1+t^2)$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{1}{16384}(-1+s)^2(2912+1984s+564s^2-405s^3-335s^4-110s^5-6s^6+3s^7+s^8)$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", (1 + v[1] v[2] v[3] v[6]) (1 + v[4] v[5])
"B CYCLES", (1 + v[2] v[4] v[6]) (1 + v[1] v[3] v[5])

Eigenvalues

R: [1. I, −1. I, 1., −1., 1., −1.]

B: [1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 6

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{9}$ ($2v[1]v[2] + 3v[1]v[3] + 2v[1]v[4] + 3v[1]v[5] + 2v[1]v[6] + 2v[2]v[3] + 3v[2]v[4] + 2v[2]v[5] + 3v[2]v[6] + 2v[3]v[4] + 3v[3]v[5] + 2v[3]v[6] + 2v[4]v[5] + 3v[4]v[6] + 2v[6]v[5]$)
degree 3 : $\frac{1}{18}$ ($v[1]v[2]v[3] + v[1]v[2]v[4] + v[1]v[2]v[5] + v[1]v[2]v[6] + v[1]v[3]v[4] + 9v[1]v[3]v[5] + v[1]v[3]v[6] + v[1]v[4]v[5] + v[1]v[4]v[6] + v[1]v[6]v[5] + v[2]v[3]v[4] + v[2]v[3]v[5] + v[2]v[3]v[6] + v[2]v[4]v[5] + 9v[2]v[4]v[6] + v[2]v[6]v[5] + v[3]v[4]v[5] + v[3]v[4]v[6] + v[3]v[6]v[5] + v[4]v[6]v[5]$)
degree 4 : $\frac{1}{9}$ ($2v[1]v[2]v[3]v[4] + 3v[1]v[2]v[3]v[5] + 2v[1]v[2]v[3]v[6] + 2v[1]v[2]v[4]v[5] + 3v[1]v[2]v[4]v[6] + 2v[1]v[2]v[6]v[5] + 3v[1]v[3]v[4]v[5] + 2v[1]v[3]v[4]v[6] + 3v[1]v[3]v[6]v[5] + 2v[1]v[4]v[6]v[5] + 2v[2]v[3]v[4]v[5] + 3v[2]v[3]v[4]v[6] + 2v[2]v[3]v[6]v[5] + 3v[2]v[4]v[6]v[5] + 2v[3]v[4]v[6]v[5]$)
degree 5 : $\frac{1}{6}$ ($v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[6] + v[1]v[2]v[3]v[6]v[5] + v[1]v[2]v[4]v[6]v[5] + v[1]v[3]v[4]v[5]v[6] + v[2]v[3]v[4]v[5]v[6]$)
degree 6 : 1 ($v[5]$) ($v[6]$) ($v[2]$) ($v[4]$) ($v[3]$) ($v[1]$)

Group spectrum $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

KERNEL STRUCTURE

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$\pi 6 = [1]$$

supp $\pi 6$ = {1}

$$u6 = [1]$$

supp u6 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[3, 4, 1, 5, 6, 2]

B–BLOCKS,

[6, 3, 5, 1, 2, 4]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

$b_1 = \{4\}$
$b_2 = \{1\}$
$b_3 = \{5\}$
$b_4 = \{6\}$
$b_5 = \{3\}$
$b_6 = \{2\}$

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[3] & h[2] & h[1] & h[2] & h[1] & h[2] \\ h[2] & h[3] & h[2] & h[1] & h[2] & h[1] \\ h[1] & h[2] & h[3] & h[2] & h[1] & h[2] \\ h[2] & h[1] & h[2] & h[3] & h[2] & h[1] \\ h[1] & h[2] & h[1] & h[2] & h[3] & h[2] \\ h[2] & h[1] & h[2] & h[1] & h[2] & h[3] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 12, Shape: 10 ⊕ 2/0

$$\text{CLB} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{4, 5}, {1, 2, 3, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 5}, {2, 4, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{5}{12} & \frac{-1}{3} & \frac{1}{6} & \frac{-1}{12} & \frac{-1}{12} \\ 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 \\ \frac{-1}{24} & \frac{5}{24} & \frac{1}{12} & \frac{-1}{6} & \frac{-13}{24} & \frac{11}{24} \\ \frac{1}{24} & \frac{-5}{24} & \frac{-1}{12} & \frac{1}{6} & \frac{13}{24} & \frac{-11}{24} \\ \frac{-3}{8} & \frac{-1}{8} & \frac{-1}{4} & \frac{1}{2} & \frac{1}{8} & \frac{1}{8} \\ \frac{11}{24} & \frac{-7}{24} & \frac{1}{12} & \frac{-1}{6} & \frac{-1}{24} & \frac{-1}{24} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4}, {1}, {5}, {6}, {3}, {2}}

1, "range", [1, 2, 3, 4, 5, 6], [[6, 5, 4, 1, 2, 3], [6, 5, 2, 3, 4, 1], [6, 3, 4, 5, 2, 1], [6, 3, 2, 1, 4, 5], [6, 1, 4, 3, 2, 5], [6, 1, 2, 5, 4, 3], [5, 6, 3, 4, 1, 2], [5, 6, 1, 2, 3, 4], [5, 4, 3, 2, 1, 6], [5, 4, 1, 6, 3, 2], [5, 2, 3, 6, 1, 4], [5, 2, 1, 4, 3, 6], [4, 5, 6, 3, 2, 1], [4, 5, 2, 1, 6, 3], [4, 3, 6, 1, 2, 5], [4, 3, 2, 5, 6, 1], [4, 1, 6, 5, 2, 3], [4, 1, 2, 3, 6, 5], [3, 6, 5, 2, 1, 4], [3, 6, 1, 4, 5, 2], [3, 4, 5, 6, 1, 2], [3, 4, 1, 2, 5, 6], [3, 2, 5, 4, 1, 6], [3, 2, 1, 6, 5, 4], [2, 5, 6, 1, 4, 3], [2, 5, 4, 3, 6, 1], [2, 3, 6, 5, 4, 1], [2, 3, 4, 1, 6, 5], [2, 1, 6, 3, 4, 5], [2, 1, 4, 5, 6, 3], [1, 6, 5, 4, 3, 2], [1, 6, 3, 2, 5, 4], [1, 4, 5, 2, 3, 6], [1, 4, 3, 6, 5, 2], [1, 2, 5, 6, 3, 4], [1, 2, 3, 4, 5, 6]] "group has", 36, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 6, 3, 4], [2, 5]]

$g_2 =$ [[1, 6], [2, 5, 4, 3]]

$g_3 =$ [[1, 6], [2, 3, 4, 5]]

$g_4 =$ [[1, 6, 5, 4], [2, 3]]

$g_5 =$ [[1, 6, 5, 2], [3, 4]]

linear dimension, 18

"Symmetric?", true

Is Z in Vec(K)? true
$\left(-2h[2] \;\; 0 \;\; 2h[2] \;\; 0 \;\; 2h[2] \;\; 0 \;\; -6h[1] \;\; 0 \;\; 6h[1] \;\; 3h[3]-6h[1] \;\; 6h[1] \;\; 0 \;\; 0 \;\; 2h[2] \;\; 3h[3]-6h[1] \;\; 6h[1] \;\; 2\right.$

"Basis for Z(G)"

1, "coeff", 6

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 2

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 3

$$Z[3] = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 0 & 0 & 0 & 0 & 3. & -3. \\ 2. & 2. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 2t^3 + 2t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 15t^5 + 26t^6 + 38t^7 + 59t^8 + 84t^9 + 121t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}

KERNEL HIERARCHY

$\pi 6 = \begin{pmatrix} 1 \end{pmatrix}$

{1}

$u6 = \begin{pmatrix} 1 \end{pmatrix}$

{1}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 5 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u5 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

picheck $\begin{pmatrix} 5 & 5 & 5 & 5 & 5 & 5 \end{pmatrix}$

$\pi 4 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

$u4 = \begin{pmatrix} \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} \end{pmatrix}$

picheck $\begin{pmatrix}20 & 20 & 20 & 20 & 20 & 20\end{pmatrix}$

$\pi 3 = \begin{pmatrix}6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6\end{pmatrix}$

$u3 = \begin{pmatrix}\frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36}\end{pmatrix}$

picheck $\begin{pmatrix}60 & 60 & 60 & 60 & 60 & 60\end{pmatrix}$

$\pi 2 = \begin{pmatrix}24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24\end{pmatrix}$

$u2 = \begin{pmatrix}\frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54}\end{pmatrix}$

picheck $\begin{pmatrix}120 & 120 & 120 & 120 & 120 & 120\end{pmatrix}$

$\pi 1 = \begin{pmatrix}120 & 120 & 120 & 120 & 120 & 120\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324}\end{pmatrix}$

picheck $\begin{pmatrix}120 & 120 & 120 & 120 & 120 & 120\end{pmatrix}$

Column Projections

$P_1 = \begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$ idem-checks N0-checks

Row Projections

$\mathrm{PP}_1 = \begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$ idem-checks

$$\mathrm{T} = \begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix} \quad \mathrm{NM} = \begin{pmatrix}5 & 4 & 4 & 4 & 4 & 4\\ 4 & 5 & 4 & 4 & 4 & 4\\ 4 & 4 & 5 & 4 & 4 & 4\\ 4 & 4 & 4 & 5 & 4 & 4\\ 4 & 4 & 4 & 4 & 5 & 4\\ 4 & 4 & 4 & 4 & 4 & 5\end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta}$ = [0, 0, 0, 0, 0, 0]

ker $N_C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} t+s & t+s & t+s & t+s & t+s & t+s \end{pmatrix}$ RB checks

ker $M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} s & 0 & 0 & 0 & t \\ 0 & 0 & t & 0 & s \\ 0 & t & 0 & s & 0 \\ 0 & s & 0 & t & 0 \\ -t & -t & -t+s & -t & -t \\ -s+t & -s & -s & -s & -s \end{pmatrix}$ RB checks

ker $M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s & t & 0 & 0 & 0 \\ 0 & 0 & s & t & 0 & 0 \\ s & 0 & 0 & 0 & t & 0 \\ t & 0 & 0 & 0 & s & 0 \\ 0 & 0 & 0 & s & 0 & t \\ 0 & t & 0 & 0 & 0 & s \end{pmatrix}$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 1T + 24\Omega$$

$\Omega \left(\frac{5}{12} \;\; \frac{1}{12} \;\; \frac{-1}{6} \;\; \frac{1}{3} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \right)$

$T\,(2 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1)$

"IS NM in Vec(K)?", true

$\text{NM}\,(12 \;\; 2 \;\; -4 \;\; 8 \;\; 4 \;\; 5 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 5 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 5)$

"IS MN in Vec(K)?", true

$\text{MN}\,(12 \;\; 2 \;\; -4 \;\; 8 \;\; 4 \;\; 5 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 5 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 4 \;\; 5)$

$$\boldsymbol{\tau} = 6/1 \text{ , rank} = 6 \text{ , ratio} = 1/1 \text{ , } n^2 / r = 6/1$$

$$\boldsymbol{\tau}' = 30/1 \text{ , } r' = 5/6 \text{ , } \boldsymbol{\tau} / n^2 = 1/6$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\text{max r} = 6/1 \text{ , } r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 6\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 36

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 36

dim span idems 1 vs no. of idems 1

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$M_C = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{5}{6}&\frac{-1}{6}\\ \frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{-1}{6}&\frac{5}{6} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&1&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&1&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&1&\frac{-1}{5}&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&1&\frac{-1}{5}\\ \frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&\frac{-1}{5}&1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.200000000, 1.200000000, 1.200000000, 1.200000000, 1.200000000]

NullSpace M_C

{[0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0], [0, 0, 0, 1, 0, 0], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0]}

NullSpace N_C

{[1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 6.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 1, 0, 0, 0, 0], [−1, 0, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

NullSpace N_0

{}

Eigenvalues M

[5., −1., −1., −1., −1., −1.]

Eigenvalues N

[5., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

==

{4, 5}

R: [2, 3, 5, 6, 4, 1]
B: [3, 4, 6, 5, 1, 2]

TRACE TWO = 1

det AT = $\frac{-1}{2} (t)^2 (1 + t^2)$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 6

Level 2 det = $\frac{-1}{16384} (2912 + 1568s + 548s^2 - 911s^3 - 694s^4 - 273s^5 - 80s^6 - s^7 + 2s^8 + s^9) (-1 + s)$

RANK of R is 6

R ranking is 1, "vs", 6

RBAR ranking 1, "vs", 6

RANK of B is 6

B ranking is 1, "vs", 6

BBAR ranking 1, "vs", 6

"R CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6]
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6]

Eigenvalues

R: [-1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

B: [-1., 1., -0.5000000000 - 0.8660254040 I, 0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO–M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 6, "RANK of M is ", 6

"RANK of the KERNEL is ", 6

"IdemSolvability Check", 3 "Trace mark", 6, "Rank mark", 6, "for kernel rank", 6

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{15}$ ($v[1]v[2] + v[1]v[3] + v[1]v[4] + v[1]v[5] + v[1]v[6] + v[2]v[3] + v[2]v[4] + v[2]v[5] + v[2]v[6] + v[3]v[4] + v[3]v[5] + v[3]v[6] + v[4]v[5] + v[4]v[6] + v[6]v[5]$)
degree 3 : $\frac{1}{20}$ ($v[1]v[2]v[3] + v[1]v[2]v[4] + v[1]v[2]v[5] + v[1]v[2]v[6] + v[1]v[3]v[4] + v[1]v[3]v[5] + v[1]v[3]v[6] + v[1]v[4]v[5] + v[1]v[4]v[6] + v[1]v[6]v[5] + v[2]v[3]v[4] + v[2]v[3]v[5] + v[2]v[3]v[6] + v[2]v[4]v[5] + v[2]v[4]v[6] + v[2]v[6]v[5] + v[3]v[4]v[5] + v[3]v[4]v[6] + v[3]v[6]v[5] + v[4]v[6]v[5]$)
degree 4 : $\frac{1}{15}$ ($v[1]v[2]v[3]v[4] + v[1]v[2]v[3]v[5] + v[1]v[2]v[3]v[6] + v[1]v[2]v[4]v[5] + v[1]v[2]v[4]v[6] + v[1]v[2]v[6]v[5] + v[1]v[3]v[4]v[5] + v[1]v[3]v[4]v[6] + v[1]v[3]v[6]v[5] + v[1]v[4]v[6]v[5] + v[2]v[3]v[4]v[5] + v[2]v[3]v[4]v[6] + v[2]v[3]v[6]v[5] + v[2]v[4]v[6]v[5] + v[3]v[4]v[6]v[5]$)
degree 5 : $\frac{1}{6}$ ($v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[6] + v[1]v[2]v[3]v[6]v[5] + v[1]v[2]v[4]v[6]v[5] + v[1]v[3]v[4]v[5]v[6] + v[2]v[3]v[4]v[5]v[6]$)
degree 6 : 1 ($v[5]$) ($v[6]$) ($v[2]$) ($v[4]$) ($v[3]$) ($v[1]$)

Group spectrum $1 + t + t^2 + t^3 + t^4 + t^5 + t^6$

KERNEL STRUCTURE

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$\pi 6 = [1]$$

supp $\pi 6$ = {1}

$$u6 = [1]$$

supp u6 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[3, 4, 5, 1, 6, 2]

B–BLOCKS,

[6, 3, 1, 5, 2, 4]

with invariant measure, [1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

$b_1 = \{4\}$
$b_2 = \{1\}$
$b_3 = \{5\}$
$b_4 = \{6\}$
$b_5 = \{3\}$
$b_6 = \{2\}$

dim(span of partition vectors), rank(N_0), rank(N): 6, 6, 6

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[1] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[1] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[1] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[1] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 25, Shape: $24 \oplus 1/0$

$$\text{CLB} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4, 5, 6}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 4, 5, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{5}{12} & \frac{-1}{3} & \frac{1}{6} & \frac{-1}{12} & \frac{-1}{12} \\ 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 \\ \frac{1}{24} & \frac{-5}{24} & \frac{-1}{12} & \frac{1}{6} & \frac{13}{24} & \frac{-11}{24} \\ \frac{-1}{24} & \frac{5}{24} & \frac{1}{12} & \frac{-1}{6} & \frac{-13}{24} & \frac{11}{24} \\ \frac{-3}{8} & \frac{-1}{8} & \frac{-1}{4} & \frac{1}{2} & \frac{1}{8} & \frac{1}{8} \\ \frac{11}{24} & \frac{-7}{24} & \frac{1}{12} & \frac{-1}{6} & \frac{-1}{24} & \frac{-1}{24} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4}, {1}, {5}, {6}, {3}, {2}}

1, "range", [1, 2, 3, 4, 5, 6], [[6, 5, 4, 3, 2, 1], [6, 5, 4, 3, 1, 2], [6, 5, 4, 2, 3, 1], [6, 5, 4, 2, 1, 3], [6, 5, 4, 1, 3, 2], [6, 5, 4, 1, 2, 3], [6, 5, 3, 4, 2, 1], [6, 5, 3, 4, 1, 2], [6, 5, 3, 2, 4, 1], [6, 5, 3, 2, 1, 4], [6, 5, 3, 1, 4, 2], [6, 5, 3, 1, 2, 4], [6, 5, 2, 4, 3, 1], [6, 5, 2, 4, 1, 3], [6, 5, 2, 3, 4, 1], [6, 5, 2, 3, 1, 4], [6, 5, 2, 1, 4, 3], [6, 5, 2, 1, 3, 4], [6, 5, 1, 4, 3, 2], [6, 5, 1, 4, 2, 3], [6, 5, 1, 3, 4, 2], [6, 5, 1, 3, 2, 4], [6, 5, 1, 2, 4, 3], [6, 5, 1, 2, 3, 4], [6, 4, 5, 3, 2, 1], [6, 4, 5, 3, 1, 2], [6, 4, 5, 2, 3, 1], [6, 4, 5, 2, 1, 3], [6, 4, 5, 1, 3, 2], [6, 4, 5, 1, 2, 3], [6, 4, 3, 5, 2, 1], [6, 4, 3, 5, 1, 2], [6, 4, 3, 2, 5, 1], [6, 4, 3, 2, 1, 5], [6, 4, 3, 1, 5, 2], [6, 4, 3, 1, 2, 5], [6, 4, 2, 5, 3, 1], [6, 4, 2, 5, 1, 3], [6, 4, 2, 3, 5, 1], [6, 4, 2, 3, 1, 5], [6, 4, 2, 1, 5, 3], [6, 4, 2, 1, 3, 5], [6, 4, 1, 5, 3, 2], [6, 4, 1, 5, 2, 3], [6, 4, 1, 3, 5, 2], [6, 4, 1, 3, 2, 5], [6, 4, 1, 2, 5, 3], [6, 4, 1, 2, 3, 5], [6, 3, 5, 4, 2, 1], [6, 3, 5, 4, 1, 2], [6, 3, 5, 2, 4, 1], [6, 3, 5, 2, 1, 4], [6, 3, 5, 1, 4, 2], [6, 3, 5, 1, 2, 4], [6, 3, 4, 5, 2, 1], [6, 3, 4, 5, 1, 2], [6, 3, 4, 2, 5, 1], [6, 3, 4, 2, 1, 5], [6, 3, 4, 1, 5, 2], [6, 3, 4, 1, 2, 5], [6, 3, 2, 5, 4, 1], [6, 3, 2, 5, 1, 4], [6, 3, 2, 4, 5, 1], [6, 3, 2, 4, 1, 5], [6, 3, 2, 1, 5, 4], [6, 3, 2, 1, 4, 5], [6, 3, 1, 5, 4, 2], [6, 3, 1, 5, 2, 4], [6, 3, 1, 4, 5, 2], [6, 3, 1, 4, 2, 5], [6, 3, 1, 2, 5, 4], [6, 3, 1, 2, 4, 5], [6, 2, 5, 4, 3, 1], [6, 2, 5, 4, 1, 3], [6, 2, 5, 3, 4, 1], [6, 2, 5, 3, 1, 4], [6, 2, 5, 1, 4, 3], [6, 2, 5, 1, 3, 4], [6, 2, 4, 5, 3, 1], [6, 2, 4, 5, 1, 3], [6, 2, 4, 3, 5, 1], [6, 2, 4, 3, 1, 5], [6, 2, 4, 1, 5, 3], [6, 2, 4, 1, 3, 5], [6, 2, 3, 5, 4, 1], [6, 2, 3, 5, 1, 4], [6, 2, 3, 4, 5, 1], [6, 2, 3, 4, 1, 5], [6, 2, 3, 1, 5, 4], [6, 2, 3, 1, 4, 5], [6, 2, 1, 5, 4, 3], [6, 2, 1, 5, 3, 4], [6, 2, 1, 4, 5, 3], [6, 2, 1, 4, 3, 5], [6, 2, 1, 3, 5, 4], [6, 2, 1, 3, 4, 5], [6, 1, 5, 4, 3, 2], [6, 1, 5, 4, 2, 3], [6, 1, 5, 3, 4, 2], [6, 1, 5, 3, 2, 4], [6, 1, 5, 2, 4, 3], [6, 1, 5, 2, 3, 4], [6, 1, 4, 5, 3, 2], [6, 1, 4, 5, 2, 3], [6, 1, 4, 3, 5, 2], [6, 1, 4, 3, 2, 5], [6, 1, 4, 2, 5, 3], [6, 1, 4, 2, 3, 5], [6, 1, 3, 5, 4, 2], [6, 1, 3, 5, 2, 4], [6, 1, 3, 4, 5, 2], [6, 1, 3, 4, 2, 5], [6, 1, 3, 2, 5, 4], [6, 1, 3, 2, 4, 5], [6, 1, 2, 5, 4, 3], [6, 1, 2, 5, 3, 4], [6, 1, 2, 4, 5, 3], [6, 1, 2, 4, 3, 5], [6, 1, 2, 3, 5, 4], [6, 1, 2, 3, 4, 5], [5, 6, 4, 3, 2, 1], [5, 6, 4, 3, 1, 2], [5, 6, 4, 2, 3, 1], [5, 6, 4, 2, 1, 3], [5, 6, 4, 1, 3, 2], [5, 6, 4, 1, 2, 3], [5, 6, 3, 4, 2, 1], [5, 6, 3, 4, 1, 2], [5, 6, 3, 2, 4, 1], [5, 6, 3, 2, 1, 4], [5, 6, 3, 1, 4, 2], [5, 6, 3, 1, 2, 4], [5, 6, 2, 4, 3, 1], [5, 6, 2, 4, 1, 3], [5, 6, 2, 3, 4, 1], [5, 6, 2, 3, 1, 4], [5, 6, 2, 1, 4, 3], [5, 6, 2, 1, 3, 4], [5, 6, 1, 4, 3, 2], [5, 6, 1, 4, 2, 3], [5, 6, 1, 3, 4, 2], [5, 6, 1, 3, 2, 4], [5, 6, 1, 2, 4, 3], [5, 6, 1, 2, 3, 4], [5, 4, 6, 3, 2, 1], [5, 4, 6, 3, 1, 2], [5, 4, 6, 2, 3, 1], [5, 4, 6, 2, 1, 3], [5, 4, 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1, 5, 6, 3], [2, 4, 1, 5, 3, 6], [2, 4, 1, 3, 6, 5], [2, 4, 1, 3, 5, 6], [2, 3, 6, 5, 4, 1], [2, 3, 6, 5, 1, 4], [2, 3, 6, 4, 5, 1], [2, 3, 6, 4, 1, 5], [2, 3, 6, 1, 5, 4], [2, 3, 6, 1, 4, 5], [2, 3, 5, 6, 4, 1], [2, 3, 5, 6, 1, 4], [2, 3, 5, 4, 6, 1], [2, 3, 5, 4, 1, 6], [2, 3, 5, 1, 6, 4], [2, 3, 5, 1, 4, 6], [2, 3, 4, 6, 5, 1], [2, 3, 4, 6, 1, 5], [2, 3, 4, 5, 6, 1], [2, 3, 4, 5, 1, 6], [2, 3, 4, 1, 6, 5], [2, 3, 4, 1, 5, 6], [2, 3, 1, 6, 5, 4], [2, 3, 1, 6, 4, 5], [2, 3, 1, 5, 6, 4], [2, 3, 1, 5, 4, 6], [2, 3, 1, 4, 6, 5], [2, 3, 1, 4, 5, 6], [2, 1, 6, 5, 4, 3], [2, 1, 6, 5, 3, 4], [2, 1, 6, 4, 5, 3], [2, 1, 6, 4, 3, 5], [2, 1, 6, 3, 5, 4], [2, 1, 6, 3, 4, 5], [2, 1, 5, 6, 4, 3], [2, 1, 5, 6, 3, 4], [2, 1, 5, 4, 6, 3], [2, 1, 5, 4, 3, 6], [2, 1, 5, 3, 6, 4], [2, 1, 5, 3, 4, 6], [2, 1, 4, 6, 5, 3], [2, 1, 4, 6, 3, 5], [2, 1, 4, 5, 6, 3], [2, 1, 4, 5, 3, 6], [2, 1, 4, 3, 6, 5], [2, 1, 4, 3, 5, 6], [2, 1, 3, 6, 5, 4], [2, 1, 3, 6, 4, 5], [2, 1, 3, 5, 6, 4], [2, 1, 3, 5, 4, 6], [2, 1, 3, 4, 6, 5], [2, 1, 3, 4, 5, 6], [1, 6, 5, 4, 3, 2], [1, 6, 5, 4, 2, 3], [1, 6, 5, 3, 4, 2], [1, 6, 5, 3, 2, 4], [1, 6, 5, 2, 4, 3], [1, 6, 5, 2, 3, 4], [1, 6, 4, 5, 3, 2], [1, 6, 4, 5, 2, 3], [1, 6, 4, 3, 5, 2], [1, 6, 4, 3, 2, 5], [1, 6, 4, 2, 5, 3], [1, 6, 4, 2, 3, 5], [1, 6, 3, 5, 4, 2], [1, 6, 3, 5, 2, 4], [1, 6, 3, 4, 5, 2], [1, 6, 3, 4, 2, 5], [1, 6, 3, 2, 5, 4], [1, 6, 3, 2, 4, 5], [1, 6, 2, 5, 4, 3], [1, 6, 2, 5, 3, 4], [1, 6, 2, 4, 5, 3], [1, 6, 2, 4, 3, 5], [1, 6, 2, 3, 5, 4], [1, 6, 2, 3, 4, 5], [1, 5, 6, 4, 3, 2], [1, 5, 6, 4, 2, 3], [1, 5, 6, 3, 4, 2], [1, 5, 6, 3, 2, 4], [1, 5, 6, 2, 4, 3], [1, 5, 6, 2, 3, 4], [1, 5, 4, 6, 3, 2], [1, 5, 4, 6, 2, 3], [1, 5, 4, 3, 6, 2], [1, 5, 4, 3, 2, 6], [1, 5, 4, 2, 6, 3], [1, 5, 4, 2, 3, 6], [1, 5, 3, 6, 4, 2], [1, 5, 3, 6, 2, 4], [1, 5, 3, 4, 6, 2], [1, 5, 3, 4, 2, 6], [1, 5, 3, 2, 6, 4], [1, 5, 3, 2, 4, 6], [1, 5, 2, 6, 4, 3], [1, 5, 2, 6, 3, 4], [1, 5, 2, 4, 6, 3], [1, 5, 2, 4, 3, 6], [1, 5, 2, 3, 6, 4], [1, 5, 2, 3, 4, 6], [1, 4, 6, 5, 3, 2], [1, 4, 6, 5, 2, 3], [1, 4, 6, 3, 5, 2], [1, 4, 6, 3, 2, 5], [1, 4, 6, 2, 5, 3], [1, 4, 6, 2, 3, 5], [1, 4, 5, 6, 3, 2], [1, 4, 5, 6, 2, 3], [1, 4, 5, 3, 6, 2], [1, 4, 5, 3, 2, 6], [1, 4, 5, 2, 6, 3], [1, 4, 5, 2, 3, 6], [1, 4, 3, 6, 5, 2], [1, 4, 3, 6, 2, 5], [1, 4, 3, 5, 6, 2], [1, 4, 3, 5, 2, 6], [1, 4, 3, 2, 6, 5], [1, 4, 3, 2, 5, 6], [1, 4, 2, 6, 5, 3], [1, 4, 2, 6, 3, 5], [1, 4, 2, 5, 6, 3], [1, 4, 2, 5, 3, 6], [1, 4, 2, 3, 6, 5], [1, 4, 2, 3, 5, 6], [1, 3, 6, 5, 4, 2], [1, 3, 6, 5, 2, 4], [1, 3, 6, 4, 5, 2], [1, 3, 6, 4, 2, 5], [1, 3, 6, 2, 5, 4], [1, 3, 6, 2, 4, 5], [1, 3, 5, 6, 4, 2], [1, 3, 5, 6, 2, 4], [1, 3, 5, 4, 6, 2], [1, 3, 5, 4, 2, 6], [1, 3, 5, 2, 6, 4], [1, 3, 5, 2, 4, 6], [1, 3, 4, 6, 5, 2], [1, 3, 4, 6, 2, 5], [1, 3, 4, 5, 6, 2], [1, 3, 4, 5, 2, 6], [1, 3, 4, 2, 6, 5], [1, 3, 4, 2, 5, 6], [1, 3, 2, 6, 5, 4], [1, 3, 2, 6, 4, 5], [1, 3, 2, 5, 6, 4], [1, 3, 2, 5, 4, 6], [1, 3, 2, 4, 6, 5], [1, 3, 2, 4, 5, 6], [1, 2, 6, 5, 4, 3], [1, 2, 6, 5, 3, 4], [1, 2, 6, 4, 5, 3], [1, 2, 6, 4, 3, 5], [1, 2, 6, 3, 5, 4], [1, 2, 6, 3, 4, 5], [1, 2, 5, 6, 4, 3], [1, 2, 5, 6, 3, 4], [1, 2, 5, 4, 6, 3], [1, 2, 5, 4, 3, 6], [1, 2, 5, 3, 6, 4], [1, 2, 5, 3, 4, 6], [1, 2, 4, 6, 5, 3], [1, 2, 4, 6, 3, 5], [1, 2, 4, 5, 6, 3], [1, 2, 4, 5, 3, 6], [1, 2, 4, 3, 6, 5], [1, 2, 4, 3, 5, 6], [1, 2, 3, 6, 5, 4], [1, 2, 3, 6, 4, 5], [1, 2, 3, 5, 6, 4], [1, 2, 3, 5, 4, 6], [1, 2, 3, 4, 6, 5], [1, 2, 3, 4, 5, 6]]

"group has", 720, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 6], [2, 5], [3, 4]]

$g_2 =$ [[1, 6, 2, 5], [3, 4]]

$g_3 =$ [[1, 6], [2, 5, 3, 4]]

$g_4 =$ [[1, 6, 3, 4, 2, 5]]

$g_5 =$ [[1, 6, 2, 5, 3, 4]]

linear dimension, 26

"Symmetric?", true

Is Z in Vec(K)? true
$\left(168h[2] + 480h[1] \quad -48h[2] - 120h[1] \quad -96h[2] - 240h[1] \quad 24h[2] \quad 24h[2] \quad -144h[2] - 120h[1] \quad 24h[2]\right.$

"Basis for Z(G)"

1, "coeff", 120

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 24

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. \\ 5. & -1. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3 + t^4 + t^5 + t^6$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 5t^4 + 7t^5 + 11t^6 + 14t^7 + 20t^8 + 26t^9 + 35t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6]}

KERNEL HIERARCHY

$\pi 6 = \begin{pmatrix} 1 \end{pmatrix}$

{1}

$u6 = \begin{pmatrix} 1 \end{pmatrix}$

{1}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 5 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u5 = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

picheck $\begin{pmatrix} 5 & 5 & 5 & 5 & 5 & 5 \end{pmatrix}$

$\pi 4 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

$u4 = \begin{pmatrix} \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} \end{pmatrix}$

picheck $\begin{pmatrix} 20 & 20 & 20 & 20 & 20 & 20 \end{pmatrix}$

$\pi 3 = \begin{pmatrix} 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \end{pmatrix}$

$u3 = \begin{pmatrix} \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} \end{pmatrix}$

picheck $\begin{pmatrix} 60 & 60 & 60 & 60 & 60 & 60 \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 & 24 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} & \frac{1}{54} \end{pmatrix}$

picheck $\begin{pmatrix} 120 & 120 & 120 & 120 & 120 & 120 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 120 & 120 & 120 & 120 & 120 & 120 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} & \frac{5}{324} \end{pmatrix}$

picheck $\begin{pmatrix} 120 & 120 & 120 & 120 & 120 & 120 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks

$$\mathrm{T} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathrm{NM} = \begin{pmatrix} 5 & 4 & 4 & 4 & 4 & 4 \\ 4 & 5 & 4 & 4 & 4 & 4 \\ 4 & 4 & 5 & 4 & 4 & 4 \\ 4 & 4 & 4 & 5 & 4 & 4 \\ 4 & 4 & 4 & 4 & 5 & 4 \\ 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0]$

$\ker N_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} t+s & t+s & t+s & t+s & t+s & t+s \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & 0 & t \\ 0 & 0 & 0 & t & s \\ s & t & 0 & 0 & 0 \\ t & s & 0 & 0 & 0 \\ -t & -t & -t & -t+s & -t \\ -s & -s & -s+t & -s & -s \end{pmatrix}$$ RB checks

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & s & t & 0 & 0 & 0 \\ 0 & 0 & s & 0 & 0 & t \\ 0 & 0 & 0 & s & t & 0 \\ 0 & 0 & 0 & t & s & 0 \\ t & 0 & 0 & 0 & 0 & s \\ s & t & 0 & 0 & 0 & 0 \end{pmatrix}$$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 6

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\ T} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \mathrm{Skew\ Omega} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 1T + 24\Omega$$

$\Omega \begin{pmatrix} \frac{13}{6} & \frac{5}{3} & \frac{5}{6} & \frac{1}{3} & \frac{1}{6} & \frac{5}{3} & \frac{7}{6} & \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{7}{6} & \frac{2}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$T \begin{pmatrix} 4 & 2 & 1 & 0 & 0 & 3 & 2 & 1 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

"IS NM in Vec(K)?", true

$\mathrm{NM} \begin{pmatrix} 56 & 42 & 21 & 8 & 4 & 43 & 30 & 13 & 4 & 4 & 30 & 17 & 5 & 4 & 4 & 17 & 4 & 4 & 5 & 4 & 4 & 4 & 4 & 4 & 4 & 5 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN (56 42 21 8 4 43 30 13 4 4 30 17 5 4 4 17 4 4 5 4 4 4 4 4 4 5)

$$\boldsymbol{\tau} = 6/1 \text{ , rank} = 6 \text{ , ratio} = 1/1 \text{ , } n^2 / r = 6/1$$

$$\boldsymbol{\tau}' = 30/1 \text{ , } r' = 5/6 \text{ , } \boldsymbol{\tau} / n^2 = 1/6$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\text{max r} = 6/1 \text{ , } r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 6\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 720

KERNEL HAS LINEAR DIMENSION 26
out of total no. of elements equal to 720

dim span idems 1 vs no. of idems 1

"PT1" = {{4}, {1}, {5}, {6}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5, 6}

$$M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1.]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 1.200000000, 1.200000000, 1.200000000, 1.200000000, 1.200000000]

NullSpace M_C

{[0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0]}

NullSpace N_C

{[1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 6.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 0, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0], [−1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1]}

NullSpace N_0

{}

Eigenvalues M

[5., −1., −1., −1., −1., −1.]

Eigenvalues N

[5., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{4, 6}

R: [2, 3, 5, 6, 1, 2]
B: [3, 4, 6, 5, 4, 1]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & \frac{5}{6} & 0 & 0 \\ \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 5

Level 2 det = $\frac{1}{16384}(-1+s)(-2912-2432s-1232s^2-265s^3-104s^4-13s^5+16s^6+21s^7+8s^8+s^9)$

RANK of R is 5

R ranking is 4, "vs", 5

RBAR ranking 3, "vs", 4

RANK of B is 5

B ranking is 2, "vs", 5

BBAR ranking 2, "vs", 5

"R CYCLES", 1 + v[1] v[2] v[3] v[5]
"B CYCLES", (1 + v[4] v[5]) (1 + v[1] v[3] v[6])

Eigenvalues

R: [0., 0., −1., 1., 1. I, −1. I]

B: [0., −1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., 1.]

NullSpace of R

{[0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[−1, 0, 0, 0, 0, 1]}

NullSpace of B^*

{[0, −1, 0, 0, 1, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 2

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3   "Trace mark", 3, "Rank mark", 3, "for kernel rank", 2

degree 1: $\frac{1}{6}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] \right)$
degree 2: $\frac{1}{9}\left(v[2] + v[4] + v[5] \right)\left(v[1] + v[3] + v[6] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"RG1" = {5, 6}

"RG2" = {4, 6}

"RG3" = {2, 6}

"RG4" = {3, 5}

"RG5" = {3, 4}

"RG6" = {2, 3}

"RG7" = {1, 5}

"RG8" = {1, 4}

"RG9" = {1, 2}

$$\pi 2 = [1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1]$$

supp π2 = {1, 3, 4, 6, 9, 10, 11, 14, 15}

$$u2 = [1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1]$$

supp u2 = {1, 3, 4, 6, 9, 10, 11, 14, 15}

Action of R on ranges, [[9], [3], [6], [7], [1], [4], [9], [3], [6]]
Action of B on ranges, [[8], [7], [8], [2], [1], [2], [5], [4], [5]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 1]

B–BLOCKS,

[1, 2]

with invariant measure, [1, 1]

N by blocks, N – check: true

b_1 = {1, 3, 6}
b_2 = {2, 4, 5}

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 21, Shape: 8 ⊕ 13/11

$$\text{CLB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 5}}, true

Ω_B in Vec(K)? , {{4, 5}, {1, 3, 6}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{5}{12} & \frac{-1}{3} & \frac{1}{6} & \frac{-1}{12} & \frac{-1}{12} \\ 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 \\ \frac{1}{24} & \frac{-5}{24} & \frac{-1}{12} & \frac{1}{6} & \frac{13}{24} & \frac{-11}{24} \\ \frac{-1}{24} & \frac{5}{24} & \frac{1}{12} & \frac{-1}{6} & \frac{-13}{24} & \frac{11}{24} \\ \frac{3}{8} & \frac{1}{8} & \frac{1}{4} & \frac{-1}{2} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-11}{24} & \frac{7}{24} & \frac{-1}{12} & \frac{1}{6} & \frac{1}{24} & \frac{1}{24} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{4} & \frac{1}{4} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6}, {2, 4, 5}}

1, "range", [5, 6], [[6, 5, 6, 5, 5, 6], [5, 6, 5, 6, 6, 5]]

2, "range", [4, 6], [[6, 4, 6, 4, 4, 6], [4, 6, 4, 6, 6, 4]]

3, "range", [2, 6], [[6, 2, 6, 2, 2, 6], [2, 6, 2, 6, 6, 2]]

4, "range", [3, 5], [[5, 3, 5, 3, 3, 5], [3, 5, 3, 5, 5, 3]]

5, "range", [3, 4], [[4, 3, 4, 3, 3, 4], [3, 4, 3, 4, 4, 3]]

6, "range", [2, 3], [[3, 2, 3, 2, 2, 3], [2, 3, 2, 3, 3, 2]]

7, "range", [1, 5], [[5, 1, 5, 1, 1, 5], [1, 5, 1, 5, 5, 1]]

8, "range", [1, 4], [[4, 1, 4, 1, 1, 4], [1, 4, 1, 4, 4, 1]]

9, "range", [1, 2], [[2, 1, 2, 1, 1, 2], [1, 2, 1, 2, 2, 1]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [2, 3]}, {7, [2, 4]}, {8, [2, 5]}, {[2, 6], 9}, {10, [3, 4]}, {11, [3, 5]}, {12, [3, 6]}, {13, [4, 5]}, {14, [4, 6]}, {15, [5, 6]}

KERNEL HIERARCHY

$\pi 2 = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$

{1, 3, 4, 6, 9, 10, 11, 14, 15}

$u2 = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$

{1, 3, 4, 6, 9, 10, 11, 14, 15}

picheck $\begin{pmatrix} 3 & 3 & 3 & 3 & 3 & 3 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 3 & 3 & 3 & 3 & 3 & 3 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 3 & 3 & 3 & 3 & 3 & 3 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_4 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_5 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_6 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_8 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem–checks} \quad \text{N0–checks}$$

$$P_9 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem–checks} \quad \text{N0–checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \text{idem–checks}$$

$$T = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad NM = \begin{pmatrix} 3 & 0 & 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 3 & 3 & 0 \\ 3 & 0 & 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 3 & 3 & 0 \\ 0 & 3 & 0 & 3 & 3 & 0 \\ 3 & 0 & 3 & 0 & 0 & 3 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 1, 0, -1, 0, 0]$

$$\ker N_c = \begin{pmatrix} 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -s & -t & t & s \\ s & 0 & -s & 0 & 0 & 0 \\ t & 0 & -t & 0 & 0 & 0 \\ 0 & -s & -t & 0 & s & t \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 0 & 0 & 0 \end{pmatrix}$

$$\text{ker } M_0 = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} s-t \\ -s+t \\ s-t \\ -s+t \\ -s+t \\ s-t \end{pmatrix} \quad \text{RB checks}$$

$$\text{ker } M_C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} s & t \\ t & s \\ s & t \\ t & s \\ t & s \\ s & t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 3 & 3 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 3 & 1 & 0 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 1 & 0 & 1 & 3 & 0 & 1 \\ 1 & 0 & 1 & 0 & 3 & 1 \\ 0 & 1 & 0 & 1 & 1 & 3 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 9T + 0\Omega$$

$\Omega\left(\frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6} \ \frac{1}{6}\right)$

$T\left(\frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM}\,(3 \ 0 \ 3 \ 0 \ 3 \ 0 \ 0 \ 3 \ 0 \ 3)$

"IS MN in Vec(K)?", true

$\mathrm{MN}\,(3 \ 0 \ 3 \ 0 \ 3 \ 0 \ 0 \ 3 \ 0 \ 3)$

$$\boldsymbol{\tau} = 18/1 \text{ , rank } = 2 \text{ , ratio } = 9/1 \text{ , } n^2 / r = 18/1$$

$$\boldsymbol{\tau}' = 18/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 12/1$$

$$\text{max r} = 6/1 \text{ , } r\text{-check is positive? } 2/3$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 18\Omega$$

There are, 1, partitions and, 9, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 10
out of total no. of elements equal to 18

dim span idems 5 vs no. of idems 9

"PT1" = {{1, 3, 6}, {2, 4, 5}}

"RG1" = {5, 6}

"RG2" = {4, 6}

"RG3" = {2, 6}

"RG4" = {3, 5}

"RG5" = {3, 4}

"RG6" = {2, 3}

"RG7" = {1, 5}

"RG8" = {1, 4}

"RG9" = {1, 2}

$$M_C = \begin{pmatrix} 2 & 0 & -1 & 0 & 0 & -1 \\ 0 & 2 & 0 & -1 & -1 & 0 \\ -1 & 0 & 2 & 0 & 0 & -1 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 2 & 0 \\ -1 & 0 & -1 & 0 & 0 & 2 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & 0 & \frac{-1}{2} & 0 & 0 & \frac{-1}{2} \\ 0 & 1 & 0 & \frac{-1}{2} & \frac{-1}{2} & 0 \\ \frac{-1}{2} & 0 & 1 & 0 & 0 & \frac{-1}{2} \\ 0 & \frac{-1}{2} & 0 & 1 & \frac{-1}{2} & 0 \\ 0 & \frac{-1}{2} & 0 & \frac{-1}{2} & 1 & 0 \\ \frac{-1}{2} & 0 & \frac{-1}{2} & 0 & 0 & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 \\ \frac{-1}{5} & 1 & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} \\ 1 & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 \\ \frac{-1}{5} & 1 & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} \\ \frac{-1}{5} & 1 & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} \\ 1 & \frac{-1}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 3., 3., 3., 3.]

Eigenvalues N_C

[0., 0., 0., 0., 3., 2.]

Eigenvalues M_C–scaled

[0., 0., 1.500000000, 1.500000000, 1.500000000, 1.500000000]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 3.600000000, 2.400000000]

NullSpace M_c

{[1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0]}

NullSpace N_c

{[0, 0, 1, 0, 0, −1], [0, 0, 0, 1, −1, 0], [0, 1, 0, 0, −1, 0], [1, 0, 0, 0, 0, −1]}

Eigenvalues M_0

[0., 6., 3., 3., 3., 3.]

Eigenvalues N_0

[3., 3., 0., 0., 0., 0.]

NullSpace M_0

{[1, −1, 1, −1, −1, 1]}

NullSpace N_0

{[0, 1, 0, 0, −1, 0], [0, 0, 0, 1, −1, 0], [0, 0, 1, 0, 0, −1], [1, 0, 0, 0, 0, −1]}

Eigenvalues M

[0., 0., 0., 0., 3., −3.]

Eigenvalues N

[0., 0., 0., 0., 3., −3.]

NullSpace M

{[1, 0, 0, 0, 0, −1], [0, 1, 0, 0, −1, 0], [0, 0, 1, 0, 0, −1], [0, 0, 0, 1, −1, 0]}

NullSpace N

{[0, 0, 1, 0, 0, −1], [0, 0, 0, −1, 1, 0], [1, 0, 0, 0, 0, −1], [0, 1, 0, −1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

===

20, [1, −1, 1, −1, −1, 1]

==

{3, 4, 5, 6}

R: [2, 3, 6, 6, 4, 2]

B: [3, 4, 5, 5, 1, 1]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 5

Level 2 det = $\frac{5}{512}(-91 - 92s - 42s^2 - 4s^3 + 5s^4)(-1 + s)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 1, "vs", 3

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 3

"R CYCLES", 1 + v[2] v[3] v[6]
"B CYCLES", 1 + v[1] v[3] v[5]

Eigenvalues

R: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[1, 0, 0, 0, 0, −1], [0, 0, −1, 1, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 1], [0, 0, −1, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 2 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{2} & 1 & 1 & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 & 1 & \frac{1}{2} & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & \frac{1}{2} & 1 & 1 & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & 1 & 1 & \frac{1}{2} & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 5

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 1 "Trace mark", 1, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}\,(v[1] + v[2] + v[3] + v[4] + v[5] + v[6])$
degree 2: $\frac{1}{12}\,(v[1]v[3] + v[1]v[4] + 2v[1]v[5] + v[2]v[3] + v[2]v[4] + 2v[2]v[6] + v[3]v[5] + v[3]v[6] + v[4]v[5] + v[4]v[6])$
degree 3 : $\frac{1}{4}\,(v[3] + v[4])\,(v[1]v[5] + v[2]v[6])$

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {3, 4}}

"PT2" = {{1, 2}, {5, 6}, {3, 4}}

"RG1" = {2, 4, 6}

"RG2" = {2, 3, 6}

"RG3" = {1, 4, 5}

"RG4" = {1, 3, 5}

$$\pi 3 = [0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0]$$

supp $\pi 3$ = {6, 8, 13, 15}

$$u3 = [1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 0, 0, 0, 1, 1]$$

supp u3 = {1, 2, 6, 7, 8, 9, 12, 13, 14, 15, 19, 20}

Action of R on ranges, [[2], [2], [1], [1]]
Action of B on ranges, [[3], [3], [4], [4]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 5, 5, 2, 4]

B–BLOCKS,

[3, 3, 5, 5, 1]

with invariant measure, [1, 1, 1, 1, 2]

N by blocks, N – check: true

$b_1 = \{1, 2\}$
$b_2 = \{1, 6\}$
$b_3 = \{5, 6\}$
$b_4 = \{2, 5\}$
$b_5 = \{3, 4\}$

dim(span of partition vectors), rank(N_0), rank(N): 4, 4, 4

LIE STRUCTURE

Dimension of Lie algebra: 18, Shape: 8 ⊕ 10/8

$$\text{CLB} = \begin{pmatrix} 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{5}{12} & \frac{-1}{3} & \frac{1}{6} & \frac{-1}{12} & \frac{-1}{12} \\ 0 & 0 & \frac{1}{2} & \frac{-1}{2} & 0 & 0 \\ \frac{-1}{24} & \frac{5}{24} & \frac{1}{12} & \frac{-1}{6} & \frac{-13}{24} & \frac{11}{24} \\ \frac{-1}{24} & \frac{5}{24} & \frac{1}{12} & \frac{-1}{6} & \frac{-13}{24} & \frac{11}{24} \\ \frac{-3}{8} & \frac{-1}{8} & \frac{-1}{4} & \frac{1}{2} & \frac{1}{8} & \frac{1}{8} \\ \frac{-11}{24} & \frac{7}{24} & \frac{-1}{12} & \frac{1}{6} & \frac{1}{24} & \frac{1}{24} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {3, 4}}

1, "range", [2, 4, 6], [[6, 4, 2, 2, 4, 6], [6, 2, 4, 4, 2, 6], [4, 6, 2, 2, 6, 4], [4, 2, 6, 6, 2, 4], [2, 6, 4, 4, 6, 2], [2, 4, 6, 6, 4, 2]]

2, "range", [2, 3, 6], [[6, 3, 2, 2, 3, 6], [6, 2, 3, 3, 2, 6], [3, 6, 2, 2, 6, 3], [3, 2, 6, 6, 2, 3], [2, 6, 3, 3, 6, 2], [2, 3, 6, 6, 3, 2]]

3, "range", [1, 4, 5], [[5, 4, 1, 1, 4, 5], [5, 1, 4, 4, 1, 5], [4, 5, 1, 1, 5, 4], [4, 1, 5, 5, 1, 4], [1, 5, 4, 4, 5, 1], [1, 4, 5, 5, 4, 1]]

4, "range", [1, 3, 5], [[5, 3, 1, 1, 3, 5], [5, 1, 3, 3, 1, 5], [3, 5, 1, 1, 5, 3], [3, 1, 5, 5, 1, 3], [1, 5, 3, 3, 5, 1], [1, 3, 5, 5, 3, 1]]

2, "partition", {{1, 2}, {5, 6}, {3, 4}}

1, "range", [2, 4, 6], [[6, 6, 4, 4, 2, 2], [6, 6, 2, 2, 4, 4], [4, 4, 6, 6, 2, 2], [4, 4, 2, 2, 6, 6], [2, 2, 6, 6, 4, 4], [2, 2, 4, 4, 6, 6]]

2, "range", [2, 3, 6], [[6, 6, 3, 3, 2, 2], [6, 6, 2, 2, 3, 3], [3, 3, 6, 6, 2, 2], [3, 3, 2, 2, 6, 6], [2, 2, 6, 6, 3, 3], [2, 2, 3, 3, 6, 6]]

3, "range", [1, 4, 5], [[5, 5, 4, 4, 1, 1], [5, 5, 1, 1, 4, 4], [4, 4, 5, 5, 1, 1], [4, 4, 1, 1, 5, 5], [1, 1, 5, 5, 4, 4], [1, 1, 4, 4, 5, 5]]

4, "range", [1, 3, 5], [[5, 5, 3, 3, 1, 1], [5, 5, 1, 1, 3, 3], [3, 3, 5, 5, 1, 1], [3, 3, 1, 1, 5, 5], [1, 1, 5, 5, 3, 3], [1, 1, 3, 3, 5, 5]]

"group has", 6, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

$g_3 =$ [[1, 3, 2]]

$g_4 =$ [[2, 3]]

$g_5 =$ [[1, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 2h[1] - h[2] & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{6, 8, 13, 15}

$u3 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 2 & 1 & 2 & 1 & 0 & 0 & 1 & 2 & 1 & 2 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

{1, 2, 6, 7, 8, 9, 12, 13, 14, 15, 19, 20}

picheck $\begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 0 & 1 & 1 & 2 & 0 & 1 & 1 & 0 & 2 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 4 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 4 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{4}{9} & \frac{4}{9} & \frac{4}{9} & \frac{4}{9} & \frac{4}{9} & \frac{4}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 4 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem−checks

$$PP_2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$ idem−checks

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 3 & 2 & 2 & 2 & 3 \\ 3 & 4 & 2 & 2 & 3 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 3 & 2 & 2 & 4 & 3 \\ 3 & 2 & 2 & 2 & 3 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, 0, 0, -1, 1]$

$\ker N_C = \begin{pmatrix} 0 & 0 & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & -1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -t+s & t-s & 0 & 0 \end{pmatrix}$ RB checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 1 \end{pmatrix}$

$\ker M_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ -1 & 0 & -1 \\ 0 & -1 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & t \\ 0 & 0 & s+t \\ -t & -s & -t-s \\ -t & -s & -t-s \\ t & 0 & s \\ t & s & 0 \end{pmatrix}$ RB checks

$\ker M_C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & s & t & 0 \\ 0 & 0 & s+t & 0 \\ -t & t & 0 & s+t \\ -t & t & 0 & s+t \\ t & 0 & s & 0 \\ t & s & 0 & 0 \end{pmatrix}$ RB checks

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 2 & 2 & 2 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \; RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 3

$$CNM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 0 & 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 0 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 4T + 12\Omega$$

$$\Omega \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$T \begin{pmatrix} 0 & 0 & 0 & \frac{3}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

"IS NM in Vec(K)?", true

$$NM \begin{pmatrix} 2 & 2 & 2 & 5 & 4 & 2 & 2 & 1 & 2 & 2 & 4 & 3 & 3 & 2 & 2 & 2 & 3 & 4 \end{pmatrix}$$

"IS MN in Vec(K)?", true

$$MN \begin{pmatrix} 2 & 2 & 2 & 5 & 4 & 2 & 2 & 1 & 2 & 2 & 4 & 3 & 3 & 2 & 2 & 2 & 3 & 4 \end{pmatrix}$$

$\boldsymbol{\tau} = 12/1$, rank = 3 , ratio = 4/1 , $n^2 / r = 12/1$

$\boldsymbol{\tau}' = 24/1$, $r' = 2/3$, $\boldsymbol{\tau} / n^2 = 1/3$

$p^2 = 1/6$, min $\boldsymbol{\tau} = 6/1$, $\boldsymbol{\tau}$–check is positive? 6/1

max r = 6/1 , r–check is positive? 1/2

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 18
out of total no. of elements equal to 48

dim span idems 5 vs no. of idems 8

"PT1" = {{1, 6}, {2, 5}, {3, 4}}

"PT2" = {{1, 2}, {5, 6}, {3, 4}}

"RG1" = {2, 4, 6}

"RG2" = {2, 3, 6}

"RG3" = {1, 4, 5}

"RG4" = {1, 3, 5}

$$M_C = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 0 & -1 & 1 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{1}{3} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{3} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{1}{3} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{3} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 0 & -1 & 1 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{2}{5} \\ \frac{2}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{2}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 4., 2.]

Eigenvalues N_C

[2., 0., 0., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 4., 2.]

Eigenvalues N_C–scaled

[2.400000000, 0., 0., 1.200000000, 1.200000000, 1.200000000]

NullSpace M_C

{[−1, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 1], [0, 0, 1, 1, 0, 0], [1, 1, 0, 0, 0, 0]}

NullSpace N_C

{[1, −1, 0, 0, 1, −1], [0, 0, 1, −1, 0, 0]}

Eigenvalues M_0

[0., 0., 0., 2., 4., 6.]

Eigenvalues N_0

[2., 1., 2., 1., 0., 0.]

NullSpace M_0

{[0, 0, 1, 1, −1, −1], [1, 0, 0, 0, −1, 0], [0, 1, 0, 0, 0, −1]}

NullSpace N_0

{[0, 0, 1, −1, 0, 0], [−1, 1, 0, 0, −1, 1]}

Eigenvalues M

[0., 4., 2., −2., −2., −2.]

Eigenvalues N

[4., −2., −1., −1., 0., 0.]

NullSpace M

{[0, 0, 1, −1, 0, 0]}

NullSpace N

{[1, −1, 0, 0, 1, −1], [0, 0, 1, −1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 2 & 2 & 1 \\ 1 & 0 & 2 & 2 & 1 & 2 \\ 2 & 2 & 0 & 0 & 2 & 2 \\ 2 & 2 & 0 & 0 & 2 & 2 \\ 2 & 1 & 2 & 2 & 0 & 1 \\ 1 & 2 & 2 & 2 & 1 & 0 \end{pmatrix}$$

===

{2, 3, 4, 5, 6}

R: [2, 4, 6, 6, 4, 2]
B: [3, 3, 5, 5, 1, 1]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 4

Level 2 det = $\frac{-3}{512}(-1+s)(91+29s-11s^2+3s^3)(1+s)$

RANK of R is 3

R ranking is 1, "vs", 3

RBAR ranking 1, "vs", 3

RANK of B is 3

B ranking is 1, "vs", 3

BBAR ranking 1, "vs", 3

"R CYCLES", 1 + v[2] v[4] v[6]

"B CYCLES", 1 + v[1] v[3] v[5]

Eigenvalues

R: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

B: [0., 0., 0., 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I]

NullSpace of R

{[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]}

NullSpace of R^*

{[0, −1, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 1], [0, 0, −1, 1, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 1], [−1, 1, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 \\ 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 \\ 2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{1}{2} & 1 & 1 & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 & 1 & \frac{1}{2} & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & \frac{1}{2} & 1 & 1 & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & 1 & 1 & \frac{1}{2} & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 6

"RANK of the KERNEL is ", 3

"IdemSolvability Check", 1 "Trace mark", 1, "Rank mark", 3, "for kernel rank", 3

degree 1: $\frac{1}{6}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6]$)
degree 2: $\frac{1}{6}$ ($v[1]v[3] + v[1]v[5] + v[2]v[4] + v[2]v[6] + v[3]v[5] + v[4]v[6]$)
degree 3 : $\frac{1}{2}$ ($v[1]v[3]v[5] + v[2]v[4]v[6]$)

Group spectrum $1 + t + t^2 + t^3$

KERNEL STRUCTURE

"PT1" = {{1, 6}, {2, 5}, {3, 4}}

"PT2" = {{1, 2}, {5, 6}, {3, 4}}

"RG1" = {2, 4, 6}

"RG2" = {1, 3, 5}

$$\pi 3 = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]$$

supp $\pi 3$ = {6, 15}

$$u3 = [1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 0, 0, 0, 1, 1]$$

supp u3 = {1, 2, 6, 7, 8, 9, 12, 13, 14, 15, 19, 20}

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [1, 1]

BPARTS [2, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 5, 5, 2, 4]

B–BLOCKS,

[3, 3, 5, 5, 1]

with invariant measure, [1, 1, 1, 1, 2]

N by blocks, N – check: true

b_1 = {1, 2}
b_2 = {1, 6}
b_3 = {5, 6}
b_4 = {2, 5}
b_5 = {3, 4}

dim(span of partition vectors), rank(N_0), rank(N): 4, 4, 4

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & h[1] & 0 & h[1] & 0 \\ 0 & h[2] & 0 & h[1] & 0 & h[1] \\ h[1] & 0 & h[2] & 0 & h[1] & 0 \\ 0 & h[1] & 0 & h[2] & 0 & h[1] \\ h[1] & 0 & h[1] & 0 & h[2] & 0 \\ 0 & h[1] & 0 & h[1] & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 13, Shape: 8 ⊕ 5/4

$$\mathrm{CLB} = \begin{pmatrix} -1 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 4, 6}}, true

Ω_B in Vec(K)? , {{1, 3, 5}}, true

$$V = \begin{pmatrix} \frac{-1}{12} & \frac{5}{12} & \frac{-1}{3} & \frac{1}{6} & \frac{-1}{12} & \frac{-1}{12} \\ 0 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{-1}{24} & \frac{5}{24} & \frac{1}{12} & \frac{-1}{6} & \frac{-13}{24} & \frac{11}{24} \\ \frac{-1}{24} & \frac{5}{24} & \frac{1}{12} & \frac{-1}{6} & \frac{-13}{24} & \frac{11}{24} \\ \frac{-3}{8} & \frac{-1}{8} & \frac{-1}{4} & \frac{1}{2} & \frac{1}{8} & \frac{1}{8} \\ \frac{-11}{24} & \frac{7}{24} & \frac{-1}{12} & \frac{1}{6} & \frac{1}{24} & \frac{1}{24} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 6}, {2, 5}, {3, 4}}

1, "range", [2, 4, 6], [[6, 4, 2, 2, 4, 6], [6, 2, 4, 4, 2, 6], [4, 6, 2, 2, 6, 4], [4, 2, 6, 6, 2, 4], [2, 6, 4, 4, 6, 2], [2, 4, 6, 6, 4, 2]]

2, "range", [1, 3, 5], [[5, 3, 1, 1, 3, 5], [5, 1, 3, 3, 1, 5], [3, 5, 1, 1, 5, 3], [3, 1, 5, 5, 1, 3], [1, 5, 3, 3, 5, 1], [1, 3, 5, 5, 3, 1]]

2, "partition", {{1, 2}, {5, 6}, {3, 4}}

1, "range", [2, 4, 6], [[6, 6, 4, 4, 2, 2], [6, 6, 2, 2, 4, 4], [4, 4, 6, 6, 2, 2], [4, 4, 2, 2, 6, 6], [2, 2, 6, 6, 4, 4], [2, 2, 4, 4, 6, 6]]

2, "range", [1, 3, 5], [[5, 5, 3, 3, 1, 1], [5, 5, 1, 1, 3, 3], [3, 3, 5, 5, 1, 1], [3, 3, 1, 1, 5, 5], [1, 1, 5, 5, 3, 3], [1, 1, 3, 3, 5, 5]]

"group has", 6, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

$g_3 =$ [[1, 3, 2]]

$g_4 =$ [[2, 3]]

$g_5 =$ [[1, 3]]

linear dimension, 5

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & 2h[1] - h[2] & 0 & h[2] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 2

Z[1] = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. & 1. \\ 2. & -1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + 14t^{10}$

n-choose-rank

{1, [1, 2, 3]}, {2, [1, 2, 4]}, {3, [1, 2, 5]}, {4, [1, 2, 6]}, {5, [1, 3, 4]}, {6, [1, 3, 5]}, {7, [1, 3, 6]}, {8, [1, 4, 5]}, {9, [1, 4, 6]}, {10, [1, 5, 6]}, {11, [2, 3, 4]}, {12, [2, 3, 5]}, {13, [2, 3, 6]}, {14, [2, 4, 5]}, {[2, 4, 6], 15}, {16, [2, 5, 6]}, {17, [3, 4, 5]}, {18, [3, 4, 6]}, {19, [3, 5, 6]}, {20, [4, 5, 6]}

KERNEL HIERARCHY

$\pi 3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

{6, 15}

$u3 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 2 & 1 & 2 & 1 & 0 & 0 & 1 & 2 & 1 & 2 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

{1, 2, 6, 7, 8, 9, 12, 13, 14, 15, 19, 20}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$

$u2 = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{4}{9} & \frac{4}{9} & \frac{4}{9} & \frac{4}{9} & \frac{4}{9} & \frac{4}{9} \end{pmatrix}$

picheck $\begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$\mathrm{PP}_2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 3 & 2 & 2 & 2 & 3 \\ 3 & 4 & 2 & 2 & 3 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 3 & 2 & 2 & 4 & 3 \\ 3 & 2 & 2 & 2 & 3 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, -1, 1, -1, 1]$

$$\ker N_C = \begin{pmatrix} -1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & t & s & 0 \\ -s & t & -s & 0 \\ s & -t & 0 & -t \\ s & -t & 0 & -t \\ -s & 0 & -s & t \\ 0 & 0 & s & t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} s+t & -s & -s & s & s \\ t & s & 0 & 0 & 0 \\ 0 & 0 & s & t & 0 \\ 0 & 0 & s & t & 0 \\ 0 & s & 0 & 0 & t \\ s & -s & -s & s & s+t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 2 & 0 & 0 & 2 & 2 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 3, 3, "vs", 3

$$\text{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 4T + 12\Omega$$

$\Omega \left(0 \;\; 0 \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6} \;\; \frac{1}{6}\right)$

$T \left(\frac{1}{4} \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{2} \;\; \frac{1}{4} \;\; \frac{1}{4} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{1}{4} \;\; \frac{1}{2}\right)$

"IS NM in Vec(K)?", true

$\text{NM} \left(1 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 4 \;\; 3 \;\; 3 \;\; 2 \;\; 2 \;\; 2 \;\; 3 \;\; 4\right)$

"IS MN in Vec(K)?", true

$\text{MN} \left(1 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 2 \;\; 4 \;\; 3 \;\; 3 \;\; 2 \;\; 2 \;\; 2 \;\; 3 \;\; 4\right)$

$$\boldsymbol{\tau} = 12/1 \text{ , rank} = 3 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 12/1$$

$$\boldsymbol{\tau}' = 24/1 \text{ , } r' = 2/3 \text{ , } \boldsymbol{\tau} / n^2 = 1/3$$

$$p^2 = 1/6 \text{ , min } \boldsymbol{\tau} = 6/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 6/1$$

$$\text{max r} = 6/1 \text{ , } r\text{–check is positive? } 1/2$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 12\Omega$$

There are, 2, partitions and, 2, ranges, with a group size of, 6

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 24

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 6}, {2, 5}, {3, 4}}

"PT2" = {{1, 2}, {5, 6}, {3, 4}}

"RG1" = {2, 4, 6}

"RG2" = {1, 3, 5}

$$M_C = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{5}{6} & \frac{1}{3} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{3} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{5}{6} & \frac{-1}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{1}{3} & \frac{-1}{6} & \frac{-1}{6} & \frac{5}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{-1}{6} & \frac{-1}{6} & \frac{-1}{6} & \frac{1}{3} & \frac{5}{6} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix} \quad N_C\text{–scaled} = \begin{pmatrix} 1 & \frac{2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{2}{5} \\ \frac{2}{5} & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{2}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & 1 & 1 & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-1}{5} & 1 & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{2}{5} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 6.]

Eigenvalues N_C

[2., 0., 0., 1., 1., 1.]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 6.]

Eigenvalues N_C−scaled

[2.400000000, 0., 0., 1.200000000, 1.200000000, 1.200000000]

NullSpace M_C

{[1, 1, 0, 0, 0, 0], [0, 1, 1, 0, 0, 0], [0, −1, 0, 1, 0, 0], [0, 1, 0, 0, 1, 0], [0, −1, 0, 0, 0, 1]}

NullSpace N_C

{[−1, 1, 0, 0, −1, 1], [0, 0, −1, 1, 0, 0]}

Eigenvalues M_0

[6., 6., 0., 0., 0., 0.]

Eigenvalues N_0

[2., 1., 2., 1., 0., 0.]

NullSpace M_0

{[0, −1, 0, 1, 0, 0], [−1, 0, 0, 0, 1, 0], [−1, 0, 1, 0, 0, 0], [0, −1, 0, 0, 0, 1]}

NullSpace N_0

{[1, −1, 0, 0, 1, −1], [0, 0, 1, −1, 0, 0]}

Eigenvalues M

[4., 4., −2., −2., −2., −2.]

Eigenvalues N

[4., −2., −1., −1., 0., 0.]

NullSpace M

{}

NullSpace N

{[0, 0, 1, −1, 0, 0], [1, −1, 0, 0, 1, −1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 2 & 2 & 2 & 1 \\ 1 & 0 & 2 & 2 & 1 & 2 \\ 2 & 2 & 0 & 0 & 2 & 2 \\ 2 & 2 & 0 & 0 & 2 & 2 \\ 2 & 1 & 2 & 2 & 0 & 1 \\ 1 & 2 & 2 & 2 & 1 & 0 \end{pmatrix}$$