T-Run

[2, 3, 2, 3, 4, 4, 1, 1], [6, 7, 6, 7, 8, 8, 5,

$\tilde{\pi}$ = [1, 1, 1, 1, 1, 1, 1, 1]
$\boldsymbol{\delta}$ = [2, 2, 2, 2, 2, 2, 2, 2]

POSSIBLE RANKS

1 x 8
2 x 4

BASE DETERMINANT 505/4096, .1232910156

NullSpace of Δ

{1, 5}, {3, 7}, {2, 6}, {4, 8}

Nullspace of A

[{7},{3}] `,` [{5},{1}] `,` [{6},{2}] `,` [{8},{4}]

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[5] v[8] + v[2] v[3]

Degree 3
v[4] v[5] v[7] + v[1] v[6] v[8] + v[3] v[4] v[6] + v[1] v[2] v[7]

Degree 4
v[1] v[4] v[6] v[7] + v[2] v[3] v[5] v[8]

Degree 5
2 v[3] v[4] v[5] v[6] v[8] + 2 v[2] v[3] v[4] v[5] v[7] + 2 v[1] v[2] v[3] v[6] v[8] + 2 v[1]

v[2] v[5] v[7] v[8]

Degree 6
4 v[1] v[4] v[5] v[6] v[7] v[8] + 4 v[1] v[2] v[3] v[4] v[6] v[7]

Degree 7
0

Degree 8
16 v[1] v[2] v[3] v[4] v[5] v[6] v[7] v[8]

==

{}

R: [2, 3, 2, 3, 4, 4, 1, 1]
B: [6, 7, 6, 7, 8, 8, 5, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 4

Level 2 det = $\frac{1}{4096}(-5+s^2)(1+s)^2(-1+s)(101-44s+10s^2-4s^3+s^4)$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 1, "vs", 2

RANK of B is 4

B ranking is 2, "vs", 4

BBAR ranking 1, "vs", 2

"R CYCLES", 1 + v[2] v[3]
"B CYCLES", 1 + v[5] v[8]

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [0, −1, 0, 1, 0, 0, 0, 0]}

NullSpace of B^*

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0&0&0&4&0&0&0&0\\ 0&0&4&0&0&0&0&0\\ 0&4&0&0&0&0&0&0\\ 4&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&4\\ 0&0&0&0&0&0&4&0\\ 0&0&0&0&0&4&0&0\\ 0&0&0&0&4&0&0&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&1&0&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 1&0&1&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 0&1&0&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 1&0&1&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&0&1&1\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&0&1&1\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&1&0&0\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&1&0&0 \end{pmatrix}$$

"RANK of N is ", 3, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 1   "Trace mark", 0, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}\,(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8])$
degree 2: $\frac{1}{4}\,(v[1]v[4] + v[2]v[3] + v[5]v[8] + v[6]v[7])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 7, 8}, {2, 4, 5, 6}}

"PT2" = {{1, 3, 5, 6}, {2, 4, 7, 8}}

"RG1" = {5, 8}

"RG2" = {6, 7}

"RG3" = {2, 3}

"RG4" = {1, 4}

$\pi 2$ = [0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0] supp $\pi 2$ = {3, 8, 25, 26}

$u2$ = [2, 0, 2, 1, 1, 1, 1, 2, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 0, 2, 2, 2, 2, 0]

supp u2 = {1, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27}

Action of R on ranges, [[4], [4], [3], [3]]
Action of B on ranges, [[1], [1], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [2, 2]

BPARTS [1, 1]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[4, 3, 4, 3]

B−BLOCKS,

[2, 1, 1, 2]

with invariant measure, [1, 1, 1, 1]

N by blocks, N − check: true

b_1 = {1, 3, 7, 8}
b_2 = {2, 4, 5, 6}
b_3 = {1, 3, 5, 6}
b_4 = {2, 4, 7, 8}

dim(span of partition vectors), rank(N_0), rank(N): 3, 3, 3

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & h[1] & 0 & 0 & 0 & 0 \\ 0 & h[2] & h[1] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[1] & h[2] & 0 & 0 & 0 & 0 & 0 \\ h[1] & 0 & 0 & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & 0 & 0 & h[1] \\ 0 & 0 & 0 & 0 & 0 & h[2] & h[1] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[1] & h[2] & 0 \\ 0 & 0 & 0 & 0 & h[1] & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 12, Shape: $3 \oplus 9/7$

$$\text{CLB} = \begin{pmatrix} -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3}}, true

Ω_B in Vec(K)? , {{5, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \end{pmatrix}$$

Omega−checks true, true

$\pi_R = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 7, 8}, {2, 4, 5, 6}}

1, "range", [5, 8], [[8, 5, 8, 5, 5, 5, 8, 8], [5, 8, 5, 8, 8, 8, 5, 5]]

2, "range", [6, 7], [[7, 6, 7, 6, 6, 6, 7, 7], [6, 7, 6, 7, 7, 7, 6, 6]]

3, "range", [2, 3], [[3, 2, 3, 2, 2, 2, 3, 3], [2, 3, 2, 3, 3, 3, 2, 2]]

4, "range", [1, 4], [[4, 1, 4, 1, 1, 1, 4, 4], [1, 4, 1, 4, 4, 4, 1, 1]]

2, "partition", {{1, 3, 5, 6}, {2, 4, 7, 8}}

1, "range", [5, 8], [[8, 5, 8, 5, 8, 8, 5, 5], [5, 8, 5, 8, 5, 5, 8, 8]]

2, "range", [6, 7], [[7, 6, 7, 6, 7, 7, 6, 6], [6, 7, 6, 7, 6, 6, 7, 7]]

3, "range", [2, 3], [[3, 2, 3, 2, 3, 3, 2, 2], [2, 3, 2, 3, 2, 2, 3, 3]]

4, "range", [1, 4], [[4, 1, 4, 1, 4, 4, 1, 1], [1, 4, 1, 4, 1, 1, 4, 4]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2$ =
$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$

{3, 8, 25, 26}

$u2$ =
$\begin{pmatrix} 2 & 0 & 2 & 1 & 1 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 2 & 2 & 2 & 0 \end{pmatrix}$

{1, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27}

picheck $\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

$\pi = \begin{pmatrix}\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8}\end{pmatrix}$

$\pi 1 = \begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

$u1 = \begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

picheck $\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2}\\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2}\\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2}\\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2}\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix}0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 0 & 4 & 0 & 2 & 2 & 2 & 2 \\ 0 & 4 & 0 & 4 & 2 & 2 & 2 & 2 \\ 4 & 0 & 4 & 0 & 2 & 2 & 2 & 2 \\ 0 & 4 & 0 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 4 & 0 & 0 \\ 2 & 2 & 2 & 2 & 4 & 4 & 0 & 0 \\ 2 & 2 & 2 & 2 & 0 & 0 & 4 & 4 \\ 2 & 2 & 2 & 2 & 0 & 0 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 1, 1, -1, -1, -1, -1]$

$$\ker N_C = \begin{pmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & -s & -s & s & t & -t & -t & t \\ s & -s & -s & s & t & -t & -t & t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB}$$

checks

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & -1 & -1 & 1 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -s & 0 & -t & 0 \\ s & 0 & t & 0 \\ -s & 0 & -t & 0 \\ s & 0 & t & 0 \\ 0 & s & 0 & t \\ 0 & s & 0 & t \\ 0 & -s & 0 & -t \\ 0 & -s & 0 & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & s & t & s & -s \\ 0 & t & -t & t & s \\ 0 & s & t & s & -s \\ 0 & t & -t & t & s \\ s & 0 & 0 & t & 0 \\ s & 0 & 0 & t & 0 \\ -s & s+t & 0 & s & 0 \\ -s & s+t & 0 & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 0 & 4 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&4&0&0&0&0\\ 0&4&4&0&0&0&0&0\\ 0&4&4&0&0&0&0&0\\ 4&0&0&4&0&0&0&0\\ 0&0&0&0&4&0&0&4\\ 0&0&0&0&0&4&4&0\\ 0&0&0&0&0&4&4&0\\ 0&0&0&0&4&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&1&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 0&1&0&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 1&0&1&0&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ 0&1&0&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&1&0&0\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&1&1&0&0\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&0&1&1\\ \frac{1}{2}&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&0&1&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ 0\ \frac{1}{4}\ 0\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

NM $\left(4\ \ 4\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 0\ \ 4\ \ 0\ \ 4\right)$

"IS MN in Vec(K)?", true

MN $\left(4\ \ 4\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 0\ \ 4\ \ 0\ \ 4\right)$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 1/8$, min $\boldsymbol{\tau} = 8/1$, $\boldsymbol{\tau}$-check is positive? 24/1

max r = 8/1 , r-check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 2, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 12
out of total no. of elements equal to 16

dim span idems 8 vs no. of idems 8

"PT1" = {{1, 3, 7, 8}, {2, 4, 5, 6}}

"PT2" = {{1, 3, 5, 6}, {2, 4, 7, 8}}

"RG1" = {5, 8}

"RG2" = {6, 7}

"RG3" = {2, 3}

"RG4" = {1, 4}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & 3 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & 3 & -1 & -1 & -1 & -1 & -1 \\ 3 & -1 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & -1 & 3 \\ -1 & -1 & -1 & -1 & -1 & 3 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & 3 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{3}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 \\ \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[3., 2., 2., 0., 0., 0., 0., 0.]

Eigenvalues M_C−scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C−scaled

[3.428571429, 2.285714286, 2.285714286, 0., 0., 0., 0., 0.]

NullSpace M_C

{[0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [1, 1, 0, 0, 1, 1, 0, 0], [1, 1, 0, 0, 0, 1, 0, 1], [0, 0, 0, 0, 0, −1, 1, 0] }

NullSpace N_C

{[0, 0, 1, 1, −1, 0, 0, −1], [0, 1, 0, −1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1], [1, 0, 0, 1, −1, 0, 0, −1]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[4., 2., 2., 0., 0., 0., 0., 0.]

NullSpace M_0

{[0, 0, 0, 0, 0, −1, 1, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

NullSpace N_0

{[−1, −1, 0, 0, 1, 0, 0, 1], [0, −1, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [−1, −1, 0, 0, 1, 0, 1, 0], [−1, 0, 1, 0, 0, 0, 0, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[4., −2., −2., 0., 0., 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [−1, −1, 0, 0, 1, 0, 1, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [−1, −1, 0, 0, 1, 0, 0, 1]}

Harmonic Basis

$$\begin{pmatrix} 0 & 2 & 0 & 2 & 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & 0 & 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2 & 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 & 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 2 & 2 & 0 & 0 \end{pmatrix}$$

==

{2, 3}

R: [2, 7, 6, 3, 4, 4, 1, 1]
B: [6, 3, 2, 7, 8, 8, 5, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{3}{2048}(-5+3s)(-1+s)(101+25s+s^2-9s^3+2s^4)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 3, "vs", 6

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 1, "vs", 4

"R CYCLES", (1 + v[3] v[4] v[6]) (1 + v[1] v[2] v[7])
"B CYCLES", (1 + v[5] v[8]) (1 + v[2] v[3])

Eigenvalues

R: [1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 0., 0.]

B: [1., -1., 1., -1., 0., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, -1, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, -1]}

NullSpace of B^*

{[0, 0, 0, 0, -1, 1, 0, 0], [0, 0, 0, 0, 0, 0, -1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[4] + v[2]v[3] + v[5]v[8] + v[6]v[7]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"RG1" = {5, 8}

"RG2" = {6, 7}

"RG3" = {2, 3}

"RG4" = {1, 4}

$\pi 2$ = [0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0]

supp $\pi 2$ = {3, 8, 25, 26}

$u2$ = [0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1, 1, 1, 0]

supp u2 = {2, 3, 4, 5, 8, 9, 10, 11, 17, 18, 21, 22, 24, 25, 26, 27}

Action of R on ranges, [[4], [4], [2], [3]]
Action of B on ranges, [[1], [1], [3], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[1, 2]

B−BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N − check: true

$b_1 = \{1, 2, 7, 8\}$
$b_2 = \{3, 4, 5, 6\}$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & h[1] & 0 & 0 & 0 & 0 \\ 0 & h[2] & h[1] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[1] & h[2] & 0 & 0 & 0 & 0 & 0 \\ h[1] & 0 & 0 & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & 0 & 0 & h[1] \\ 0 & 0 & 0 & 0 & 0 & h[2] & h[1] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[1] & h[2] & 0 \\ 0 & 0 & 0 & 0 & h[1] & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 18, Shape: $6 \oplus 12/10$

$$CLB = \begin{pmatrix} 1 & 1 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 4, 6}, {1, 2, 7}}, true

Ω_B in Vec(K)? , {{2, 3}, {5, 8}}, false

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{8} & \frac{1}{8} & 0 & \frac{3}{8} & 0 & 0 & \frac{3}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 7, 8}, {3, 4, 5, 6}}

1, "range", [5, 8], [[8, 8, 5, 5, 5, 5, 8, 8], [5, 5, 8, 8, 8, 8, 5, 5]]

2, "range", [6, 7], [[7, 7, 6, 6, 6, 6, 7, 7], [6, 6, 7, 7, 7, 7, 6, 6]]

3, "range", [2, 3], [[3, 3, 2, 2, 2, 2, 3, 3], [2, 2, 3, 3, 3, 3, 2, 2]]

4, "range", [1, 4], [[4, 4, 1, 1, 1, 1, 4, 4], [1, 1, 4, 4, 4, 4, 1, 1]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

{3, 8, 25, 26}

$u2 =$

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

{2, 3, 4, 5, 8, 9, 10, 11, 17, 18, 21, 22, 24, 25, 26, 27}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

idem-checks   N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 4 & 0 & 0 & 0 & 0 & 4 & 4 \\ 4 & 4 & 0 & 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 4 & 4 & 4 & 4 & 0 & 0 \\ 0 & 0 & 4 & 4 & 4 & 4 & 0 & 0 \\ 0 & 0 & 4 & 4 & 4 & 4 & 0 & 0 \\ 0 & 0 & 4 & 4 & 4 & 4 & 0 & 0 \\ 4 & 4 & 0 & 0 & 0 & 0 & 4 & 4 \\ 4 & 4 & 0 & 0 & 0 & 0 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 0, 0, 1, -1, 0, 0, -1]$

$$\ker N_c = \begin{pmatrix} 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -t & s & 0 & 0 & -s & t & 0 \\ 0 & -t & 0 & s & 0 & -s & 0 & t \\ s & -s & 0 & 0 & t & -t & 0 & 0 \\ 0 & -s & t & 0 & 0 & -t & s & 0 \\ 0 & -t & 0 & s & 0 & -s & 0 & t \\ s & -s & 0 & 0 & t & -t & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & -1 & 0 & 0 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -s & -t & 0 \\ 0 & t & s & 0 \\ 0 & -t & -s & 0 \\ 0 & s & t & 0 \\ -t & 0 & 0 & s \\ -t & 0 & 0 & s \\ t & 0 & 0 & -s \\ t & 0 & 0 & -s \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & t & s & 0 \\ 0 & t+s & -s & s & 0 \\ 0 & 0 & s & t & 0 \\ 0 & t+s & -t & t & 0 \\ -t & t+s & 0 & t+s & -s \\ -t & t+s & 0 & t+s & -s \\ t & 0 & 0 & 0 & s \\ t & 0 & 0 & 0 & s \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 0 & 4 & 0 & 4 & 0 \end{pmatrix}$

$$R N_0 R^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew } T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$T \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

"IS NM in Vec(K)?", true

NM $\begin{pmatrix} 4 & 4 & 0 & 0 & 0 & 0 & 4 & 4 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 4 & 4 & 0 & 0 & 0 & 0 & 4 & 4 \end{pmatrix}$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 24/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 8
out of total no. of elements equal to 8

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"RG1" = {5, 8}

"RG2" = {6, 7}

"RG3" = {2, 3}

"RG4" = {1, 4}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & 3 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & 3 & -1 & -1 & -1 & -1 & -1 \\ 3 & -1 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & -1 & 3 \\ -1 & -1 & -1 & -1 & -1 & 3 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & 3 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 \\ 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 \\ \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} \\ 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 \\ 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_c

[0., 0., 0., 0., 0., 0., 4., 3.]

Eigenvalues M_c−scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_c−scaled

[0., 0., 0., 0., 0., 0., 4.571428571, 3.428571429]

NullSpace M_c

{[0, 0, 0, 0, −1, 0, 0, 1], [1, 0, 0, −1, 0, 0, 0, 0], [0, 1, 0, 1, 1, 1, 0, 0], [0, 0, 1, 1, 1, 1, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0] }

NullSpace N_c

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 0, 1, 0, 0, 0], [0, 0, −1, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, −1, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace M_0

{[0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1], [0, 0, 0, 0, 0, −1, 1, 0]}

NullSpace N_0

{[0, 0, −1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [0, 0, −1, 0, 0, 1, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{}

NullSpace N

{[−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1], [0, 0, −1, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

==

20, [1, 1, 1, −1, −1, 1, 1, 1]

==

{4, 6}

R: [2, 3, 2, 7, 4, 8, 1, 1]
B: [6, 7, 6, 3, 8, 4, 5, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 6

Level 2 det = $\frac{1}{16384}(-1+s)(-2020-1745s-799s^2-43s^3+79s^4+37s^5+19s^6-9s^7+s^8)$

RANK of R is 6

R ranking is 4, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 3, "vs", 5

"R CYCLES", 1 + v[2] v[3]
"B CYCLES", (1 + v[5] v[8]) (1 + v[3] v[4] v[6])

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [−1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., 1., 0., 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1]}

NullSpace of B^*

{[0, 0, 0, 0, 0, 0, 1, −1], [1, 0, −1, 0, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 12 & 0 & 0 & 0 & 0 & 8 & 8 \\ 12 & 0 & 12 & 4 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 8 & 0 & 4 & 4 \\ 0 & 4 & 0 & 0 & 8 & 0 & 4 & 12 \\ 0 & 0 & 8 & 8 & 0 & 12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12 & 0 & 12 & 4 \\ 8 & 0 & 4 & 4 & 0 & 12 & 0 & 0 \\ 8 & 0 & 4 & 12 & 0 & 4 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8] \right)$

degree 2: $\frac{3}{28}\left(3v[1]v[2] + 2v[1]v[7] + 2v[1]v[8] + 3v[2]v[3] + v[2]v[4] + 2v[3]v[5] + v[3]v[7] + v[3]v[8] + 2v[4]v[5] + v[4]v[7] + 3v[4]v[8] + 3v[5]v[6] + 3v[6]v[7] + v[6]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 4, 6}, {2, 5, 7, 8}}

"RG1" = {6, 8}

"RG2" = {6, 7}

"RG3" = {5, 6}

"RG4" = {4, 8}

"RG5" = {4, 7}

"RG6" = {4, 5}

"RG7" = {2, 4}

"RG8" = {3, 8}

"RG9" = {3, 7}

"RG10" = {3, 5}

"RG11" = {2, 3}

"RG12" = {1, 8}

"RG13" = {1, 7}

"RG14" = {1, 2}

$\pi 2$ = [3, 0, 0, 0, 0, 2, 2, 3, 1, 0, 0, 0, 0, 0, 2, 0, 1, 1, 2, 0, 1, 3, 3, 0, 0, 3, 1, 0]

supp $\pi 2$ = {1, 6, 7, 8, 9, 15, 17, 18, 19, 21, 22, 23, 26, 27}

$u2$ = [1, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0]

supp u2 = {1, 4, 6, 7, 8, 9, 11, 15, 17, 18, 19, 21, 22, 23, 26, 27}

Action of R on ranges, [[12], [12], [4], [13], [13], [5], [9], [14], [14], [7], [11], [14], [14], [11]]
Action of B on ranges, [[6], [6], [4], [10], [10], [8], [9], [3], [3], [1], [2], [3], [3], [2]]

$$\boldsymbol{\beta} = \left(\frac{1}{28}\ \frac{3}{28}\ \frac{3}{28}\ \frac{3}{28}\ \frac{1}{28}\ \frac{1}{14}\ \frac{1}{28}\ \frac{1}{28}\ \frac{1}{28}\ \frac{1}{14}\ \frac{3}{28}\ \frac{1}{14}\ \frac{1}{14}\ \frac{3}{28}\right)$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 1]

B-BLOCKS,

[1, 2]

with invariant measure, [1, 1]

N by blocks, N - check: true

$b_1 = \{1, 3, 4, 6\}$
$b_2 = \{2, 5, 7, 8\}$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 37, Shape: 15 ⊕ 22/20

$$\mathrm{CLB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3}}, true

Ω_B in Vec(K)? , {{3, 4, 6}, {5, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{4} & \frac{1}{6} & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 4, 6}, {2, 5, 7, 8}}

1, "range", [6, 8], [[8, 6, 8, 8, 6, 8, 6, 6], [6, 8, 6, 6, 8, 6, 8, 8]]

2, "range", [6, 7], [[7, 6, 7, 7, 6, 7, 6, 6], [6, 7, 6, 6, 7, 6, 7, 7]]

3, "range", [5, 6], [[6, 5, 6, 6, 5, 6, 5, 5], [5, 6, 5, 5, 6, 5, 6, 6]]

4, "range", [4, 8], [[8, 4, 8, 8, 4, 8, 4, 4], [4, 8, 4, 4, 8, 4, 8, 8]]

5, "range", [4, 7], [[7, 4, 7, 7, 4, 7, 4, 4], [4, 7, 4, 4, 7, 4, 7, 7]]

6, "range", [4, 5], [[5, 4, 5, 5, 4, 5, 4, 4], [4, 5, 4, 4, 5, 4, 5, 5]]

7, "range", [2, 4], [[4, 2, 4, 4, 2, 4, 2, 2], [2, 4, 2, 2, 4, 2, 4, 4]]

8, "range", [3, 8], [[8, 3, 8, 8, 3, 8, 3, 3], [3, 8, 3, 3, 8, 3, 8, 8]]

9, "range", [3, 7], [[7, 3, 7, 7, 3, 7, 3, 3], [3, 7, 3, 3, 7, 3, 7, 7]]

10, "range", [3, 5], [[5, 3, 5, 5, 3, 5, 3, 3], [3, 5, 3, 3, 5, 3, 5, 5]]

11, "range", [2, 3], [[3, 2, 3, 3, 2, 3, 2, 2], [2, 3, 2, 2, 3, 2, 3, 3]]

12, "range", [1, 8], [[8, 1, 8, 8, 1, 8, 1, 1], [1, 8, 1, 1, 8, 1, 8, 8]]

13, "range", [1, 7], [[7, 1, 7, 7, 1, 7, 1, 1], [1, 7, 1, 1, 7, 1, 7, 7]]

14, "range", [1, 2], [[2, 1, 2, 2, 1, 2, 1, 1], [1, 2, 1, 1, 2, 1, 2, 2]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 2 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 1 & 3 & 3 & 0 & 0 & 3 & 1 & 0 \end{pmatrix}$

{1, 6, 7, 8, 9, 15, 17, 18, 19, 21, 22, 23, 26, 27}

$u2 =$
$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$

{1, 4, 6, 7, 8, 9, 11, 15, 17, 18, 19, 21, 22, 23, 26, 27}

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_8 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_9 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_{10} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_{11} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 0 & 4 & 4 & 0 & 4 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 & 0 & 4 & 4 \\ 4 & 0 & 4 & 4 & 0 & 4 & 0 & 0 \\ 4 & 0 & 4 & 4 & 0 & 4 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 & 0 & 4 & 4 \\ 4 & 0 & 4 & 4 & 0 & 4 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 & 0 & 4 & 4 \\ 0 & 4 & 0 & 0 & 4 & 0 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, 1, 0, 0, -1, -1, 0, 0]$

$$\ker N_C = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s & 0 & -s & 0 & t & 0 & -t & 0 \\ s & 0 & -s & 0 & t & 0 & -t & 0 \\ 0 & -s & t & 0 & 0 & -t & s & 0 \\ 0 & 0 & -s & s & 0 & 0 & -t & t \\ 0 & -s & 0 & t & 0 & -t & 0 & s \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 0 & 0 & 0 & -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} -s+t \\ s-t \\ -s+t \\ -s+t \\ s-t \\ -s+t \\ s-t \\ s-t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} t & s \\ s & t \\ t & s \\ t & s \\ s & t \\ t & s \\ s & t \\ s & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4 & \frac{12}{7} & 0 & 0 & 0 & 0 & \frac{8}{7} & \frac{8}{7} \\ \frac{12}{7} & 4 & \frac{12}{7} & \frac{4}{7} & 0 & 0 & 0 & 0 \\ 0 & \frac{12}{7} & 4 & 0 & \frac{8}{7} & 0 & \frac{4}{7} & \frac{4}{7} \\ 0 & \frac{4}{7} & 0 & 4 & \frac{8}{7} & 0 & \frac{4}{7} & \frac{12}{7} \\ 0 & 0 & \frac{8}{7} & \frac{8}{7} & 4 & \frac{12}{7} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{12}{7} & 4 & \frac{12}{7} & \frac{4}{7} \\ \frac{8}{7} & 0 & \frac{4}{7} & \frac{4}{7} & 0 & \frac{12}{7} & 4 & 0 \\ \frac{8}{7} & 0 & \frac{4}{7} & \frac{12}{7} & 0 & \frac{4}{7} & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&1&1&0&1&0&0\\ 0&1&0&0&1&0&1&1\\ 1&0&1&1&0&1&0&0\\ 1&0&1&1&0&1&0&0\\ 0&1&0&0&1&0&1&1\\ 1&0&1&1&0&1&0&0\\ 0&1&0&0&1&0&1&1\\ 0&1&0&0&1&0&1&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{1}{4}\ 0\ 0\ \frac{1}{4}\ 0\ 0\ 0\ \frac{1}{4}\ 0\ \frac{1}{4}\ \frac{1}{4}\ 0\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

NM $(4\ 4\ 0\ 0\ 4\ 0\ 0\ 0\ 4\ 0\ 4\ 4\ 0\ 4)$

"IS MN in Vec(K)?", true

MN $(4\ 4\ 0\ 0\ 4\ 0\ 0\ 0\ 4\ 0\ 4\ 4\ 0\ 4)$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 24/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 14, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 28

dim span idems 7 vs no. of idems 14

"PT1" = {{1, 3, 4, 6}, {2, 5, 7, 8}}

"RG1" = {6, 8}

"RG2" = {6, 7}

"RG3" = {5, 6}

"RG4" = {4, 8}

"RG5" = {4, 7}

"RG6" = {4, 5}

"RG7" = {2, 4}

"RG8" = {3, 8}

"RG9" = {3, 7}

"RG10" = {3, 5}

"RG11" = {2, 3}

"RG12" = {1, 8}

"RG13" = {1, 7}

"RG14" = {1, 2}

$$M_c = \begin{pmatrix} 3 & \frac{5}{7} & -1 & -1 & -1 & -1 & \frac{1}{7} & \frac{1}{7} \\ \frac{5}{7} & 3 & \frac{5}{7} & \frac{-3}{7} & -1 & -1 & -1 & -1 \\ -1 & \frac{5}{7} & 3 & -1 & \frac{1}{7} & -1 & \frac{-3}{7} & \frac{-3}{7} \\ -1 & \frac{-3}{7} & -1 & 3 & \frac{1}{7} & -1 & \frac{-3}{7} & \frac{5}{7} \\ -1 & -1 & \frac{1}{7} & \frac{1}{7} & 3 & \frac{5}{7} & -1 & -1 \\ -1 & -1 & -1 & -1 & \frac{5}{7} & 3 & \frac{5}{7} & \frac{-3}{7} \\ \frac{1}{7} & -1 & \frac{-3}{7} & \frac{-3}{7} & -1 & \frac{5}{7} & 3 & -1 \\ \frac{1}{7} & -1 & \frac{-3}{7} & \frac{5}{7} & -1 & \frac{-3}{7} & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{5}{21} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{1}{21} & \frac{1}{21} \\ \frac{5}{21} & 1 & \frac{5}{21} & \frac{-1}{7} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{5}{21} & 1 & \frac{-1}{3} & \frac{1}{21} & \frac{-1}{3} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{3} & \frac{-1}{7} & \frac{-1}{3} & 1 & \frac{1}{21} & \frac{-1}{3} & \frac{-1}{7} & \frac{5}{21} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{1}{21} & \frac{1}{21} & 1 & \frac{5}{21} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{5}{21} & 1 & \frac{5}{21} & \frac{-1}{7} \\ \frac{1}{21} & \frac{-1}{3} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{3} & \frac{5}{21} & 1 & \frac{-1}{3} \\ \frac{1}{21} & \frac{-1}{3} & \frac{-1}{7} & \frac{5}{21} & \frac{-1}{3} & \frac{-1}{7} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 \\ 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 \\ 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_c

[0., 0., 5.082918009, 2.118627666, 3.084168612, 5.881372334, 2.917081990, 4.915831388]

Eigenvalues N_C

[0., 0., 0., 0., 0., 0., 4., 3.]

Eigenvalues M_C−scaled

[0., 0., 1.694306003, 0.7062092219, 1.028056204, 1.960457444, 0.9723606643, 1.638610464]

Eigenvalues N_C−scaled

[0., 0., 0., 0., 0., 0., 4.571428571, 3.428571429]

NullSpace M_C

{[1, 0, 1, 1, 0, 1, 0, 0], [0, 1, 0, 0, 1, 0, 1, 1]}

NullSpace N_C

{[0, 0, 0, 1, 0, −1, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1], [1, 0, 0, 0, 0, −1, 0, 0], [0, 0, 0, 0, −1, 0, 1, 0], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 1, 0, 0, −1, 0, 0]}

Eigenvalues M_0

[0., 8., 5.082918009, 2.118627666, 3.084168612, 5.881372334, 2.917081990, 4.915831388]

Eigenvalues N_0

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace M_0

{[1, −1, 1, 1, −1, 1, −1, −1]}

NullSpace N_0

{[0, 1, 0, 0, 0, 0, 0, −1], [0, 0, 0, 1, 0, −1, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1], [0, 0, 0, 0, 1, 0, 0, −1], [1, 0, 0, 0, 0, −1, 0, 0], [0, 0, 1, 0, 0, −1, 0, 0]}

Eigenvalues M

[−4., 4., 1.881372335, −1.082918009, 0.9158313890, 1.082918009, −1.881372335, −0.9158313892]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 0, 0, 0, 1, 0], [0, −1, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

==

{4, 7}

R: [2, 3, 2, 7, 4, 4, 5, 1]
B: [6, 7, 6, 3, 8, 8, 1, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 6

Level 2 det = $\frac{1}{16384}\left(-2020 - 1745s - 799s^2 - 43s^3 + 79s^4 + 37s^5 + 19s^6 - 9s^7 + s^8\right)\left(-1 + s\right)$

RANK of R is 6

R ranking is 4, "vs", 6

RBAR ranking 3, "vs", 5

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 1, "vs", 2

"R CYCLES", (1 + v[2] v[3]) (1 + v[4] v[5] v[7])
"B CYCLES", 1 + v[5] v[8]

Eigenvalues

R: [−1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., 1., 0., 0., 0.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of R^*

{[0, 0, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 12 & 4 & 8 & 4 & 0 \\ 0 & 0 & 0 & 12 & 8 & 0 & 8 & 0 \\ 0 & 0 & 0 & 4 & 4 & 8 & 12 & 0 \\ 12 & 12 & 4 & 0 & 0 & 0 & 0 & 0 \\ 4 & 8 & 4 & 0 & 0 & 0 & 0 & 12 \\ 8 & 0 & 8 & 0 & 0 & 0 & 0 & 12 \\ 4 & 8 & 12 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 12 & 12 & 4 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8] \right)$

degree 2: $\frac{3}{28}\left(3v[1]v[4] + v[1]v[5] + 2v[1]v[6] + v[1]v[7] + 3v[2]v[4] + 2v[2]v[5] + 2v[2]v[7] + v[3]v[4] + v[3]v[5] + 2v[3]v[6] + 3v[3]v[7] + 3v[5]v[8] + 3v[6]v[8] + v[7]v[8] \right)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{4, 5, 6, 7}, {1, 2, 3, 8}}

"RG1" = {7, 8}

"RG2" = {6, 8}

"RG3" = {5, 8}

"RG4" = {3, 7}

"RG5" = {3, 6}

"RG6" = {3, 5}

"RG7" = {3, 4}

"RG8" = {2, 7}

"RG9" = {2, 5}

"RG10" = {2, 4}

"RG11" = {1, 7}

"RG12" = {1, 6}

"RG13" = {1, 5}

"RG14" = {1, 4}

$\pi 2$ = [0, 0, 3, 1, 2, 1, 0, 0, 3, 2, 0, 2, 0, 1, 1, 2, 3, 0, 0, 0, 0, 0, 0, 0, 3, 0, 3, 1]

supp $\pi 2$ = {3, 4, 5, 6, 9, 10, 12, 14, 15, 16, 17, 25, 27, 28}

$u2$ = [0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1]

supp u2 = {3, 4, 5, 6, 9, 10, 11, 12, 14, 15, 16, 17, 22, 25, 27, 28}

Action of R on ranges, [[13], [14], [14], [9], [10], [10], [8], [6], [7], [4], [9], [10], [10], [8]]
Action of B on ranges, [[13], [3], [3], [12], [2], [2], [5], [11], [1], [4], [12], [2], [2], [5]]

$$\boldsymbol{\beta} = \left(\frac{1}{28} \;\; \frac{3}{28} \;\; \frac{3}{28} \;\; \frac{3}{28} \;\; \frac{1}{14} \;\; \frac{1}{28} \;\; \frac{1}{28} \;\; \frac{1}{14} \;\; \frac{1}{14} \;\; \frac{3}{28} \;\; \frac{1}{28} \;\; \frac{1}{14} \;\; \frac{1}{28} \;\; \frac{3}{28}\right)$$

RPARTS [1]

BPARTS [1]

$$\boldsymbol{\alpha} = (1)$$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[1, 2]

B–BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N – check: true

b_1 = {4, 5, 6, 7}
b_2 = {1, 2, 3, 8}

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 37, Shape: 15 ⊕ 22/20

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{4, 5, 7}, {2, 3}}, true

Ω_B in Vec(K)? , {{5, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4, 5, 6, 7}, {1, 2, 3, 8}}

1, "range", [7, 8], [[8, 8, 8, 7, 7, 7, 7, 8], [7, 7, 7, 8, 8, 8, 8, 7]]

2, "range", [6, 8], [[8, 8, 8, 6, 6, 6, 6, 8], [6, 6, 6, 8, 8, 8, 8, 6]]

3, "range", [5, 8], [[8, 8, 8, 5, 5, 5, 5, 8], [5, 5, 5, 8, 8, 8, 8, 5]]

4, "range", [3, 7], [[7, 7, 7, 3, 3, 3, 3, 7], [3, 3, 3, 7, 7, 7, 7, 3]]

5, "range", [3, 6], [[6, 6, 6, 3, 3, 3, 3, 6], [3, 3, 3, 6, 6, 6, 6, 3]]

6, "range", [3, 5], [[5, 5, 5, 3, 3, 3, 3, 5], [3, 3, 3, 5, 5, 5, 5, 3]]

7, "range", [3, 4], [[4, 4, 4, 3, 3, 3, 3, 4], [3, 3, 3, 4, 4, 4, 4, 3]]

8, "range", [2, 7], [[7, 7, 7, 2, 2, 2, 2, 7], [2, 2, 2, 7, 7, 7, 7, 2]]

9, "range", [2, 5], [[5, 5, 5, 2, 2, 2, 2, 5], [2, 2, 2, 5, 5, 5, 5, 2]]

10, "range", [2, 4], [[4, 4, 4, 2, 2, 2, 2, 4], [2, 2, 2, 4, 4, 4, 4, 2]]

11, "range", [1, 7], [[7, 7, 7, 1, 1, 1, 1, 7], [1, 1, 1, 7, 7, 7, 7, 1]]

12, "range", [1, 6], [[6, 6, 6, 1, 1, 1, 1, 6], [1, 1, 1, 6, 6, 6, 6, 1]]

13, "range", [1, 5], [[5, 5, 5, 1, 1, 1, 1, 5], [1, 1, 1, 5, 5, 5, 5, 1]]

14, "range", [1, 4], [[4, 4, 4, 1, 1, 1, 1, 4], [1, 1, 1, 4, 4, 4, 4, 1]]

"group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 3 & 1 & 2 & 1 & 0 & 0 & 3 & 2 & 0 & 2 & 0 & 1 & 1 & 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 & 1 \end{pmatrix}$

{3, 4, 5, 6, 9, 10, 12, 14, 15, 16, 17, 25, 27, 28}

$u2 =$
$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$

{3, 4, 5, 6, 9, 10, 11, 12, 14, 15, 16, 17, 22, 25, 27, 28}

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_6 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_7 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_8 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_9 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{10} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

idem-checks N0-checks

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 4 & 4 & 0 & 0 & 0 & 0 & 4 \\ 4 & 4 & 4 & 0 & 0 & 0 & 0 & 4 \\ 4 & 4 & 4 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 4 & 4 & 4 & 4 & 0 \\ 0 & 0 & 0 & 4 & 4 & 4 & 4 & 0 \\ 0 & 0 & 0 & 4 & 4 & 4 & 4 & 0 \\ 0 & 0 & 0 & 4 & 4 & 4 & 4 & 0 \\ 4 & 4 & 4 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 1, 0, 1, 0, -1, 0, -1]$

$$\ker N_C = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -t & 0 & t & 0 & -s & 0 & s & 0 \\ -t & 0 & 0 & s & -s & 0 & 0 & t \\ s & -s & 0 & 0 & t & -t & 0 & 0 \\ -t & 0 & 0 & s & -s & 0 & 0 & t \\ 0 & -s & s & 0 & 0 & -t & t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & -1 & -1 & 0 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} s-t \\ s-t \\ s-t \\ t-s \\ t-s \\ t-s \\ t-s \\ s-t \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} s & t \\ s & t \\ s & t \\ t & s \\ t & s \\ t & s \\ t & s \\ s & t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 \end{pmatrix}$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{B}\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&\frac{12}{7}&\frac{4}{7}&\frac{8}{7}&\frac{4}{7}&0\\ 0&4&0&\frac{12}{7}&\frac{8}{7}&0&\frac{8}{7}&0\\ 0&0&4&\frac{4}{7}&\frac{4}{7}&\frac{8}{7}&\frac{12}{7}&0\\ \frac{12}{7}&\frac{12}{7}&\frac{4}{7}&4&0&0&0&0\\ \frac{4}{7}&\frac{8}{7}&\frac{4}{7}&0&4&0&0&\frac{12}{7}\\ \frac{8}{7}&0&\frac{8}{7}&0&0&4&0&\frac{12}{7}\\ \frac{4}{7}&\frac{8}{7}&\frac{12}{7}&0&0&0&4&\frac{4}{7}\\ 0&0&0&0&\frac{12}{7}&\frac{12}{7}&\frac{4}{7}&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&1&1&0&0&0&0&1\\ 1&1&1&0&0&0&0&1\\ 1&1&1&0&0&0&0&1\\ 0&0&0&1&1&1&1&0\\ 0&0&0&1&1&1&1&0\\ 0&0&0&1&1&1&1&0\\ 0&0&0&1&1&1&1&0\\ 1&1&1&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ 0\ \ 0\ \ 0\ \ \frac{1}{4}\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

NM $(4\ \ 4\ \ 4\ \ 0\ \ 0\ \ 0\ \ 4\ \ 0\ \ 0\ \ 0\ \ 0\ \ 4\ \ 4\ \ 4)$

"IS MN in Vec(K)?", true

MN $(4\ \ 4\ \ 4\ \ 0\ \ 0\ \ 0\ \ 4\ \ 0\ \ 0\ \ 0\ \ 0\ \ 4\ \ 4\ \ 4)$

$\boldsymbol{\tau}$ = 32/1 , rank = 2 , ratio = 16/1 , n^2 / r = 32/1

$\boldsymbol{\tau}'$ = 32/1 , r' = 1/2 , $\boldsymbol{\tau}$ / n^2 = 1/2

p^2 = 1/8 , min $\boldsymbol{\tau}$ = 8/1 , $\boldsymbol{\tau}$–check is positive? 24/1

max r = 8/1 , r–check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 14, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 28

dim span idems 7 vs no. of idems 14

"PT1" = {{4, 5, 6, 7}, {1, 2, 3, 8}}

"RG1" = {7, 8}

"RG2" = {6, 8}

"RG3" = {5, 8}

"RG4" = {3, 7}

"RG5" = {3, 6}

"RG6" = {3, 5}

"RG7" = {3, 4}

"RG8" = {2, 7}

"RG9" = {2, 5}

"RG10" = {2, 4}

"RG11" = {1, 7}

"RG12" = {1, 6}

"RG13" = {1, 5}

"RG14" = {1, 4}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & \frac{5}{7} & \frac{-3}{7} & \frac{1}{7} & \frac{-3}{7} & -1 \\ -1 & 3 & -1 & \frac{5}{7} & \frac{1}{7} & -1 & \frac{1}{7} & -1 \\ -1 & -1 & 3 & \frac{-3}{7} & \frac{-3}{7} & \frac{1}{7} & \frac{5}{7} & -1 \\ \frac{5}{7} & \frac{5}{7} & \frac{-3}{7} & 3 & -1 & -1 & -1 & -1 \\ \frac{-3}{7} & \frac{1}{7} & \frac{-3}{7} & -1 & 3 & -1 & -1 & \frac{5}{7} \\ \frac{1}{7} & -1 & \frac{1}{7} & -1 & -1 & 3 & -1 & \frac{5}{7} \\ \frac{-3}{7} & \frac{1}{7} & \frac{5}{7} & -1 & -1 & -1 & 3 & \frac{-3}{7} \\ -1 & -1 & -1 & -1 & \frac{5}{7} & \frac{5}{7} & \frac{-3}{7} & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{5}{21} & \frac{-1}{7} & \frac{1}{21} & \frac{-1}{7} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{5}{21} & \frac{1}{21} & \frac{-1}{3} & \frac{1}{21} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{1}{21} & \frac{5}{21} & \frac{-1}{3} \\ \frac{5}{21} & \frac{5}{21} & \frac{-1}{7} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{7} & \frac{1}{21} & \frac{-1}{7} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{5}{21} \\ \frac{1}{21} & \frac{-1}{3} & \frac{1}{21} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{5}{21} \\ \frac{-1}{7} & \frac{1}{21} & \frac{5}{21} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{7} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{5}{21} & \frac{5}{21} & \frac{-1}{7} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \\ 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \\ 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} \\ 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_c

[0., 0., 5.082918009, 2.118627666, 3.084168612, 5.881372334, 2.917081990, 4.915831388]

Eigenvalues N_C

[0., 0., 0., 0., 0., 0., 4., 3.]

Eigenvalues M_C–scaled

[0., 0., 1.694306003, 0.7062092219, 1.028056204, 1.960457444, 0.9723606643, 1.638610464]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 0., 0., 4.571428571, 3.428571429]

NullSpace M_C

{[0, 0, 0, 1, 1, 1, 1, 0], [1, 1, 1, 0, 0, 0, 0, 1]}

NullSpace N_C

{[0, 0, 0, 0, 0, 1, −1, 0], [0, 0, 0, 0, 1, 0, −1, 0], [0, 0, 0, 1, 0, 0, −1, 0], [0, 0, 1, 0, 0, 0, 0, −1], [0, 1, 0, 0, 0, 0, 0, −1], [1, 0, 0, 0, 0, 0, 0, −1]}

Eigenvalues M_0

[0., 8., 5.082918009, 2.118627666, 3.084168612, 5.881372334, 2.917081990, 4.915831388]

Eigenvalues N_0

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace M_0

{[1, 1, 1, −1, −1, −1, −1, 1]}

NullSpace N_0

{[0, 0, 0, −1, 0, 0, 1, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0], [0, 0, 0, −1, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1], [−1, 1, 0, 0, 0, 0, 0, 0]}

Eigenvalues M

[−4., 4., 1.082918009, −1.881372335, −0.9158313892, 1.881372335, −1.082918009, 0.9158313890]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{}

NullSpace N

{[−1, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, −1, 1, 0, 0, 0], [0, 0, 0, −1, 0, 1, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{5, 8}

R: [2, 3, 2, 3, 8, 4, 1, 5]
B: [6, 7, 6, 7, 4, 8, 5, 1]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{3}{2048}(-5 + 3s)(101 + 25s + s^2 - 9s^3 + 2s^4)(-1 + s)$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 1, "vs", 4

RANK of B is 6

B ranking is 3, "vs", 6

BBAR ranking 3, "vs", 6

"R CYCLES", (1 + v[5] v[8]) (1 + v[2] v[3])
"B CYCLES", (1 + v[4] v[5] v[7]) (1 + v[1] v[6] v[8])

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 0., 0.]

NullSpace of R

{[0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0]}

NullSpace of B^*

{[0, −1, 0, 1, 0, 0, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3   "Trace mark", 4, "Rank mark", 4, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[4] + v[2]v[3] + v[5]v[8] + v[6]v[7]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"RG1" = {5, 8}

"RG2" = {6, 7}

"RG3" = {2, 3}

"RG4" = {1, 4}

$\pi 2$ = [0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0]

supp $\pi 2$ = {3, 8, 25, 26}

$u2$ = [1, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1]

supp u2 = {1, 3, 4, 6, 8, 11, 13, 14, 15, 17, 20, 22, 23, 25, 26, 28}

Action of R on ranges, [[1], [4], [3], [3]]
Action of B on ranges, [[4], [1], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 1]

B-BLOCKS,

[1, 2]

with invariant measure, [1, 1]

N by blocks, N - check: true

b_1 = {1, 3, 6, 8}
b_2 = {2, 4, 5, 7}

dim(span of partition vectors), rank(N_0), rank(N):   2, 2, 2

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & h[1] & 0 & 0 & 0 & 0 \\ 0 & h[2] & h[1] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[1] & h[2] & 0 & 0 & 0 & 0 & 0 \\ h[1] & 0 & 0 & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & 0 & 0 & h[1] \\ 0 & 0 & 0 & 0 & 0 & h[2] & h[1] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[1] & h[2] & 0 \\ 0 & 0 & 0 & 0 & h[1] & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 18, Shape: $6 \oplus 12/10$

$$\text{CLB} = \begin{pmatrix} 0 & -1 & 1 & 1 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3}, {5, 8}}, false

Ω_B in Vec(K)? , {{1, 6, 8}, {4, 5, 7}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{3}{8} & \frac{3}{8} & 0 & \frac{1}{8} & 0 & 0 & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [5, 8], [[8, 5, 8, 5, 5, 8, 5, 8], [5, 8, 5, 8, 8, 5, 8, 5]]

2, "range", [6, 7], [[7, 6, 7, 6, 6, 7, 6, 7], [6, 7, 6, 7, 7, 6, 7, 6]]

3, "range", [2, 3], [[3, 2, 3, 2, 2, 3, 2, 3], [2, 3, 2, 3, 3, 2, 3, 2]]

4, "range", [1, 4], [[4, 1, 4, 1, 1, 4, 1, 4], [1, 4, 1, 4, 4, 1, 4, 1]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

{3, 8, 25, 26}

$u2 =$
$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

{1, 3, 4, 6, 8, 11, 13, 14, 15, 17, 20, 22, 23, 25, 26, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks N0–checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem–checks N0–checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 0 & 4 & 0 & 0 & 4 & 0 & 4 \\ 0 & 4 & 0 & 4 & 4 & 0 & 4 & 0 \\ 4 & 0 & 4 & 0 & 0 & 4 & 0 & 4 \\ 0 & 4 & 0 & 4 & 4 & 0 & 4 & 0 \\ 0 & 4 & 0 & 4 & 4 & 0 & 4 & 0 \\ 4 & 0 & 4 & 0 & 0 & 4 & 0 & 4 \\ 0 & 4 & 0 & 4 & 4 & 0 & 4 & 0 \\ 4 & 0 & 4 & 0 & 0 & 4 & 0 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 1, 1, 0, 0, -1, -1, 0]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} t & -s & 0 & 0 & s & -t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -s & t & 0 & 0 & -t & s \\ 0 & -s & 0 & s & 0 & -t & 0 & t \\ s & 0 & -s & 0 & t & 0 & -t & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 0 & 1 & 0 & 0 & -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -s & -t & 0 \\ 0 & s & t & 0 \\ 0 & -s & -t & 0 \\ 0 & s & t & 0 \\ t & 0 & 0 & s \\ s & 0 & 0 & t \\ -s & 0 & 0 & -t \\ -t & 0 & 0 & -s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & t & 0 & s \\ s+t & 0 & -t & 0 & t \\ 0 & 0 & t & 0 & s \\ s+t & 0 & -t & 0 & t \\ s+t & -s & 0 & -t & s+t \\ s+t & -t & 0 & -s & s+t \\ 0 & t & 0 & s & 0 \\ 0 & s & 0 & t & 0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 0 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&4&0&0&0&0\\ 0&4&4&0&0&0&0&0\\ 0&4&4&0&0&0&0&0\\ 4&0&0&4&0&0&0&0\\ 0&0&0&0&4&0&0&4\\ 0&0&0&0&0&4&4&0\\ 0&0&0&0&0&4&4&0\\ 0&0&0&0&4&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&1&0&0&1&0&1\\ 0&1&0&1&1&0&1&0\\ 1&0&1&0&0&1&0&1\\ 0&1&0&1&1&0&1&0\\ 0&1&0&1&1&0&1&0\\ 1&0&1&0&0&1&0&1\\ 0&1&0&1&1&0&1&0\\ 1&0&1&0&0&1&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$$\Omega \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$T \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$

"IS NM in Vec(K)?", true

NM $\begin{pmatrix} 4 & 0 & 4 & 0 & 0 & 4 & 0 & 4 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 4 & 0 & 4 & 0 & 0 & 4 & 0 & 4 \end{pmatrix}$

$$\boldsymbol{\tau} = 32/1 \text{ , rank} = 2 \text{ , ratio} = 16/1 \text{ , } n^2 / r = 32/1$$

$$\boldsymbol{\tau}' = 32/1 \text{ , } r' = 1/2 \text{ , } \boldsymbol{\tau} / n^2 = 1/2$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 24/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 8
out of total no. of elements equal to 8

dim span idems 4 vs no. of idems 4

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"RG1" = {5, 8}

"RG2" = {6, 7}

"RG3" = {2, 3}

"RG4" = {1, 4}

$$M_C = \begin{pmatrix} 3 & -1 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & 3 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & 3 & -1 & -1 & -1 & -1 & -1 \\ 3 & -1 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & -1 & 3 \\ -1 & -1 & -1 & -1 & -1 & 3 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & 3 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & -1 & 3 \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 \\ \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} \\ 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 \\ \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} \\ 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 \\ \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} \\ 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_c

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_c

[0., 0., 0., 0., 0., 0., 4., 3.]

Eigenvalues M_c−scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_c−scaled

[0., 0., 0., 0., 0., 0., 4.571428571, 3.428571429]

NullSpace M_c

{[1, 1, 0, 0, 1, 1, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0], [1, 1, 0, 0, 0, 1, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0]}

NullSpace N_c

{[0, −1, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 0, 0, 1], [−1, 0, 1, 0, 0, 0, 0, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace M_0

{[0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, −1], [0, 0, 0, 0, 0, −1, 1, 0]}

NullSpace N_0

{[−1, 0, 0, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 1, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1], [0, −1, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{}

NullSpace N

{[0, 0, 1, 0, 0, −1, 0, 0], [0, 0, 0, 1, −1, 0, 0, 0], [0, 0, 0, 0, −1, 0, 1, 0], [0, 0, 0, 0, 0, −1, 0, 1], [1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, −1, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

==

{6, 7}

R: [2, 3, 2, 3, 4, 8, 5, 1]
B: [6, 7, 6, 7, 8, 4, 1, 5]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{-3}{2048}(-1+s)(5+3s)(101-25s+9s^2-7s^3+2s^4)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", 1 + v[2] v[3]
"B CYCLES", (1 + v[5] v[8]) (1 + v[1] v[4] v[6] v[7])

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [1. I, −1. I, 0., 0., 1., −1., 1., −1.]

NullSpace of R

{[0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1, 0, 0]}

NullSpace of B

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0]}

NullSpace of R^*

{[0, −1, 0, 1, 0, 0, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0]}

NullSpace of B^*

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 0 & 1 & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} & \frac{2}{3} \\ 1 & 0 & 1 & 0 & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} & \frac{1}{3} \\ 0 & 1 & 0 & 1 & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} & \frac{2}{3} \\ 1 & 0 & 1 & 0 & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & 1 \\ \frac{4}{9} & \frac{5}{9} & \frac{4}{9} & \frac{5}{9} & \frac{1}{3} & 0 & 1 & \frac{2}{3} \\ \frac{5}{9} & \frac{4}{9} & \frac{5}{9} & \frac{4}{9} & \frac{2}{3} & 1 & 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3   "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}\,(\,v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]\,)$
degree 2: $\frac{1}{4}\,(\,v[1]v[4] + v[2]v[3] + v[5]v[8] + v[6]v[7]\,)$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 3, 7, 8}, {2, 4, 5, 6}}

"PT3" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"PT4" = {{1, 3, 5, 6}, {2, 4, 7, 8}}

"RG1" = {5, 8}

"RG2" = {6, 7}

"RG3" = {2, 3}

"RG4" = {1, 4}

$$\pi 2 = [0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0]$$

supp $\pi 2$ = {3, 8, 25, 26}

$$u2 = [9, 0, 9, 3, 4, 5, 6, 9, 0, 6, 5, 4, 3, 9, 3, 4, 5, 6, 6, 5, 4, 3, 3, 6, 9, 9, 6, 3]$$

supp u2 = {1, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

Action of R on ranges, [[4], [1], [3], [3]]
Action of B on ranges, [[1], [4], [2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

RPARTS [3, 3, 4, 4]

BPARTS [3, 1, 4, 2]

$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{4}{9} \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[8, 3, 2, 7, 8, 7, 3, 2]

B-BLOCKS,

[7, 5, 6, 8, 4, 1, 3, 2]

with invariant measure, [1, 4, 4, 1, 2, 2, 2, 2]

N by blocks, N – check: true

b_1 = {1, 3, 6, 8}
b_2 = {1, 3, 5, 6}
b_3 = {2, 4, 7, 8}
b_4 = {2, 4, 5, 7}
b_5 = {1, 3, 7, 8}
b_6 = {2, 4, 5, 6}
b_7 = {1, 3, 5, 7}
b_8 = {2, 4, 6, 8}

dim(span of partition vectors), rank(N_0), rank(N): 4, 4, 4

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & h[1] & 0 & 0 & 0 & 0 \\ 0 & h[2] & h[1] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[1] & h[2] & 0 & 0 & 0 & 0 & 0 \\ h[1] & 0 & 0 & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & 0 & 0 & h[1] \\ 0 & 0 & 0 & 0 & 0 & h[2] & h[1] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[1] & h[2] & 0 \\ 0 & 0 & 0 & 0 & h[1] & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 20, Shape: 11 ⊕ 9/6

$$\text{CLB} = \begin{pmatrix} 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3}}, true

Ω_B in Vec(K)? , {{1, 4, 6, 7}, {5, 8}}, false

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{3}{16} & 0 & 0 & \frac{3}{16} & \frac{1}{8} & \frac{3}{16} & \frac{3}{16} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 3, 6, 8}, {2, 4, 5, 7}}

1, "range", [5, 8], [[8, 5, 8, 5, 5, 8, 5, 8], [5, 8, 5, 8, 8, 5, 8, 5]]

2, "range", [6, 7], [[7, 6, 7, 6, 6, 7, 6, 7], [6, 7, 6, 7, 7, 6, 7, 6]]

3, "range", [2, 3], [[3, 2, 3, 2, 2, 3, 2, 3], [2, 3, 2, 3, 3, 2, 3, 2]]

4, "range", [1, 4], [[4, 1, 4, 1, 1, 4, 1, 4], [1, 4, 1, 4, 4, 1, 4, 1]]

2, "partition", {{1, 3, 7, 8}, {2, 4, 5, 6}}

1, "range", [5, 8], [[8, 5, 8, 5, 5, 5, 8, 8], [5, 8, 5, 8, 8, 8, 5, 5]]

2, "range", [6, 7], [[7, 6, 7, 6, 6, 6, 7, 7], [6, 7, 6, 7, 7, 7, 6, 6]]

3, "range", [2, 3], [[3, 2, 3, 2, 2, 2, 3, 3], [2, 3, 2, 3, 3, 3, 2, 2]]

4, "range", [1, 4], [[4, 1, 4, 1, 1, 1, 4, 4], [1, 4, 1, 4, 4, 4, 1, 1]]

3, "partition", {{1, 3, 5, 7}, {2, 4, 6, 8}}

1, "range", [5, 8], [[8, 5, 8, 5, 8, 5, 8, 5], [5, 8, 5, 8, 5, 8, 5, 8]]

2, "range", [6, 7], [[7, 6, 7, 6, 7, 6, 7, 6], [6, 7, 6, 7, 6, 7, 6, 7]]

3, "range", [2, 3], [[3, 2, 3, 2, 3, 2, 3, 2], [2, 3, 2, 3, 2, 3, 2, 3]]

4, "range", [1, 4], [[4, 1, 4, 1, 4, 1, 4, 1], [1, 4, 1, 4, 1, 4, 1, 4]]

4, "partition", {{1, 3, 5, 6}, {2, 4, 7, 8}}

1, "range", [5, 8], [[8, 5, 8, 5, 8, 8, 5, 5], [5, 8, 5, 8, 5, 5, 8, 8]]

2, "range", [6, 7], [[7, 6, 7, 6, 7, 7, 6, 6], [6, 7, 6, 7, 6, 6, 7, 7]]

3, "range", [2, 3], [[3, 2, 3, 2, 3, 3, 2, 2], [2, 3, 2, 3, 2, 2, 3, 3]]

4, "range", [1, 4], [[4, 1, 4, 1, 4, 4, 1, 1], [1, 4, 1, 4, 1, 1, 4, 4]] "group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5$

$t^9 + 6t^{10}$

n−choose−rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$

{3, 8, 25, 26}

$u2 =$
$\begin{pmatrix} 9 & 0 & 9 & 3 & 4 & 5 & 6 & 9 & 0 & 6 & 5 & 4 & 3 & 9 & 3 & 4 & 5 & 6 & 6 & 5 & 4 & 3 & 3 & 6 & 9 & 9 & 6 & 3 \end{pmatrix}$

{1, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{5}{9} & \frac{4}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{4}{9} & \frac{5}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{5}{9} & \frac{4}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{4}{9} & \frac{5}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{4}{9} & \frac{5}{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{5}{9} & \frac{4}{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{5}{9} & 0 & 0 & \frac{4}{9} & 0 & 0 & 0 & 0 \\ \frac{4}{9} & 0 & 0 & \frac{5}{9} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{6} & \frac{5}{36} & \frac{1}{9} & \frac{1}{12} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{12} & \frac{1}{9} & \frac{5}{36} & \frac{1}{6} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{6} & \frac{5}{36} & \frac{1}{9} & \frac{1}{12} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{12} & \frac{1}{9} & \frac{5}{36} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{4} & \frac{1}{6} & \frac{1}{12} & 0 \\ \frac{5}{36} & \frac{1}{9} & \frac{5}{36} & \frac{1}{9} & \frac{1}{6} & \frac{1}{4} & 0 & \frac{1}{12} \\ \frac{1}{9} & \frac{5}{36} & \frac{1}{9} & \frac{5}{36} & \frac{1}{12} & 0 & \frac{1}{4} & \frac{1}{6} \\ \frac{1}{12} & \frac{1}{6} & \frac{1}{12} & \frac{1}{6} & 0 & \frac{1}{12} & \frac{1}{6} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 0 & 4 & 0 & \frac{8}{3} & \frac{20}{9} & \frac{16}{9} & \frac{4}{3} \\ 0 & 4 & 0 & 4 & \frac{4}{3} & \frac{16}{9} & \frac{20}{9} & \frac{8}{3} \\ 4 & 0 & 4 & 0 & \frac{8}{3} & \frac{20}{9} & \frac{16}{9} & \frac{4}{3} \\ 0 & 4 & 0 & 4 & \frac{4}{3} & \frac{16}{9} & \frac{20}{9} & \frac{8}{3} \\ \frac{8}{3} & \frac{4}{3} & \frac{8}{3} & \frac{4}{3} & 4 & \frac{8}{3} & \frac{4}{3} & 0 \\ \frac{20}{9} & \frac{16}{9} & \frac{20}{9} & \frac{16}{9} & \frac{8}{3} & 4 & 0 & \frac{4}{3} \\ \frac{16}{9} & \frac{20}{9} & \frac{16}{9} & \frac{20}{9} & \frac{4}{3} & 0 & 4 & \frac{8}{3} \\ \frac{4}{3} & \frac{8}{3} & \frac{4}{3} & \frac{8}{3} & 0 & \frac{4}{3} & \frac{8}{3} & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 1, 1, 0, 0, -1, -1, 0]$

$$\ker N_c = \begin{pmatrix} -1 & -1 & 0 & 0 & 0 & 1 & 1 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} t & -s & -s & t & s & -t & -t & s \\ s & -s & -s & s & t & -t & -t & t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 0 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -s & 0 & -t & 0 \\ s & 0 & t & 0 \\ -s & 0 & -t & 0 \\ s & 0 & t & 0 \\ 0 & s & 0 & t \\ 0 & t & 0 & s \\ 0 & -t & 0 & -s \\ 0 & -s & 0 & -t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} s+t & s & 0 & -s & 0 \\ 0 & t & 0 & s & 0 \\ s+t & s & 0 & -s & 0 \\ 0 & t & 0 & s & 0 \\ 0 & 0 & s & 0 & t \\ 0 & 0 & t & 0 & s \\ s+t & s+t & -t & 0 & -s \\ s+t & s+t & -s & 0 & -t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 1 & 0 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 1 \\ 1 & 0 & 1 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 1 & 0 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 1 \\ 1 & 0 & 1 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 1 & \frac{2}{3} \\ 0 & 1 & 0 & 1 & 0 & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 1 & 0 & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} & \frac{2}{3} \\ 0 & 1 & 0 & 1 & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} & \frac{1}{3} \\ 1 & 0 & 1 & 0 & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} & \frac{2}{3} \\ 0 & 1 & 0 & 1 & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{4}{9} & \frac{5}{9} & \frac{4}{9} & \frac{5}{9} & \frac{2}{3} & 1 & 0 & \frac{1}{3} \\ \frac{5}{9} & \frac{4}{9} & \frac{5}{9} & \frac{4}{9} & \frac{1}{3} & 0 & 1 & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}, \; BB^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 1 & 0 & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} & \frac{1}{3} \\ 0 & 1 & 0 & 1 & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} & \frac{2}{3} \\ 1 & 0 & 1 & 0 & \frac{2}{3} & \frac{5}{9} & \frac{4}{9} & \frac{1}{3} \\ 0 & 1 & 0 & 1 & \frac{1}{3} & \frac{4}{9} & \frac{5}{9} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 & \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{5}{9} & \frac{4}{9} & \frac{5}{9} & \frac{4}{9} & \frac{2}{3} & 1 & 0 & \frac{1}{3} \\ \frac{4}{9} & \frac{5}{9} & \frac{4}{9} & \frac{5}{9} & \frac{1}{3} & 0 & 1 & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 16T + 0\Omega$$

$\Omega \left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T \left(\frac{1}{4}\ \frac{1}{6}\ \frac{1}{9}\ \frac{5}{36}\ \frac{1}{6}\ \frac{1}{4}\ \frac{1}{12}\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{9}\ \frac{5}{36}\ \frac{1}{6}\ 0\ \frac{1}{4}\ 0\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM} \left(4\ \frac{8}{3}\ \frac{16}{9}\ \frac{20}{9}\ \frac{8}{3}\ 4\ \frac{4}{3}\ \frac{8}{3}\ \frac{4}{3}\ \frac{16}{9}\ \frac{20}{9}\ \frac{8}{3}\ 0\ 4\ 0\ 4\right)$

"IS MN in Vec(K)?", true

$\mathrm{MN} \left(4\ \frac{8}{3}\ \frac{16}{9}\ \frac{20}{9}\ \frac{8}{3}\ 4\ \frac{4}{3}\ \frac{8}{3}\ \frac{4}{3}\ \frac{16}{9}\ \frac{20}{9}\ \frac{8}{3}\ 0\ 4\ 0\ 4\right)$

$\boldsymbol{\tau} = 32/1$, rank = 2 , ratio = 16/1 , $n^2 / r = 32/1$

$\boldsymbol{\tau}' = 32/1$, $r' = 1/2$, $\boldsymbol{\tau} / n^2 = 1/2$

$p^2 = 1/8$, min $\boldsymbol{\tau} = 8/1$, $\boldsymbol{\tau}$–check is positive? 24/1

max r = 8/1 , r–check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 4, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 16
out of total no. of elements equal to 32

dim span idems 12 vs no. of idems 16

"PT1" = {{1, 3, 6, 8}, {2, 4, 5, 7}}

"PT2" = {{1, 3, 7, 8}, {2, 4, 5, 6}}

"PT3" = {{1, 3, 5, 7}, {2, 4, 6, 8}}

"PT4" = {{1, 3, 5, 6}, {2, 4, 7, 8}}

"RG1" = {5, 8}

"RG2" = {6, 7}

"RG3" = {2, 3}

"RG4" = {1, 4}

$$M_C = \begin{pmatrix} 3 & -1 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & 3 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & 3 & -1 & -1 & -1 & -1 & -1 \\ 3 & -1 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & -1 & 3 \\ -1 & -1 & -1 & -1 & -1 & 3 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & 3 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & -1 & 3 \end{pmatrix} \quad N_C =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{13}{24} & \frac{31}{72} & \frac{23}{72} & \frac{5}{24} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{5}{24} & \frac{23}{72} & \frac{31}{72} & \frac{13}{24} \\ \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{13}{24} & \frac{31}{72} & \frac{23}{72} & \frac{5}{24} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{5}{24} & \frac{23}{72} & \frac{31}{72} & \frac{13}{24} \\ \frac{13}{24} & \frac{5}{24} & \frac{13}{24} & \frac{5}{24} & \frac{7}{8} & \frac{13}{24} & \frac{5}{24} & \frac{-1}{8} \\ \frac{31}{72} & \frac{23}{72} & \frac{31}{72} & \frac{23}{72} & \frac{13}{24} & \frac{7}{8} & \frac{-1}{8} & \frac{5}{24} \\ \frac{23}{72} & \frac{31}{72} & \frac{23}{72} & \frac{31}{72} & \frac{5}{24} & \frac{-1}{8} & \frac{7}{8} & \frac{13}{24} \\ \frac{5}{24} & \frac{13}{24} & \frac{5}{24} & \frac{13}{24} & \frac{-1}{8} & \frac{5}{24} & \frac{13}{24} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{13}{21} & \frac{31}{63} & \frac{23}{63} & \frac{5}{21} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & \frac{5}{21} & \frac{23}{63} & \frac{31}{63} & \frac{13}{21} \\ 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{13}{21} & \frac{31}{63} & \frac{23}{63} & \frac{5}{21} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & \frac{5}{21} & \frac{23}{63} & \frac{31}{63} & \frac{13}{21} \\ \frac{13}{21} & \frac{5}{21} & \frac{13}{21} & \frac{5}{21} & 1 & \frac{13}{21} & \frac{5}{21} & \frac{-1}{7} \\ \frac{31}{63} & \frac{23}{63} & \frac{31}{63} & \frac{23}{63} & \frac{13}{21} & 1 & \frac{-1}{7} & \frac{5}{21} \\ \frac{23}{63} & \frac{31}{63} & \frac{23}{63} & \frac{31}{63} & \frac{5}{21} & \frac{-1}{7} & 1 & \frac{13}{21} \\ \frac{5}{21} & \frac{13}{21} & \frac{5}{21} & \frac{13}{21} & \frac{-1}{7} & \frac{5}{21} & \frac{13}{21} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 3., 2.247353463, 0.6217973830, 1.130849154]

Eigenvalues M_C−scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 0., 0., 0., 3.428571429, 2.568403958, 0.7106255807, 1.292399033]

NullSpace M_C

{[0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [1, 1, 0, 0, 1, 1, 0, 0], [1, 1, 0, 0, 1, 0, 1, 0], [0, 0, 0, 0, −1, 0, 0, 1] }

NullSpace N_C

{[−1, −1, 0, 0, 1, 0, 0, 1], [−1, −1, 0, 0, 0, 1, 1, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 0., 4., 2.247353463, 0.6217973830, 1.130849154]

NullSpace M_0

{[0, 0, 0, 0, −1, 0, 0, 1], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0]}

NullSpace N_0

{[−1, 0, 1, 0, 0, 0, 0, 0], [−1, 0, 0, −1, 0, 1, 1, 0], [0, 1, 0, −1, 0, 0, 0, 0], [−1, 0, 0, −1, 1, 0, 0, 1]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 0., 4., −0.6217973825, −2.247353464, −1.130849154]

NullSpace M

{}

NullSpace N

{[0, −1, 0, 1, 0, 0, 0, 0], [−1, −1, 0, 0, 1, 0, 0, 1], [−1, −1, 0, 0, 0, 1, 1, 0], [−1, 0, 1, 0, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 9 & 0 & 9 & 3 & 4 & 5 & 6 \\ 9 & 0 & 9 & 0 & 6 & 5 & 4 & 3 \\ 0 & 9 & 0 & 9 & 3 & 4 & 5 & 6 \\ 9 & 0 & 9 & 0 & 6 & 5 & 4 & 3 \\ 3 & 6 & 3 & 6 & 0 & 3 & 6 & 9 \\ 4 & 5 & 4 & 5 & 3 & 0 & 9 & 6 \\ 5 & 4 & 5 & 4 & 6 & 9 & 0 & 3 \\ 6 & 3 & 6 & 3 & 9 & 6 & 3 & 0 \end{pmatrix}$$

==

40, [1, −1, 1, 1, −1, 1, −1, 1]
==

60, [1, 1, 1, −1, 1, 1, −1, −1]

==

{2, 3, 5, 7}

R: [2, 7, 6, 3, 8, 4, 5, 1]
B: [6, 3, 2, 7, 4, 8, 1, 5]

TRACE TWO = 1

det AT = $1\,(t)^4$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 8

Level 2 det = $\frac{5}{8192}(-1+s)(-1010-81s-349s^2-63s^3+s^4+21s^5+25s^6+11s^7+5s^8)$

RANK of R is 8

R ranking is 1, "vs", 8

RBAR ranking 1, "vs", 8

RANK of B is 8

B ranking is 1, "vs", 8

BBAR ranking 1, "vs", 8

"R CYCLES", (1 + v[3] v[4] v[6]) (1 + v[1] v[2] v[5] v[7] v[8])
"B CYCLES", (1 + v[2] v[3]) (1 + v[1] v[4] v[5] v[6] v[7] v[8])

Eigenvalues

R: [-0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 0.3090169942 + 0.9510565160 I, -0.8090169942 + 0.5877852520 I, -0.8090169942 - 0.5877852520 I, 0.3090169942 - 0.9510565160 I, 1., 1.]

B: [0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I, -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., -1., 1.,

−1.]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 8

"RANK of the KERNEL is ", 8

"IdemSolvability Check", 3 "Trace mark", 8, "Rank mark", 8, "for kernel rank", 8

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{28}$ ($v[1]v[8] + v[5]v[7] + v[1]v[4] + v[3]v[8] + v[3]v[6] + v[6]v[7] + v[4]v[7] + v[5]v[6] + v[2]v[4] + v[2]v[6] + v[4]v[5] + v[2]v[7] + v[1]v[3] + v[1]v[7] + v[6]v[8] + v[5]v[8] + v[2]v[3] + v[1]v[5] + v[4]v[8] + v[1]v[2] + v[1]v[6] + v[3]v[4] + v[2]v[8] + v[7]v[8] + v[4]v[6] + v[2]v[5] + v[3]v[7] + v[3]v[5]$)

degree 3 : $\frac{1}{56}$ ($v[2]v[3]v[8] + v[6]v[7]v[8] + v[1]v[2]v[6] + v[1]v[4]v[5] + v[2]v[4]v[6] + v[2]v[7]v[8] + v[3]v[6]v[7] + v[4]v[7]v[8] + v[3]v[4]v[8] + v[1]v[2]v[3] + v[2]v[$

3]v[4] + v[2]v[3]v[6] + v[3]v[4]v[7] + v[4]v[5]v[6] + v[3]v[5]v[6] + v[1]v[5]v[7] + v[
2]v[3]v[7] + v[5]v[6]v[8] + v[1]v[3]v[5] + v[1]v[4]v[6] + v[2]v[4]v[8] + v[1]v[7]v[8]
+ v[1]v[2]v[8] + v[1]v[2]v[4] + v[2]v[4]v[7] + v[4]v[5]v[7] + v[3]v[4]v[6] + v[1]v[2]v
[7] + v[1]v[6]v[8] + v[4]v[5]v[8] + v[1]v[4]v[8] + v[3]v[4]v[5] + v[5]v[6]v[7] + v[1]v[
6]v[7] + v[3]v[6]v[8] + v[2]v[5]v[7] + v[2]v[5]v[6] + v[2]v[6]v[7] + v[1]v[5]v[8] + v[
1]v[5]v[6] + v[5]v[7]v[8] + v[1]v[3]v[4] + v[2]v[3]v[5] + v[1]v[3]v[6] + v[1]v[4]v[7]
+ v[1]v[2]v[5] + v[2]v[4]v[5] + v[2]v[5]v[8] + v[3]v[7]v[8] + v[4]v[6]v[7] + v[3]v[5]
v[7] + v[2]v[6]v[8] + v[3]v[5]v[8] + v[1]v[3]v[8] + v[1]v[3]v[7] + v[4]v[6]v[8])

degree 4 : $\frac{1}{70}$ (v[1]v[2]v[4]v[5] + v[1]v[3]v[4]v[7] + v[1]v[2]v[5]v[8] + v[1]v[4]v[6]
v[8] + v[1]v[6]v[7]v[8] + v[1]v[3]v[6]v[8] + v[3]v[6]v[7]v[8] + v[3]v[4]v[6]v[7] + v[2]
v[5]v[6]v[8] + v[2]v[4]v[5]v[7] + v[2]v[3]v[5]v[7] + v[2]v[4]v[6]v[8] + v[1]v[2]v[6]v[
7] + v[1]v[4]v[6]v[7] + v[2]v[3]v[5]v[8] + v[3]v[4]v[7]v[8] + v[1]v[2]v[4]v[6] + v[3]v
[4]v[5]v[8] + v[2]v[4]v[7]v[8] + v[2]v[3]v[4]v[8] + v[5]v[6]v[7]v[8] + v[2]v[4]v[5]v[6
] + v[1]v[3]v[5]v[8] + v[1]v[4]v[5]v[8] + v[1]v[2]v[4]v[8] + v[3]v[4]v[5]v[6] + v[4]v[
6]v[7]v[8] + v[1]v[3]v[4]v[5] + v[2]v[3]v[4]v[7] + v[2]v[5]v[6]v[7] + v[1]v[3]v[5]v[7]
+ v[2]v[6]v[7]v[8] + v[2]v[4]v[6]v[7] + v[2]v[4]v[5]v[8] + v[1]v[2]v[4]v[7] + v[1]v[2]
v[6]v[8] + v[1]v[5]v[6]v[8] + v[1]v[3]v[4]v[6] + v[4]v[5]v[7]v[8] + v[1]v[3]v[7]v[8] +
v[3]v[4]v[5]v[7] + v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[8] + v[1]v[2]v[5]v[6] + v[2]v[3]v
[4]v[6] + v[3]v[5]v[6]v[8] + v[1]v[5]v[7]v[8] + v[2]v[3]v[7]v[8] + v[3]v[5]v[7]v[8] +
v[1]v[2]v[5]v[7] + v[4]v[5]v[6]v[8] + v[2]v[3]v[5]v[6] + v[4]v[5]v[6]v[7] + v[1]v[4]v[
5]v[6] + v[2]v[5]v[7]v[8] + v[1]v[5]v[6]v[7] + v[1]v[3]v[5]v[6] + v[2]v[3]v[4]v[5] + v
[2]v[3]v[6]v[8] + v[1]v[4]v[7]v[8] + v[1]v[4]v[5]v[7] + v[3]v[4]v[6]v[8] + v[1]v[2]v[7
]v[8] + v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[4] + v[1]v[3]v[6]v[7] + v[1]v[2]v[3]v[8] + v[
1]v[2]v[3]v[6] + v[1]v[2]v[3]v[7] + v[1]v[2]v[3]v[5])

degree 5 : $\frac{1}{56}$ (v[1]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[8] + v[2]v[3]v[4]v[5]v[8] +
v[2]v[4]v[5]v[7]v[8] + v[1]v[5]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]
v[8] + v[2]v[3]v[4]v[5]v[7] + v[1]v[2]v[5]v[7]v[8] + v[1]v[2]v[3]v[6]v[8] + v[2]v[4]v[
5]v[6]v[8] + v[1]v[4]v[6]v[7]v[8] + v[2]v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[5]v[7] + v[1]
v[2]v[4]v[6]v[8] + v[2]v[4]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6] + v[1]v[2]v[4]v[6]v[7]
+ v[2]v[3]v[5]v[6]v[8] + v[1]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[5]v[
6]v[8] + v[1]v[3]v[5]v[6]v[8] + v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[4]v[5]v[6] + v[1]v[3]
v[4]v[6]v[7] + v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[7]v[8] + v[
4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[8] + v[2]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[
8] + v[1]v[2]v[4]v[7]v[8] + v[1]v[3]v[6]v[7]v[8] + v[1]v[2]v[5]v[6]v[7] + v[2]v[3]v[4]
v[6]v[8] + v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6] + v[1]v[2]v[3]v[5]v[6] + v[3]v[
4]v[5]v[6]v[7] + v[2]v[4]v[5]v[6]v[7] + v[1]v[4]v[5]v[6]v[8] + v[1]v[2]v[4]v[5]v[7] +
v[3]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7] + v[1]v[2]v[3]v[5]v[7] + v[1]v[2]v[3]v[4]
v[7] + v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[8] + v[1]v[2]v[6]v[7]v[8] + v[3]v[4]v[
5]v[7]v[8] + v[1]v[2]v[4]v[5]v[8] + v[1]v[2]v[3]v[4]v[6] + v[1]v[3]v[4]v[7]v[8] + v[1]

$v[3]v[5]v[7]v[8]$)
degree 6 : $\frac{1}{28}$ ($v[1]v[2]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[5]v[6]v[8] + v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7] + v[1]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[7] + v[2]v[3]v[4]v[5]v[7]v[8] + v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[8] + v[1]v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[8] + v[1]v[2]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[5]v[6]v[7] + v[1]v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[6]$)
degree 7 : $\frac{1}{8}$ ($v[1]v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[7]v[8]$)
degree 8 : 1 ($v[1]$) ($v[2]$) ($v[3]$) ($v[8]$) ($v[7]$) ($v[6]$) ($v[5]$) ($v[4]$)

Group spectrum $1 + t + t^2 + t^3 + t^4 + t^5 + t^6 + t^7 + t^8$

KERNEL STRUCTURE

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$\pi 8 = [1]$$

supp $\pi 8$ = {1}

$$u8 = [1]$$

supp u8 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[8, 1, 4, 6, 7, 3, 2, 5]

B−BLOCKS,

[7, 3, 2, 5, 8, 1, 4, 6]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{1\}$
$b_2 = \{2\}$
$b_3 = \{3\}$
$b_4 = \{4\}$
$b_5 = \{5\}$
$b_6 = \{6\}$
$b_7 = \{7\}$
$b_8 = \{8\}$

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[1] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[1] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[1] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[1] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[1] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[1] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 49, Shape: $48 \oplus 1/0$

$$CLB = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{5} & \frac{1}{5} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{5} & \frac{1}{5} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{5} & \frac{1}{5} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 5, 7, 8}, {3, 4, 6}}, true

Ω_B in Vec(K)? , {{2, 3}, {1, 4, 5, 6, 7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

1, "range", [1, 2, 3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 4, 3, 2, 1], [8, 7, 6, 5, 4, 2, 1, 3], [8, 7, 6, 5, 4, 1, 3, 2], [8, 7, 6, 5, 3, 4, 1, 2], [8, 7, 6, 5, 3, 2, 4, 1], [8, 7, 6, 5, 3, 1, 2, 4], [8, 7, 6, 5, 2, 4, 3, 1], [8, 7, 6, 5, 2, 3, 1, 4], [8, 7, 6, 5, 2, 1, 4, 3], [8, 7, 6, 5, 1, 4, 2, 3], [...20140 terms...], [1, 2, 3, 4, 8, 5, 7, 6], [1, 2, 3, 4, 7, 8, 5, 6], [1, 2, 3, 4, 7, 6, 8, 5], [1, 2, 3, 4, 7, 5, 6, 8], [1, 2, 3, 4, 6, 8, 7, 5], [1, 2, 3, 4, 6, 7, 5, 8], [1, 2, 3, 4, 6, 5, 8, 7], [1, 2, 3, 4, 5, 8, 6, 7], [1, 2, 3, 4, 5, 7, 8, 6], [1, 2, 3, 4, 5, 6, 7, 8]]

"group has", 20160, "elements"   Group element 1,1 = $\begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0 \end{pmatrix}$

g_1 =   [[1, 8], [2, 7], [3, 6], [4, 5]]

g_2 =   [[1, 8, 3, 6, 2, 7], [4, 5]]

g_3 =   [[1, 8, 2, 7, 3, 6], [4, 5]]

g_4 =   [[1, 8, 2, 7], [3, 6, 4, 5]]

g_5 =   [[1, 8], [2, 7, 4, 5, 3, 6]]

linear dimension, 50

"Symmetric?", true

Is Z in Vec(K)? true
$\big(1440h[2] + 7560h[1] \quad -1080h[2] - 2520h[1] \quad 5760h[2] + 10080h[1] \quad 3600h[2] + 2$

"Basis for Z(G)"

1, "coeff", 2520

Z[1] = $\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$

2, "coeff", 360

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 7. & -1. & -1. & -1. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3 + t^4 + t^5 + t^6 + t^7 + t^8$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 5t^4 + 7t^5 + 11t^6 + 15t^7 + 22t^8 + 29t^9 + 40t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 8 = (1)$

{1}

$u8 = (1)$

{1}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 7 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u7 = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$\pi 6 =$
$(2 \;\; 2$

$u6 =$
$\left(\frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32} \;\; \frac{1}{32}\right.$

picheck $\begin{pmatrix} 42 & 42 & 42 & 42 & 42 & 42 & 42 & 42 \end{pmatrix}$

$\pi 5 =$
$(6 \;\; 6$

$u5 =$
$\left(\frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256}\right.$

picheck $\begin{pmatrix} 210 & 210 & 210 & 210 & 210 & 210 & 210 & 210 \end{pmatrix}$

$\pi 4 =$
$(24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24$

$u4 =$
$\left(\frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512}\right.$

picheck $\begin{pmatrix} 840 & 840 & 840 & 840 & 840 & 840 & 840 & 840 \end{pmatrix}$

$\pi 3 =$
$(120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120$

$u3 =$
$\left(\frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096}\right.$

picheck $\begin{pmatrix} 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 \end{pmatrix}$

$\pi 2 =$

$(720 \quad 720 \quad 720 \quad 720 \quad 720 \quad 720 \quad 720 \quad 720 \quad 720 \quad 720 \quad 720 \quad 720 \quad 720 \quad 720 \quad 720$

$u2 =$
$\left(\frac{45}{16384} \quad \frac{45}{16384} \quad \frac{45}{16384} \quad \frac{45}{16384} \quad \frac{45}{16384} \quad \frac{45}{16384} \quad \frac{45}{16384} \quad \frac{45}{16384} \quad \frac{45}{16384} \quad \frac{45}{16384} \quad \frac{45}{16384}\right.$

picheck $(5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040)$

$\pi 1 = (5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040)$

$u1 = \left(\frac{315}{131072} \quad \frac{315}{131072} \quad \frac{315}{131072} \quad \frac{315}{131072} \quad \frac{315}{131072} \quad \frac{315}{131072} \quad \frac{315}{131072} \quad \frac{315}{131072}\right)$

picheck $(5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040 \quad 5040)$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0, 0, 0]$

$\ker N_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} t+s & t+s & t+s & t+s & t+s & t+s & t+s & t+s \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & s & 0 & 0 & t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s & t \\ 0 & t & 0 & 0 & s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & t & s \\ s & 0 & t & 0 & 0 & 0 & 0 \\ t & 0 & s & 0 & 0 & 0 & 0 \\ -t & -t & -t & -t+s & -t & -t & -t \\ -s & -s & -s & -s+t & -s & -s & -s \end{pmatrix}$$

RB checks

$$\ker M_c = \begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&1&0&0&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&0&0&s&t&0&0\\ 0&t&0&0&0&0&s&0\\ 0&0&0&0&t&s&0&0\\ 0&s&0&0&0&0&t&0\\ t&0&s&0&0&0&0&0\\ s&0&t&0&0&0&0&0\\ 0&0&0&s&0&0&0&t\\ 0&0&0&t&0&0&0&s \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 1&1&1&1&1&1&1&1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 8, "vs", 8

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 1T + 48\Omega$$

Ω

$\left(\frac{31}{8} \;\; \frac{5}{8} \;\; \frac{17}{8} \;\; \frac{3}{8} \;\; \frac{5}{16} \;\; \frac{-7}{8} \;\; \frac{5}{8} \;\; \frac{13}{4} \;\; \frac{21}{8} \;\; \frac{13}{8} \;\; \frac{7}{8} \;\; \frac{3}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{21}{8} \;\; 2 \;\; \frac{9}{8} \;\; \frac{1}{2} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; 2 \;\; \frac{11}{8} \;\; \frac{5}{8} \right.$

T

$\left(6 \;\; 0 \;\; 2 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 5 \;\; 4 \;\; 2 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 4 \;\; 3 \;\; 2 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 3 \;\; 2 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \right.$

"IS NM in Vec(K)?", true

NM
(192 30 104 18 15 −42 30 161 130 80 43 18 6 6 130 99 56 25 6

"IS MN in Vec(K)?", true

MN
(192 30 104 18 15 −42 30 161 130 80 43 18 6 6 130 99 56 25 6

$\boldsymbol{\tau} = 8/1$, rank = 8 , ratio = 1/1 , $n^2 / r = 8/1$

$\boldsymbol{\tau}' = 56/1$, $r' = 7/8$, $\boldsymbol{\tau} / n^2 = 1/8$

$p^2 = 1/8$, min $\boldsymbol{\tau} = 8/1$, $\boldsymbol{\tau}$–check is positive? 0/1

max r = 8/1 , r–check is positive? 0/1

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 20160

KERNEL HAS LINEAR DIMENSION 50
out of total no. of elements equal to 20160

dim span idems 1 vs no. of idems 1

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0] }

NullSpace N_C

{[1, 1, 1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[0, 0, −1, 0, 1, 0, 0, 0], [0, 0, −1, 1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, 1, −1, 0, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 1, 0], [1, 0, −1, 0, 0, 0, 0, 0], [0, 0, −1, 0, 0, 1, 0, 0]}

NullSpace N_0

{}

Eigenvalues M

[7., −1., −1., −1., −1., −1., −1., −1.]

Eigenvalues N

[7., −1., −1., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

===

{2, 3, 5, 8}

R: [2, 7, 6, 3, 8, 4, 1, 5]
B: [6, 3, 2, 7, 4, 8, 5, 1]

TRACE TWO = 2

det AT = $-1\,(t)^4$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 8

Level 2 det = $\frac{-5}{4096}\left(101 - 62s - 6s^2 + 2s^3 + 5s^4\right)\left(-5 + s^2\right)\left(1 + s\right)\left(-1 + s\right)^2$

RANK of R is 8

R ranking is 1, "vs", 8

RBAR ranking 1, "vs", 8

RANK of B is 8

B ranking is 1, "vs", 8

BBAR ranking 1, "vs", 8

"R CYCLES", (1 + v[3] v[4] v[6]) (1 + v[1] v[2] v[7]) (1 + v[5] v[8])
"B CYCLES", (1 + v[2] v[3]) (1 + v[4] v[5] v[7]) (1 + v[1] v[6] v[8])

Eigenvalues

R: [-1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., 1., 1.]

B: [-1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., 1., 1.]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0&1&1&1&1&1&1&1\\ 1&0&1&1&1&1&1&1\\ 1&1&0&1&1&1&1&1\\ 1&1&1&0&1&1&1&1\\ 1&1&1&1&0&1&1&1\\ 1&1&1&1&1&0&1&1\\ 1&1&1&1&1&1&0&1\\ 1&1&1&1&1&1&1&0 \end{pmatrix} \quad N = \begin{pmatrix} 0&1&1&1&1&1&1&1\\ 1&0&1&1&1&1&1&1\\ 1&1&0&1&1&1&1&1\\ 1&1&1&0&1&1&1&1\\ 1&1&1&1&0&1&1&1\\ 1&1&1&1&1&0&1&1\\ 1&1&1&1&1&1&0&1\\ 1&1&1&1&1&1&1&0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 8

"RANK of the KERNEL is ", 8

"IdemSolvability Check", 3 "Trace mark", 8, "Rank mark", 8, "for kernel rank", 8

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{24}$ ($v[1]v[8] + v[5]v[7] + 6v[1]v[4] + v[3]v[8] + v[3]v[6] + 6v[6]v[7] + v[4]v[7] + v[5]v[6] + v[2]v[4] + v[2]v[6] + v[4]v[5] + v[2]v[7] + v[1]v[3] + v[1]v[7] + v[6]v[8] + 6v[5]v[8] + 6v[2]v[3] + v[1]v[5] + v[4]v[8] + v[1]v[2] + v[1]v[6] + v[3]v[4] + v[2]v[8] + v[7]v[8] + v[4]v[6] + v[2]v[5] + v[3]v[7] + v[3]v[5]$)

degree 3 : $\frac{1}{24}$ ($4v[2]v[3]v[8] + 4v[6]v[7]v[8] + 3v[1]v[2]v[6] + 4v[1]v[4]v[5] + 3v[2]v[4]v[6] + 3v[2]v[7]v[8] + 4v[3]v[6]v[7] + 3v[4]v[7]v[8] + 3v[3]v[4]v[8] + 4v[1]v[2]v[3] + 4v[2]v[3]v[4] + 4v[2]v[3]v[6] + 3v[3]v[4]v[7] + 3v[4]v[5]v[6] + 3v[3]v[5]v[6] + 3v[1]v[5]v[7] + 4v[2]v[3]v[7] + 4v[5]v[6]v[8] + 3v[1]v[3]v[5] + 4v[1]v[4]v[6] + 3v[2]v[4]v[8] + 3v[1]v[7]v[8] + 3v[1]v[2]v[8] + 4v[1]v[2]v[4] + 3v[2]v[4]v[7] + 3$

v[4]v[5]v[7] + 3 v[3]v[4]v[6] + 3 v[1]v[2]v[7] + 3 v[1]v[6]v[8] + 4 v[4]v[5]v[8] + 4 v[1]
v[4]v[8] + 3 v[3]v[4]v[5] + 4 v[5]v[6]v[7] + 4 v[1]v[6]v[7] + 3 v[3]v[6]v[8] + 3 v[2]v[5]
v[7] + 3 v[2]v[5]v[6] + 4 v[2]v[6]v[7] + 4 v[1]v[5]v[8] + 3 v[1]v[5]v[6] + 4 v[5]v[7]v[8]
+ 4 v[1]v[3]v[4] + 4 v[2]v[3]v[5] + 3 v[1]v[3]v[6] + 4 v[1]v[4]v[7] + 3 v[1]v[2]v[5] + 3
v[2]v[4]v[5] + 4 v[2]v[5]v[8] + 3 v[3]v[7]v[8] + 4 v[4]v[6]v[7] + 3 v[3]v[5]v[7] + 3 v[2]
v[6]v[8] + 4 v[3]v[5]v[8] + 3 v[1]v[3]v[8] + 3 v[1]v[3]v[7] + 3 v[4]v[6]v[8])

degree 4 : $\frac{1}{6}$ (v[1]v[2]v[4]v[5] + v[1]v[3]v[4]v[7] + v[1]v[2]v[5]v[8] + v[1]v[4]v[6]v[
8] + v[1]v[6]v[7]v[8] + 3 v[1]v[3]v[6]v[8] + v[3]v[6]v[7]v[8] + v[3]v[4]v[6]v[7] + v[2]
v[5]v[6]v[8] + 3 v[2]v[4]v[5]v[7] + v[2]v[3]v[5]v[7] + 3 v[2]v[4]v[6]v[8] + v[1]v[2]v[
6]v[7] + 8v[1]v[4]v[6]v[7] + 8v[2]v[3]v[5]v[8] + 3 v[3]v[4]v[7]v[8] + v[1]v[2]v[4]v[6]
+ v[3]v[4]v[5]v[8] + 3 v[2]v[4]v[7]v[8] + v[2]v[3]v[4]v[8] + 8v[5]v[6]v[7]v[8] + 3 v[2]
v[4]v[5]v[6] + v[1]v[3]v[5]v[8] + 8v[1]v[4]v[5]v[8] + v[1]v[2]v[4]v[8] + 3 v[3]v[4]v[
5]v[6] + v[4]v[6]v[7]v[8] + v[1]v[3]v[4]v[5] + v[2]v[3]v[4]v[7] + v[2]v[5]v[6]v[7] + 3
v[1]v[3]v[5]v[7] + v[2]v[6]v[7]v[8] + v[2]v[4]v[6]v[7] + v[2]v[4]v[5]v[8] + v[1]v[2]v[
4]v[7] + 3 v[1]v[2]v[6]v[8] + v[1]v[5]v[6]v[8] + v[1]v[3]v[4]v[6] + v[4]v[5]v[7]v[8] +
3 v[1]v[3]v[7]v[8] + 3 v[3]v[4]v[5]v[7] + v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[8] + 3 v[1]v[
2]v[5]v[6] + v[2]v[3]v[4]v[6] + v[3]v[5]v[6]v[8] + v[1]v[5]v[7]v[8] + v[2]v[3]v[7]v[8]
+ v[3]v[5]v[7]v[8] + 3 v[1]v[2]v[5]v[7] + v[4]v[5]v[6]v[8] + v[2]v[3]v[5]v[6] + v[4]v[
5]v[6]v[7] + v[1]v[4]v[5]v[6] + v[2]v[5]v[7]v[8] + v[1]v[5]v[6]v[7] + 3 v[1]v[3]v[5]v[
6] + v[2]v[3]v[4]v[5] + v[2]v[3]v[6]v[8] + v[1]v[4]v[7]v[8] + v[1]v[4]v[5]v[7] + 3 v[3]
v[4]v[6]v[8] + 3 v[1]v[2]v[7]v[8] + 8v[2]v[3]v[6]v[7] + 8v[1]v[2]v[3]v[4] + v[1]v[3]v[
6]v[7] + v[1]v[2]v[3]v[8] + v[1]v[2]v[3]v[6] + v[1]v[2]v[3]v[7] + v[1]v[2]v[3]v[5])

degree 5 : $\frac{1}{24}$ (4 v[1]v[4]v[5]v[7]v[8] + 4 v[1]v[2]v[3]v[5]v[8] + 4 v[2]v[3]v[4]v[5]v[8
] + 3 v[2]v[4]v[5]v[7]v[8] + 4 v[1]v[5]v[6]v[7]v[8] + 3 v[1]v[3]v[5]v[6]v[7] + 3 v[3]v[4]
v[5]v[6]v[8] + 3 v[2]v[3]v[4]v[5]v[7] + 3 v[1]v[2]v[5]v[7]v[8] + 3 v[1]v[2]v[3]v[6]v[8]
+ 3 v[2]v[4]v[5]v[6]v[8] + 4 v[1]v[4]v[6]v[7]v[8] + 4 v[2]v[3]v[5]v[6]v[7] + 3 v[1]v[3]v
[4]v[5]v[7] + 3 v[1]v[2]v[4]v[6]v[8] + 3 v[2]v[4]v[6]v[7]v[8] + 3 v[2]v[3]v[4]v[5]v[6]
+ 4 v[1]v[2]v[4]v[6]v[7] + 4 v[2]v[3]v[5]v[6]v[8] + 4 v[1]v[4]v[5]v[6]v[7] + 3 v[2]v[3]
v[4]v[7]v[8] + 3 v[1]v[2]v[5]v[6]v[8] + 3 v[1]v[3]v[5]v[6]v[8] + 4 v[2]v[3]v[5]v[7]v[8]
+ 3 v[1]v[2]v[4]v[5]v[6] + 4 v[1]v[3]v[4]v[6]v[7] + 3 v[3]v[4]v[6]v[7]v[8] + 4 v[1]v[2]v
[3]v[6]v[7] + 3 v[1]v[2]v[3]v[7]v[8] + 4 v[4]v[5]v[6]v[7]v[8] + 4 v[1]v[3]v[4]v[5]v[8]
+ 4 v[2]v[5]v[6]v[7]v[8] + 3 v[1]v[3]v[4]v[6]v[8] + 3 v[1]v[2]v[4]v[7]v[8] + 3 v[1]v[3]v
[6]v[7]v[8] + 3 v[1]v[2]v[5]v[6]v[7] + 3 v[2]v[3]v[4]v[6]v[8] + 4 v[2]v[3]v[6]v[7]v[8]
+ 3 v[1]v[3]v[4]v[5]v[6] + 3 v[1]v[2]v[3]v[5]v[6] + 3 v[3]v[4]v[5]v[6]v[7] + 3 v[2]v[4]
v[5]v[6]v[7] + 4 v[1]v[4]v[5]v[6]v[8] + 3 v[1]v[2]v[4]v[5]v[7] + 4 v[3]v[5]v[6]v[7]v[8]
+ 4 v[2]v[3]v[4]v[6]v[7] + 3 v[1]v[2]v[3]v[5]v[7] + 4 v[1]v[2]v[3]v[4]v[7] + 4 v[1]v[2]
v[3]v[4]v[5] + 4 v[1]v[2]v[3]v[4]v[8] + 3 v[1]v[2]v[6]v[7]v[8] + 3 v[3]v[4]v[5]v[7]v[8]
+ 4 v[1]v[2]v[4]v[5]v[8] + 4 v[1]v[2]v[3]v[4]v[6] + 3 v[1]v[3]v[4]v[7]v[8] + 3 v[1]v[3]
v[5]v[7]v[8])

degree 6 : $\frac{1}{24}$ ($v[1]v[2]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[5]v[6]v[8] + v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[7]v[8] + 6v[1]v[2]v[3]v[4]v[6]v[7] + 6v[1]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[7] + v[2]v[3]v[4]v[5]v[7]v[8] + 6v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[7]v[8] + 6v[1]v[2]v[3]v[4]v[5]v[8] + v[1]v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[8] + v[1]v[2]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[5]v[6]v[7] + v[1]v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[6]$)

degree 7 : $\frac{1}{8}$ ($v[1]v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[7]v[8]$)

degree 8 : 1 ($v[1]$) ($v[2]$) ($v[3]$) ($v[8]$) ($v[7]$) ($v[6]$) ($v[5]$) ($v[4]$)

Group spectrum $1 + t + 2t^2 + 2t^3 + 3t^4 + 2t^5 + 2t^6 + t^7 + t^8$

KERNEL STRUCTURE

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$\pi 8 = [1]$$

supp $\pi 8$ = {1}

$$u8 = [1]$$

supp u8 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[7, 1, 4, 6, 8, 3, 2, 5]

B−BLOCKS,

[8, 3, 2, 5, 7, 1, 4, 6]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{1\}$
$b_2 = \{2\}$
$b_3 = \{3\}$
$b_4 = \{4\}$
$b_5 = \{5\}$
$b_6 = \{6\}$
$b_7 = \{7\}$
$b_8 = \{8\}$

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[3] & h[3] & h[2] & h[3] & h[3] & h[3] & h[3] \\ h[3] & h[1] & h[2] & h[3] & h[3] & h[3] & h[3] & h[3] \\ h[3] & h[2] & h[1] & h[3] & h[3] & h[3] & h[3] & h[3] \\ h[2] & h[3] & h[3] & h[1] & h[3] & h[3] & h[3] & h[3] \\ h[3] & h[3] & h[3] & h[3] & h[1] & h[3] & h[3] & h[2] \\ h[3] & h[3] & h[3] & h[3] & h[3] & h[1] & h[2] & h[3] \\ h[3] & h[3] & h[3] & h[3] & h[3] & h[2] & h[1] & h[3] \\ h[3] & h[3] & h[3] & h[3] & h[2] & h[3] & h[3] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 24, Shape: 23 ⊕ 1/0

$$CLB = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{3, 4, 6}, {1, 2, 7}, {5, 8}}, true

Ω_B in Vec(K)? , {{2, 3}, {1, 6, 8}, {4, 5, 7}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

1, "range", [1, 2, 3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 4, 3, 2, 1], [8, 7, 6, 5, 4, 2, 3, 1], [8, 7, 6, 5, 1, 3, 2, 4], [8, 7, 6, 5, 1, 2, 3, 4], [8, 6, 7, 5, 4, 3, 2, 1], [8, 6, 7, 5, 4, 2, 3, 1], [8, 6, 7, 5, 1, 3, 2, 4], [8, 6, 7, 5, 1, 2, 3, 4], [8, 4, 1, 5, 3, 7, 6, 2], [8, 4, 1, 5, 3, 6, 7, 2], [8, 4, 1, 5, 2, 7, 6, 3], [8, 4, 1, 5, 2, 6, 7, 3], [8, 3, 2, 5, 7, 4, 1, 6], [8, 3, 2, 5, 7, 1, 4, 6], [8, 3, 2, 5, 6, 4, 1, 7], [8, 3, 2, 5, 6, 1, 4, 7], [8, 2, 3, 5, 7, 4, 1, 6], [8, 2, 3, 5, 7, 1, 4, 6], [8, 2, 3, 5, 6, 4, 1, 7], [8, 2, 3, 5, 6, 1, 4, 7], [8, 1, 4, 5, 3, 7, 6, 2], [8, 1, 4, 5, 3, 6, 7, 2], [8, 1, 4, 5, 2, 7, 6, 3], [8, 1, 4, 5, 2, 6, 7, 3], [7, 8, 5, 6, 3, 4, 1, 2], [7, 8, 5, 6, 3, 1, 4, 2], [7, 8, 5, 6, 2, 4, 1, 3], [7, 8, 5, 6, 2, 1, 4, 3], [7, 5, 8, 6, 3, 4, 1, 2], [7, 5, 8, 6, 3, 1, 4, 2], [7, 5, 8, 6, 2, 4, 1, 3], [7, 5, 8, 6, 2, 1, 4, 3], [7, 4, 1, 6, 8, 3, 2, 5], [7, 4, 1, 6, 8, 2, 3, 5], [7, 4, 1, 6, 5, 3, 2, 8], [7, 4, 1, 6, 5, 2, 3, 8], [7, 3, 2, 6, 4, 8, 5, 1], [7, 3, 2, 6, 4, 5, 8, 1], [7, 3, 2, 6, 1, 8, 5, 4], [7, 3, 2, 6, 1, 5, 8, 4], [7, 2, 3, 6, 4, 8, 5, 1], [7, 2, 3, 6, 4, 5, 8, 1], [7, 2, 3, 6, 1, 8, 5, 4], [7, 2, 3, 6, 1, 5, 8, 4], [7, 1, 4, 6, 8, 3, 2, 5], [7, 1, 4, 6, 8, 2, 3, 5], [7, 1, 4, 6, 5, 3, 2, 8], [7, 1, 4, 6, 5, 2, 3, 8], [6, 8, 5, 7, 3, 4, 1, 2], [6, 8, 5, 7, 3, 1, 4, 2], [6, 8, 5, 7, 2, 4, 1, 3], [6, 8, 5, 7, 2, 1, 4, 3], [6, 5, 8, 7, 3, 4, 1, 2], [6, 5, 8, 7, 3, 1, 4, 2], [6, 5, 8, 7, 2, 4, 1, 3], [6, 5, 8, 7, 2, 1, 4, 3], [6, 4, 1, 7, 8, 3, 2, 5], [6, 4, 1, 7, 8, 2, 3, 5], [6, 4, 1, 7, 5, 3, 2, 8], [6, 4, 1, 7, 5, 2, 3, 8], [6, 3, 2, 7, 4, 8, 5, 1], [6, 3, 2, 7, 4, 5, 8, 1], [6, 3, 2, 7, 1, 8, 5, 4], [6, 3, 2, 7, 1, 5, 8, 4], [6, 2, 3, 7, 4, 8, 5, 1], [6, 2, 3, 7, 4, 5, 8, 1], [6, 2, 3, 7, 1, 8, 5, 4], [6, 2, 3, 7, 1, 5, 8, 4], [6, 1, 4, 7, 8, 3, 2, 5], [6, 1, 4, 7, 8, 2, 3, 5], [6, 1, 4, 7, 5, 3, 2, 8], [6, 1, 4, 7, 5, 2, 3, 8], [5, 7, 6, 8, 4, 3, 2, 1], [5, 7, 6, 8, 4, 2, 3, 1], [5, 7, 6, 8, 1, 3, 2, 4], [5, 7, 6, 8, 1, 2, 3, 4], [5, 6, 7, 8, 4, 3, 2, 1], [5, 6, 7, 8, 4, 2, 3, 1], [5, 6, 7, 8, 1, 3, 2, 4], [5, 6, 7, 8, 1, 2, 3, 4], [5, 4, 1, 8, 3, 7, 6, 2], [5, 4, 1, 8, 3, 6, 7, 2], [5, 4, 1, 8, 2, 7, 6, 3], [5, 4, 1, 8, 2, 6, 7, 3], [5, 3, 2, 8, 7, 4, 1, 6], [5, 3, 2, 8, 7, 1, 4, 6], [5, 3, 2, 8, 6, 4, 1, 7], [5, 3, 2, 8, 6, 1, 4, 7], [5, 2, 3, 8, 7, 4, 1, 6], [5, 2, 3, 8, 7, 1, 4, 6], [5, 2, 3, 8, 6, 4, 1, 7], [5, 2, 3, 8, 6, 1, 4, 7], [5, 1, 4, 8, 3, 7, 6, 2], [5, 1, 4, 8, 3, 6, 7, 2], [5, 1, 4, 8, 2, 7, 6, 3], [5, 1, 4, 8, 2, 6, 7, 3], [4, 8, 5, 1, 7, 3, 2, 6], [4, 8, 5, 1, 7, 2, 3, 6], [4, 8, 5, 1, 6, 3, 2, 7], [4, 8, 5, 1, 6, 2, 3, 7], [4, 7, 6, 1, 3, 8, 5, 2], [4, 7, 6, 1, 3, 5, 8, 2], [4, 7, 6, 1, 2, 8, 5, 3], [4, 7, 6, 1, 2, 5, 8, 3], [4, 6, 7, 1, 3, 8, 5, 2], [4, 6, 7, 1, 3, 5, 8, 2], [4, 6, 7, 1, 2, 8, 5, 3], [4, 6, 7, 1, 2, 5, 8, 3], [4, 5, 8, 1, 7, 3, 2, 6], [4, 5, 8, 1, 7, 2, 3, 6], [4, 5, 8, 1, 6, 3, 2, 7], [4, 5, 8, 1, 6, 2, 3, 7], [4, 3, 2, 1, 8, 7, 6, 5], [4, 3, 2, 1, 8, 6, 7, 5], [4, 3, 2, 1, 5, 7, 6, 8], [4, 3, 2, 1, 5, 6, 7, 8], [4, 2, 3, 1, 8, 7, 6, 5], [4, 2, 3, 1, 8, 6, 7, 5], [4, 2, 3, 1, 5, 7, 6, 8], [4, 2, 3, 1, 5, 6, 7, 8], [3, 8, 5, 2, 4, 7, 6, 1], [3, 8, 5, 2, 4, 6, 7, 1], [3, 8, 5, 2, 1, 7, 6, 4], [3, 8, 5, 2, 1, 6, 7, 4], [3, 7, 6, 2, 8, 4, 1, 5], [3, 7, 6, 2, 8, 1, 4, 5], [3, 7, 6, 2, 5, 4, 1, 8], [3, 7, 6, 2, 5, 1, 4, 8], [3, 6, 7, 2, 8, 4, 1, 5], [3, 6, 7, 2, 8, 1, 4, 5], [3, 6, 7, 2, 5, 4, 1, 8], [3, 6, 7, 2, 5, 1, 4, 8], [3, 5, 8, 2, 4, 7, 6, 1], [3, 5, 8, 2, 4, 6, 7, 1], [3, 5, 8, 2, 1, 7, 6, 4], [3, 5, 8, 2, 1, 6, 7, 4], [3, 4, 1, 2, 7, 8, 5, 6], [3, 4, 1, 2, 7, 5, 8, 6], [3, 4, 1, 2, 6, 8, 5, 7], [3, 4, 1, 2, 6, 5, 8, 7], [3, 1, 4, 2, 7, 8, 5, 6], [3, 1, 4, 2, 7, 5, 8, 6], [3, 1, 4, 2, 6, 8, 5, 7], [3, 1, 4, 2, 6, 5, 8, 7], [2, 8, 5, 3, 4, 7, 6, 1], [2, 8, 5, 3, 4, 6, 7, 1], [2, 8, 5, 3, 1, 7, 6, 4], [2, 8, 5, 3, 1, 6, 7, 4], [2, 7, 6, 3, 8, 4, 1, 5], [2, 7, 6, 3, 8, 1, 4, 5], [2, 7, 6, 3, 5, 4, 1, 8], [2, 7, 6, 3, 5, 1, 4, 8], [2, 6, 7, 3, 8, 4, 1, 5], [2, 6, 7, 3, 8, 1, 4, 5], [2, 6, 7, 3, 5, 4, 1, 8], [2, 6, 7, 3, 5, 1, 4, 8], [2, 5, 8, 3, 4, 7, 6, 1], [2, 5, 8, 3, 4, 6, 7, 1], [2, 5, 8, 3, 1, 7, 6, 4], [2, 5, 8, 3, 1, 6, 7, 4], [2, 4, 1, 3, 7, 8, 5, 6], [2, 4, 1, 3, 7, 5, 8, 6], [2, 4, 1, 3, 6, 8, 5, 7], [2, 4, 1, 3, 6, 5, 8, 7], [2, 1, 4, 3, 7, 8, 5, 6], [2, 1, 4, 3, 7, 5, 8, 6], [2, 1, 4, 3, 6, 8, 5, 7], [2, 1, 4, 3, 6, 5, 8, 7], [1, 8, 5, 4, 7, 3, 2, 6], [1, 8, 5, 4, 7, 2, 3, 6], [1, 8, 5, 4, 6, 3, 2, 7], [1, 8, 5, 4, 6, 2, 3, 7], [1, 7, 6, 4, 3, 8, 5, 2], [1, 7, 6, 4, 3, 5, 8, 2], [1, 7, 6, 4, 2, 8, 5, 3], [1, 7, 6, 4, 2, 5, 8, 3], [1, 6, 7, 4, 3, 8, 5, 2], [1, 6, 7, 4, 3, 5, 8, 2], [1, 6, 7, 4, 2, 8, 5, 3], [1, 6, 7, 4, 2, 5, 8, 3], [1, 5, 8, 4, 7, 3, 2, 6], [1, 5, 8, 4, 7, 2, 3, 6], [1, 5, 8, 4, 6, 3, 2, 7], [1, 5, 8, 4, 6, 2, 3, 7], [1, 3, 2, 4, 8, 7, 6, 5], [1, 3, 2, 4, 8, 6, 7, 5], [1, 3, 2, 4, 5, 7, 6, 8], [1, 3, 2, 4, 5, 6, 7, 8], [1, 2, 3, 4, 8, 7, 6, 5], [1, 2, 3, 4, 8, 6, 7, 5], [1, 2, 3, 4, 5, 7, 6, 8], [1, 2, 3, 4, 5, 6, 7, 8]]

"group has", 192, "elements"   Group element 1,1 = $\begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0 \end{pmatrix}$

$g_1 =$ [[1, 8], [2, 7], [3, 6], [4, 5]]

$g_2 =$ [[1, 8], [2, 7, 3, 6], [4, 5]]

$g_3 =$ [[1, 8, 4, 5], [2, 7], [3, 6]]

$g_4 =$ [[1, 8, 4, 5], [2, 7, 3, 6]]

$g_5 =$ [[1, 8], [2, 6, 3, 7], [4, 5]]

linear dimension, 26

"Symmetric?", true

Is Z in Vec(K)? true
$\big(-16h[2] \quad 4h[2] \quad 4h[2] \quad 4h[2] \quad -8h[2]-24h[1] \quad 24h[1] \quad 4h[2] \quad -24h[1] \quad 4h[2] \quad 4h$

"Basis for Z(G)"

1, "coeff", 24

Z[1] = $\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$

2, "coeff", 4

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

3, "coeff", 24

$$Z[\,3\,] = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
2, 3, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 6. & -2. & -2. & -2. & 0 & 0 & 0 & 0 \\ 1. & -1. & 1. & -1. & 1. & -1. & 1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + 2t^3 + 3t^4 + 2t^5 + 2t^6 + t^7 + t^8$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 11t^4 + 18t^5 + 36t^6 + 58t^7 + 102 t^8 + 160t^9 + 258t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 8 = \begin{pmatrix} 1 \end{pmatrix}$

{1}

$u8 = \begin{pmatrix} 1 \end{pmatrix}$

{1}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 7 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u7 = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$\pi 6 =$
$(2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2 \ 2$

$u6 =$
$\left(\frac{1}{32} \ \frac{1}{32}\right.$

picheck $\begin{pmatrix} 42 & 42 & 42 & 42 & 42 & 42 & 42 & 42 \end{pmatrix}$

$\pi 5 =$
$(6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6$

$u5 =$
$\left(\frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256} \ \frac{3}{256}\right.$

picheck $\begin{pmatrix} 210 & 210 & 210 & 210 & 210 & 210 & 210 & 210 \end{pmatrix}$

$\pi 4 =$
$(24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24 \ 24$

$u4 =$
$\left(\frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\ \frac{3}{512}\right.$

picheck $(840\ \ 840\ \ 840\ \ 840\ \ 840\ \ 840\ \ 840\ \ 840)$

$\pi 3 =$
$(120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120$

$u3 =$
$\left(\frac{15}{4096}\ \frac{15}{4096}\ \frac{15}{4096}\ \frac{15}{4096}\ \frac{15}{4096}\ \frac{15}{4096}\ \frac{15}{4096}\ \frac{15}{4096}\ \frac{15}{4096}\ \frac{15}{4096}\ \frac{15}{4096}\ \frac{15}{4096}\right.$

picheck $(2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520)$

$\pi 2 =$
$(720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720$

$u2 =$
$\left(\frac{45}{16384}\ \frac{45}{16384}\ \frac{45}{16384}\ \frac{45}{16384}\ \frac{45}{16384}\ \frac{45}{16384}\ \frac{45}{16384}\ \frac{45}{16384}\ \frac{45}{16384}\ \frac{45}{16384}\ \frac{45}{16384}\right.$

picheck $(5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

$\pi 1 = (5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

$u1 = \left(\frac{315}{131072}\ \frac{315}{131072}\ \frac{315}{131072}\ \frac{315}{131072}\ \frac{315}{131072}\ \frac{315}{131072}\ \frac{315}{131072}\ \frac{315}{131072}\right)$

picheck $(5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem–checks

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0, 0, 0]$

$\ker N_C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} s+t & s+t & s+t & s+t & s+t & s+t & s+t & s+t \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & 0 & t & 0 & 0 \\ 0 & t & 0 & 0 & 0 & s & 0 \\ t & 0 & 0 & 0 & s & 0 & 0 \\ 0 & s & 0 & 0 & 0 & t & 0 \\ 0 & 0 & t & 0 & 0 & 0 & s \\ 0 & 0 & s & 0 & 0 & 0 & t \\ -s & -s & -s & -s+t & -s & -s & -s \\ -t & -t & -t & -t+s & -t & -t & -t \end{pmatrix}$$

RB checks

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & s & 0 & t & 0 \\ 0 & 0 & 0 & t & 0 & 0 & 0 & s \\ 0 & 0 & 0 & 0 & t & 0 & s & 0 \\ 0 & 0 & 0 & s & 0 & 0 & 0 & t \\ t & 0 & s & 0 & 0 & 0 & 0 & 0 \\ s & 0 & t & 0 & 0 & 0 & 0 & 0 \\ 0 & t & 0 & 0 & 0 & s & 0 & 0 \\ 0 & s & 0 & 0 & 0 & t & 0 & 0 \end{pmatrix}$$ RB checks

$$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 8, "vs", 8

$$CNM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 1T + 48\Omega$$

Ω
$$\begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{-3}{4} & \frac{9}{8} & \frac{1}{8} & \frac{1}{8} & \frac{5}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$T\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

"IS NM in Vec(K)?", true

NM
$\begin{pmatrix} 7 & 6 & 6 & 6 & 12 & 6 & 7 & -38 & 56 & 6 & 6 & 31 & 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN
$\begin{pmatrix} 7 & 6 & 6 & 6 & 12 & 6 & 7 & -38 & 56 & 6 & 6 & 31 & 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$

$$\boldsymbol{\tau} = 8/1 \text{ , rank} = 8 \text{ , ratio} = 1/1 \text{ , } n^2 / r = 8/1$$

$$\boldsymbol{\tau}' = 56/1 \text{ , } r' = 7/8 \text{ , } \boldsymbol{\tau} / n^2 = 1/8$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 192

KERNEL HAS LINEAR DIMENSION 26
out of total no. of elements equal to 192

dim span idems 1 vs no. of idems 1

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8} \end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_c

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0] }

NullSpace N_C

{[1, 1, 1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[0, 0, 0, −1, 0, 1, 0, 0], [0, 0, 1, −1, 0, 0, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, 1, 0, −1, 0, 0, 0, 0], [0, 0, 0, −1, 0, 0, 0, 1], [1, 0, 0, −1, 0, 0, 0, 0], [0, 0, 0, −1, 1, 0, 0, 0]}

NullSpace N_0

{}

Eigenvalues M

[7., −1., −1., −1., −1., −1., −1., −1.]

Eigenvalues N

[7., −1., −1., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0 \end{pmatrix} \quad \begin{pmatrix} 0&1&1&0&1&1&1&1\\ 1&0&0&1&1&1&1&1\\ 1&0&0&1&1&1&1&1\\ 0&1&1&0&1&1&1&1\\ 1&1&1&1&0&1&1&0\\ 1&1&1&1&1&0&0&1\\ 1&1&1&1&1&0&0&1\\ 1&1&1&1&0&1&1&0 \end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

==

{2, 3, 6, 7}

R: [2, 7, 6, 3, 4, 8, 5, 1]
B: [6, 3, 2, 7, 8, 4, 1, 5]

TRACE TWO = 3

det AT = $-1\,(t)^4$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 8

Level 2 det = $\frac{-5}{4096} (-1 + s)^3 (101 + 58s + 26s^2 + 10s^3 + 5s^4) (5 + 2s + s^2)$

RANK of R is 8

R ranking is 1, "vs", 8

RBAR ranking 1, "vs", 8

RANK of B is 8

B ranking is 1, "vs", 8

BBAR ranking 1, "vs", 8

"R CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6] v[7] v[8]
"B CYCLES", (1 + v[5] v[8]) (1 + v[2] v[3]) (1 + v[1] v[4] v[6] v[7])

Eigenvalues

R: [−1. I, 1. I, −1., 1., −0.7071067810 − 0.7071067810 I, 0.7071067810 + 0.7071067810 I, −0.7071067810 + 0.7071067810 I, 0.7071067810 − 0.7071067810 I]

B: [1. I, −1. I, 1., −1., 1., −1., 1., −1.]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 3

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 8

"RANK of the KERNEL is ", 8

"IdemSolvability Check", 3 "Trace mark", 8, "Rank mark", 8, "for kernel rank", 8

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{16}$ ($v[1]v[8] + v[5]v[7] + 4v[1]v[4] + 2v[3]v[8] + v[3]v[6] + 4v[6]v[7] + 2v[4]v[7] + v[5]v[6] + v[2]v[4] + v[2]v[6] + v[4]v[5] + v[2]v[7] + v[1]v[3] + 2v[1]v[7] + v[6]v[8] + 4v[5]v[8] + 4v[2]v[3] + v[1]v[5] + v[4]v[8] + v[1]v[2] + 2v[1]v[6] + v[3]v[4] + 2v[2]v[8] + v[7]v[8] + 2v[4]v[6] + 2v[2]v[5] + v[3]v[7] + 2v[3]v[5]$)

degree 3 : $\frac{1}{16}$ ($4v[2]v[3]v[8] + 2v[6]v[7]v[8] + v[1]v[2]v[6] + 2v[1]v[4]v[5] + v[2]v[4]v[6] + v[2]v[7]v[8] + 2v[3]v[6]v[7] + v[4]v[7]v[8] + v[3]v[4]v[8] + 2v[1]v[2]v[3] + 2v[2]v[3]v[4] + 2v[2]v[3]v[6] + v[3]v[4]v[7] + v[4]v[5]v[6] + v[3]v[5]v[6] + v[1]v[5]v[7] + 2v[2]v[3]v[7] + 2v[5]v[6]v[8] + v[1]v[3]v[5] + 4v[1]v[4]v[6] + v[2]v[4]v[8] + v[1]v[7]v[8] + v[1]v[2]v[8] + 2v[1]v[2]v[4] + v[2]v[4]v[7] + v[4]v[5]v[7] + v[3]v[4]v[$

6] + v[1]v[2]v[7] + v[1]v[6]v[8] + 2v[4]v[5]v[8] + 2v[1]v[4]v[8] + v[3]v[4]v[5] + 2v[5]v[6]v[7] + 4v[1]v[6]v[7] + v[3]v[6]v[8] + v[2]v[5]v[7] + v[2]v[5]v[6] + 2v[2]v[6]v[7] + 2v[1]v[5]v[8] + v[1]v[5]v[6] + 2v[5]v[7]v[8] + 2v[1]v[3]v[4] + 4v[2]v[3]v[5] + v[1]v[3]v[6] + 4v[1]v[4]v[7] + v[1]v[2]v[5] + v[2]v[4]v[5] + 4v[2]v[5]v[8] + v[3]v[7]v[8] + 4v[4]v[6]v[7] + v[3]v[5]v[7] + v[2]v[6]v[8] + 4v[3]v[5]v[8] + v[1]v[3]v[8] + v[1]v[3]v[7] + v[4]v[6]v[8])

degree 4 : $\frac{1}{4}$ (2v[1]v[2]v[4]v[5] + v[1]v[3]v[4]v[7] + v[1]v[2]v[5]v[8] + v[1]v[4]v[6]v[8] + v[1]v[6]v[7]v[8] + 2v[1]v[3]v[6]v[8] + 2v[3]v[6]v[7]v[8] + v[3]v[4]v[6]v[7] + v[2]v[5]v[6]v[8] + 2v[2]v[4]v[5]v[7] + v[2]v[3]v[5]v[7] + 2v[2]v[4]v[6]v[8] + v[1]v[2]v[6]v[7] + 16v[1]v[4]v[6]v[7] + 16v[2]v[3]v[5]v[8] + 2v[3]v[4]v[7]v[8] + v[1]v[2]v[4]v[6] + v[3]v[4]v[5]v[8] + 2v[2]v[4]v[7]v[8] + v[2]v[3]v[4]v[8] + 8v[5]v[6]v[7]v[8] + 2v[2]v[4]v[5]v[6] + v[1]v[3]v[5]v[8] + 8v[1]v[4]v[5]v[8] + 2v[1]v[2]v[4]v[8] + 2v[3]v[4]v[5]v[6] + v[4]v[6]v[7]v[8] + 2v[1]v[3]v[4]v[5] + 2v[2]v[3]v[4]v[7] + 2v[2]v[5]v[6]v[7] + 2v[1]v[3]v[5]v[7] + 2v[2]v[6]v[7]v[8] + v[2]v[4]v[6]v[7] + v[2]v[4]v[5]v[8] + v[1]v[2]v[4]v[7] + 2v[1]v[2]v[6]v[8] + 2v[1]v[5]v[6]v[8] + v[1]v[3]v[4]v[6] + 2v[4]v[5]v[7]v[8] + 2v[1]v[3]v[7]v[8] + 2v[3]v[4]v[5]v[7] + 2v[3]v[5]v[6]v[7] + 2v[1]v[3]v[4]v[8] + 2v[1]v[2]v[5]v[6] + 2v[2]v[3]v[4]v[6] + v[3]v[5]v[6]v[8] + 2v[1]v[5]v[7]v[8] + v[2]v[3]v[7]v[8] + v[3]v[5]v[7]v[8] + 2v[1]v[2]v[5]v[7] + 2v[4]v[5]v[6]v[8] + v[2]v[3]v[5]v[6] + v[4]v[5]v[6]v[7] + v[1]v[4]v[5]v[6] + v[2]v[5]v[7]v[8] + v[1]v[5]v[6]v[7] + 2v[1]v[3]v[5]v[6] + v[2]v[3]v[4]v[5] + v[2]v[3]v[6]v[8] + v[1]v[4]v[7]v[8] + v[1]v[4]v[5]v[7] + 2v[3]v[4]v[6]v[8] + 2v[1]v[2]v[7]v[8] + 8v[2]v[3]v[6]v[7] + 8v[1]v[2]v[3]v[4] + v[1]v[3]v[6]v[7] + v[1]v[2]v[3]v[8] + 2v[1]v[2]v[3]v[6] + 2v[1]v[2]v[3]v[7] + v[1]v[2]v[3]v[5])

degree 5 : $\frac{1}{16}$ (2v[1]v[4]v[5]v[7]v[8] + 4v[1]v[2]v[3]v[5]v[8] + 4v[2]v[3]v[4]v[5]v[8] + v[2]v[4]v[5]v[7]v[8] + 2v[1]v[5]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[7] + v[1]v[2]v[5]v[7]v[8] + v[1]v[2]v[3]v[6]v[8] + v[2]v[4]v[5]v[6]v[8] + 4v[1]v[4]v[6]v[7]v[8] + 2v[2]v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[5]v[7] + v[1]v[2]v[4]v[6]v[8] + v[2]v[4]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6] + 4v[1]v[2]v[4]v[6]v[7] + 4v[2]v[3]v[5]v[6]v[8] + 4v[1]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[5]v[6]v[8] + v[1]v[3]v[5]v[6]v[8] + 4v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[4]v[5]v[6] + 4v[1]v[3]v[4]v[6]v[7] + v[3]v[4]v[6]v[7]v[8] + 2v[1]v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[7]v[8] + 2v[4]v[5]v[6]v[7]v[8] + 2v[1]v[3]v[4]v[5]v[8] + 2v[2]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[7]v[8] + v[1]v[3]v[6]v[7]v[8] + v[1]v[2]v[5]v[6]v[7] + v[2]v[3]v[4]v[6]v[8] + 2v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6] + v[1]v[2]v[3]v[5]v[6] + v[3]v[4]v[5]v[6]v[7] + v[2]v[4]v[5]v[6]v[7] + 2v[1]v[4]v[5]v[6]v[8] + v[1]v[2]v[4]v[5]v[7] + 2v[3]v[5]v[6]v[7]v[8] + 2v[2]v[3]v[4]v[6]v[7] + v[1]v[2]v[3]v[5]v[7] + 2v[1]v[2]v[3]v[4]v[7] + 2v[1]v[2]v[3]v[4]v[5] + 2v[1]v[2]v[3]v[4]v[8] + v[1]v[2]v[6]v[7]v[8] + v[3]v[4]v[5]v[7]v[8] + 2v[1]v[2]v[4]v[5]v[8] + 2v[1]v[2]v[3]v[4]v[6] + v[1]v[3]v[4]v[7]v[8] + v[1]v[3]v[5]v[7]v[8]) degree 6 : $\frac{1}{16}\,(\,2\,v[1]v[2]v[4]v[5]v[6]v[7] + 2\,v[2]v[3]v[4]v[5]v[6]v[8] + v[2]v[4]v[5]v[6]v[7]v[8] + 2\,v[1]v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[7]v[8] + 4\,v[1]v[2]v[3]v[4]v[6]v[7] + 4\,v[1]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[7] + 2\,v[2]v[3]v[4]v[5]v[7]v[8] + 4\,v[2]v[3]v[5]v[6]v[7]v[8] + 2\,v[1]v[3]v[4]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[7]v[8] + 4\,v[1]v[2]v[3]v[4]v[5]v[8] + v[1]v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[6]v[7] + 2\,v[1]v[2]v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[7]v[8] + 2\,v[1]v[2]v[3]v[5]v[6]v[8] + v[1]v[2]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[5]v[6]v[7] + v[1]v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[7]v[8] + 2\,v[1]v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[6]\,)$

degree 7 : $\frac{1}{8}\,(\,v[1]v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[7]v[8]\,)$

degree 8 : $1\,(\,v[1]\,)\,(\,v[2]\,)\,(\,v[3]\,)\,(\,v[8]\,)\,(\,v[7]\,)\,(\,v[6]\,)\,(\,v[5]\,)\,(\,v[4]\,)$

Group spectrum $1 + t + 3\,t^2 + 3\,t^3 + 5\,t^4 + 3\,t^5 + 3\,t^6 + t^7 + t^8$

KERNEL STRUCTURE

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$\pi 8 = [1]$$

supp $\pi 8$ = {1}

$$u8 = [1]$$

supp u8 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[8, 1, 4, 5, 7, 3, 2, 6]

B–BLOCKS,

[7, 3, 2, 6, 8, 1, 4, 5]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

$b_1 = \{1\}$
$b_2 = \{2\}$
$b_3 = \{3\}$
$b_4 = \{4\}$
$b_5 = \{5\}$
$b_6 = \{6\}$
$b_7 = \{7\}$
$b_8 = \{8\}$

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[5] & h[4] & h[4] & h[3] & h[4] & h[2] & h[1] & h[4] \\ h[4] & h[5] & h[3] & h[4] & h[1] & h[4] & h[4] & h[2] \\ h[4] & h[3] & h[5] & h[4] & h[2] & h[4] & h[4] & h[1] \\ h[3] & h[4] & h[4] & h[5] & h[4] & h[1] & h[2] & h[4] \\ h[4] & h[2] & h[1] & h[4] & h[5] & h[4] & h[4] & h[3] \\ h[1] & h[4] & h[4] & h[2] & h[4] & h[5] & h[3] & h[4] \\ h[2] & h[4] & h[4] & h[1] & h[4] & h[3] & h[5] & h[4] \\ h[4] & h[1] & h[2] & h[4] & h[3] & h[4] & h[4] & h[5] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 11, Shape: $9 \oplus 2/0$

$$CLB = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4, 5, 6, 7, 8}}, true

Ω_B in Vec(K)? , {{2, 3}, {1, 4, 6, 7}, {5, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$ vs $\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$ vs $\left(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right)$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

1, "range", [1, 2, 3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 4, 3, 2, 1], [8, 6, 7, 5, 1, 3, 2, 4], [8, 4, 1, 5, 6, 3, 2, 7], [8, 1, 4, 5, 7, 3, 2, 6], [7, 8, 5, 6, 2, 1, 4, 3], [7, 5, 8, 6, 3, 1, 4, 2], [7, 3, 2, 6, 8, 1, 4, 5], [7, 2, 3, 6, 5, 1, 4, 8], [6, 8, 5, 7, 2, 4, 1, 3], [6, 5, 8, 7, 3, 4, 1, 2], [6, 3, 2, 7, 8, 4, 1, 5], [6, 2, 3, 7, 5, 4, 1, 8], [5, 7, 6, 8, 4, 2, 3, 1], [5, 6, 7, 8, 1, 2, 3, 4], [5, 4, 1, 8, 6, 2, 3, 7], [5, 1, 4, 8, 7, 2, 3, 6], [4, 8, 5, 1, 2, 7, 6, 3], [4, 5, 8, 1, 3, 7, 6, 2], [4, 3, 2, 1, 8, 7, 6, 5], [4, 2, 3, 1, 5, 7, 6, 8], [3, 7, 6, 2, 4, 5, 8, 1], [3, 6, 7, 2, 1, 5, 8, 4], [3, 4, 1, 2, 6, 5, 8, 7], [3, 1, 4, 2, 7, 5, 8, 6], [2, 7, 6, 3, 4, 8, 5, 1], [2, 6, 7, 3, 1, 8, 5, 4], [2, 4, 1, 3, 6, 8, 5, 7], [2, 1, 4, 3, 7, 8, 5, 6], [1, 8, 5, 4, 2, 6, 7, 3], [1, 5, 8, 4, 3, 6, 7, 2], [1, 3, 2, 4, 8, 6, 7, 5], [1, 2, 3, 4, 5, 6, 7, 8]]

"group has", 32, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

g_1 =   [[1, 8], [2, 7], [3, 6], [4, 5]]

g_2 =   [[1, 8, 4, 5], [2, 6, 3, 7]]

g_3 =   [[1, 8, 7, 2, 4, 5, 6, 3]]

g_4 =   [[1, 8, 6, 3, 4, 5, 7, 2]]

g_5 =   [[1, 7, 4, 6], [2, 8, 3, 5]]

linear dimension, 14

"Symmetric?", false

Is Z in Vec(K)? true
$\big(-2h[2] \quad h[2] \quad h[2] \quad h[2] \quad -4h[1]-4h[3] \quad 4h[5] \quad 4h[3] \quad 4h[1] \quad 4h[4] \quad h[2] \quad 4h[3] \quad h$

"Basis for Z(G)"

1, "coeff", 4

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

3, "coeff", 4

$$Z[\,3\,] = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

4, "coeff", 4

$$Z[\,4\,] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

5, "coeff", 4

$$Z[\,5\,] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
1, 5, true
2, 3, true
2, 4, true
2, 5, true
3, 4, true
3, 5, true
4, 5, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 0 & 0 & 0 & 0 & 0 & 0 & 4. & -4. \\ 1. & -1. & 1. & -1. & 1. & -1. & 1. & -1. \\ -1. & 1. & 1.I & -1.I & -1. & 1. & 1.I & -1.I \\ -1. & 1. & 1.I & -1.I & -1. & 1. & 1.I & -1.I \end{pmatrix}$$

Group spectrum: $1 + t + 3t^2 + 3t^3 + 5t^4 + 3t^5 + 3t^6 + t^7 + t^8$

Molien Series to order 10: $1 + t + 4t^2 + 8t^3 + 21t^4 + 39t^5 + 81t^6 + 144t^7 + 264t^8 + 438t^9 + 732t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 8 = \left(1\right)$

$\{1\}$

$u8 = \left(1\right)$

$\{1\}$

picheck $\left(1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1\right)$

$\pi = \left(\frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8}\right)$

$\pi 7 = \left(1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1\right)$

$u7 = \left(\frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8}\right)$

picheck $\left(7 \;\; 7 \;\; 7 \;\; 7 \;\; 7 \;\; 7 \;\; 7 \;\; 7\right)$

$\pi 6 =$
$(2 \;\; 2$

$u6 =$
$(\frac{1}{32} \;\; \frac{1}{32}$

picheck $\left(42 \;\; 42 \;\; 42 \;\; 42 \;\; 42 \;\; 42 \;\; 42 \;\; 42\right)$

$\pi 5 =$
$(6 \;\; 6$

$u5 =$
$(\frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256}$

picheck $\left(210 \;\; 210 \;\; 210 \;\; 210 \;\; 210 \;\; 210 \;\; 210 \;\; 210\right)$

$\pi 4 =$
$(24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24$

$u4 =$
$(\frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512}$

picheck $\begin{pmatrix} 840 & 840 & 840 & 840 & 840 & 840 & 840 & 840 \end{pmatrix}$

$\pi 3 =$
$(120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120$

$u3 =$
$\left(\frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; 4\right.$

picheck $\begin{pmatrix} 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 \end{pmatrix}$

$\pi 2 =$
$(720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720$

$u2 =$
$\left(\frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384}\right.$

picheck $\begin{pmatrix} 5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} & \frac{315}{131072} \end{pmatrix}$

picheck $\begin{pmatrix} 5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040 & 5040 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0, 0, 0]$

$\ker N_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} t+s & t+s & t+s & t+s & t+s & t+s & t+s & t+s \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & t & s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s & t & 0 & 0 \\ 0 & s & t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t & s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & t & s \\ 0 & 0 & 0 & 0 & 0 & s & t \\ -t+s & -t & -t & -t & -t & -t & -t \\ -s+t & -s & -s & -s & -s & -s & -s \end{pmatrix}$$

RB checks

$$\ker M_c = \begin{pmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} 0&t&0&0&0&0&0&s \\ 0&0&s&0&0&t&0&0 \\ 0&s&0&0&0&0&0&t \\ 0&0&t&0&0&s&0&0 \\ 0&0&0&t&0&0&s&0 \\ 0&0&0&s&0&0&t&0 \\ s&0&0&0&t&0&0&0 \\ t&0&0&0&s&0&0&0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 1&1&1&1&1&1&1&1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 8, "vs", 8

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 1T + 48\Omega$$

$$\Omega \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM in Vec(K)?", true

NM $\begin{pmatrix} 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$

$$\boldsymbol{\tau} = 8/1 \text{ , rank } = 8 \text{ , ratio } = 1/1 \text{ , } n^2 / r = 8/1$$

$$\boldsymbol{\tau}' = 56/1 \text{ , } r' = 7/8 \text{ , } \boldsymbol{\tau} / n^2 = 1/8$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 32

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 32

dim span idems 1 vs no. of idems 1

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1., 1., 1.]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C−scaled

[0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0] }

NullSpace N_C

{[1, 1, 1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0]}

NullSpace N_0

{}

Eigenvalues M

[7., −1., −1., −1., −1., −1., −1., −1.]

Eigenvalues N

[7., −1., −1., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix}0&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&1&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&1&0&0\\0&0&0&0&1&0&0&0\end{pmatrix} \quad \begin{pmatrix}0&1&1&0&1&0&0&1\\1&0&0&1&0&1&1&0\\1&0&0&1&0&1&1&0\\0&1&1&0&1&0&0&1\\1&0&0&1&0&1&1&0\\0&1&1&0&1&0&0&1\\0&1&1&0&1&0&0&1\\1&0&0&1&0&1&1&0\end{pmatrix} \quad \begin{pmatrix}0&0&0&0&0&1&1&0\\0&0&0&0&1&0&0&1\\0&0&0&0&1&0&0&1\\0&0&0&0&0&1&1&0\\0&1&1&0&0&0&0&0\\1&0&0&1&0&0&0&0\\1&0&0&1&0&0&0&0\\0&1&1&0&0&0&0&0\end{pmatrix}$$

Commutator(s)

$$1, 2: \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$1, 3: \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$2, 3: \quad \text{commutator} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

==

{2, 3, 6, 8}

R: [2, 7, 6, 3, 4, 8, 1, 5]
B: [6, 3, 2, 7, 8, 4, 5, 1]

TRACE TWO = 1

det AT = $1\,(t)^4$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 8

Level 2 det = $\frac{5}{8192}(-1+s)(-1010-81s-349s^2-63s^3+s^4+21s^5+25s^6+11s^7+5s^8)$

RANK of R is 8

R ranking is 1, "vs", 8

RBAR ranking 1, "vs", 8

RANK of B is 8

B ranking is 1, "vs", 8

BBAR ranking 1, "vs", 8

"R CYCLES", (1 + v[3] v[4] v[5] v[6] v[8]) (1 + v[1] v[2] v[7])
"B CYCLES", (1 + v[1] v[4] v[5] v[6] v[7] v[8]) (1 + v[2] v[3])

Eigenvalues

R: [-0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 0.3090169942 + 0.9510565160 I, -0.8090169942 + 0.5877852520 I, -0.8090169942 - 0.5877852520 I, 0.3090169942 - 0.9510565160 I, 1., 1.]

B: [0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I, -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., -1., 1.,

−1.]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 8

"RANK of the KERNEL is ", 8

"IdemSolvability Check", 3 "Trace mark", 8, "Rank mark", 8, "for kernel rank", 8

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{28}$ ($v[1]v[8] + v[5]v[7] + v[1]v[4] + v[3]v[8] + v[3]v[6] + v[6]v[7] + v[4]v[7] + v[5]v[6] + v[2]v[4] + v[2]v[6] + v[4]v[5] + v[2]v[7] + v[1]v[3] + v[1]v[7] + v[6]v[8] + v[5]v[8] + v[2]v[3] + v[1]v[5] + v[4]v[8] + v[1]v[2] + v[1]v[6] + v[3]v[4] + v[2]v[8] + v[7]v[8] + v[4]v[6] + v[2]v[5] + v[3]v[7] + v[3]v[5]$)

degree 3 : $\frac{1}{56}$ ($v[2]v[3]v[8] + v[6]v[7]v[8] + v[1]v[2]v[6] + v[1]v[4]v[5] + v[2]v[4]v[6] + v[2]v[7]v[8] + v[3]v[6]v[7] + v[4]v[7]v[8] + v[3]v[4]v[8] + v[1]v[2]v[3] + v[2]v[$

3]v[4] + v[2]v[3]v[6] + v[3]v[4]v[7] + v[4]v[5]v[6] + v[3]v[5]v[6] + v[1]v[5]v[7] + v[
2]v[3]v[7] + v[5]v[6]v[8] + v[1]v[3]v[5] + v[1]v[4]v[6] + v[2]v[4]v[8] + v[1]v[7]v[8]
+ v[1]v[2]v[8] + v[1]v[2]v[4] + v[2]v[4]v[7] + v[4]v[5]v[7] + v[3]v[4]v[6] + v[1]v[2]v
[7] + v[1]v[6]v[8] + v[4]v[5]v[8] + v[1]v[4]v[8] + v[3]v[4]v[5] + v[5]v[6]v[7] + v[1]v[
6]v[7] + v[3]v[6]v[8] + v[2]v[5]v[7] + v[2]v[5]v[6] + v[2]v[6]v[7] + v[1]v[5]v[8] + v[
1]v[5]v[6] + v[5]v[7]v[8] + v[1]v[3]v[4] + v[2]v[3]v[5] + v[1]v[3]v[6] + v[1]v[4]v[7]
+ v[1]v[2]v[5] + v[2]v[4]v[5] + v[2]v[5]v[8] + v[3]v[7]v[8] + v[4]v[6]v[7] + v[3]v[5]
v[7] + v[2]v[6]v[8] + v[3]v[5]v[8] + v[1]v[3]v[8] + v[1]v[3]v[7] + v[4]v[6]v[8])

degree 4 : $\frac{1}{70}$ (v[1]v[2]v[4]v[5] + v[1]v[3]v[4]v[7] + v[1]v[2]v[5]v[8] + v[1]v[4]v[6]
v[8] + v[1]v[6]v[7]v[8] + v[1]v[3]v[6]v[8] + v[3]v[6]v[7]v[8] + v[3]v[4]v[6]v[7] + v[2]
v[5]v[6]v[8] + v[2]v[4]v[5]v[7] + v[2]v[3]v[5]v[7] + v[2]v[4]v[6]v[8] + v[1]v[2]v[6]v[
7] + v[1]v[4]v[6]v[7] + v[2]v[3]v[5]v[8] + v[3]v[4]v[7]v[8] + v[1]v[2]v[4]v[6] + v[3]v
[4]v[5]v[8] + v[2]v[4]v[7]v[8] + v[2]v[3]v[4]v[8] + v[5]v[6]v[7]v[8] + v[2]v[4]v[5]v[6
] + v[1]v[3]v[5]v[8] + v[1]v[4]v[5]v[8] + v[1]v[2]v[4]v[8] + v[3]v[4]v[5]v[6] + v[4]v[
6]v[7]v[8] + v[1]v[3]v[4]v[5] + v[2]v[3]v[4]v[7] + v[2]v[5]v[6]v[7] + v[1]v[3]v[5]v[7]
+ v[2]v[6]v[7]v[8] + v[2]v[4]v[6]v[7] + v[2]v[4]v[5]v[8] + v[1]v[2]v[4]v[7] + v[1]v[2]
v[6]v[8] + v[1]v[5]v[6]v[8] + v[1]v[3]v[4]v[6] + v[4]v[5]v[7]v[8] + v[1]v[3]v[7]v[8] +
v[3]v[4]v[5]v[7] + v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[8] + v[1]v[2]v[5]v[6] + v[2]v[3]v
[4]v[6] + v[3]v[5]v[6]v[8] + v[1]v[5]v[7]v[8] + v[2]v[3]v[7]v[8] + v[3]v[5]v[7]v[8] +
v[1]v[2]v[5]v[7] + v[4]v[5]v[6]v[8] + v[2]v[3]v[5]v[6] + v[4]v[5]v[6]v[7] + v[1]v[4]v[
5]v[6] + v[2]v[5]v[7]v[8] + v[1]v[5]v[6]v[7] + v[1]v[3]v[5]v[6] + v[2]v[3]v[4]v[5] + v
[2]v[3]v[6]v[8] + v[1]v[4]v[7]v[8] + v[1]v[4]v[5]v[7] + v[3]v[4]v[6]v[8] + v[1]v[2]v[7
]v[8] + v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[4] + v[1]v[3]v[6]v[7] + v[1]v[2]v[3]v[8] + v[
1]v[2]v[3]v[6] + v[1]v[2]v[3]v[7] + v[1]v[2]v[3]v[5])

degree 5 : $\frac{1}{56}$ (v[1]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[8] + v[2]v[3]v[4]v[5]v[8] +
v[2]v[4]v[5]v[7]v[8] + v[1]v[5]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]
v[8] + v[2]v[3]v[4]v[5]v[7] + v[1]v[2]v[5]v[7]v[8] + v[1]v[2]v[3]v[6]v[8] + v[2]v[4]v[
5]v[6]v[8] + v[1]v[4]v[6]v[7]v[8] + v[2]v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[5]v[7] + v[1]
v[2]v[4]v[6]v[8] + v[2]v[4]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6] + v[1]v[2]v[4]v[6]v[7]
+ v[2]v[3]v[5]v[6]v[8] + v[1]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[5]v[
6]v[8] + v[1]v[3]v[5]v[6]v[8] + v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[4]v[5]v[6] + v[1]v[3]
v[4]v[6]v[7] + v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[7]v[8] + v[
4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[8] + v[2]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[
8] + v[1]v[2]v[4]v[7]v[8] + v[1]v[3]v[6]v[7]v[8] + v[1]v[2]v[5]v[6]v[7] + v[2]v[3]v[4]
v[6]v[8] + v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6] + v[1]v[2]v[3]v[5]v[6] + v[3]v[
4]v[5]v[6]v[7] + v[2]v[4]v[5]v[6]v[7] + v[1]v[4]v[5]v[6]v[8] + v[1]v[2]v[4]v[5]v[7] +
v[3]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7] + v[1]v[2]v[3]v[5]v[7] + v[1]v[2]v[3]v[4]
v[7] + v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[8] + v[1]v[2]v[6]v[7]v[8] + v[3]v[4]v[
5]v[7]v[8] + v[1]v[2]v[4]v[5]v[8] + v[1]v[2]v[3]v[4]v[6] + v[1]v[3]v[4]v[7]v[8] + v[1]

$v[3]v[5]v[7]v[8]$)

degree 6 : $\frac{1}{28}$ ($v[1]v[2]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[5]v[6]v[8] + v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7] + v[1]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[7] + v[2]v[3]v[4]v[5]v[7]v[8] + v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[8] + v[1]v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[8] + v[1]v[2]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[5]v[6]v[7] + v[1]v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[6]$)

degree 7 : $\frac{1}{8}$ ($v[1]v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[7]v[8]$)

degree 8 : 1 ($v[1]$) ($v[2]$) ($v[3]$) ($v[8]$) ($v[7]$) ($v[6]$) ($v[5]$) ($v[4]$)

Group spectrum $1 + t + t^2 + t^3 + t^4 + t^5 + t^6 + t^7 + t^8$

KERNEL STRUCTURE

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$\pi 8 = [1]$$

supp $\pi 8$ = {1}

$$u8 = [1]$$

supp u8 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[7, 1, 4, 5, 8, 3, 2, 6]

B−BLOCKS,

[8, 3, 2, 6, 7, 1, 4, 5]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{1\}$
$b_2 = \{2\}$
$b_3 = \{3\}$
$b_4 = \{4\}$
$b_5 = \{5\}$
$b_6 = \{6\}$
$b_7 = \{7\}$
$b_8 = \{8\}$

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[1] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[1] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[1] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[1] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[1] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[1] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 49, Shape: $48 \oplus 1/0$

$$CLB = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 7}, {3, 4, 5, 6, 8}}, true

Ω_B in Vec(K)? , {{2, 3}, {1, 4, 5, 6, 7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

1, "range", [1, 2, 3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 4, 3, 2, 1], [8, 7, 6, 5, 4, 2, 1, 3], [8, 7, 6, 5, 4, 1, 3, 2], [8, 7, 6, 5, 3, 4, 1, 2], [8, 7, 6, 5, 3, 2, 4, 1], [8, 7, 6, 5, 3, 1, 2, 4], [8, 7, 6, 5, 2, 4, 3, 1], [8, 7, 6, 5, 2, 3, 1, 4], [8, 7, 6, 5, 2, 1, 4, 3], [8, 7, 6, 5, 1, 4, 2, 3], [...20140 terms...], [1, 2, 3, 4, 8, 5, 7, 6], [1, 2, 3, 4, 7, 8, 5, 6], [1, 2, 3, 4, 7, 6, 8, 5], [1, 2, 3, 4, 7, 5, 6, 8], [1, 2, 3, 4, 6, 8, 7, 5], [1, 2, 3, 4, 6, 7, 5, 8], [1, 2, 3, 4, 6, 5, 8, 7], [1, 2, 3, 4, 5, 8, 6, 7], [1, 2, 3, 4, 5, 7, 8, 6], [1, 2, 3, 4, 5, 6, 7, 8]]

"group has", 20160, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

g_1 =   [[1, 8], [2, 7], [3, 6], [4, 5]]

g_2 =   [[1, 8, 3, 6, 2, 7], [4, 5]]

g_3 =   [[1, 8, 2, 7, 3, 6], [4, 5]]

g_4 =   [[1, 8, 2, 7], [3, 6, 4, 5]]

g_5 =   [[1, 8], [2, 7, 4, 5, 3, 6]]

linear dimension, 50

"Symmetric?", true

Is Z in Vec(K)? true

$(1440h[2] + 7560h[1] \quad -1080h[2] - 2520h[1] \quad 5760h[2] + 10080h[1] \quad 3600h[2] + 2$

"Basis for Z(G)"

1, "coeff", 2520

Z[1] = $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 360

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 7. & -1. & -1. & -1. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3 + t^4 + t^5 + t^6 + t^7 + t^8$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 5t^4 + 7t^5 + 11t^6 + 15t^7 + 22t^8 + 29t^9 + 40t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 8 = (1)$

{1}

$u8 = (1)$

{1}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 7 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u7 = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$\pi 6 =$
$\left(2 \quad 2 \right.$

$u6 =$
$\left(\frac{1}{32} \quad \frac{1}{32} \right.$

picheck $\begin{pmatrix} 42 & 42 & 42 & 42 & 42 & 42 & 42 & 42 \end{pmatrix}$

$\pi 5 =$
$\left(6 \quad 6 \right.$

$u5 =$
$\left(\frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \quad \frac{3}{256} \right.$

picheck $\begin{pmatrix} 210 & 210 & 210 & 210 & 210 & 210 & 210 & 210 \end{pmatrix}$

$\pi 4 =$
$\left(24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \quad 24 \right.$

$u4 =$
$\left(\frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \quad \frac{3}{512} \right.$

picheck $\begin{pmatrix} 840 & 840 & 840 & 840 & 840 & 840 & 840 & 840 \end{pmatrix}$

$\pi 3 =$
$\left(120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \quad 120 \right.$

$u3 =$
$\left(\frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \quad \frac{15}{4096} \right.$

picheck $\begin{pmatrix} 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 \end{pmatrix}$

$\pi 2 =$

$(720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720$

$u2 =$
$\left(\frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384}\right.$

picheck $(5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040)$

$\pi 1 = (5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040)$

$u1 = \left(\frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072}\right)$

picheck $(5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040)$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0, 0, 0]$

$\ker N_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} s+t & s+t & s+t & s+t & s+t & s+t & s+t & s+t \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & t & s & 0 & 0 \\ t & 0 & 0 & 0 & 0 & 0 & s \\ 0 & 0 & 0 & s & t & 0 & 0 \\ s & 0 & 0 & 0 & 0 & 0 & t \\ 0 & s & 0 & 0 & 0 & t & 0 \\ 0 & t & 0 & 0 & 0 & s & 0 \\ -s & -s & -s+t & -s & -s & -s & -s \\ -t & -t & -t+s & -t & -t & -t & -t \end{pmatrix}$$

RB checks

$$\ker M_c = \begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0 \end{pmatrix} \begin{pmatrix} t&0&0&0&0&0&s&0\\ 0&s&0&0&t&0&0&0\\ s&0&0&0&0&0&t&0\\ 0&t&0&0&s&0&0&0\\ 0&0&0&s&0&t&0&0\\ 0&0&0&t&0&s&0&0\\ 0&0&t&0&0&0&0&s\\ 0&0&s&0&0&0&0&t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 1&1&1&1&1&1&1&1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 8, "vs", 8

$$\text{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 1T + 48\Omega$$

Ω

$$\left(\frac{31}{8} \quad \frac{5}{8} \quad \frac{17}{8} \quad \frac{3}{8} \quad \frac{5}{16} \quad \frac{-7}{8} \quad \frac{5}{8} \quad \frac{13}{4} \quad \frac{21}{8} \quad \frac{13}{8} \quad \frac{7}{8} \quad \frac{3}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{21}{8} \quad 2 \quad \frac{9}{8} \quad \frac{1}{2} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad 2 \quad \frac{11}{8} \quad \frac{5}{8}\right.$$

T

$$(6 \quad 0 \quad 2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 5 \quad 4 \quad 2 \quad 1 \quad 0 \quad 0 \quad 0 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \quad 0 \quad 0 \quad 3 \quad 2 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 2$$

"IS NM in Vec(K)?", true

NM
(192 30 104 18 15 −42 30 161 130 80 43 18 6 6 130 99 56 25 6

"IS MN in Vec(K)?", true

MN
(192 30 104 18 15 −42 30 161 130 80 43 18 6 6 130 99 56 25 6

$\boldsymbol{\tau}$ = 8/1 , rank = 8 , ratio = 1/1 , n^2 / r = 8/1

$\boldsymbol{\tau}'$ = 56/1 , r' = 7/8 , $\boldsymbol{\tau} / n^2$ = 1/8

p^2 = 1/8 , min $\boldsymbol{\tau}$ = 8/1 , $\boldsymbol{\tau}$−check is positive? 0/1

max r = 8/1 , r−check is positive? 0/1

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 20160

KERNEL HAS LINEAR DIMENSION 50
out of total no. of elements equal to 20160

dim span idems 1 vs no. of idems 1

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_c

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0] }

NullSpace N_C

{[1, 1, 1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[0, 1, 0, 0, 0, 0, 0, −1], [0, 0, 0, 1, 0, 0, 0, −1], [1, 0, 0, 0, 0, 0, 0, −1], [0, 0, 0, 0, 0, 1, 0, −1], [0, 0, 0, 0, 1, 0, 0, −1], [0, 0, 1, 0, 0, 0, 0, −1], [0, 0, 0, 0, 0, 0, 1, −1]}

NullSpace N_0

{}

Eigenvalues M

[7., −1., −1., −1., −1., −1., −1., −1.]

Eigenvalues N

[7., −1., −1., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

80, [1, -1, 1, -1, 1, 1, -1, -1]

==

{3, 4, 5, 7}

R: [2, 3, 6, 7, 8, 4, 5, 1]
B: [6, 7, 2, 3, 4, 8, 1, 5]

TRACE TWO = 1

det AT = $-1\,(t)^4$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 8

Level 2 det = $\frac{5}{4096} (-1 + s) (-101 + 2s + 8s^2 + 6s^3 + 5s^4) (5 + s^2) (1 + s)$

RANK of R is 8

R ranking is 1, "vs", 8

RBAR ranking 1, "vs", 8

RANK of B is 8

B ranking is 1, "vs", 8

BBAR ranking 1, "vs", 8

"R CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6] v[7] v[8]
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6] v[7] v[8]

Eigenvalues

R: [-1. I, 1. I, -1., 1., -0.7071067810 - 0.7071067810 I, 0.7071067810 + 0.7071067810 I, -0.7071067810 + 0.7071067810 I, 0.7071067810 - 0.7071067810 I]

B: [-1. I, 1. I, -1., 1., -0.7071067810 - 0.7071067810 I, 0.7071067810 + 0.7071067810 I, -0.7071067810 + 0.7071067810 I, 0.7071067810 - 0.7071067810 I]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 8

"RANK of the KERNEL is ", 8

"IdemSolvability Check", 3   "Trace mark", 8, "Rank mark", 8, "for kernel rank", 8

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{28}$ ($v[1]v[8] + v[5]v[7] + v[1]v[4] + v[3]v[8] + v[3]v[6] + v[6]v[7] + v[4]v[7] + v[5]v[6] + v[2]v[4] + v[2]v[6] + v[4]v[5] + v[2]v[7] + v[1]v[3] + v[1]v[7] + v[6]v[8] + v[5]v[8] + v[2]v[3] + v[1]v[5] + v[4]v[8] + v[1]v[2] + v[1]v[6] + v[3]v[4] + v[2]v[8] + v[7]v[8] + v[4]v[6] + v[2]v[5] + v[3]v[7] + v[3]v[5]$)

degree 3 : $\frac{1}{56}$ ($v[2]v[3]v[8] + v[6]v[7]v[8] + v[1]v[2]v[6] + v[1]v[4]v[5] + v[2]v[4]v[6] + v[2]v[7]v[8] + v[3]v[6]v[7] + v[4]v[7]v[8] + v[3]v[4]v[8] + v[1]v[2]v[3] + v[2]v[3]v[4] + v[2]v[3]v[6] + v[3]v[4]v[7] + v[4]v[5]v[6] + v[3]v[5]v[6] + v[1]v[5]v[7] + v[$

2]v[3]v[7] + v[5]v[6]v[8] + v[1]v[3]v[5] + v[1]v[4]v[6] + v[2]v[4]v[8] + v[1]v[7]v[8]
+ v[1]v[2]v[8] + v[1]v[2]v[4] + v[2]v[4]v[7] + v[4]v[5]v[7] + v[3]v[4]v[6] + v[1]v[2]v
[7] + v[1]v[6]v[8] + v[4]v[5]v[8] + v[1]v[4]v[8] + v[3]v[4]v[5] + v[5]v[6]v[7] + v[1]v[
6]v[7] + v[3]v[6]v[8] + v[2]v[5]v[7] + v[2]v[5]v[6] + v[2]v[6]v[7] + v[1]v[5]v[8] + v[
1]v[5]v[6] + v[5]v[7]v[8] + v[1]v[3]v[4] + v[2]v[3]v[5] + v[1]v[3]v[6] + v[1]v[4]v[7]
+ v[1]v[2]v[5] + v[2]v[4]v[5] + v[2]v[5]v[8] + v[3]v[7]v[8] + v[4]v[6]v[7] + v[3]v[5]
v[7] + v[2]v[6]v[8] + v[3]v[5]v[8] + v[1]v[3]v[8] + v[1]v[3]v[7] + v[4]v[6]v[8])

degree 4 : $\frac{1}{28}$ (2 v[1]v[2]v[4]v[5] + 2 v[1]v[3]v[4]v[7] + 2 v[1]v[2]v[5]v[8] + 3 v[1]v[4
]v[6]v[8] + 2 v[1]v[6]v[7]v[8] + 2 v[1]v[3]v[6]v[8] + 2 v[3]v[6]v[7]v[8] + 2 v[3]v[4]v[6]
v[7] + 2 v[2]v[5]v[6]v[8] + 2 v[2]v[4]v[5]v[7] + 3 v[2]v[3]v[5]v[7] + 2 v[2]v[4]v[6]v[8]
+ 2 v[1]v[2]v[6]v[7] + 2 v[1]v[4]v[6]v[7] + 2 v[2]v[3]v[5]v[8] + 3 v[3]v[4]v[7]v[8] + 2 v
[1]v[2]v[4]v[6] + 2 v[3]v[4]v[5]v[8] + 2 v[2]v[4]v[7]v[8] + 3 v[2]v[3]v[4]v[8] + 3 v[5]v
[6]v[7]v[8] + 3 v[2]v[4]v[5]v[6] + 3 v[1]v[3]v[5]v[8] + 3 v[1]v[4]v[5]v[8] + 2 v[1]v[2]v[
4]v[8] + 3 v[3]v[4]v[5]v[6] + 3 v[4]v[6]v[7]v[8] + 2 v[1]v[3]v[4]v[5] + 2 v[2]v[3]v[4]v[
7] + 2 v[2]v[5]v[6]v[7] + 2 v[1]v[3]v[5]v[7] + 2 v[2]v[6]v[7]v[8] + 3 v[2]v[4]v[6]v[7] +
3 v[2]v[4]v[5]v[8] + 3 v[1]v[2]v[4]v[7] + 3 v[1]v[2]v[6]v[8] + 2 v[1]v[5]v[6]v[8] + 3 v[1
]v[3]v[4]v[6] + 2 v[4]v[5]v[7]v[8] + 3 v[1]v[3]v[7]v[8] + 3 v[3]v[4]v[5]v[7] + 2 v[3]v[5
]v[6]v[7] + 2 v[1]v[3]v[4]v[8] + 3 v[1]v[2]v[5]v[6] + 2 v[2]v[3]v[4]v[6] + 3 v[3]v[5]v[6
]v[8] + 2 v[1]v[5]v[7]v[8] + 2 v[2]v[3]v[7]v[8] + 2 v[3]v[5]v[7]v[8] + 2 v[1]v[2]v[5]v[7]
+ 2 v[4]v[5]v[6]v[8] + 2 v[2]v[3]v[5]v[6] + 2 v[4]v[5]v[6]v[7] + 2 v[1]v[4]v[5]v[6] + 3
v[2]v[5]v[7]v[8] + 3 v[1]v[5]v[6]v[7] + 2 v[1]v[3]v[5]v[6] + 2 v[2]v[3]v[4]v[5] + 3 v[2]
v[3]v[6]v[8] + 2 v[1]v[4]v[7]v[8] + 3 v[1]v[4]v[5]v[7] + 2 v[3]v[4]v[6]v[8] + 3 v[1]v[2]
v[7]v[8] + 3 v[2]v[3]v[6]v[7] + 3 v[1]v[2]v[3]v[4] + 3 v[1]v[3]v[6]v[7] + 2 v[1]v[2]v[3]
v[8] + 2 v[1]v[2]v[3]v[6] + 2 v[1]v[2]v[3]v[7] + 3 v[1]v[2]v[3]v[5])

degree 5 : $\frac{1}{56}$ (v[1]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[8] + v[2]v[3]v[4]v[5]v[8] +
v[2]v[4]v[5]v[7]v[8] + v[1]v[5]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]
v[8] + v[2]v[3]v[4]v[5]v[7] + v[1]v[2]v[5]v[7]v[8] + v[1]v[2]v[3]v[6]v[8] + v[2]v[4]v[
5]v[6]v[8] + v[1]v[4]v[6]v[7]v[8] + v[2]v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[5]v[7] + v[1]
v[2]v[4]v[6]v[8] + v[2]v[4]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6] + v[1]v[2]v[4]v[6]v[7]
+ v[2]v[3]v[5]v[6]v[8] + v[1]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[5]v[
6]v[8] + v[1]v[3]v[5]v[6]v[8] + v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[4]v[5]v[6] + v[1]v[3]
v[4]v[6]v[7] + v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[7]v[8] + v[
4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[8] + v[2]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[
8] + v[1]v[2]v[4]v[7]v[8] + v[1]v[3]v[6]v[7]v[8] + v[1]v[2]v[5]v[6]v[7] + v[2]v[3]v[4]
v[6]v[8] + v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6] + v[1]v[2]v[3]v[5]v[6] + v[3]v[
4]v[5]v[6]v[7] + v[2]v[4]v[5]v[6]v[7] + v[1]v[4]v[5]v[6]v[8] + v[1]v[2]v[4]v[5]v[7] +
v[3]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7] + v[1]v[2]v[3]v[5]v[7] + v[1]v[2]v[3]v[4]
v[7] + v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[8] + v[1]v[2]v[6]v[7]v[8] + v[3]v[4]v[
5]v[7]v[8] + v[1]v[2]v[4]v[5]v[8] + v[1]v[2]v[3]v[4]v[6] + v[1]v[3]v[4]v[7]v[8] + v[1]

$v[3]v[5]v[7]v[8]$)

degree 6 : $\frac{1}{28}$ ($v[1]v[2]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[5]v[6]v[8] + v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7] + v[1]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[7] + v[2]v[3]v[4]v[5]v[7]v[8] + v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[8] + v[1]v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[8] + v[1]v[2]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[5]v[6]v[7] + v[1]v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[6]$)

degree 7 : $\frac{1}{8}$ ($v[1]v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[7]v[8]$)

degree 8 : 1 ($v[1]$) ($v[2]$) ($v[3]$) ($v[8]$) ($v[7]$) ($v[6]$) ($v[5]$) ($v[4]$)

Group spectrum $1 + t + t^2 + t^3 + 2t^4 + t^5 + t^6 + t^7 + t^8$

KERNEL STRUCTURE

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$\pi 8 = [1]$$

supp $\pi 8$ = {1}

$$u8 = [1]$$

supp u8 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R−BLOCKS,

[8, 1, 2, 6, 7, 3, 4, 5]

B−BLOCKS,

[7, 3, 4, 5, 8, 1, 2, 6]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N − check: true

$b_1 = \{1\}$
$b_2 = \{2\}$
$b_3 = \{3\}$
$b_4 = \{4\}$
$b_5 = \{5\}$
$b_6 = \{6\}$
$b_7 = \{7\}$
$b_8 = \{8\}$

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & h[1] & h[1] & h[1] & h[1] & h[1] & h[1] \\ h[1] & h[2] & h[1] & h[1] & h[1] & h[1] & h[1] & h[1] \\ h[1] & h[1] & h[2] & h[1] & h[1] & h[1] & h[1] & h[1] \\ h[1] & h[1] & h[1] & h[2] & h[1] & h[1] & h[1] & h[1] \\ h[1] & h[1] & h[1] & h[1] & h[2] & h[1] & h[1] & h[1] \\ h[1] & h[1] & h[1] & h[1] & h[1] & h[2] & h[1] & h[1] \\ h[1] & h[1] & h[1] & h[1] & h[1] & h[1] & h[2] & h[1] \\ h[1] & h[1] & h[1] & h[1] & h[1] & h[1] & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 49, Shape: $48 \oplus 1/0$

$$CLB = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4, 5, 6, 7, 8}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 4, 5, 6, 7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

1, "range", [1, 2, 3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 4, 3, 2, 1], [8, 7, 5, 2, 6, 4, 1, 3], [8, 7, 4, 6, 3, 1, 5, 2], [8, 7, 3, 4, 1, 2, 6, 5], [8, 7, 2, 1, 5, 6, 3, 4], [8, 7, 1, 3, 2, 5, 4, 6], [8, 6, 7, 4, 5, 3, 1, 2], [8, 6, 5, 7, 3, 2, 4, 1], [8, 6, 4, 1, 7, 5, 2, 3], [8, 6, 3, 5, 2, 1, 7, 4], [8, 6, 2, 3, 1, 4, 5, 7], [8, 6, 1, 2, 4, 7, 3, 5], [8, 5, 7, 6, 2, 4, 3, 1], [8, 5, 6, 3, 7, 2, 1, 4], [8, 5, 4, 2, 1, 3, 7, 6], [8, 5, 3, 1, 6, 7, 4, 2], [8, 5, 2, 7, 4, 1, 6, 3], [8, 5, 1, 4, 3, 6, 2, 7], [8, 4, 7, 3, 6, 1, 2, 5], [8, 4, 6, 7, 1, 5, 3, 2], [8, 4, 5, 1, 2, 3, 6, 7], [8, 4, 3, 2, 7, 6, 5, 1], [8, 4, 2, 5, 3, 7, 1, 6], [8, 4, 1, 6, 5, 2, 7, 3], [8, 3, 7, 1, 4, 2, 5, 6], [8, 3, 6, 2, 5, 1, 4, 7], [8, 3, 5, 6, 1, 7, 2, 4], [8, 3, 4, 7, 2, 6, 1, 5], [8, 3, 2, 4, 6, 5, 7, 1], [8, 3, 1, 5, 7, 4, 6, 2], [8, 2, 7, 5, 1, 6, 4, 3], [8, 2, 6, 1, 3, 4, 7, 5], [8, 2, 5, 4, 7, 1, 3, 6], [8, 2, 4, 3, 5, 7, 6, 1], [8, 2, 3, 6, 4, 5, 1, 7], [8, 2, 1, 7, 6, 3, 5, 4], [8, 1, 7, 2, 3, 5, 6, 4], [8, 1, 6, 4, 2, 7, 5, 3], [8, 1, 5, 3, 4, 6, 7, 2], [8, 1, 4, 5, 6, 2, 3, 7], [8, 1, 3, 7, 5, 4, 2, 6], [8, 1, 2, 6, 7, 3, 4, 5], [7, 8, 6, 4, 5, 2, 3, 1], [7, 8, 5, 6, 2, 1, 4, 3], [7, 8, 4, 3, 6, 5, 1, 2], [7, 8, 3, 1, 4, 6, 2, 5], [7, 8, 2, 5, 1, 3, 6, 4], [7, 8, 1, 2, 3, 4, 5, 6], [7, 6, 8, 5, 4, 2, 1, 3], [7, 6, 5, 1, 8, 4, 3, 2], [7, 6, 4, 8, 2, 3, 5, 1], [7, 6, 3, 2, 1, 5, 4, 8], [7, 6, 2, 4, 3, 1, 8, 5], [7, 6, 1, 3, 5, 8, 2, 4], [7, 5, 8, 2, 6, 1, 3, 4], [7, 5, 6, 8, 1, 4, 2, 3], [7, 5, 4, 1, 3, 2, 6, 8], [7, 5, 3, 4, 2, 8, 1, 6], [7, 5, 2, 3, 8, 6, 4, 1], [7, 5, 1, 6, 4, 3, 8, 2], [7, 4, 8, 6, 3, 5, 2, 1], [7, 4, 6, 2, 8, 3, 1, 5], [7, 4, 5, 3, 1, 2, 8, 6], [7, 4, 3, 8, 5, 1, 6, 2], [7, 4, 2, 1, 6, 8, 5, 3], [7, 4, 1, 5, 2, 6, 3, 8], [7, 3, 8, 4, 1, 6, 5, 2], [7, 3, 6, 1, 2, 5, 8, 4], [7, 3, 5, 2, 4, 8, 6, 1], [7, 3, 4, 5, 8, 1, 2, 6], [7, 3, 2, 6, 5, 4, 1, 8], [7, 3, 1, 8, 6, 2, 4, 5], [7, 2, 8, 1, 5, 3, 4, 6], [7, 2, 6, 3, 4, 1, 5, 8], [7, 2, 5, 8, 3, 6, 1, 4], [7, 2, 4, 6, 1, 8, 3, 5], [7, 2, 3, 5, 6, 4, 8, 1], [7, 2, 1, 4, 8, 5, 6, 3], [7, 1, 8, 3, 2, 4, 6, 5], [7, 1, 6, 5, 3, 8, 4, 2], [7, 1, 5, 4, 6, 3, 2, 8], [7, 1, 4, 2, 5, 6, 8, 3], [7, 1, 3, 6, 8, 2, 5, 4], [7, 1, 2, 8, 4, 5, 3, 6], [6, 8, 7, 5, 4, 1, 3, 2], [6, 8, 5, 3, 7, 4, 2, 1], [6, 8, 4, 7, 1, 2, 5, 3], [6, 8, 3, 2, 5, 7, 1, 4], [6, 8, 2, 1, 3, 5, 4, 7], [6, 8, 1, 4, 2, 3, 7, 5], [6, 7, 8, 4, 5, 1, 2, 3], [6, 7, 5, 8, 1, 3, 4, 2], [6, 7, 4, 2, 8, 5, 3, 1], [6, 7, 3, 1, 2, 4, 5, 8], [6, 7, 2, 3, 4, 8, 1, 5], [6, 7, 1, 5, 3, 2, 8, 4], [6, 5, 8, 7, 3, 4, 1, 2], [6, 5, 7, 1, 8, 3, 2, 4], [6, 5, 4, 3, 2, 1, 8, 7], [6, 5, 3, 8, 4, 2, 7, 1], [6, 5, 2, 4, 1, 7, 3, 8], [6, 5, 1, 2, 7, 8, 4, 3], [6, 4, 8, 1, 7, 2, 3, 5], [6, 4, 7, 8, 2, 5, 1, 3], [6, 4, 5, 2, 3, 1, 7, 8], [6, 4, 3, 5, 1, 8, 2, 7], [6, 4, 2, 7, 5, 3, 8, 1], [6, 4, 1, 3, 8, 7, 5, 2], [6, 3, 8, 5, 2, 7, 4, 1], [6, 3, 7, 2, 1, 4, 8, 5], [6, 3, 5, 4, 8, 2, 1, 7], [6, 3, 4, 1, 5, 8, 7, 2], [6, 3, 2, 8, 7, 1, 5, 4], [6, 3, 1, 7, 4, 5, 2, 8], [6, 2, 8, 3, 1, 5, 7, 4], [6, 2, 7, 4, 3, 8, 5, 1], [6, 2, 5, 1, 4, 7, 8, 3], [6, 2, 4, 5, 7, 3, 1, 8], [6, 2, 3, 7, 8, 1, 4, 5], [6, 2, 1, 8, 5, 4, 3, 7], [6, 1, 8, 2, 4, 3, 5, 7], [6, 1, 7, 3, 5, 2, 4, 8], [6, 1, 5, 7, 2, 8, 3, 4], [6, 1, 4, 8, 3, 7, 2, 5], [6, 1, 3, 4, 7, 5, 8, 2], [6, 1, 2, 5, 8, 4, 7, 3], [5, 8, 7, 2, 6, 3, 4, 1], [5, 8, 6, 7, 3, 1, 2, 4], [5, 8, 4, 1, 2, 7, 3, 6], [5, 8, 3, 6, 1, 4, 7, 2], [5, 8, 2, 4, 7, 6, 1, 3], [5, 8, 1, 3, 4, 2, 6, 7], [5, 7, 8, 6, 2, 3, 1, 4], [5, 7, 6, 1, 8, 2, 4, 3], [5, 7, 4, 3, 1, 6, 2, 8], [5, 7, 3, 2, 4, 1, 8, 6], [5, 7, 2, 8, 3, 4, 6, 1], [5, 7, 1, 4, 6, 8, 3, 2], [5, 6, 8, 3, 7, 1, 4, 2], [5, 6, 7, 8, 1, 2, 3, 4], [5, 6, 4, 2, 3, 8, 1, 7], [5, 6, 3, 4, 8, 7, 2, 1], [5, 6, 2, 1, 4, 3, 7, 8], [5, 6, 1, 7, 2, 4, 8, 3], [5, 4, 8, 2, 1, 7, 6, 3], [5, 4, 7, 1, 3, 6, 8, 2], [5, 4, 6, 3, 2, 8, 7, 1], [5, 4, 3, 7, 6, 2, 1, 8], [5, 4, 2, 6, 8, 1, 3, 7], [5, 4, 1, 8, 7, 3, 2, 6], [5, 3, 8, 1, 6, 4, 2, 7], [5, 3, 7, 4, 2, 1, 6, 8], [5, 3, 6, 8, 4, 7, 1, 2], [5, 3, 4, 6, 7, 2, 8, 1], [5, 3, 2, 7, 1, 8, 4, 6], [5, 3, 1, 2, 8, 6, 7, 4], [5, 2, 8, 7, 4, 6, 3, 1], [5, 2, 7, 3, 8, 4, 1, 6], [5, 2, 6, 4, 1, 3, 8, 7], [5, 2, 4, 8, 6, 1, 7, 3], [5, 2, 3, 1, 7, 8, 6, 4], [5, 2, 1, 6, 3, 7, 4, 8], [5, 1, 8, 4, 3, 2, 7, 6], [5, 1, 7, 6, 4, 8, 2, 3], [5, 1, 6, 2, 7, 4, 3, 8], [5, 1, 4, 7, 8, 3, 6, 2], [5, 1, 3, 8, 2, 6, 4, 7], [5, 1, 2, 3, 6, 7, 8, 4], [4, 8, 7, 6, 3, 2, 1, 5], [4, 8, 6, 1, 7, 3, 5, 2], [4, 8, 5, 2, 1, 6, 3, 7], [4, 8, 3, 7, 2, 5, 6, 1], [4, 8, 2, 3, 5, 1, 7, 6], [4, 8, 1, 5, 6, 7, 2, 3], [4, 7, 8, 3, 6, 2, 5, 1], [4, 7, 6, 8, 2, 1, 3, 5], [4, 7, 5, 1, 3, 8, 2, 6], [4, 7, 3, 5, 8, 6, 1, 2], [4, 7, 2, 6, 1, 5, 8, 3], [4, 7, 1, 2, 5, 3, 6, 8], [4, 6, 8, 7, 1, 3, 2, 5], [4, 6, 7, 2, 8, 1, 5, 3], [4, 6, 5, 3, 2, 7, 1, 8], [4, 6, 3, 1, 5, 2, 8, 7], [4, 6, 2, 5, 7, 8, 3, 1], [4, 6, 1, 8, 3, 5, 7, 2], [4, 5, 8, 1, 2, 6, 7, 3], [4, 5, 7, 3, 1, 8, 6, 2], [4, 5, 6, 2, 3, 7, 8, 1], [4, 5, 3, 6, 7, 1, 2, 8], [4, 5, 2, 8, 6, 3, 1, 7], [4, 5, 1, 7, 8, 2, 3, 6], [4, 3, 8, 2, 7, 5, 1, 6], [4, 3, 7, 8, 5, 6, 2, 1], [4, 3, 6, 5, 1, 2, 7, 8], [4, 3, 5, 7, 6, 1, 8, 2], [4, 3, 2, 1, 8, 7, 6, 5], [4, 3, 1, 6, 2, 8, 5, 7], [4, 2, 8, 5, 3, 1, 6, 7], [4, 2, 7, 1, 6, 5, 3, 8], [4, 2, 6, 7, 5, 8, 1, 3], [4, 2, 5, 6, 8, 3, 7, 1], [4, 2, 3, 8, 1, 7, 5, 6], [4, 2, 1, 3, 7, 6, 8, 5], [4, 1, 8, 6, 5, 7, 3, 2], [4, 1, 7, 5, 2, 3, 8, 6], [4, 1, 6, 3, 8, 5, 2, 7], [4, 1, 5, 8, 7, 2, 6, 3], [4, 1, 3, 2, 6, 8, 7, 5], [4, 1, 2, 7, 3, 6, 5, 8], [3, 8, 7, 4, 1, 5, 2, 6], [3, 8, 6, 5, 2, 4, 1, 7], [3, 8, 5, 1, 6, 2, 7, 4], [3, 8, 4, 2, 7, 1, 6, 5], [3, 8, 2, 6, 4, 7, 5, 1], [3, 8, 1, 7, 5, 6, 4, 2], [3, 7, 8, 1, 4, 5, 6, 2], [3, 7, 6, 2, 1, 8, 5, 4], [3, 7, 5, 4, 2, 6, 8, 1], [3, 7, 4, 8, 5, 2, 1, 6], [3, 7, 2, 5, 6, 1, 4, 8], [3, 7, 1, 6, 8, 4, 2, 5], [3, 6, 8, 2, 5, 4, 7, 1], [3, 6, 7, 1, 2, 8, 4, 5], [3, 6, 5, 8, 4, 1, 2, 7], [3, 6, 4, 5, 1, 7, 8, 2], [3, 6, 2, 7, 8, 5, 1, 4], [3, 6, 1, 4, 7, 2, 5, 8], [3, 5, 8, 6, 1, 2, 4, 7], [3, 5, 7, 2, 4, 6, 1, 8], [3, 5, 6, 4, 8, 1, 7, 2], [3, 5, 4, 7, 6, 8, 2, 1], [3, 5, 2, 1, 7, 4, 8, 6], [3, 5, 1, 8, 2, 7, 6, 4], [3, 4, 8, 7, 2, 1, 5, 6], [3, 4, 7, 5, 8, 2, 6, 1], [3, 4, 6, 1, 5, 7, 2, 8], [3, 4, 5, 6, 7, 8, 1, 2], [3, 4, 2, 8, 1, 6, 7, 5], [3, 4, 1, 2, 6, 5, 8, 7], [3, 2, 8, 4, 6, 7, 1, 5], [3, 2, 7, 6, 5, 1, 8, 4], [3, 2, 6, 8, 7, 5, 4, 1], [3, 2, 5, 7, 1, 4, 6, 8], [3, 2, 4, 1, 8, 6, 5, 7], [3, 2, 1, 5, 4, 8, 7, 6], [3, 1, 8, 5, 7, 6, 2, 4], [3, 1, 7, 8, 6, 4, 5, 2], [3, 1, 6, 7, 4, 2, 8, 5], [3, 1, 5, 2, 8, 7, 4, 6], [3, 1, 4, 6, 2, 5, 7, 8], [3, 1, 2, 4, 5, 8, 6, 7], [2, 8, 7, 1, 5, 4, 6, 3], [2, 8, 6, 3, 1, 7, 4, 5], [2, 8, 5, 7, 4, 3, 1, 6], [2, 8, 4, 5, 3, 6, 7, 1], [2, 8, 3, 4, 6, 1, 5, 7], [2, 8, 1, 6, 7, 5, 3, 4], [2, 7, 8, 5, 1, 4, 3, 6], [2, 7, 6, 4, 3, 5, 1, 8], [2, 7, 5, 3, 8, 1, 6, 4], [2, 7, 4, 1, 6, 3, 8, 5], [2, 7, 3, 6, 5, 8, 4, 1], [2, 7, 1, 8, 4, 6, 5, 3], [2, 6, 8, 1, 3, 7, 5, 4], [2, 6, 7, 3, 4, 5, 8, 1], [2, 6, 5, 4, 1, 8, 7, 3], [2, 6, 4, 7, 5, 1, 3, 8], [2, 6, 3, 8, 7, 4, 1, 5], [2, 6, 1, 5, 8, 3, 4, 7], [2, 5, 8, 4, 7, 3, 6, 1], [2, 5, 7, 8, 3, 1, 4, 6], [2, 5, 6, 1, 4, 8, 3, 7], [2, 5, 4, 6, 8, 7, 1, 3], [2, 5, 3, 7, 1, 6, 8, 4], [2, 5, 1, 3, 6, 4, 7, 8], [2, 4, 8, 3, 5, 6, 1, 7], [2, 4, 7, 6, 1, 3, 5, 8], [2, 4, 6, 5, 7, 1, 8, 3], [2, 4, 5, 8, 6, 7, 3, 1], [2, 4, 3, 1, 8, 5, 7, 6], [2, 4, 1, 7, 3, 8, 6, 5], [2, 3, 8, 6, 4, 1, 7, 5], [2, 3, 7, 5, 6, 8, 1, 4], [2, 3, 6, 7, 8, 4, 5, 1], [2, 3, 5, 1, 7, 6, 4, 8], [2, 3, 4, 8, 1, 5, 6, 7], [2, 3, 1, 4, 5, 7, 8, 6], [2, 1, 8, 7, 6, 5, 4, 3], [2, 1, 7, 4, 8, 6, 3, 5], [2, 1, 6, 8, 5, 3, 7, 4], [2, 1, 5, 6, 3, 4, 8, 7], [2, 1, 4, 3, 7, 8, 5, 6], [2, 1, 3, 5, 4, 7, 6, 8], [1, 8, 7, 3, 2, 6, 5, 4], [1, 8, 6, 2, 4, 5, 7, 3], [1, 8, 5, 4, 3, 7, 6, 2], [1, 8, 4, 6, 5, 3, 2, 7], [1, 8, 3, 5, 7, 2, 4, 6], [1, 8, 2, 7, 6, 4, 3, 5], [1, 7, 8, 2, 3, 6, 4, 5], [1, 7, 6, 3, 5, 4, 8, 2], [1, 7, 5, 6, 4, 2, 3, 8], [1, 7, 4, 5, 2, 8, 6, 3], [1, 7, 3, 8, 6, 5, 2, 4], [1, 7, 2, 4, 8, 3, 5, 6], [1, 6, 8, 4, 2, 5, 3, 7], [1, 6, 7, 5, 3, 4, 2, 8], [1, 6, 5, 2, 7, 3, 8, 4], [1, 6, 4, 3, 8, 2, 7, 5], [1, 6, 3, 7, 4, 8, 5, 2], [1, 6, 2, 8, 5, 7, 4, 3], [1, 5, 8, 3, 4, 7, 2, 6], [1, 5, 7, 4, 6, 2, 8, 3], [1, 5, 6, 7, 2, 3, 4, 8], [1, 5, 4, 8, 7, 6, 3, 2], [1, 5, 3, 2, 8, 4, 6, 7], [1, 5, 2, 6, 3, 8, 7, 4], [1, 4, 8, 5, 6, 3, 7, 2], [1, 4, 7, 2, 5, 8, 3, 6], [1, 4, 6, 8, 3, 2, 5, 7], [1, 4, 5, 7, 8, 6, 2, 3], [1, 4, 3, 6, 2, 7, 8, 5], [1, 4, 2, 3, 7, 5, 6, 8], [1, 3, 8, 7, 5, 2, 6, 4], [1, 3, 7, 6, 8, 5, 4, 2], [1, 3, 6, 4, 7, 8, 2, 5], [1, 3, 5, 8, 2, 4, 7, 6], [1, 3, 4, 2, 6, 7, 5, 8], [1, 3, 2, 5, 4, 6, 8, 7], [1, 2, 8, 6, 7, 4, 5, 3], [1, 2, 7, 8, 4, 3, 6, 5], [1, 2, 6, 5, 8, 7, 3, 4], [1, 2, 5, 3, 6, 8, 4, 7], [1, 2, 4, 7, 3, 5, 8, 6], [1, 2, 3, 4, 5, 6, 7, 8]]

"group has", 336, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

g_1 =   [[1, 8], [2, 7], [3, 6], [4, 5]]

g_2 =   [[1, 8, 3, 5, 6, 4, 2, 7]]

g_3 =   [[1, 8, 2, 7, 5, 3, 4, 6]]

$g_4 =$ [[1, 8, 5], [2, 7, 6]]

$g_5 =$ [[1, 8, 4], [2, 7, 3]]

linear dimension, 50

"Symmetric?", true

Is Z in Vec(K)? true

$$\left(\frac{||(96h[2]+252h[1])||}{||7||} \quad \frac{||(-12h[2])||}{||7||} \quad \frac{||(-24h[2]-126h[1])||}{||7||} \quad \frac{||(36h[2]+84h[1])||}{||7||} \quad \frac{||(-}{}\right.$$

"Basis for Z(G)"

1, "coeff", 42

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 6

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 7. & -1. & -1. & -1. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3 + 2t^4 + t^5 + t^6 + t^7 + t^8$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 6t^4 + 8t^5 + 15t^6 + 22t^7 + 38t^8 + 55t^9 + 89t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 8 = (1)$

{1}

$u8 = (1)$

{1}

picheck $(1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1)$

$\pi = \left(\frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8}\right)$

$\pi 7 = (1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1)$

$u7 = \left(\frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8}\right)$

picheck $(7 \;\; 7 \;\; 7 \;\; 7 \;\; 7 \;\; 7 \;\; 7 \;\; 7)$

$\pi 6 =$
$(2 \;\; 2$

$u6 =$
$\left(\frac{1}{32} \;\; \frac{1}{32}\right.$

picheck $(42 \;\; 42 \;\; 42 \;\; 42 \;\; 42 \;\; 42 \;\; 42 \;\; 42)$

$\pi 5$ =
$(6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6$

$u5$ =
$\left(\frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\right.$

picheck $(210\ \ 210\ \ 210\ \ 210\ \ 210\ \ 210\ \ 210\ \ 210)$

$\pi 4$ =
$(24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24$

$u4$ =
$\left(\frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\right.$

picheck $(840\ \ 840\ \ 840\ \ 840\ \ 840\ \ 840\ \ 840\ \ 840)$

$\pi 3$ =
$(120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120$

$u3$ =
$\left(\frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ 4\right.$

picheck $(2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520)$

$\pi 2$ =
$(720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720$

$u2$ =
$\left(\frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\right.$

picheck $(5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

$\pi 1 = (5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

$u1 = \left(\frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\right)$

picheck $(5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

Column Projections

$$P_1 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix} \quad NM = \begin{pmatrix} 7&6&6&6&6&6&6&6\\ 6&7&6&6&6&6&6&6\\ 6&6&7&6&6&6&6&6\\ 6&6&6&7&6&6&6&6\\ 6&6&6&6&7&6&6&6\\ 6&6&6&6&6&7&6&6\\ 6&6&6&6&6&6&7&6\\ 6&6&6&6&6&6&6&7 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0, 0, 0]$

ker $N_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} t+s & t+s & t+s & t+s & t+s & t+s & t+s & t+s \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & 0 & 0 & 0 & t \\ 0 & s & 0 & 0 & 0 & t & 0 \\ t & 0 & 0 & 0 & 0 & 0 & s \\ 0 & t & 0 & 0 & 0 & s & 0 \\ 0 & 0 & t & s & 0 & 0 & 0 \\ 0 & 0 & s & t & 0 & 0 & 0 \\ -t & -t & -t & -t & -t+s & -t & -t \\ -s & -s & -s & -s & -s+t & -s & -s \end{pmatrix}$$

RB checks

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & t & s & 0 & 0 & 0 & 0 \\ t & 0 & 0 & 0 & 0 & 0 & 0 & s \\ 0 & 0 & s & t & 0 & 0 & 0 & 0 \\ s & 0 & 0 & 0 & 0 & 0 & 0 & t \\ 0 & s & 0 & 0 & 0 & t & 0 & 0 \\ 0 & t & 0 & 0 & 0 & s & 0 & 0 \\ 0 & 0 & 0 & 0 & s & 0 & t & 0 \\ 0 & 0 & 0 & 0 & t & 0 & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, BB^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 8, "vs", 8

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 1T + 48\Omega$$

Ω

$$\left(\frac{5}{8}\ \frac{1}{8}\ \frac{-41}{16}\ \frac{13}{8}\ \frac{23}{8}\ \frac{-3}{4}\ \frac{-19}{8}\ \frac{5}{8}\ 5\ 5\ \frac{41}{8}\ \frac{-3}{4}\ 2\ \frac{-1}{8}\ \frac{21}{8}\ \frac{-1}{4}\ \frac{7}{4}\ \frac{5}{8}\ \frac{-1}{16}\ \frac{15}{32}\ -\right.$$

T

$$\left(1\ \ -1\ \ \frac{-9}{2}\ \frac{1}{3}\ \frac{17}{2}\ -3\ \ -4\ \ 1\ \ 11\ \ 15\ \ \frac{33}{2}\ \frac{-5}{2}\ \frac{29}{7}\ -1\ \ 4\ \ -1\ \ 3\ \ 1\ \ \frac{-1}{2}\ \frac{3}{4}\ \frac{-}{2}\right.$$

"IS NM in Vec(K)?", true

NM

$$\left(31\ \ 5\ \ \frac{-255}{2}\ \frac{235}{3}\ \frac{293}{2}\ -39\ \ -118\ \ 31\ \ 251\ \ 255\ \ \frac{525}{2}\ \frac{-77}{2}\ \frac{701}{7}\ -7\ \ 130\ \ -\right.$$

"IS MN in Vec(K)?", true

MN

$$\left(31\ \ 5\ \ \frac{-255}{2}\ \frac{235}{3}\ \frac{293}{2}\ -39\ \ -118\ \ 31\ \ 251\ \ 255\ \ \frac{525}{2}\ \frac{-77}{2}\ \frac{701}{7}\ -7\ \ 130\ \ -\right.$$

$$\boldsymbol{\tau} = 8/1 \text{ , rank } = 8 \text{ , ratio } = 1/1 \text{ , } n^2 / r = 8/1$$

$$\boldsymbol{\tau}' = 56/1 \text{ , } r' = 7/8 \text{ , } \boldsymbol{\tau} / n^2 = 1/8$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 336

KERNEL HAS LINEAR DIMENSION 50
out of total no. of elements equal to 336

dim span idems 1 vs no. of idems 1

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_c

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0] }

NullSpace N_C

{[1, 1, 1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[0, 0, 0, 0, 0, −1, 0, 1], [1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, 0, 0, −1, 0, 0], [0, 0, 0, 0, 1, −1, 0, 0], [0, 0, 0, 1, 0, −1, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0], [0, 0, 1, 0, 0, −1, 0, 0]}

NullSpace N_0

{}

Eigenvalues M

[7., −1., −1., −1., −1., −1., −1., −1.]

Eigenvalues N

[7., −1., −1., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{3, 4, 5, 8}

R: [2, 3, 6, 7, 8, 4, 1, 5]
B: [6, 7, 2, 3, 4, 8, 5, 1]

TRACE TWO = 1

det AT = $1\,(t)^4$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 8

Level 2 det = $\frac{5}{8192}(-1+s)(-1010-81s-349s^2-63s^3+s^4+21s^5+25s^6+11s^7+5s^8)$

RANK of R is 8

R ranking is 1, "vs", 8

RBAR ranking 1, "vs", 8

RANK of B is 8

B ranking is 1, "vs", 8

BBAR ranking 1, "vs", 8

"R CYCLES", (1 + v[5] v[8]) (1 + v[1] v[2] v[3] v[4] v[6] v[7])
"B CYCLES", (1 + v[2] v[3] v[4] v[5] v[7]) (1 + v[1] v[6] v[8])

Eigenvalues

R: [−0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 0.5000000000 + 0.8660254040 I, 0.5000000000 − 0.8660254040 I, 1., −1., 1., −1.]

B: [−0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 0.3090169942 + 0.9510565160 I, −0.8090169942 + 0.5877852520 I, −0.8090169942 − 0.5877852520 I, 0.3090169942 − 0.9510565160 I, 1., 1.]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 8

"RANK of the KERNEL is ", 8

"IdemSolvability Check", 3 "Trace mark", 8, "Rank mark", 8, "for kernel rank", 8

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{28}$ ($v[1]v[8] + v[5]v[7] + v[1]v[4] + v[3]v[8] + v[3]v[6] + v[6]v[7] + v[4]v[7] + v[5]v[6] + v[2]v[4] + v[2]v[6] + v[4]v[5] + v[2]v[7] + v[1]v[3] + v[1]v[7] + v[6]v[8] + v[5]v[8] + v[2]v[3] + v[1]v[5] + v[4]v[8] + v[1]v[2] + v[1]v[6] + v[3]v[4] + v[2]v[8] + v[7]v[8] + v[4]v[6] + v[2]v[5] + v[3]v[7] + v[3]v[5]$)

degree 3 : $\frac{1}{56}$ ($v[2]v[3]v[8] + v[6]v[7]v[8] + v[1]v[2]v[6] + v[1]v[4]v[5] + v[2]v[4]v[6] + v[2]v[7]v[8] + v[3]v[6]v[7] + v[4]v[7]v[8] + v[3]v[4]v[8] + v[1]v[2]v[3] + v[2]v[3]v[4] + v[2]v[3]v[6] + v[3]v[4]v[7] + v[4]v[5]v[6] + v[3]v[5]v[6] + v[1]v[5]v[7] + v[$

2]v[3]v[7] + v[5]v[6]v[8] + v[1]v[3]v[5] + v[1]v[4]v[6] + v[2]v[4]v[8] + v[1]v[7]v[8] + v[1]v[2]v[8] + v[1]v[2]v[4] + v[2]v[4]v[7] + v[4]v[5]v[7] + v[3]v[4]v[6] + v[1]v[2]v[7] + v[1]v[6]v[8] + v[4]v[5]v[8] + v[1]v[4]v[8] + v[3]v[4]v[5] + v[5]v[6]v[7] + v[1]v[6]v[7] + v[3]v[6]v[8] + v[2]v[5]v[7] + v[2]v[5]v[6] + v[2]v[6]v[7] + v[1]v[5]v[8] + v[1]v[5]v[6] + v[5]v[7]v[8] + v[1]v[3]v[4] + v[2]v[3]v[5] + v[1]v[3]v[6] + v[1]v[4]v[7] + v[1]v[2]v[5] + v[2]v[4]v[5] + v[2]v[5]v[8] + v[3]v[7]v[8] + v[4]v[6]v[7] + v[3]v[5]v[7] + v[2]v[6]v[8] + v[3]v[5]v[8] + v[1]v[3]v[8] + v[1]v[3]v[7] + v[4]v[6]v[8])

degree 4 : $\frac{1}{70}$ (v[1]v[2]v[4]v[5] + v[1]v[3]v[4]v[7] + v[1]v[2]v[5]v[8] + v[1]v[4]v[6]v[8] + v[1]v[6]v[7]v[8] + v[1]v[3]v[6]v[8] + v[3]v[6]v[7]v[8] + v[3]v[4]v[6]v[7] + v[2]v[5]v[6]v[8] + v[2]v[4]v[5]v[7] + v[2]v[3]v[5]v[7] + v[2]v[4]v[6]v[8] + v[1]v[2]v[6]v[7] + v[1]v[4]v[6]v[7] + v[2]v[3]v[5]v[8] + v[3]v[4]v[7]v[8] + v[1]v[2]v[4]v[6] + v[3]v[4]v[5]v[8] + v[2]v[4]v[7]v[8] + v[2]v[3]v[4]v[8] + v[5]v[6]v[7]v[8] + v[2]v[4]v[5]v[6] + v[1]v[3]v[5]v[8] + v[1]v[4]v[5]v[8] + v[1]v[2]v[4]v[8] + v[3]v[4]v[5]v[6] + v[4]v[6]v[7]v[8] + v[1]v[3]v[4]v[5] + v[2]v[3]v[4]v[7] + v[2]v[5]v[6]v[7] + v[1]v[3]v[5]v[7] + v[2]v[6]v[7]v[8] + v[2]v[4]v[6]v[7] + v[2]v[4]v[5]v[8] + v[1]v[2]v[4]v[7] + v[1]v[2]v[6]v[8] + v[1]v[5]v[6]v[8] + v[1]v[3]v[4]v[6] + v[4]v[5]v[7]v[8] + v[1]v[3]v[7]v[8] + v[3]v[4]v[5]v[7] + v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[8] + v[1]v[2]v[5]v[6] + v[2]v[3]v[4]v[6] + v[3]v[5]v[6]v[8] + v[1]v[5]v[7]v[8] + v[2]v[3]v[7]v[8] + v[3]v[5]v[7]v[8] + v[1]v[2]v[5]v[7] + v[4]v[5]v[6]v[8] + v[2]v[3]v[5]v[6] + v[4]v[5]v[6]v[7] + v[1]v[4]v[5]v[6] + v[2]v[5]v[7]v[8] + v[1]v[5]v[6]v[7] + v[1]v[3]v[5]v[6] + v[2]v[3]v[4]v[5] + v[2]v[3]v[6]v[8] + v[1]v[4]v[7]v[8] + v[1]v[4]v[5]v[7] + v[3]v[4]v[6]v[8] + v[1]v[2]v[7]v[8] + v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[4] + v[1]v[3]v[6]v[7] + v[1]v[2]v[3]v[8] + v[1]v[2]v[3]v[6] + v[1]v[2]v[3]v[7] + v[1]v[2]v[3]v[5])

degree 5 : $\frac{1}{56}$ (v[1]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[8] + v[2]v[3]v[4]v[5]v[8] + v[2]v[4]v[5]v[7]v[8] + v[1]v[5]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[7] + v[1]v[2]v[5]v[7]v[8] + v[1]v[2]v[3]v[6]v[8] + v[2]v[4]v[5]v[6]v[8] + v[1]v[4]v[6]v[7]v[8] + v[2]v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[5]v[7] + v[1]v[2]v[4]v[6]v[8] + v[2]v[4]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6] + v[1]v[2]v[4]v[6]v[7] + v[2]v[3]v[5]v[6]v[8] + v[1]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[5]v[6]v[8] + v[1]v[3]v[5]v[6]v[8] + v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[4]v[5]v[6] + v[1]v[3]v[4]v[6]v[7] + v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[7]v[8] + v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[8] + v[2]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[7]v[8] + v[1]v[3]v[6]v[7]v[8] + v[1]v[2]v[5]v[6]v[7] + v[2]v[3]v[4]v[6]v[8] + v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6] + v[1]v[2]v[3]v[5]v[6] + v[3]v[4]v[5]v[6]v[7] + v[2]v[4]v[5]v[6]v[7] + v[1]v[4]v[5]v[6]v[8] + v[1]v[2]v[4]v[5]v[7] + v[3]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7] + v[1]v[2]v[3]v[5]v[7] + v[1]v[2]v[3]v[4]v[7] + v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[8] + v[1]v[2]v[6]v[7]v[8] + v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[4]v[5]v[8] + v[1]v[2]v[3]v[4]v[6] + v[1]v[3]v[4]v[7]v[8] + v[1]v[3]v[5]v[7]v[8])

degree 6 : $\frac{1}{28}$ ($v[1]v[2]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[5]v[6]v[8] + v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7] + v[1]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[7] + v[2]v[3]v[4]v[5]v[7]v[8] + v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[8] + v[1]v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[8] + v[1]v[2]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[5]v[6]v[7] + v[1]v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[6]$)

degree 7 : $\frac{1}{8}$ ($v[1]v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[7]v[8]$)

degree 8 : 1 ($v[1]$) ($v[2]$) ($v[3]$) ($v[8]$) ($v[7]$) ($v[6]$) ($v[5]$) ($v[4]$)

Group spectrum $1 + t + t^2 + t^3 + t^4 + t^5 + t^6 + t^7 + t^8$

KERNEL STRUCTURE

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$\pi 8 = [1]$$

supp $\pi 8$ = {1}

$$u8 = [1]$$

supp u8 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[7, 1, 2, 6, 8, 3, 4, 5]

B–BLOCKS,

[8, 3, 4, 5, 7, 1, 2, 6]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

$b_1 = \{1\}$
$b_2 = \{2\}$
$b_3 = \{3\}$
$b_4 = \{4\}$
$b_5 = \{5\}$
$b_6 = \{6\}$
$b_7 = \{7\}$
$b_8 = \{8\}$

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[1] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[1] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[1] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[1] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[1] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[1] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 49, Shape: $48 \oplus 1/0$

$$CLB = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} & 0 \\ 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} & 0 \\ 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} & 0 \\ 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4, 6, 7}, {5, 8}}, true

Ω_B in Vec(K)? , {{1, 6, 8}, {2, 3, 4, 5, 7}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \left(\frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8}\right)$ vs $(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \left(\frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8}\right)$ vs $(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

1, "range", [1, 2, 3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 4, 3, 2, 1], [8, 7, 6, 5, 4, 2, 1, 3], [8, 7, 6, 5, 4, 1, 3, 2], [8, 7, 6, 5, 3, 4, 1, 2], [8, 7, 6, 5, 3, 2, 4, 1], [8, 7, 6, 5, 3, 1, 2, 4], [8, 7, 6, 5, 2, 4, 3, 1], [8, 7, 6, 5, 2, 3, 1, 4], [8, 7, 6, 5, 2, 1, 4, 3], [8, 7, 6, 5, 1, 4, 2, 3], [...20140 terms...], [1, 2, 3, 4, 8, 5, 7, 6], [1, 2, 3, 4, 7, 8, 5, 6], [1, 2, 3, 4, 7, 6, 8, 5], [1, 2, 3, 4, 7, 5, 6, 8], [1, 2, 3, 4, 6, 8, 7, 5], [1, 2, 3, 4, 6, 7, 5, 8], [1, 2, 3, 4, 6, 5, 8, 7], [1, 2, 3, 4, 5, 8, 6, 7], [1, 2, 3, 4, 5, 7, 8, 6], [1, 2, 3, 4, 5, 6, 7, 8]]

"group has", 20160, "elements"   Group element 1,1 = $\begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0 \end{pmatrix}$

$g_1 =$ [[1, 8], [2, 7], [3, 6], [4, 5]]

$g_2 =$ [[1, 8, 3, 6, 2, 7], [4, 5]]

$g_3 =$ [[1, 8, 2, 7, 3, 6], [4, 5]]

$g_4 =$ [[1, 8, 2, 7], [3, 6, 4, 5]]

$g_5 =$ [[1, 8], [2, 7, 4, 5, 3, 6]]

linear dimension, 50

"Symmetric?", true

Is Z in Vec(K)? true
$(1440h[2] + 7560h[1] \quad -1080h[2] - 2520h[1] \quad 5760h[2] + 10080h[1] \quad 3600h[2] + 2$

"Basis for Z(G)"

1, "coeff", 2520

Z[1] = $\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$

2, "coeff", 360

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 7. & -1. & -1. & -1. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3 + t^4 + t^5 + t^6 + t^7 + t^8$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 5t^4 + 7t^5 + 11t^6 + 15t^7 + 22t^8 + 29t^9 + 40t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 8 = (1)$

{1}

$u8 = (1)$

{1}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$\pi 7 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u7 = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$\pi 6 =$
$(2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2$

$u6 =$
$\left(\frac{1}{32}\ \ \frac{1}{32}\right.$

picheck $\begin{pmatrix} 42 & 42 & 42 & 42 & 42 & 42 & 42 & 42 \end{pmatrix}$

$\pi 5 =$
$(6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6$

$u5 =$
$\left(\frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\right.$

picheck $\begin{pmatrix} 210 & 210 & 210 & 210 & 210 & 210 & 210 & 210 \end{pmatrix}$

$\pi 4 =$
$(24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24$

$u4 =$
$\left(\frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\right.$

picheck $\begin{pmatrix} 840 & 840 & 840 & 840 & 840 & 840 & 840 & 840 \end{pmatrix}$

$\pi 3 =$
$(120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120$

$u3 =$
$\left(\frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\right.$

picheck $\begin{pmatrix} 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 \end{pmatrix}$

$\pi 2 =$

$(720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720 \;\; 720$

$u2 =$
$\left(\frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384} \;\; \frac{45}{16384}\right.$

picheck $(5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040)$

$\pi 1 = (5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040)$

$u1 = \left(\frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072} \;\; \frac{315}{131072}\right)$

picheck $(5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040 \;\; 5040)$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0, 0, 0]$

$\ker N_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} s+t & s+t & s+t & s+t & s+t & s+t & s+t & s+t \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & 0 & 0 & t & 0 \\ 0 & s & 0 & 0 & t & 0 & 0 \\ 0 & 0 & t & 0 & 0 & s & 0 \\ 0 & t & 0 & 0 & s & 0 & 0 \\ t & 0 & 0 & s & 0 & 0 & 0 \\ s & 0 & 0 & t & 0 & 0 & 0 \\ -s & -s & -s & -s & -s & -s & -s+t \\ -t & -t & -t & -t & -t & -t & -t+s \end{pmatrix}$$

RB checks

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & 0 & 0 & t & 0 & 0 \\ 0 & 0 & 0 & 0 & s & 0 & 0 & t \\ t & 0 & 0 & 0 & 0 & s & 0 & 0 \\ 0 & 0 & 0 & 0 & t & 0 & 0 & s \\ 0 & s & 0 & t & 0 & 0 & 0 & 0 \\ 0 & t & 0 & s & 0 & 0 & 0 & 0 \\ 0 & 0 & t & 0 & 0 & 0 & s & 0 \\ 0 & 0 & s & 0 & 0 & 0 & t & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 8, "vs", 8

$$\mathrm{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 1T + 48\Omega$$

Ω

$$\left(\frac{31}{8} \;\; \frac{5}{8} \;\; \frac{17}{8} \;\; \frac{3}{8} \;\; \frac{5}{16} \;\; \frac{-7}{8} \;\; \frac{5}{8} \;\; \frac{13}{4} \;\; \frac{21}{8} \;\; \frac{13}{8} \;\; \frac{7}{8} \;\; \frac{3}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{21}{8} \;\; 2 \;\; \frac{9}{8} \;\; \frac{1}{2} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; \frac{1}{8} \;\; 2 \;\; \frac{11}{8} \;\; \frac{5}{8}\right.$$

T

$$\left(6 \;\; 0 \;\; 2 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 5 \;\; 4 \;\; 2 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 4 \;\; 3 \;\; 2 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 3 \;\; 2 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 2\right.$$

"IS NM in Vec(K)?", true

NM
(192 30 104 18 15 −42 30 161 130 80 43 18 6 6 130 99 56 25 6

"IS MN in Vec(K)?", true

MN
(192 30 104 18 15 −42 30 161 130 80 43 18 6 6 130 99 56 25 6

$$\boldsymbol{\tau} = 8/1 \text{ , rank} = 8 \text{ , ratio} = 1/1 \text{ , } n^2 / r = 8/1$$

$$\boldsymbol{\tau}' = 56/1 \text{ , } r' = 7/8 \text{ , } \boldsymbol{\tau} / n^2 = 1/8$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{–check is positive? } 0/1$$

$$\text{max r} = 8/1 \text{ , } r\text{–check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 20160

KERNEL HAS LINEAR DIMENSION 50
out of total no. of elements equal to 20160

dim span idems 1 vs no. of idems 1

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0] }

NullSpace N_C

{[1, 1, 1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1]}

NullSpace N_0

{}

Eigenvalues M

[7., −1., −1., −1., −1., −1., −1., −1.]

Eigenvalues N

[7., −1., −1., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{3, 4, 6, 7}

R: [2, 3, 6, 7, 4, 8, 5, 1]
B: [6, 7, 2, 3, 8, 4, 1, 5]

TRACE TWO = 1

det AT = $1 (t)^4$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 8

Level 2 det = $\frac{5}{8192}\left(-1010 - 81s - 349s^2 - 63s^3 + s^4 + 21s^5 + 25s^6 + 11s^7 + 5s^8\right)(-1 + s)$

RANK of R is 8

R ranking is 1, "vs", 8

RBAR ranking 1, "vs", 8

RANK of B is 8

B ranking is 1, "vs", 8

BBAR ranking 1, "vs", 8

"R CYCLES", (1 + v[4] v[5] v[7]) (1 + v[1] v[2] v[3] v[6] v[8])
"B CYCLES", (1 + v[1] v[2] v[3] v[4] v[6] v[7]) (1 + v[5] v[8])

Eigenvalues

R: [-0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 0.3090169942 + 0.9510565160 I, -0.8090169942 + 0.5877852520 I, -0.8090169942 - 0.5877852520 I, 0.3090169942 - 0.9510565160 I, 1., 1.]

B: [0.5000000000 + 0.8660254040 I, 0.5000000000 - 0.8660254040 I, -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., -1., 1.,

−1.]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 8

"RANK of the KERNEL is ", 8

"IdemSolvability Check", 3 "Trace mark", 8, "Rank mark", 8, "for kernel rank", 8

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{28}$ ($v[1]v[8] + v[5]v[7] + v[1]v[4] + v[3]v[8] + v[3]v[6] + v[6]v[7] + v[4]v[$
$7] + v[5]v[6] + v[2]v[4] + v[2]v[6] + v[4]v[5] + v[2]v[7] + v[1]v[3] + v[1]v[7] + v[6]v$
$[8] + v[5]v[8] + v[2]v[3] + v[1]v[5] + v[4]v[8] + v[1]v[2] + v[1]v[6] + v[3]v[4] + v[2]$
$v[8] + v[7]v[8] + v[4]v[6] + v[2]v[5] + v[3]v[7] + v[3]v[5]$)

degree 3 : $\frac{1}{56}$ ($v[2]v[3]v[8] + v[6]v[7]v[8] + v[1]v[2]v[6] + v[1]v[4]v[5] + v[2]v[4]v[$
$6] + v[2]v[7]v[8] + v[3]v[6]v[7] + v[4]v[7]v[8] + v[3]v[4]v[8] + v[1]v[2]v[3] + v[2]v[$

3]v[4] + v[2]v[3]v[6] + v[3]v[4]v[7] + v[4]v[5]v[6] + v[3]v[5]v[6] + v[1]v[5]v[7] + v[
2]v[3]v[7] + v[5]v[6]v[8] + v[1]v[3]v[5] + v[1]v[4]v[6] + v[2]v[4]v[8] + v[1]v[7]v[8]
+ v[1]v[2]v[8] + v[1]v[2]v[4] + v[2]v[4]v[7] + v[4]v[5]v[7] + v[3]v[4]v[6] + v[1]v[2]v
[7] + v[1]v[6]v[8] + v[4]v[5]v[8] + v[1]v[4]v[8] + v[3]v[4]v[5] + v[5]v[6]v[7] + v[1]v[
6]v[7] + v[3]v[6]v[8] + v[2]v[5]v[7] + v[2]v[5]v[6] + v[2]v[6]v[7] + v[1]v[5]v[8] + v[
1]v[5]v[6] + v[5]v[7]v[8] + v[1]v[3]v[4] + v[2]v[3]v[5] + v[1]v[3]v[6] + v[1]v[4]v[7]
+ v[1]v[2]v[5] + v[2]v[4]v[5] + v[2]v[5]v[8] + v[3]v[7]v[8] + v[4]v[6]v[7] + v[3]v[5]
v[7] + v[2]v[6]v[8] + v[3]v[5]v[8] + v[1]v[3]v[8] + v[1]v[3]v[7] + v[4]v[6]v[8])

degree 4 : $\frac{1}{70}$ (v[1]v[2]v[4]v[5] + v[1]v[3]v[4]v[7] + v[1]v[2]v[5]v[8] + v[1]v[4]v[6]
v[8] + v[1]v[6]v[7]v[8] + v[1]v[3]v[6]v[8] + v[3]v[6]v[7]v[8] + v[3]v[4]v[6]v[7] + v[2]
v[5]v[6]v[8] + v[2]v[4]v[5]v[7] + v[2]v[3]v[5]v[7] + v[2]v[4]v[6]v[8] + v[1]v[2]v[6]v[
7] + v[1]v[4]v[6]v[7] + v[2]v[3]v[5]v[8] + v[3]v[4]v[7]v[8] + v[1]v[2]v[4]v[6] + v[3]v
[4]v[5]v[8] + v[2]v[4]v[7]v[8] + v[2]v[3]v[4]v[8] + v[5]v[6]v[7]v[8] + v[2]v[4]v[5]v[6
] + v[1]v[3]v[5]v[8] + v[1]v[4]v[5]v[8] + v[1]v[2]v[4]v[8] + v[3]v[4]v[5]v[6] + v[4]v[
6]v[7]v[8] + v[1]v[3]v[4]v[5] + v[2]v[3]v[4]v[7] + v[2]v[5]v[6]v[7] + v[1]v[3]v[5]v[7]
+ v[2]v[6]v[7]v[8] + v[2]v[4]v[6]v[7] + v[2]v[4]v[5]v[8] + v[1]v[2]v[4]v[7] + v[1]v[2]
v[6]v[8] + v[1]v[5]v[6]v[8] + v[1]v[3]v[4]v[6] + v[4]v[5]v[7]v[8] + v[1]v[3]v[7]v[8] +
v[3]v[4]v[5]v[7] + v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[8] + v[1]v[2]v[5]v[6] + v[2]v[3]v
[4]v[6] + v[3]v[5]v[6]v[8] + v[1]v[5]v[7]v[8] + v[2]v[3]v[7]v[8] + v[3]v[5]v[7]v[8] +
v[1]v[2]v[5]v[7] + v[4]v[5]v[6]v[8] + v[2]v[3]v[5]v[6] + v[4]v[5]v[6]v[7] + v[1]v[4]v[
5]v[6] + v[2]v[5]v[7]v[8] + v[1]v[5]v[6]v[7] + v[1]v[3]v[5]v[6] + v[2]v[3]v[4]v[5] + v
[2]v[3]v[6]v[8] + v[1]v[4]v[7]v[8] + v[1]v[4]v[5]v[7] + v[3]v[4]v[6]v[8] + v[1]v[2]v[7
]v[8] + v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[4] + v[1]v[3]v[6]v[7] + v[1]v[2]v[3]v[8] + v[
1]v[2]v[3]v[6] + v[1]v[2]v[3]v[7] + v[1]v[2]v[3]v[5])

degree 5 : $\frac{1}{56}$ (v[1]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[8] + v[2]v[3]v[4]v[5]v[8] +
v[2]v[4]v[5]v[7]v[8] + v[1]v[5]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]
v[8] + v[2]v[3]v[4]v[5]v[7] + v[1]v[2]v[5]v[7]v[8] + v[1]v[2]v[3]v[6]v[8] + v[2]v[4]v[
5]v[6]v[8] + v[1]v[4]v[6]v[7]v[8] + v[2]v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[5]v[7] + v[1]
v[2]v[4]v[6]v[8] + v[2]v[4]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6] + v[1]v[2]v[4]v[6]v[7]
+ v[2]v[3]v[5]v[6]v[8] + v[1]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[5]v[
6]v[8] + v[1]v[3]v[5]v[6]v[8] + v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[4]v[5]v[6] + v[1]v[3]
v[4]v[6]v[7] + v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[7]v[8] + v[
4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[8] + v[2]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[
8] + v[1]v[2]v[4]v[7]v[8] + v[1]v[3]v[6]v[7]v[8] + v[1]v[2]v[5]v[6]v[7] + v[2]v[3]v[4]
v[6]v[8] + v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6] + v[1]v[2]v[3]v[5]v[6] + v[3]v[
4]v[5]v[6]v[7] + v[2]v[4]v[5]v[6]v[7] + v[1]v[4]v[5]v[6]v[8] + v[1]v[2]v[4]v[5]v[7] +
v[3]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7] + v[1]v[2]v[3]v[5]v[7] + v[1]v[2]v[3]v[4]
v[7] + v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[8] + v[1]v[2]v[6]v[7]v[8] + v[3]v[4]v[
5]v[7]v[8] + v[1]v[2]v[4]v[5]v[8] + v[1]v[2]v[3]v[4]v[6] + v[1]v[3]v[4]v[7]v[8] + v[1]

$v[3]v[5]v[7]v[8]$)

degree 6 : $\frac{1}{28}$ ($v[1]v[2]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[5]v[6]v[8] + v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7] + v[1]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[7] + v[2]v[3]v[4]v[5]v[7]v[8] + v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[8] + v[1]v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[8] + v[1]v[2]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[5]v[6]v[7] + v[1]v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[6]$)

degree 7 : $\frac{1}{8}$ ($v[1]v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[7]v[8]$)

degree 8 : 1 ($v[1]$) ($v[2]$) ($v[3]$) ($v[8]$) ($v[7]$) ($v[6]$) ($v[5]$) ($v[4]$)

Group spectrum $1 + t + t^2 + t^3 + t^4 + t^5 + t^6 + t^7 + t^8$

KERNEL STRUCTURE

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$\pi 8 = [1]$$

supp $\pi 8$ = {1}

$$u8 = [1]$$

supp u8 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[8, 1, 2, 5, 7, 3, 4, 6]

B–BLOCKS,

[7, 3, 4, 6, 8, 1, 2, 5]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N – check: true

$b_1 = \{1\}$
$b_2 = \{2\}$
$b_3 = \{3\}$
$b_4 = \{4\}$
$b_5 = \{5\}$
$b_6 = \{6\}$
$b_7 = \{7\}$
$b_8 = \{8\}$

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[2] & h[1] & h[1] & h[1] & h[1] & h[1] & h[1] & h[1] \\ h[1] & h[2] & h[1] & h[1] & h[1] & h[1] & h[1] & h[1] \\ h[1] & h[1] & h[2] & h[1] & h[1] & h[1] & h[1] & h[1] \\ h[1] & h[1] & h[1] & h[2] & h[1] & h[1] & h[1] & h[1] \\ h[1] & h[1] & h[1] & h[1] & h[2] & h[1] & h[1] & h[1] \\ h[1] & h[1] & h[1] & h[1] & h[1] & h[2] & h[1] & h[1] \\ h[1] & h[1] & h[1] & h[1] & h[1] & h[1] & h[2] & h[1] \\ h[1] & h[1] & h[1] & h[1] & h[1] & h[1] & h[1] & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 49, Shape: 48 ⊕ 1/0

$$CLB = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{5} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 6, 8}, {4, 5, 7}}, true

Ω_B in Vec(K)? , {{5, 8}, {1, 2, 3, 4, 6, 7}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

1, "range", [1, 2, 3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 4, 3, 2, 1], [8, 7, 6, 5, 4, 2, 1, 3], [8, 7, 6, 5, 4, 1, 3, 2], [8, 7, 6, 5, 3, 4, 1, 2], [8, 7, 6, 5, 3, 2, 4, 1], [8, 7, 6, 5, 3, 1, 2, 4], [8, 7, 6, 5, 2, 4, 3, 1], [8, 7, 6, 5, 2, 3, 1, 4], [8, 7, 6, 5, 2, 1, 4, 3], [8, 7, 6, 5, 1, 4, 2, 3], [...20140 terms...], [1, 2, 3, 4, 8, 5, 7, 6], [1, 2, 3, 4, 7, 8, 5, 6], [1, 2, 3, 4, 7, 6, 8, 5], [1, 2, 3, 4, 7, 5, 6, 8], [1, 2, 3, 4, 6, 8, 7, 5], [1, 2, 3, 4, 6, 7, 5, 8], [1, 2, 3, 4, 6, 5, 8, 7], [1, 2, 3, 4, 5, 8, 6, 7], [1, 2, 3, 4, 5, 7, 8, 6], [1, 2, 3, 4, 5, 6, 7, 8]]

"group has", 20160, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

g_1 =   [[1, 8], [2, 7], [3, 6], [4, 5]]

g_2 =   [[1, 8, 3, 6, 2, 7], [4, 5]]

g_3 =   [[1, 8, 2, 7, 3, 6], [4, 5]]

g_4 =   [[1, 8, 2, 7], [3, 6, 4, 5]]

g_5 =   [[1, 8], [2, 7, 4, 5, 3, 6]]

linear dimension, 50

"Symmetric?", true

Is Z in Vec(K)? true
$(1440h[2] + 7560h[1] \quad -1080h[2] - 2520h[1] \quad 5760h[2] + 10080h[1] \quad 3600h[2] + 25$

"Basis for Z(G)"

1, "coeff", 2520

Z[1] = $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 360

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 7. & -1. & -1. & -1. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3 + t^4 + t^5 + t^6 + t^7 + t^8$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 5t^4 + 7t^5 + 11t^6 + 15t^7 + 22t^8 + 29t^9 + 40t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 8 = (1)$

{1}

$u8 = (1)$

{1}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 7 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u7 = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$\pi 6 =$
$\left(2 \;\; 2 \right.$

$u6 =$
$\left(\frac{1}{32} \;\; \frac{1}{32} \right.$

picheck $\begin{pmatrix} 42 & 42 & 42 & 42 & 42 & 42 & 42 & 42 \end{pmatrix}$

$\pi 5 =$
$\left(6 \;\; 6 \right.$

$u5 =$
$\left(\frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \;\; \frac{3}{256} \right.$

picheck $\begin{pmatrix} 210 & 210 & 210 & 210 & 210 & 210 & 210 & 210 \end{pmatrix}$

$\pi 4 =$
$\left(24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \;\; 24 \right.$

$u4 =$
$\left(\frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \;\; \frac{3}{512} \right.$

picheck $\begin{pmatrix} 840 & 840 & 840 & 840 & 840 & 840 & 840 & 840 \end{pmatrix}$

$\pi 3 =$
$\left(120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \;\; 120 \right.$

$u3 =$
$\left(\frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; \frac{15}{4096} \;\; 4 \right.$

picheck $\begin{pmatrix} 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 & 2520 \end{pmatrix}$

$\pi 2 =$

$(720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720$

$u2 =$
$\left(\frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\right.$

picheck $(5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

$\pi 1 = (5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

$u1 = \left(\frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\right)$

picheck $(5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0, 0, 0]$

$\ker N_c = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} t+s & t+s & t+s & t+s & t+s & t+s & t+s & t+s \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -s & -s & -s & -s+t & -s & -s & -s \\ t & 0 & 0 & 0 & 0 & s & 0 \\ -t & -t & -t & -t+s & -t & -t & -t \\ s & 0 & 0 & 0 & 0 & t & 0 \\ 0 & t & s & 0 & 0 & 0 & 0 \\ 0 & s & t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & t & 0 & s \\ 0 & 0 & 0 & 0 & s & 0 & t \end{pmatrix}$$

RB checks

$$\ker M_c = \begin{pmatrix} 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&s&0&0&0&0&t\\ s&0&0&0&0&0&t&0\\ 0&0&t&0&0&0&0&s\\ t&0&0&0&0&0&s&0\\ 0&s&0&0&t&0&0&0\\ 0&t&0&0&s&0&0&0\\ 0&0&0&t&0&s&0&0\\ 0&0&0&s&0&t&0&0 \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 1&1&1&1&1&1&1&1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 8, "vs", 8

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 1T + 48\Omega$$

Ω

$$\left(\frac{31}{8} \quad \frac{5}{8} \quad \frac{17}{8} \quad \frac{3}{8} \quad \frac{5}{16} \quad \frac{-7}{8} \quad \frac{5}{8} \quad \frac{13}{4} \quad \frac{21}{8} \quad \frac{13}{8} \quad \frac{7}{8} \quad \frac{3}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{21}{8} \quad 2 \quad \frac{9}{8} \quad \frac{1}{2} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad 2 \quad \frac{11}{8} \quad \frac{5}{8}\right.$$

T

$$(6 \quad 0 \quad 2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 5 \quad 4 \quad 2 \quad 1 \quad 0 \quad 0 \quad 0 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \quad 0 \quad 0 \quad 3 \quad 2 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 2$$

"IS NM in Vec(K)?", true

NM
(192 30 104 18 15 −42 30 161 130 80 43 18 6 6 130 99 56 25 6

"IS MN in Vec(K)?", true

MN
(192 30 104 18 15 −42 30 161 130 80 43 18 6 6 130 99 56 25 6

$\boldsymbol{\tau} = 8/1$, rank = 8 , ratio = 1/1 , $n^2 / r = 8/1$

$\boldsymbol{\tau}' = 56/1$, $r' = 7/8$, $\boldsymbol{\tau} / n^2 = 1/8$

$p^2 = 1/8$, min $\boldsymbol{\tau} = 8/1$, $\boldsymbol{\tau}$–check is positive? 0/1

max r = 8/1 , r–check is positive? 0/1

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 20160

KERNEL HAS LINEAR DIMENSION 50
out of total no. of elements equal to 20160

dim span idems 1 vs no. of idems 1

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1, 0, 0] }

NullSpace N_C

{[1, 1, 1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 0, 0, 1, 0, 0, 0, 0], [−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 0, 1]}

NullSpace N_0

{}

Eigenvalues M

[7., −1., −1., −1., −1., −1., −1., −1.]

Eigenvalues N

[7., −1., −1., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{3, 4, 6, 8}

R: [2, 3, 6, 7, 4, 8, 1, 5]
B: [6, 7, 2, 3, 8, 4, 5, 1]

TRACE TWO = 1

det AT = $-1\,(t)^4$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 8

Level 2 det = $\frac{5}{4096}(-1+s)(1+s)(5+s^2)(-101+2s+8s^2+6s^3+5s^4)$

RANK of R is 8

R ranking is 1, "vs", 8

RBAR ranking 1, "vs", 8

RANK of B is 8

B ranking is 1, "vs", 8

BBAR ranking 1, "vs", 8

"R CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6] v[7] v[8]
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[5] v[6] v[7] v[8]

Eigenvalues

R: [-1. I, 1. I, -1., 1., -0.7071067810 - 0.7071067810 I, 0.7071067810 + 0.7071067810 I, -0.7071067810 + 0.7071067810 I, 0.7071067810 - 0.7071067810 I]

B: [-1. I, 1. I, -1., 1., -0.7071067810 - 0.7071067810 I, 0.7071067810 + 0.7071067810 I, -0.7071067810 + 0.7071067810 I, 0.7071067810 - 0.7071067810 I]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 8, "RANK of M is ", 8

"RANK of the KERNEL is ", 8

"IdemSolvability Check", 3 "Trace mark", 8, "Rank mark", 8, "for kernel rank", 8

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{28}$ ($v[1]v[8] + v[5]v[7] + v[1]v[4] + v[3]v[8] + v[3]v[6] + v[6]v[7] + v[4]v[7] + v[5]v[6] + v[2]v[4] + v[2]v[6] + v[4]v[5] + v[2]v[7] + v[1]v[3] + v[1]v[7] + v[6]v[8] + v[5]v[8] + v[2]v[3] + v[1]v[5] + v[4]v[8] + v[1]v[2] + v[1]v[6] + v[3]v[4] + v[2]v[8] + v[7]v[8] + v[4]v[6] + v[2]v[5] + v[3]v[7] + v[3]v[5]$)

degree 3 : $\frac{1}{56}$ ($v[2]v[3]v[8] + v[6]v[7]v[8] + v[1]v[2]v[6] + v[1]v[4]v[5] + v[2]v[4]v[6] + v[2]v[7]v[8] + v[3]v[6]v[7] + v[4]v[7]v[8] + v[3]v[4]v[8] + v[1]v[2]v[3] + v[2]v[3]v[4] + v[2]v[3]v[6] + v[3]v[4]v[7] + v[4]v[5]v[6] + v[3]v[5]v[6] + v[1]v[5]v[7] + v[$

2]v[3]v[7] + v[5]v[6]v[8] + v[1]v[3]v[5] + v[1]v[4]v[6] + v[2]v[4]v[8] + v[1]v[7]v[8]
+ v[1]v[2]v[8] + v[1]v[2]v[4] + v[2]v[4]v[7] + v[4]v[5]v[7] + v[3]v[4]v[6] + v[1]v[2]v
[7] + v[1]v[6]v[8] + v[4]v[5]v[8] + v[1]v[4]v[8] + v[3]v[4]v[5] + v[5]v[6]v[7] + v[1]v[
6]v[7] + v[3]v[6]v[8] + v[2]v[5]v[7] + v[2]v[5]v[6] + v[2]v[6]v[7] + v[1]v[5]v[8] + v[
1]v[5]v[6] + v[5]v[7]v[8] + v[1]v[3]v[4] + v[2]v[3]v[5] + v[1]v[3]v[6] + v[1]v[4]v[7]
+ v[1]v[2]v[5] + v[2]v[4]v[5] + v[2]v[5]v[8] + v[3]v[7]v[8] + v[4]v[6]v[7] + v[3]v[5]
v[7] + v[2]v[6]v[8] + v[3]v[5]v[8] + v[1]v[3]v[8] + v[1]v[3]v[7] + v[4]v[6]v[8])

degree 4 : $\frac{1}{28}$ (2 v[1]v[2]v[4]v[5] + 3 v[1]v[3]v[4]v[7] + 3 v[1]v[2]v[5]v[8] + 2 v[1]v[4
]v[6]v[8] + 3 v[1]v[6]v[7]v[8] + 3 v[1]v[3]v[6]v[8] + 2 v[3]v[6]v[7]v[8] + 3 v[3]v[4]v[6]
v[7] + 3 v[2]v[5]v[6]v[8] + 3 v[2]v[4]v[5]v[7] + 2 v[2]v[3]v[5]v[7] + 3 v[2]v[4]v[6]v[8]
+ 3 v[1]v[2]v[6]v[7] + 2 v[1]v[4]v[6]v[7] + 2 v[2]v[3]v[5]v[8] + 2 v[3]v[4]v[7]v[8] + 3 v
[1]v[2]v[4]v[6] + 3 v[3]v[4]v[5]v[8] + 3 v[2]v[4]v[7]v[8] + 2 v[2]v[3]v[4]v[8] + 3 v[5]v
[6]v[7]v[8] + 2 v[2]v[4]v[5]v[6] + 2 v[1]v[3]v[5]v[8] + 3 v[1]v[4]v[5]v[8] + 2 v[1]v[2]v[
4]v[8] + 2 v[3]v[4]v[5]v[6] + 2 v[4]v[6]v[7]v[8] + 2 v[1]v[3]v[4]v[5] + 2 v[2]v[3]v[4]v[
7] + 2 v[2]v[5]v[6]v[7] + 3 v[1]v[3]v[5]v[7] + 2 v[2]v[6]v[7]v[8] + 2 v[2]v[4]v[6]v[7] +
2 v[2]v[4]v[5]v[8] + 2 v[1]v[2]v[4]v[7] + 2 v[1]v[2]v[6]v[8] + 2 v[1]v[5]v[6]v[8] + 2 v[1
]v[3]v[4]v[6] + 2 v[4]v[5]v[7]v[8] + 2 v[1]v[3]v[7]v[8] + 2 v[3]v[4]v[5]v[7] + 2 v[3]v[5
]v[6]v[7] + 2 v[1]v[3]v[4]v[8] + 2 v[1]v[2]v[5]v[6] + 2 v[2]v[3]v[4]v[6] + 2 v[3]v[5]v[6
]v[8] + 2 v[1]v[5]v[7]v[8] + 3 v[2]v[3]v[7]v[8] + 3 v[3]v[5]v[7]v[8] + 3 v[1]v[2]v[5]v[7]
+ 2 v[4]v[5]v[6]v[8] + 3 v[2]v[3]v[5]v[6] + 3 v[4]v[5]v[6]v[7] + 3 v[1]v[4]v[5]v[6] + 2
v[2]v[5]v[7]v[8] + 2 v[1]v[5]v[6]v[7] + 3 v[1]v[3]v[5]v[6] + 3 v[2]v[3]v[4]v[5] + 2 v[2]
v[3]v[6]v[8] + 3 v[1]v[4]v[7]v[8] + 2 v[1]v[4]v[5]v[7] + 3 v[3]v[4]v[6]v[8] + 2 v[1]v[2]
v[7]v[8] + 3 v[2]v[3]v[6]v[7] + 3 v[1]v[2]v[3]v[4] + 2 v[1]v[3]v[6]v[7] + 3 v[1]v[2]v[3]
v[8] + 2 v[1]v[2]v[3]v[6] + 2 v[1]v[2]v[3]v[7] + 2 v[1]v[2]v[3]v[5])

degree 5 : $\frac{1}{56}$ (v[1]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[8] + v[2]v[3]v[4]v[5]v[8] +
v[2]v[4]v[5]v[7]v[8] + v[1]v[5]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]
v[8] + v[2]v[3]v[4]v[5]v[7] + v[1]v[2]v[5]v[7]v[8] + v[1]v[2]v[3]v[6]v[8] + v[2]v[4]v[
5]v[6]v[8] + v[1]v[4]v[6]v[7]v[8] + v[2]v[3]v[5]v[6]v[7] + v[1]v[3]v[4]v[5]v[7] + v[1]
v[2]v[4]v[6]v[8] + v[2]v[4]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6] + v[1]v[2]v[4]v[6]v[7]
+ v[2]v[3]v[5]v[6]v[8] + v[1]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[5]v[
6]v[8] + v[1]v[3]v[5]v[6]v[8] + v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[4]v[5]v[6] + v[1]v[3]
v[4]v[6]v[7] + v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[6]v[7] + v[1]v[2]v[3]v[7]v[8] + v[
4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[8] + v[2]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[
8] + v[1]v[2]v[4]v[7]v[8] + v[1]v[3]v[6]v[7]v[8] + v[1]v[2]v[5]v[6]v[7] + v[2]v[3]v[4]
v[6]v[8] + v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6] + v[1]v[2]v[3]v[5]v[6] + v[3]v[
4]v[5]v[6]v[7] + v[2]v[4]v[5]v[6]v[7] + v[1]v[4]v[5]v[6]v[8] + v[1]v[2]v[4]v[5]v[7] +
v[3]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7] + v[1]v[2]v[3]v[5]v[7] + v[1]v[2]v[3]v[4]
v[7] + v[1]v[2]v[3]v[4]v[5] + v[1]v[2]v[3]v[4]v[8] + v[1]v[2]v[6]v[7]v[8] + v[3]v[4]v[
5]v[7]v[8] + v[1]v[2]v[4]v[5]v[8] + v[1]v[2]v[3]v[4]v[6] + v[1]v[3]v[4]v[7]v[8] + v[1]

v[3]v[5]v[7]v[8])
degree 6 : $\frac{1}{28}$ (v[1]v[2]v[4]v[5]v[6]v[7] + v[2]v[3]v[4]v[5]v[6]v[8] + v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7] + v[3]v[4]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7] + v[1]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[7] + v[2]v[3]v[4]v[5]v[7]v[8] + v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[8] + v[1]v[3]v[4]v[5]v[6]v[8] + v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[4]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[8] + v[1]v[2]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[8] + v[1]v[2]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[5]v[6]v[7] + v[1]v[2]v[3]v[6]v[7]v[8] + v[1]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[3]v[4]v[7]v[8] + v[1]v[2]v[3]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[5]v[6])
degree 7 : $\frac{1}{8}$ (v[1]v[2]v[3]v[4]v[5]v[6]v[7] + v[1]v[2]v[3]v[4]v[5]v[6]v[8] + v[1]v[2]v[3]v[4]v[5]v[7]v[8] + v[1]v[2]v[3]v[4]v[6]v[7]v[8] + v[1]v[2]v[3]v[5]v[6]v[7]v[8] + v[1]v[2]v[4]v[5]v[6]v[7]v[8] + v[1]v[3]v[4]v[5]v[6]v[7]v[8] + v[2]v[3]v[4]v[5]v[6]v[7]v[8])
degree 8 : 1 (v[1]) (v[2]) (v[3]) (v[8]) (v[7]) (v[6]) (v[5]) (v[4])

Group spectrum $1 + t + t^2 + t^3 + 2t^4 + t^5 + t^6 + t^7 + t^8$

KERNEL STRUCTURE

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$\pi 8 = [1]$$

supp π8 = {1}

$$u8 = [1]$$

supp u8 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[7, 1, 2, 5, 8, 3, 4, 6]

B-BLOCKS,

[8, 3, 4, 6, 7, 1, 2, 5]

with invariant measure, [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, N - check: true

$b_1 = \{1\}$
$b_2 = \{2\}$
$b_3 = \{3\}$
$b_4 = \{4\}$
$b_5 = \{5\}$
$b_6 = \{6\}$
$b_7 = \{7\}$
$b_8 = \{8\}$

dim(span of partition vectors), rank(N_0), rank(N): 8, 8, 8

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[1] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[1] & h[2] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[1] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[1] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[1] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[1] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[2] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 49, Shape: $48 \oplus 1/0$

$$CLB = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4, 5, 6, 7, 8}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 4, 5, 6, 7, 8}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

1, "range", [1, 2, 3, 4, 5, 6, 7, 8], [[8, 7, 6, 5, 4, 3, 2, 1], [8, 7, 5, 3, 1, 2, 4, 6], [8, 7, 4, 1, 3, 6, 5, 2], [8, 7, 3, 2, 6, 4, 1, 5], [8, 7, 2, 4, 5, 1, 6, 3], [8, 7, 1, 6, 2, 5, 3, 4], [8, 6, 7, 1, 2, 3, 4, 5], [8, 6, 5, 7, 4, 1, 3, 2], [8, 6, 4, 2, 1, 5, 7, 3], [8, 6, 3, 4, 7, 2, 5, 1], [8, 6, 2, 5, 3, 7, 1, 4], [8, 6, 1, 3, 5, 4, 2, 7], [8, 5, 7, 6, 4, 2, 1, 3], [8, 5, 6, 2, 3, 1, 4, 7], [8, 5, 4, 3, 2, 7, 6, 1], [8, 5, 3, 7, 1, 6, 2, 4], [8, 5, 2, 1, 7, 4, 3, 6], [8, 5, 1, 4, 6, 3, 7, 2], [8, 4, 7, 2, 5, 6, 3, 1], [8, 4, 6, 3, 7, 5, 1, 2], [8, 4, 5, 1, 6, 7, 2, 3], [8, 4, 3, 5, 2, 1, 7, 6], [8, 4, 2, 6, 1, 3, 5, 7], [8, 4, 1, 7, 3, 2, 6, 5], [8, 3, 7, 5, 1, 4, 6, 2], [8, 3, 6, 1, 5, 2, 7, 4], [8, 3, 5, 4, 2, 6, 1, 7], [8, 3, 4, 6, 7, 1, 2, 5], [8, 3, 2, 7, 6, 5, 4, 1], [8, 3, 1, 2, 4, 7, 5, 6], [8, 2, 7, 3, 6, 1, 5, 4], [8, 2, 6, 4, 1, 7, 3, 5], [8, 2, 5, 6, 3, 4, 7, 1], [8, 2, 4, 7, 5, 3, 1, 6], [8, 2, 3, 1, 4, 5, 6, 7], [8, 2, 1, 5, 7, 6, 4, 3], [8, 1, 7, 4, 3, 5, 2, 6], [8, 1, 6, 7, 2, 4, 5, 3], [8, 1, 5, 2, 7, 3, 6, 4], [8, 1, 4, 5, 6, 2, 3, 7], [8, 1, 3, 6, 5, 7, 4, 2], [8, 1, 2, 3, 4, 6, 7, 5], [7, 8, 6, 1, 3, 4, 2, 5], [7, 8, 5, 6, 2, 1, 4, 3], [7, 8, 4, 2, 6, 3, 5, 1], [7, 8, 3, 5, 4, 6, 1, 2], [7, 8, 2, 3, 1, 5, 6, 4], [7, 8, 1, 4, 5, 2, 3, 6], [7, 6, 8, 5, 2, 4, 3, 1], [7, 6, 5, 4, 1, 3, 2, 8], [7, 6, 4, 3, 8, 2, 1, 5], [7, 6, 3, 2, 5, 1, 8, 4], [7, 6, 2, 1, 4, 8, 5, 3], [7, 6, 1, 8, 3, 5, 4, 2], [7, 5, 8, 3, 4, 1, 2, 6], [7, 5, 6, 8, 2, 3, 1, 4], [7, 5, 4, 6, 1, 8, 3, 2], [7, 5, 3, 1, 6, 2, 4, 8], [7, 5, 2, 4, 3, 6, 8, 1], [7, 5, 1, 2, 8, 4, 6, 3], [7, 4, 8, 1, 5, 3, 6, 2], [7, 4, 6, 5, 1, 2, 8, 3], [7, 4, 5, 2, 3, 8, 1, 6], [7, 4, 3, 6, 8, 5, 2, 1], [7, 4, 2, 8, 6, 1, 3, 5], [7, 4, 1, 3, 2, 6, 5, 8], [7, 3, 8, 2, 1, 6, 4, 5], [7, 3, 6, 4, 8, 1, 5, 2], [7, 3, 5, 8, 4, 2, 6, 1], [7, 3, 4, 1, 2, 5, 8, 6], [7, 3, 2, 6, 5, 4, 1, 8], [7, 3, 1, 5, 6, 8, 2, 4], [7, 2, 8, 4, 6, 5, 1, 3], [7, 2, 6, 3, 5, 8, 4, 1], [7, 2, 5, 1, 8, 6, 3, 4], [7, 2, 4, 5, 3, 1, 6, 8], [7, 2, 3, 8, 1, 4, 5, 6], [7, 2, 1, 6, 4, 3, 8, 5], [7, 1, 8, 6, 3, 2, 5, 4], [7, 1, 6, 2, 4, 5, 3, 8], [7, 1, 5, 3, 6, 4, 8, 2], [7, 1, 4, 8, 5, 6, 2, 3], [7, 1, 3, 4, 2, 8, 6, 5], [7, 1, 2, 5, 8, 3, 4, 6], [6, 8, 7, 5, 3, 2, 4, 1], [6, 8, 5, 2, 1, 4, 3, 7], [6, 8, 4, 3, 5, 1, 7, 2], [6, 8, 3, 1, 2, 7, 5, 4], [6, 8, 2, 4, 7, 3, 1, 5], [6, 8, 1, 7, 4, 5, 2, 3], [6, 7, 8, 1, 4, 2, 3, 5], [6, 7, 5, 8, 3, 1, 2, 4], [6, 7, 4, 5, 2, 8, 1, 3], [6, 7, 3, 4, 1, 5, 8, 2], [6, 7, 2, 3, 8, 4, 5, 1], [6, 7, 1, 2, 5, 3, 4, 8], [6, 5, 8, 7, 3, 4, 1, 2], [6, 5, 7, 4, 2, 1, 3, 8], [6, 5, 4, 1, 8, 3, 2, 7], [6, 5, 3, 2, 4, 8, 7, 1], [6, 5, 2, 8, 1, 7, 4, 3], [6, 5, 1, 3, 7, 2, 8, 4], [6, 4, 8, 2, 7, 1, 5, 3], [6, 4, 7, 3, 1, 8, 2, 5], [6, 4, 5, 7, 2, 3, 8, 1], [6, 4, 3, 8, 5, 2, 1, 7], [6, 4, 2, 1, 3, 5, 7, 8], [6, 4, 1, 5, 8, 7, 3, 2], [6, 3, 8, 4, 5, 7, 2, 1], [6, 3, 7, 2, 8, 5, 1, 4], [6, 3, 5, 1, 7, 8, 4, 2], [6, 3, 4, 7, 1, 2, 5, 8], [6, 3, 2, 5, 4, 1, 8, 7], [6, 3, 1, 8, 2, 4, 7, 5], [6, 2, 8, 5, 1, 3, 7, 4], [6, 2, 7, 1, 5, 4, 8, 3], [6, 2, 5, 3, 4, 7, 1, 8], [6, 2, 4, 8, 7, 5, 3, 1], [6, 2, 3, 7, 8, 1, 4, 5], [6, 2, 1, 4, 3, 8, 5, 7], [6, 1, 8, 3, 2, 5, 4, 7], [6, 1, 7, 8, 4, 3, 5, 2], [6, 1, 5, 4, 8, 2, 7, 3], [6, 1, 4, 2, 3, 7, 8, 5], [6, 1, 3, 5, 7, 4, 2, 8], [6, 1, 2, 7, 5, 8, 3, 4], [5, 8, 7, 3, 2, 4, 1, 6], [5, 8, 6, 7, 1, 3, 4, 2], [5, 8, 4, 1, 7, 2, 6, 3], [5, 8, 3, 4, 6, 1, 2, 7], [5, 8, 2, 6, 4, 7, 3, 1], [5, 8, 1, 2, 3, 6, 7, 4], [5, 7, 8, 6, 1, 4, 2, 3], [5, 7, 6, 4, 3, 2, 1, 8], [5, 7, 4, 2, 8, 1, 3, 6], [5, 7, 3, 8, 2, 6, 4, 1], [5, 7, 2, 1, 6, 3, 8, 4], [5, 7, 1, 3, 4, 8, 6, 2], [5, 6, 8, 2, 4, 3, 1, 7], [5, 6, 7, 8, 1, 2, 3, 4], [5, 6, 4, 7, 3, 8, 2, 1], [5, 6, 3, 1, 8, 4, 7, 2], [5, 6, 2, 3, 7, 1, 4, 8], [5, 6, 1, 4, 2, 7, 8, 3], [5, 4, 8, 3, 6, 2, 7, 1], [5, 4, 7, 6, 3, 1, 8, 2], [5, 4, 6, 1, 2, 8, 3, 7], [5, 4, 3, 2, 1, 7, 6, 8], [5, 4, 2, 7, 8, 6, 1, 3], [5, 4, 1, 8, 7, 3, 2, 6], [5, 3, 8, 7, 2, 1, 6, 4], [5, 3, 7, 1, 4, 6, 2, 8], [5, 3, 6, 2, 7, 4, 8, 1], [5, 3, 4, 8, 6, 7, 1, 2], [5, 3, 2, 4, 1, 8, 7, 6], [5, 3, 1, 6, 8, 2, 4, 7], [5, 2, 8, 1, 3, 7, 4, 6], [5, 2, 7, 4, 8, 3, 6, 1], [5, 2, 6, 8, 4, 1, 7, 3], [5, 2, 4, 3, 1, 6, 8, 7], [5, 2, 3, 6, 7, 8, 1, 4], [5, 2, 1, 7, 6, 4, 3, 8], [5, 1, 8, 4, 7, 6, 3, 2], [5, 1, 7, 2, 6, 8, 4, 3], [5, 1, 6, 3, 8, 7, 2, 4], [5, 1, 4, 6, 2, 3, 7, 8], [5, 1, 3, 7, 4, 2, 8, 6], [5, 1, 2, 8, 3, 4, 6, 7], [4, 8, 7, 1, 6, 5, 3, 2], [4, 8, 6, 2, 5, 7, 1, 3], [4, 8, 5, 3, 7, 6, 2, 1], [4, 8, 3, 6, 1, 2, 7, 5], [4, 8, 2, 7, 3, 1, 5, 6], [4, 8, 1, 5, 2, 3, 6, 7], [4, 7, 8, 2, 3, 5, 6, 1], [4, 7, 6, 3, 2, 1, 8, 5], [4, 7, 5, 6, 8, 3, 1, 2], [4, 7, 3, 1, 5, 8, 2, 6], [4, 7, 2, 5, 1, 6, 3, 8], [4, 7, 1, 8, 6, 2, 5, 3], [4, 6, 8, 3, 1, 7, 5, 2], [4, 6, 7, 5, 8, 1, 2, 3], [4, 6, 5, 1, 3, 2, 8, 7], [4, 6, 3, 7, 2, 5, 1, 8], [4, 6, 2, 8, 5, 3, 7, 1], [4, 6, 1, 2, 7, 8, 3, 5], [4, 5, 8, 1, 2, 6, 7, 3], [4, 5, 7, 2, 1, 3, 8, 6], [4, 5, 6, 7, 8, 2, 3, 1], [4, 5, 3, 8, 7, 1, 6, 2], [4, 5, 2, 3, 6, 8, 1, 7], [4, 5, 1, 6, 3, 7, 2, 8], [4, 3, 8, 5, 7, 2, 1, 6], [4, 3, 7, 6, 2, 8, 5, 1], [4, 3, 6, 8, 1, 5, 2, 7], [4, 3, 5, 2, 6, 1, 7, 8], [4, 3, 2, 1, 8, 7, 6, 5], [4, 3, 1, 7, 5, 6, 8, 2], [4, 2, 8, 6, 5, 1, 3, 7], [4, 2, 7, 8, 3, 6, 1, 5], [4, 2, 6, 1, 7, 3, 5, 8], [4, 2, 5, 7, 1, 8, 6, 3], [4, 2, 3, 5, 6, 7, 8, 1], [4, 2, 1, 3, 8, 5, 7, 6], [4, 1, 8, 7, 6, 3, 2, 5], [4, 1, 7, 3, 5, 2, 6, 8], [4, 1, 6, 5, 3, 8, 7, 2], [4, 1, 5, 8, 2, 7, 3, 6], [4, 1, 3, 2, 8, 6, 5, 7], [4, 1, 2, 6, 7, 5, 8, 3], [3, 8, 7, 2, 4, 1, 6, 5], [3, 8, 6, 4, 2, 5, 7, 1], [3, 8, 5, 7, 6, 2, 1, 4], [3, 8, 4, 5, 1, 7, 2, 6], [3, 8, 2, 1, 5, 6, 4, 7], [3, 8, 1, 6, 7, 4, 5, 2], [3, 7, 8, 5, 6, 1, 4, 2], [3, 7, 6, 2, 1, 8, 5, 4], [3, 7, 5, 1, 2, 4, 6, 8], [3, 7, 4, 6, 5, 2, 8, 1], [3, 7, 2, 8, 4, 5, 1, 6], [3, 7, 1, 4, 8, 6, 2, 5], [3, 6, 8, 1, 7, 5, 2, 4], [3, 6, 7, 4, 5, 8, 1, 2], [3, 6, 5, 2, 8, 7, 4, 1], [3, 6, 4, 8, 2, 1, 5, 7], [3, 6, 2, 7, 1, 4, 8, 5], [3, 6, 1, 5, 4, 2, 7, 8], [3, 5, 8, 4, 1, 2, 6, 7], [3, 5, 7, 8, 6, 4, 2, 1], [3, 5, 6, 1, 4, 7, 8, 2], [3, 5, 4, 2, 7, 6, 1, 8], [3, 5, 2, 6, 8, 1, 7, 4], [3, 5, 1, 7, 2, 8, 4, 6], [3, 4, 8, 6, 2, 7, 1, 5], [3, 4, 7, 1, 8, 2, 5, 6], [3, 4, 6, 7, 5, 1, 2, 8], [3, 4, 5, 8, 1, 6, 7, 2], [3, 4, 2, 5, 7, 8, 6, 1], [3, 4, 1, 2, 6, 5, 8, 7], [3, 2, 8, 7, 4, 6, 5, 1], [3, 2, 7, 6, 1, 5, 4, 8], [3, 2, 6, 5, 8, 4, 1, 7], [3, 2, 5, 4, 7, 1, 8, 6], [3, 2, 4, 1, 6, 8, 7, 5], [3, 2, 1, 8, 5, 7, 6, 4], [3, 1, 8, 2, 5, 4, 7, 6], [3, 1, 7, 5, 2, 6, 8, 4], [3, 1, 6, 8, 7, 2, 4, 5], [3, 1, 5, 6, 4, 8, 2, 7], [3, 1, 4, 7, 8, 5, 6, 2], [3, 1, 2, 4, 6, 7, 5, 8], [2, 8, 7, 4, 1, 6, 5, 3], [2, 8, 6, 5, 7, 1, 3, 4], [2, 8, 5, 1, 4, 3, 7, 6], [2, 8, 4, 6, 3, 5, 1, 7], [2, 8, 3, 7, 5, 4, 6, 1], [2, 8, 1, 3, 6, 7, 4, 5], [2, 7, 8, 3, 5, 6, 1, 4], [2, 7, 6, 1, 8, 5, 4, 3], [2, 7, 5, 4, 6, 8, 3, 1], [2, 7, 4, 8, 1, 3, 6, 5], [2, 7, 3, 6, 4, 1, 5, 8], [2, 7, 1, 5, 3, 4, 8, 6], [2, 6, 8, 4, 3, 1, 7, 5], [2, 6, 7, 3, 4, 5, 8, 1], [2, 6, 5, 8, 7, 4, 1, 3], [2, 6, 4, 1, 5, 7, 3, 8], [2, 6, 3, 5, 1, 8, 4, 7], [2, 6, 1, 7, 8, 3, 5, 4], [2, 5, 8, 6, 7, 3, 4, 1], [2, 5, 7, 1, 3, 8, 6, 4], [2, 5, 6, 3, 1, 4, 7, 8], [2, 5, 4, 7, 6, 1, 8, 3], [2, 5, 3, 4, 8, 7, 1, 6], [2, 5, 1, 8, 4, 6, 3, 7], [2, 4, 8, 7, 1, 5, 3, 6], [2, 4, 7, 5, 6, 3, 1, 8], [2, 4, 6, 8, 3, 7, 5, 1], [2, 4, 5, 3, 8, 1, 6, 7], [2, 4, 3, 1, 7, 6, 8, 5], [2, 4, 1, 6, 5, 8, 7, 3], [2, 3, 8, 1, 6, 4, 5, 7], [2, 3, 7, 8, 5, 1, 4, 6], [2, 3, 6, 7, 4, 8, 1, 5], [2, 3, 5, 6, 1, 7, 8, 4], [2, 3, 4, 5, 8, 6, 7, 1], [2, 3, 1, 4, 7, 5, 6, 8], [2, 1, 8, 5, 4, 7, 6, 3], [2, 1, 7, 6, 8, 4, 3, 5], [2, 1, 6, 4, 5, 3, 8, 7], [2, 1, 5, 7, 3, 6, 4, 8], [2, 1, 4, 3, 7, 8, 5, 6], [2, 1, 3, 8, 6, 5, 7, 4], [1, 8, 7, 6, 5, 3, 2, 4], [1, 8, 6, 3, 4, 2, 5, 7], [1, 8, 5, 4, 3, 7, 6, 2], [1, 8, 4, 7, 2, 6, 3, 5], [1, 8, 3, 2, 7, 5, 4, 6], [1, 8, 2, 5, 6, 4, 7, 3], [1, 7, 8, 4, 2, 3, 5, 6], [1, 7, 6, 8, 5, 4, 3, 2], [1, 7, 5, 2, 4, 6, 8, 3], [1, 7, 4, 3, 6, 5, 2, 8], [1, 7, 3, 5, 8, 2, 6, 4], [1, 7, 2, 6, 3, 8, 4, 5], [1, 6, 8, 7, 5, 2, 4, 3], [1, 6, 7, 2, 3, 4, 5, 8], [1, 6, 5, 3, 2, 8, 7, 4], [1, 6, 4, 5, 7, 3, 8, 2], [1, 6, 3, 8, 4, 7, 2, 5], [1, 6, 2, 4, 8, 5, 3, 7], [1, 5, 8, 2, 6, 7, 3, 4], [1, 5, 7, 3, 8, 6, 4, 2], [1, 5, 6, 4, 7, 8, 2, 3], [1, 5, 4, 8, 3, 2, 7, 6], [1, 5, 3, 6, 2, 4, 8, 7], [1, 5, 2, 7, 4, 3, 6, 8], [1, 4, 8, 5, 3, 6, 2, 7], [1, 4, 7, 8, 2, 5, 6, 3], [1, 4, 6, 2, 8, 3, 7, 5], [1, 4, 5, 6, 7, 2, 3, 8], [1, 4, 3, 7, 6, 8, 5, 2], [1, 4, 2, 3, 5, 7, 8, 6], [1, 3, 8, 6, 4, 5, 7, 2], [1, 3, 7, 4, 6, 2, 8, 5], [1, 3, 6, 5, 2, 7, 4, 8], [1, 3, 5, 7, 8, 4, 2, 6], [1, 3, 4, 2, 5, 8, 6, 7], [1, 3, 2, 8, 7, 6, 5, 4], [1, 2, 8, 3, 7, 4, 6, 5], [1, 2, 7, 5, 4, 8, 3, 6], [1, 2, 6, 7, 3, 5, 8, 4], [1, 2, 5, 8, 6, 3, 4, 7], [1, 2, 4, 6, 8, 7, 5, 3], [1, 2, 3, 4, 5, 6, 7, 8]]

"group has", 336, "elements"   Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

g_1 =   [[1, 8], [2, 7], [3, 6], [4, 5]]

g_2 =   [[1, 8, 6, 2, 7, 4, 3, 5]]

g_3 =   [[1, 8, 2, 7, 5, 3, 4]]

$g_4 =$ [[1, 8, 5, 6, 4, 2, 7]]

$g_5 =$ [[1, 8, 3, 2, 7, 6]]

linear dimension, 50

"Symmetric?", true

Is Z in Vec(K)? true

$$\left(\frac{||(60h[2]-84h[1])||}{||7||} \quad \frac{||(-48h[2]-168h[1])||}{||7||} \quad \frac{||(6h[2]+294h[1])||}{||7||} \quad \frac{||(48h[2]+168h[1])|}{||7||} \right.$$

"Basis for Z(G)"

1, "coeff", 42

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 6

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. & 1. & 1. & 1. \\ 7. & -1. & -1. & -1. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3 + 2t^4 + t^5 + t^6 + t^7 + t^8$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 6t^4 + 8t^5 + 15t^6 + 22t^7 + 38t^8 + 55t^9 + 89t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 8 = (1)$

{1}

$u8 = (1)$

{1}

picheck $(1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1)$

$\pi = \left(\frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\right)$

$\pi 7 = (1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1)$

$u7 = \left(\frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\right)$

picheck $(7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7)$

$\pi 6 =$
$(2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2$

$u6 =$
$\left(\frac{1}{32}\ \ \frac{1}{32}\right.$

picheck $(42\ \ 42\ \ 42\ \ 42\ \ 42\ \ 42\ \ 42\ \ 42)$

$\pi 5 =$
$(6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6$

$u5 =$
$\left(\frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\ \ \frac{3}{256}\right.$

picheck $(210\ \ 210\ \ 210\ \ 210\ \ 210\ \ 210\ \ 210\ \ 210)$

$\pi 4 =$
$(24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24\ \ 24$

$u4 =$
$\left(\frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\ \ \frac{3}{512}\right.$

picheck $(840\ \ 840\ \ 840\ \ 840\ \ 840\ \ 840\ \ 840\ \ 840)$

$\pi 3 =$
$(120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120\ \ 120$

$u3 =$
$\left(\frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ \frac{15}{4096}\ \ 4\right.$

picheck $(2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520\ \ 2520)$

$\pi 2 =$
$(720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720\ \ 720$

$u2 =$
$\left(\frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\ \ \frac{45}{16384}\right.$

picheck $(5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

$\pi 1 = (5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

$u1 = \left(\frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\ \ \frac{315}{131072}\right)$

picheck $(5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040\ \ 5040)$

Column Projections

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad NM = \begin{pmatrix} 7 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 7 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 7 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 7 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 7 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 7 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 7 \end{pmatrix}$$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0, 0, 0, 0]$

$\ker N_C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} s+t & s+t & s+t & s+t & s+t & s+t & s+t & s+t \end{pmatrix}$ RB checks

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & 0 & t & 0 & 0 \\ 0 & s & 0 & 0 & 0 & t & 0 \\ t & 0 & 0 & 0 & s & 0 & 0 \\ 0 & t & 0 & 0 & 0 & s & 0 \\ 0 & 0 & s & 0 & 0 & 0 & t \\ 0 & 0 & t & 0 & 0 & 0 & s \\ -s & -s & -s & -s+t & -s & -s & -s \\ -t & -t & -t & -t+s & -t & -t & -t \end{pmatrix}$$

RB checks

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & t & 0 & 0 & 0 & 0 & s & 0 \\ 0 & 0 & 0 & t & 0 & s & 0 & 0 \\ 0 & s & 0 & 0 & 0 & 0 & t & 0 \\ 0 & 0 & 0 & s & 0 & t & 0 & 0 \\ 0 & 0 & t & 0 & 0 & 0 & 0 & s \\ 0 & 0 & s & 0 & 0 & 0 & 0 & t \\ t & 0 & 0 & 0 & s & 0 & 0 & 0 \\ s & 0 & 0 & 0 & t & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{R}\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \; B B^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 8, 8, "vs", 8

$$CNM = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 1T + 48\Omega$$

Ω

$$\left(\frac{5}{8}\ \ \frac{1}{8}\ \ \frac{41}{16}\ \ -5\ \ \frac{25}{24}\ \ \frac{53}{16}\ \ \frac{-43}{28}\ \ \frac{5}{8}\ \ \frac{-1}{4}\ \ 0\ \ \frac{-41}{8}\ \ \frac{-17}{8}\ \ \frac{1}{56}\ \ \frac{-153}{56}\ \ \frac{5}{8}\ \ \frac{83}{48}\ \ \frac{203}{208}\ \ \frac{11}{16}\ \ \frac{-}{3}\right.$$

T

$$\left(1\ \ 1\ \ \frac{7}{2}\ \ -7\ \ \frac{1}{6}\ \ \frac{9}{4}\ \ \frac{-27}{14}\ \ 1\ \ \frac{-5}{2}\ \ \frac{1}{2}\ \ -9\ \ -4\ \ \frac{-5}{7}\ \ \frac{-31}{7}\ \ 1\ \ 3\ \ \frac{30}{13}\ \ 1\ \ \frac{-8}{3}\ \ \frac{-3}{2}\ \ \frac{-3}{2}\right.$$

"IS NM in Vec(K)?", true

NM

$$\left(31\ \ 7\ \ \frac{253}{2}\ \ -247\ \ \frac{301}{6}\ \ \frac{645}{4}\ \ \frac{-1059}{14}\ \ 31\ \ \frac{-29}{2}\ \ \frac{1}{2}\ \ -255\ \ -106\ \ \frac{1}{7}\ \ \frac{-949}{7}\ \ 31\ \ 8\right.$$

"IS MN in Vec(K)?", true

MN

$$\left(31\ \ 7\ \ \frac{253}{2}\ \ -247\ \ \frac{301}{6}\ \ \frac{645}{4}\ \ \frac{-1059}{14}\ \ 31\ \ \frac{-29}{2}\ \ \frac{1}{2}\ \ -255\ \ -106\ \ \frac{1}{7}\ \ \frac{-949}{7}\ \ 31\ \ 8\right.$$

$$\boldsymbol{\tau} = 8/1 \text{ , rank } = 8 \text{ , ratio } = 1/1 \text{ , } n^2 / r = 8/1$$

$$\boldsymbol{\tau}' = 56/1 \text{ , } r' = 7/8 \text{ , } \boldsymbol{\tau} / n^2 = 1/8$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 8\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 336

KERNEL HAS LINEAR DIMENSION 50
out of total no. of elements equal to 336

dim span idems 1 vs no. of idems 1

"PT1" = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}}

"RG1" = {1, 2, 3, 4, 5, 6, 7, 8}

$$M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8}&\frac{-1}{8}\\ \frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{-1}{8}&\frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_c

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143, 1.142857143]

NullSpace M_C

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0, 0, 0] }

NullSpace N_C

{[1, 1, 1, 1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_0

[1., 1., 1., 1., 1., 1., 1., 1.]

NullSpace M_0

{[−1, 0, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 0, 0, 0, 0, 1], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0]}

NullSpace N_0

{}

Eigenvalues M

[7., −1., −1., −1., −1., −1., −1., −1.]

Eigenvalues N

[7., −1., −1., −1., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

==

{5, 6, 7, 8}

R: [2, 3, 2, 3, 8, 8, 5, 5]
B: [6, 7, 6, 7, 4, 4, 1, 1]

TRACE TWO = 2

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 4

Level 2 det = $\frac{1}{4096}(1+s)(101+48s+18s^2-8s^3+s^4)(-1+s)^2(5-2s+s^2)$

RANK of R is 4

R ranking is 1, "vs", 4

RBAR ranking 1, "vs", 4

RANK of B is 4

B ranking is 1, "vs", 4

BBAR ranking 1, "vs", 4

"R CYCLES", (1 + v[2] v[3]) (1 + v[5] v[8])
"B CYCLES", 1 + v[1] v[4] v[6] v[7]

Eigenvalues

R: [1., −1., 1., −1., 0., 0., 0., 0.]

B: [0., 0., 0., 0., −1., 1., 1. I, −1. I]

NullSpace of R

{[0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of R^*

{[0, −1, 0, 1, 0, 0, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [−1, 0, 1, 0, 0, 0, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 2

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 2 & 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 0 & 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 2 & 0 & 0 & 2 & 0 \\ 2 & 0 & 0 & 2 & 0 & 2 & 0 & 0 \\ 0 & 2 & 2 & 0 & 2 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 8

"RANK of the KERNEL is ", 4

"IdemSolvability Check", 3 "Trace mark", 4, "Rank mark", 4, "for kernel rank", 4

degree 1: $\frac{1}{8}\left(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8] \right)$

degree 2: $\frac{1}{4}\left(2v[1]v[4] + v[1]v[6] + v[1]v[7] + 2v[2]v[3] + v[2]v[5] + v[2]v[8] + v[3]v[5] + v[3]v[8] + v[4]v[6] + v[4]v[7] + 2v[5]v[8] + 2v[6]v[7] \right)$

degree 3 : $\frac{1}{8}\left(v[1]v[4]v[6] + v[1]v[4]v[7] + v[1]v[6]v[7] + v[2]v[3]v[5] + v[2]v[3]v[8] + v[2]v[5]v[8] + v[3]v[5]v[8] + v[4]v[6]v[7] \right)$

degree 4 : $\frac{1}{2}\left(v[1]v[4]v[6]v[7] + v[2]v[3]v[5]v[8] \right)$

Group spectrum $1 + t + 2t^2 + t^3 + t^4$

KERNEL STRUCTURE

"PT1" = {{2, 4}, {1, 3}, {5, 6}, {7, 8}}

"RG1" = {2, 3, 5, 8}

"RG2" = {1, 4, 6, 7}

$\pi 4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp $\pi 4$ = {29, 42}

$u4$ = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

supp u4 = {11, 12, 13, 14, 27, 28, 29, 30, 41, 42, 43, 44, 57, 58, 59, 60}

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [2]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[2, 1, 4, 3]

B-BLOCKS,

[3, 4, 2, 1]

with invariant measure, [1, 1, 1, 1]

N by blocks, N - check: true

$b_1 = \{2, 4\}$
$b_2 = \{1, 3\}$
$b_3 = \{5, 6\}$
$b_4 = \{7, 8\}$

dim(span of partition vectors), rank(N_0), rank(N): 4, 4, 4

$$\text{Centralizer} = \begin{pmatrix} h[1] & 0 & 0 & h[4] & 0 & h[3] & h[2] & 0 \\ 0 & h[1] & h[4] & 0 & h[2] & 0 & 0 & h[3] \\ 0 & h[4] & h[1] & 0 & h[3] & 0 & 0 & h[2] \\ h[4] & 0 & 0 & h[1] & 0 & h[2] & h[3] & 0 \\ 0 & h[3] & h[2] & 0 & h[1] & 0 & 0 & h[4] \\ h[2] & 0 & 0 & h[3] & 0 & h[1] & h[4] & 0 \\ h[3] & 0 & 0 & h[2] & 0 & h[4] & h[1] & 0 \\ 0 & h[2] & h[3] & 0 & h[4] & 0 & 0 & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 6, Shape: $0 \oplus 6/4$

$$\text{CLB} = \begin{pmatrix} 1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{2, 3}, {5, 8}}, true

Ω_B in Vec(K)? , {{1, 4, 6, 7}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \end{pmatrix}$$

Omega–checks true, true

"I–V is singular"

$\pi_R = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{2, 4}, {1, 3}, {5, 6}, {7, 8}}

1, "range", [2, 3, 5, 8], [[8, 5, 8, 5, 3, 3, 2, 2], [5, 8, 5, 8, 2, 2, 3, 3], [3, 2, 3, 2, 5, 5, 8, 8], [2, 3, 2, 3, 8, 8, 5, 5]]

2, "range", [1, 4, 6, 7], [[7, 6, 7, 6, 1, 1, 4, 4], [6, 7, 6, 7, 4, 4, 1, 1], [4, 1, 4, 1, 7, 7, 6, 6], [1, 4, 1, 4, 6, 6, 7, 7]]

"group has", 4, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 3, 2, 4]]

$g_2 =$ [[1, 4, 2, 3]]

$g_3 =$ []

$g_4 =$ [[1, 2], [3, 4]]

linear dimension, 4

"Symmetric?", false

Is Z in Vec(K)? true
$\begin{pmatrix} h[3] & h[4] & h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3, "coeff", 1

$$Z[\ 3\ ] = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

4, "coeff", 1

$$Z[\ 4\ ] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Check abelian

1, 2, true
1, 3, true
1, 4, true
2, 3, true
2, 4, true
3, 4, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. \\ 1. & -1. & 1. & -1. \\ -1. & 1. & 1.\,I & -1.\,I \\ -1. & 1. & 1.\,I & -1.\,I \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 6 & 2 \\ 0 & 0 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + 2t^2 + t^3 + t^4$

Molien Series to order 10: $1 + t + 3t^2 + 5t^3 + 10t^4 + 14t^5 + 22t^6 + 30t^7 + 43t^8 + 55t^9 + 73t^{10}$

n-choose-rank

{1, [1, 2, 3, 4]}, {2, [1, 2, 3, 5]}, {3, [1, 2, 3, 6]}, {4, [1, 2, 3, 7]}, {5, [1, 2, 3, 8]}, {6, [1, 2, 4, 5]}, {7, [1, 2, 4, 6]}, {8, [1, 2, 4, 7]}, {9, [1, 2, 4, 8]}, {10, [1, 2, 5, 6]}, {11, [1, 2, 5, 7]}, {12, [1, 2, 5, 8]}, {13, [1, 2, 6, 7]}, {14, [1, 2, 6, 8]}, {15, [1, 2, 7, 8]}, {16, [1, 3, 4, 5]}, {17, [1, 3, 4, 6]}, {18, [1, 3, 4, 7]}, {19, [1, 3, 4, 8]}, {20, [1, 3, 5, 6]}, {21, [1, 3, 5, 7]}, {22, [1, 3, 5, 8]}, {23, [1, 3, 6, 7]}, {24, [1, 3, 6, 8]}, {25, [1, 3, 7, 8]}, {26, [1, 4, 5, 6]}, {27, [1, 4, 5, 7]}, {28, [1, 4, 5, 8]}, {29, [1, 4, 6, 7]}, {30, [1, 4, 6, 8]}, {31, [1, 4, 7, 8]}, {32, [1, 5, 6, 7]}, {33, [1, 5, 6, 8]}, {34, [1, 5, 7, 8]}, {35, [1, 6, 7, 8]}, {36, [2, 3, 4, 5]}, {37, [2, 3, 4, 6]}, {38, [2, 3, 4, 7]}, {39, [2, 3, 4, 8]}, {40, [2, 3, 5, 6]}, {41, [2, 3, 5, 7]}, {42, [2, 3, 5, 8]}, {43, [2, 3, 6, 7]}, {44, [2, 3, 6, 8]}, {45, [2, 3, 7, 8]}, {46, [2, 4, 5, 6]}, {47, [2, 4, 5, 7]}, {48, [2, 4, 5, 8]}, {49, [2, 4, 6, 7]}, {50, [2, 4, 6, 8]}, {51, [2, 4, 7, 8]}, {52, [2, 5, 6, 7]}, {53, [2, 5, 6, 8]}, {54, [2, 5, 7, 8]}, {55, [2, 6, 7, 8]}, {56, [3, 4, 5, 6]}, {57, [3, 4, 5, 7]}, {58, [3, 4, 5, 8]}, {59, [3, 4, 6, 7]}, {60, [3, 4, 6, 8]}, {61, [3, 4, 7, 8]}, {62, [3, 5, 6, 7]}, {63, [3, 5, 6, 8]}, {64, [3, 5, 7, 8]}, {65, [3, 6, 7, 8]}, {66, [4, 5, 6, 7]}, {67, [4, 5, 6, 8]}, {68, [4, 5, 7, 8]}, {69, [4, 6, 7, 8]}, {70, [5, 6, 7, 8]}

KERNEL HIERARCHY

$\pi 4$ =
(0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1

{29, 42}

$u4$ =
(0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1

{11, 12, 13, 14, 27, 28, 29, 30, 41, 42, 43, 44, 57, 58, 59, 60}

picheck $\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$

$\pi = \begin{pmatrix}\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8}\end{pmatrix}$

$\pi 3 =$
$(0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 1 \; 1 \; 0 \; 0 \; 0 \; 0 \; 1 \; 0 \; 0 \; 0 \; 1 \; 0 \; 0 \; 1 \; 0 \; 0 \; 0$

$u3 =$
$\Big(0 \; 0 \; \frac{1}{4} \; \frac{1}{4} \; \frac{1}{4} \; \frac{1}{4} \; 0 \; 0 \; 0 \; 0 \; 0 \; \frac{1}{4} \; \frac{1}{4} \; \frac{1}{4} \; \frac{1}{4} \; 0 \; \frac{1}{4} \; \frac{1}{4} \; \frac{1}{4} \; \frac{1}{4} \; 0 \; 0 \; \frac{1}{4} \; \frac{1}{4} \; \frac{1}{4} \; \frac{1}{4} \; 0 \; 0$

picheck $\begin{pmatrix}3 & 3 & 3 & 3 & 3 & 3 & 3 & 3\end{pmatrix}$

$\pi 2 =$
$\begin{pmatrix}0 & 0 & 2 & 0 & 2 & 2 & 0 & 2 & 0 & 2 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 2 & 0 & 2 & 2 & 0 & 0 & 0 & 2 & 2 & 0 & 0\end{pmatrix}$

$u2 =$
$\Big(\frac{1}{8} \; 0 \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; 0 \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; 0 \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8} \; \frac{1}{8}$

picheck $\begin{pmatrix}6 & 6 & 6 & 6 & 6 & 6 & 6 & 6\end{pmatrix}$

$\pi 1 = \begin{pmatrix}6 & 6 & 6 & 6 & 6 & 6 & 6 & 6\end{pmatrix}$

$u1 = \begin{pmatrix}\frac{3}{32} & \frac{3}{32} & \frac{3}{32} & \frac{3}{32} & \frac{3}{32} & \frac{3}{32} & \frac{3}{32} & \frac{3}{32}\end{pmatrix}$

picheck $\begin{pmatrix}6 & 6 & 6 & 6 & 6 & 6 & 6 & 6\end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix}0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem−checks N0−checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$ idem−checks

$$T = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad NM = \begin{pmatrix} 6 & 4 & 6 & 4 & 4 & 4 & 4 & 4 \\ 4 & 6 & 4 & 6 & 4 & 4 & 4 & 4 \\ 6 & 4 & 6 & 4 & 4 & 4 & 4 & 4 \\ 4 & 6 & 4 & 6 & 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 & 6 & 6 & 4 & 4 \\ 4 & 4 & 4 & 4 & 6 & 6 & 4 & 4 \\ 4 & 4 & 4 & 4 & 4 & 4 & 6 & 6 \\ 4 & 4 & 4 & 4 & 4 & 4 & 6 & 6 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 1, 1, -1, 1, -1, -1, 1]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 & -1 & -1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t & 0 & 0 & -s & -s & -s \\ 0 & t & 0 & 0 & s & 0 \\ t & 0 & 0 & -s & -s & -s \\ 0 & t & 0 & 0 & s & 0 \\ 0 & 0 & t & s & 0 & 0 \\ 0 & 0 & t & s & 0 & 0 \\ -t & -t & -t & 0 & 0 & s \\ -t & -t & -t & 0 & 0 & s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} -1 & 1 & -1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & t & 0 & 0 & 0 & s \\ 0 & 0 & 0 & t & 0 & s & 0 \\ 0 & 0 & t & 0 & 0 & 0 & s \\ 0 & 0 & 0 & t & 0 & s & 0 \\ t & 0 & 0 & 0 & s & 0 & 0 \\ t & 0 & 0 & 0 & s & 0 & 0 \\ -t & t+s & -t & -t & t & t & t \\ -t & t+s & -t & -t & t & t & t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 0 & 2 & 0 & 0 & 2 & 2 & 2 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 4

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 2&0&0&2&0&2&2&0\\ 0&2&2&0&2&0&0&2\\ 0&2&2&0&2&0&0&2\\ 2&0&0&2&0&2&2&0\\ 0&2&2&0&2&0&0&2\\ 2&0&0&2&0&2&2&0\\ 2&0&0&2&0&2&2&0\\ 0&2&2&0&2&0&0&2 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&1&0&0&0&0&0\\ 0&1&0&1&0&0&0&0\\ 1&0&1&0&0&0&0&0\\ 0&1&0&1&0&0&0&0\\ 0&0&0&0&1&1&0&0\\ 0&0&0&0&1&1&0&0\\ 0&0&0&0&0&0&1&1\\ 0&0&0&0&0&0&1&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 4T + 32\Omega$$

$$\Omega \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$T \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

"IS NM in Vec(K)?", true

NM $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 6 & 4 & 6 \end{pmatrix}$

"IS MN in Vec(K)?", true

MN $\begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 6 & 4 & 6 \end{pmatrix}$

$$\boldsymbol{\tau} = 16/1 \text{ , rank} = 4 \text{ , ratio} = 4/1 \text{ , } n^2 / r = 16/1$$

$$\boldsymbol{\tau}' = 48/1 \text{ , } r' = 3/4 \text{ , } \boldsymbol{\tau} / n^2 = 1/4$$

$$p^2 = 1/8 \text{ , min } \boldsymbol{\tau} = 8/1 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 8/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 1/2$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 16\Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 4

KERNEL HAS LINEAR DIMENSION 8
out of total no. of elements equal to 8

dim span idems 2 vs no. of idems 2

"PT1" = {{2, 4}, {1, 3}, {5, 6}, {7, 8}}

"RG1" = {2, 3, 5, 8}

"RG2" = {1, 4, 6, 7}

$$M_c = \begin{pmatrix} 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_c

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_C

[1., 2., 2., 2., 0., 0., 0., 0.]

Eigenvalues M_C−scaled

[0., 0., 0., 0., 0., 0., 0., 8.]

Eigenvalues N_C−scaled

[1.142857143, 2.285714286, 2.285714286, 2.285714286, 0., 0., 0., 0.]

NullSpace M_C

{[0, 0, 0, 0, 0, 1, 0, 1], [0, 0, 0, 1, 0, 0, 0, 1], [0, 0, 0, 0, 0, 0, 1, 1], [0, 0, 1, 0, 0, 0, 0, −1], [0, 0, 0, 0, 1, 0, 0, −1], [0, 1, 0, 0, 0, 0, 0, −1], [1, 0, 0, 0, 0, 0, 0, 1]}

NullSpace N_C

{[0, −1, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [0, 0, 0, 0, −1, 1, 0, 0], [−1, 0, 1, 0, 0, 0, 0, 0]}

Eigenvalues M_0

[8., 8., 0., 0., 0., 0., 0., 0.]

Eigenvalues N_0

[2., 2., 2., 2., 0., 0., 0., 0.]

NullSpace M_0

{[−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 0, 0, 0, 0, 1]}

NullSpace N_0

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

Eigenvalues M

[6., 6., −2., −2., −2., −2., −2., −2.]

Eigenvalues N

[6., −2., −2., −2., 0., 0., 0., 0.]

NullSpace M

{}

NullSpace N

{[−1, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1], [0, −1, 0, 1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Commutator(s)

$$1, 2 : \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

==

100, [1, −1, −1, −1, −1, −1, 1, 1]
==

120, [1, 1, 1, −1, −1, −1, −1, −1]

==

{2, 3, 4, 5, 6, 8}

R: [2, 7, 6, 7, 8, 8, 1, 5]
B: [6, 3, 2, 3, 4, 4, 5, 1]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 6

Level 2 det = $\frac{1}{16384}(-1+s)(-2020 - 1745s - 799s^2 - 43s^3 + 79s^4 + 37s^5 + 19s^6 - 9s^7 + s^8)$

RANK of R is 6

R ranking is 4, "vs", 6

RBAR ranking 3, "vs", 5

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 1, "vs", 2

"R CYCLES", (1 + v[5] v[8]) (1 + v[1] v[2] v[7])
"B CYCLES", 1 + v[2] v[3]

Eigenvalues

R: [−1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., 1., 0., 0., 0.]

B: [0., 0., 0., 0., 0., 0., 1., −1.]

NullSpace of R

{[0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of R^*

{[0, 0, 0, 0, −1, 1, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 1, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 4 & 0 & 12 & 4 & 0 & 8 \\ 0 & 0 & 12 & 0 & 4 & 4 & 0 & 8 \\ 4 & 12 & 0 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 8 & 8 & 0 & 0 \\ 12 & 4 & 0 & 8 & 0 & 0 & 4 & 0 \\ 4 & 4 & 0 & 8 & 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 0 & 4 & 12 & 0 & 12 \\ 8 & 8 & 0 & 0 & 0 & 0 & 12 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}\,(v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8])$

degree 2: $\frac{1}{28}\,(v[1]v[3] + 3\,v[1]v[5] + v[1]v[6] + 2\,v[1]v[8] + 3\,v[2]v[3] + v[2]v[5] + v[2]v[6] + 2\,v[2]v[8] + 3\,v[3]v[4] + 2\,v[4]v[5] + 2\,v[4]v[6] + v[5]v[7] + 3\,v[6]v[7] + 3\,v[7]v[8])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{3, 5, 6, 8}, {1, 2, 4, 7}}

"RG1" = {7, 8}

"RG2" = {6, 7}

"RG3" = {5, 7}

"RG4" = {4, 6}

"RG5" = {4, 5}

"RG6" = {3, 4}

"RG7" = {2, 8}

"RG8" = {2, 6}

"RG9" = {2, 5}

"RG10" = {2, 3}

"RG11" = {1, 8}

"RG12" = {1, 6}

"RG13" = {1, 5}

"RG14" = {1, 3}

$\pi 2 = [0, 1, 0, 3, 1, 0, 2, 3, 0, 1, 1, 0, 2, 3, 0, 0, 0, 0, 2, 2, 0, 0, 0, 1, 0, 3, 0, 3]$

supp $\pi 2$ = {2, 4, 5, 7, 8, 10, 11, 13, 14, 19, 20, 24, 26, 28}

$u2 = [0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1]$

supp u2 = {2, 4, 5, 7, 8, 10, 11, 13, 14, 17, 19, 20, 22, 24, 26, 28}

Action of R on ranges, [[13], [11], [11], [1], [1], [2], [3], [1], [1], [2], [9], [7], [7], [8]]
Action of B on ranges, [[13], [5], [5], [6], [6], [10], [14], [6], [6], [10], [12], [4], [4], [8]]

$$\boldsymbol{\beta} = \left(\frac{3}{28} \;\; \frac{3}{28} \;\; \frac{1}{28} \;\; \frac{1}{14} \;\; \frac{1}{14} \;\; \frac{3}{28} \;\; \frac{1}{14} \;\; \frac{1}{28} \;\; \frac{1}{28} \;\; \frac{3}{28} \;\; \frac{1}{14} \;\; \frac{1}{28} \;\; \frac{3}{28} \;\; \frac{1}{28}\right)$$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[1, 2]

B–BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N – check: true

$b_1 = \{3, 5, 6, 8\}$
$b_2 = \{1, 2, 4, 7\}$

dim(span of partition vectors), rank(N_0), rank(N):   2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 37, Shape: 15 ⊕ 22/20

$$\mathrm{CLB} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 8}, {1, 2, 7}}, true

Ω_B in Vec(K)? , {{2, 3}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{17}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{9}{40} & \frac{1}{40} & \frac{-7}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \end{pmatrix}$$

Omega−checks true, true

"I−V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{6} & \frac{1}{4} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{3, 5, 6, 8}, {1, 2, 4, 7}}

1, "range", [7, 8], [[8, 8, 7, 8, 7, 7, 8, 7], [7, 7, 8, 7, 8, 8, 7, 8]]

2, "range", [6, 7], [[7, 7, 6, 7, 6, 6, 7, 6], [6, 6, 7, 6, 7, 7, 6, 7]]

3, "range", [5, 7], [[7, 7, 5, 7, 5, 5, 7, 5], [5, 5, 7, 5, 7, 7, 5, 7]]

4, "range", [4, 6], [[6, 6, 4, 6, 4, 4, 6, 4], [4, 4, 6, 4, 6, 6, 4, 6]]

5, "range", [4, 5], [[5, 5, 4, 5, 4, 4, 5, 4], [4, 4, 5, 4, 5, 5, 4, 5]]

6, "range", [3, 4], [[4, 4, 3, 4, 3, 3, 4, 3], [3, 3, 4, 3, 4, 4, 3, 4]]

7, "range", [2, 8], [[8, 8, 2, 8, 2, 2, 8, 2], [2, 2, 8, 2, 8, 8, 2, 8]]

8, "range", [2, 6], [[6, 6, 2, 6, 2, 2, 6, 2], [2, 2, 6, 2, 6, 6, 2, 6]]

9, "range", [2, 5], [[5, 5, 2, 5, 2, 2, 5, 2], [2, 2, 5, 2, 5, 5, 2, 5]]

10, "range", [2, 3], [[3, 3, 2, 3, 2, 2, 3, 2], [2, 2, 3, 2, 3, 3, 2, 3]]

11, "range", [1, 8], [[8, 8, 1, 8, 1, 1, 8, 1], [1, 1, 8, 1, 8, 8, 1, 8]]

12, "range", [1, 6], [[6, 6, 1, 6, 1, 1, 6, 1], [1, 1, 6, 1, 6, 6, 1, 6]]

13, "range", [1, 5], [[5, 5, 1, 5, 1, 1, 5, 1], [1, 1, 5, 1, 5, 5, 1, 5]]

14, "range", [1, 3], [[3, 3, 1, 3, 1, 1, 3, 1], [1, 1, 3, 1, 3, 3, 1, 3]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$g_1 =$ [[1, 2]]

$g_2 =$ []

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[2] & h[1] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2$ =
$\begin{pmatrix} 0 & 1 & 0 & 3 & 1 & 0 & 2 & 3 & 0 & 1 & 1 & 0 & 2 & 3 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 3 & 0 & 3 \end{pmatrix}$

{2, 4, 5, 7, 8, 10, 11, 13, 14, 19, 20, 24, 26, 28}

$u2$ =
$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$

{2, 4, 5, 7, 8, 10, 11, 13, 14, 17, 19, 20, 22, 24, 26, 28}

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_7 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_8 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_9 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{10} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$ idem-checks

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 4 & 0 & 4 & 0 & 0 & 4 & 0 \\ 4 & 4 & 0 & 4 & 0 & 0 & 4 & 0 \\ 0 & 0 & 4 & 0 & 4 & 4 & 0 & 4 \\ 4 & 4 & 0 & 4 & 0 & 0 & 4 & 0 \\ 0 & 0 & 4 & 0 & 4 & 4 & 0 & 4 \\ 0 & 0 & 4 & 0 & 4 & 4 & 0 & 4 \\ 4 & 4 & 0 & 4 & 0 & 0 & 4 & 0 \\ 0 & 0 & 4 & 0 & 4 & 4 & 0 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [0, 0, -1, -1, 0, 0, 1, 1]$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & -s & 0 & 0 & t & -t & 0 & 0 \\ t & -t & 0 & 0 & s & -s & 0 & 0 \\ 0 & -t & 0 & t & 0 & -s & 0 & s \\ 0 & -t & 0 & t & 0 & -s & 0 & s \\ 0 & -s & t & 0 & 0 & -t & s & 0 \\ 0 & -s & t & 0 & 0 & -t & s & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} 1 & 1 & 0 & 0 & -1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 \\ -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} -s+t \\ -s+t \\ -t+s \\ -s+t \\ -t+s \\ -t+s \\ -s+t \\ -t+s \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} t & s \\ t & s \\ s & t \\ t & s \\ s & t \\ s & t \\ t & s \\ s & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4 & 0 & \frac{4}{7} & 0 & \frac{12}{7} & \frac{4}{7} & 0 & \frac{8}{7} \\ 0 & 4 & \frac{12}{7} & 0 & \frac{4}{7} & \frac{4}{7} & 0 & \frac{8}{7} \\ \frac{4}{7} & \frac{12}{7} & 4 & \frac{12}{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12}{7} & 4 & \frac{8}{7} & \frac{8}{7} & 0 & 0 \\ \frac{12}{7} & \frac{4}{7} & 0 & \frac{8}{7} & 4 & 0 & \frac{4}{7} & 0 \\ \frac{4}{7} & \frac{4}{7} & 0 & \frac{8}{7} & 0 & 4 & \frac{12}{7} & 0 \\ 0 & 0 & 0 & 0 & \frac{4}{7} & \frac{12}{7} & 4 & \frac{12}{7} \\ \frac{8}{7} & \frac{8}{7} & 0 & 0 & 0 & 0 & \frac{12}{7} & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&1&0&1&0&0&1&0\\ 1&1&0&1&0&0&1&0\\ 0&0&1&0&1&1&0&1\\ 1&1&0&1&0&0&1&0\\ 0&0&1&0&1&1&0&1\\ 0&0&1&0&1&1&0&1\\ 1&1&0&1&0&0&1&0\\ 0&0&1&0&1&1&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$\Omega \left(\frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \ \frac{1}{8} \right)$

$T \left(\frac{1}{4} \ \frac{1}{4} \ 0 \ \frac{1}{4} \ 0 \ 0 \ 0 \ \frac{1}{4} \ 0 \ 0 \ \frac{1}{4} \ 0 \ \frac{1}{4} \ \frac{1}{4} \right)$

"IS NM in Vec(K)?", true

NM $(4 \ 4 \ 0 \ 4 \ 0 \ 0 \ 0 \ 4 \ 0 \ 0 \ 4 \ 0 \ 4 \ 4)$

"IS MN in Vec(K)?", true

MN $(4 \ 4 \ 0 \ 4 \ 0 \ 0 \ 0 \ 4 \ 0 \ 0 \ 4 \ 0 \ 4 \ 4)$

$$\tau = 32/1 \text{ , rank } = 2 \text{ , ratio } = 16/1 \text{ , } n^2 / r = 32/1$$

$$\tau' = 32/1 \text{ , } r' = 1/2 \text{ , } \tau / n^2 = 1/2$$

$$p^2 = 1/8 \text{ , min } \tau = 8/1 \text{ , } \tau\text{-check is positive? } 24/1$$

$$\text{max r} = 8/1 \text{ , } r\text{-check is positive? } 3/4$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 14, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 28

dim span idems 7 vs no. of idems 14

"PT1" = {{3, 5, 6, 8}, {1, 2, 4, 7}}

"RG1" = {7, 8}

"RG2" = {6, 7}

"RG3" = {5, 7}

"RG4" = {4, 6}

"RG5" = {4, 5}

"RG6" = {3, 4}

"RG7" = {2, 8}

"RG8" = {2, 6}

"RG9" = {2, 5}

"RG10" = {2, 3}

"RG11" = {1, 8}

"RG12" = {1, 6}

"RG13" = {1, 5}

"RG14" = {1, 3}

$$M_c = \begin{pmatrix} 3 & -1 & \frac{-3}{7} & -1 & \frac{5}{7} & \frac{-3}{7} & -1 & \frac{1}{7} \\ -1 & 3 & \frac{5}{7} & -1 & \frac{-3}{7} & \frac{-3}{7} & -1 & \frac{1}{7} \\ \frac{-3}{7} & \frac{5}{7} & 3 & \frac{5}{7} & -1 & -1 & -1 & -1 \\ -1 & -1 & \frac{5}{7} & 3 & \frac{1}{7} & \frac{1}{7} & -1 & -1 \\ \frac{5}{7} & \frac{-3}{7} & -1 & \frac{1}{7} & 3 & -1 & \frac{-3}{7} & -1 \\ \frac{-3}{7} & \frac{-3}{7} & -1 & \frac{1}{7} & -1 & 3 & \frac{5}{7} & -1 \\ -1 & -1 & -1 & -1 & \frac{-3}{7} & \frac{5}{7} & 3 & \frac{5}{7} \\ \frac{1}{7} & \frac{1}{7} & -1 & -1 & -1 & -1 & \frac{5}{7} & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix}
\frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\
\frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\
\frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} \\
\frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\
\frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} \\
\frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} \\
\frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} \\
\frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{7}{8}
\end{pmatrix}$$

$$M_C\text{–scaled} = \begin{pmatrix}
1 & \frac{-1}{3} & \frac{-1}{7} & \frac{-1}{3} & \frac{5}{21} & \frac{-1}{7} & \frac{-1}{3} & \frac{1}{21} \\
\frac{-1}{3} & 1 & \frac{5}{21} & \frac{-1}{3} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{3} & \frac{1}{21} \\
\frac{-1}{7} & \frac{5}{21} & 1 & \frac{5}{21} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\
\frac{-1}{3} & \frac{-1}{3} & \frac{5}{21} & 1 & \frac{1}{21} & \frac{1}{21} & \frac{-1}{3} & \frac{-1}{3} \\
\frac{5}{21} & \frac{-1}{7} & \frac{-1}{3} & \frac{1}{21} & 1 & \frac{-1}{3} & \frac{-1}{7} & \frac{-1}{3} \\
\frac{-1}{7} & \frac{-1}{7} & \frac{-1}{3} & \frac{1}{21} & \frac{-1}{3} & 1 & \frac{5}{21} & \frac{-1}{3} \\
\frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{7} & \frac{5}{21} & 1 & \frac{5}{21} \\
\frac{1}{21} & \frac{1}{21} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{5}{21} & 1
\end{pmatrix} \quad N_C\text{–scaled} =$$

$$\begin{pmatrix} 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 \\ 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 \\ 1 & 1 & \frac{-1}{7} & 1 & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} \\ \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{7} & 1 & 1 & \frac{-1}{7} & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_c

[0., 0., 5.082918009, 2.118627666, 3.084168612, 5.881372334, 2.917081990, 4.915831388]

Eigenvalues N_C

[0., 0., 0., 0., 0., 0., 4., 3.]

Eigenvalues M_C−scaled

[0., 0., 1.694306003, 0.7062092219, 1.028056204, 1.960457444, 0.9723606643, 1.638610464]

Eigenvalues N_C−scaled

[0., 0., 0., 0., 0., 0., 4.571428571, 3.428571429]

NullSpace M_C

{[1, 1, 0, 1, 0, 0, 1, 0], [0, 0, 1, 0, 1, 1, 0, 1]}

NullSpace N_C

{[0, −1, 0, 1, 0, 0, 0, 0], [0, 0, −1, 0, 1, 0, 0, 0], [0, 0, −1, 0, 0, 1, 0, 0], [0, −1, 0, 0, 0, 0, 1, 0], [1, −1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1]}

Eigenvalues M_0

[0., 8., 5.082918009, 2.118627666, 3.084168612, 5.881372334, 2.917081990, 4.915831388]

Eigenvalues N_0

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace M_0

{[1, 1, −1, 1, −1, −1, 1, −1]}

NullSpace N_0

{[−1, 0, 0, 0, 0, 0, 1, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, −1, 0, 0, 1, 0, 0], [−1, 1, 0, 0, 0, 0, 0, 0], [0, 0, −1, 0, 0, 0, 0, 1], [0, 0, −1, 0, 1, 0, 0, 0]}

Eigenvalues M

[−4., 4., 1.082918009, −1.881372335, −0.9158313892, 1.881372335, −1.082918009, 0.9158313890]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{}

NullSpace N

{[0, 1, 0, −1, 0, 0, 0, 0], [1, 0, 0, −1, 0, 0, 0, 0], [0, 0, 0, −1, 0, 0, 1, 0], [0, 0, 0, 0, 0, −1, 0, 1], [0, 0, 0, 0, 1, −1, 0, 0], [0, 0, 1, 0, 0, −1, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

==

{2, 3, 4, 5, 7, 8}

R: [2, 7, 6, 7, 8, 4, 5, 5]
B: [6, 3, 2, 3, 4, 8, 1, 1]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 5, "vs", 6

Level 2 det = $\frac{1}{16384}\left(-2020 - 1745 s - 799 s^2 - 43 s^3 + 79 s^4 + 37 s^5 + 19 s^6 - 9 s^7 + s^8\right)\left(-1 + s\right)$

RANK of R is 6

R ranking is 4, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 4, "vs", 6

BBAR ranking 3, "vs", 5

"R CYCLES", 1 + v[5] v[8]
"B CYCLES", (1 + v[2] v[3]) (1 + v[1] v[6] v[8])

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., -1.]

B: [-1., -0.5000000000 + 0.8660254040 I, -0.5000000000 - 0.8660254040 I, 1., 1., 0., 0., 0.]

NullSpace of R

{[1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0]}

NullSpace of R^*

{[0, −1, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1]}

NullSpace of B^*

{[0, −1, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, −1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 4 & 12 & 12 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 12 & 8 & 4 \\ 12 & 0 & 0 & 0 & 0 & 8 & 0 & 8 \\ 12 & 0 & 0 & 0 & 0 & 4 & 8 & 4 \\ 0 & 0 & 0 & 0 & 0 & 4 & 12 & 12 \\ 0 & 12 & 8 & 4 & 4 & 0 & 0 & 0 \\ 0 & 8 & 0 & 8 & 12 & 0 & 0 & 0 \\ 0 & 4 & 8 & 4 & 12 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)

degree 2: $\frac{1}{28}$ ($v[1]v[2] + 3v[1]v[3] + 3v[1]v[4] + 3v[2]v[6] + 2v[2]v[7] + v[2]v[8] + 2v[3]v[6] + 2v[3]v[8] + v[4]v[6] + 2v[4]v[7] + v[4]v[8] + v[5]v[6] + 3v[5]v[7] + 3v[5]v[8]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"RG1" = {5, 8}

"RG2" = {4, 8}

"RG3" = {3, 8}

"RG4" = {2, 8}

"RG5" = {5, 7}

"RG6" = {4, 7}

"RG7" = {2, 7}

"RG8" = {5, 6}

"RG9" = {4, 6}

"RG10" = {3, 6}

"RG11" = {2, 6}

"RG12" = {1, 4}

"RG13" = {1, 3}

"RG14" = {1, 2}

$\pi 2$ = [1, 3, 3, 0, 0, 0, 0, 0, 0, 0, 3, 2, 1, 0, 0, 2, 0, 2, 0, 1, 2, 1, 1, 3, 3, 0, 0, 0]

supp $\pi 2$ = {1, 2, 3, 11, 12, 13, 16, 18, 20, 21, 22, 23, 24, 25}

$u2$ = [1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0]

supp u2 = {1, 2, 3, 4, 11, 12, 13, 16, 17, 18, 20, 21, 22, 23, 24, 25}

Action of R on ranges, [[1], [5], [8], [5], [1], [5], [5], [2], [6], [9], [6], [7], [11], [7]]
Action of B on ranges, [[12], [13], [14], [13], [12], [13], [13], [2], [3], [4], [3], [10], [11], [10]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{3}{28} & \frac{1}{28} & \frac{1}{14} & \frac{1}{28} & \frac{3}{28} & \frac{1}{14} & \frac{1}{14} & \frac{1}{28} & \frac{1}{28} & \frac{1}{14} & \frac{3}{28} & \frac{3}{28} & \frac{3}{28} & \frac{1}{28} \end{pmatrix}$$

RPARTS [1]

BPARTS [1]

$$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R–BLOCKS,

[2, 1]

B–BLOCKS,

[1, 2]

with invariant measure, [1, 1]

N by blocks, N – check: true

$b_1 = \{1, 6, 7, 8\}$
$b_2 = \{2, 3, 4, 5\}$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 37, Shape: 15 ⊕ 22/20

$$\mathrm{CLB} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 8}}, true

Ω_B in Vec(K)? , {{1, 6, 8}, {2, 3}}, true

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-23}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{6} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 6, 7, 8}, {2, 3, 4, 5}}

1, "range", [5, 8], [[8, 5, 5, 5, 5, 8, 8, 8], [5, 8, 8, 8, 8, 5, 5, 5]]

2, "range", [4, 8], [[8, 4, 4, 4, 4, 8, 8, 8], [4, 8, 8, 8, 8, 4, 4, 4]]

3, "range", [3, 8], [[8, 3, 3, 3, 3, 8, 8, 8], [3, 8, 8, 8, 8, 3, 3, 3]]

4, "range", [2, 8], [[8, 2, 2, 2, 2, 8, 8, 8], [2, 8, 8, 8, 8, 2, 2, 2]]

5, "range", [5, 7], [[7, 5, 5, 5, 5, 7, 7, 7], [5, 7, 7, 7, 7, 5, 5, 5]]

6, "range", [4, 7], [[7, 4, 4, 4, 4, 7, 7, 7], [4, 7, 7, 7, 7, 4, 4, 4]]

7, "range", [2, 7], [[7, 2, 2, 2, 2, 7, 7, 7], [2, 7, 7, 7, 7, 2, 2, 2]]

8, "range", [5, 6], [[6, 5, 5, 5, 5, 6, 6, 6], [5, 6, 6, 6, 6, 5, 5, 5]]

9, "range", [4, 6], [[6, 4, 4, 4, 4, 6, 6, 6], [4, 6, 6, 6, 6, 4, 4, 4]]

10, "range", [3, 6], [[6, 3, 3, 3, 3, 6, 6, 6], [3, 6, 6, 6, 6, 3, 3, 3]]

11, "range", [2, 6], [[6, 2, 2, 2, 2, 6, 6, 6], [2, 6, 6, 6, 6, 2, 2, 2]]

12, "range", [1, 4], [[4, 1, 1, 1, 1, 4, 4, 4], [1, 4, 4, 4, 4, 1, 1, 1]]

13, "range", [1, 3], [[3, 1, 1, 1, 1, 3, 3, 3], [1, 3, 3, 3, 3, 1, 1, 1]]

14, "range", [1, 2], [[2, 1, 1, 1, 1, 2, 2, 2], [1, 2, 2, 2, 2, 1, 1, 1]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

$Z[\ 1\ ] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

$Z[\ 2\ ] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

$EIGS = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 1 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 2 & 1 & 0 & 0 & 2 & 0 & 2 & 0 & 1 & 2 & 1 & 1 & 3 & 3 & 0 & 0 & 0 \end{pmatrix}$

{1, 2, 3, 11, 12, 13, 16, 18, 20, 21, 22, 23, 24, 25}

$u2 =$
$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$

{1, 2, 3, 4, 11, 12, 13, 16, 17, 18, 20, 21, 22, 23, 24, 25}

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem−checks N0−checks

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ idem-checks N0-checks

$$P_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_9 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{10} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem-checks N0-checks

$$P_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_{13} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ idem−checks N0−checks

$$P_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 4 & 4 & 4 \\ 0 & 4 & 4 & 4 & 4 & 0 & 0 & 0 \\ 0 & 4 & 4 & 4 & 4 & 0 & 0 & 0 \\ 0 & 4 & 4 & 4 & 4 & 0 & 0 & 0 \\ 0 & 4 & 4 & 4 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 4 & 4 & 4 \\ 4 & 0 & 0 & 0 & 0 & 4 & 4 & 4 \\ 4 & 0 & 0 & 0 & 0 & 4 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 0, -1, 0, 1, 0, 1, 0]$

$$\ker N_C = \begin{pmatrix} 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & t & -t & 0 & 0 & s & -s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -t & t & 0 & 0 & -s & s \\ 0 & -s & 0 & s & 0 & -t & 0 & t \\ t & -s & 0 & 0 & s & -t & 0 & 0 \\ t & -s & 0 & 0 & s & -t & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} -s+t \\ -t+s \\ -t+s \\ -t+s \\ -t+s \\ -s+t \\ -s+t \\ -s+t \end{pmatrix} \quad \text{RB checks}$$

$$\ker M_c = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} s & t \\ t & s \\ t & s \\ t & s \\ t & s \\ s & t \\ s & t \\ s & t \end{pmatrix} \quad \text{RB checks}$$

$$n\pi x^{\dagger} = \begin{pmatrix} 4 & 4 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$\text{CNM} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4 & \frac{4}{7} & \frac{12}{7} & \frac{12}{7} & 0 & 0 & 0 & 0 \\ \frac{4}{7} & 4 & 0 & 0 & 0 & \frac{12}{7} & \frac{8}{7} & \frac{4}{7} \\ \frac{12}{7} & 0 & 4 & 0 & 0 & \frac{8}{7} & 0 & \frac{8}{7} \\ \frac{12}{7} & 0 & 0 & 4 & 0 & \frac{4}{7} & \frac{8}{7} & \frac{4}{7} \\ 0 & 0 & 0 & 0 & 4 & \frac{4}{7} & \frac{12}{7} & \frac{12}{7} \\ 0 & \frac{12}{7} & \frac{8}{7} & \frac{4}{7} & \frac{4}{7} & 4 & 0 & 0 \\ 0 & \frac{8}{7} & 0 & \frac{8}{7} & \frac{12}{7} & 0 & 4 & 0 \\ 0 & \frac{4}{7} & \frac{8}{7} & \frac{4}{7} & \frac{12}{7} & 0 & 0 & 4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&0&0&0&0&1&1&1\\ 0&1&1&1&1&0&0&0\\ 0&1&1&1&1&0&0&0\\ 0&1&1&1&1&0&0&0\\ 0&1&1&1&1&0&0&0\\ 1&0&0&0&0&1&1&1\\ 1&0&0&0&0&1&1&1\\ 1&0&0&0&0&1&1&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\text{NM} = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\ \ \frac{1}{8}\right)$

$T\left(0\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ 0\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ \frac{1}{4}\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

NM $(0\ \ 0\ \ 4\ \ 4\ \ 4\ \ 0\ \ 4\ \ 4\ \ 4\ \ 0\ \ 0\ \ 0\ \ 0\ \ 4)$

"IS MN in Vec(K)?", true

MN $(0\ \ 0\ \ 4\ \ 4\ \ 4\ \ 0\ \ 4\ \ 4\ \ 4\ \ 0\ \ 0\ \ 0\ \ 0\ \ 4)$

$\boldsymbol{\tau}$ = 32/1 , rank = 2 , ratio = 16/1 , n^2 / r = 32/1

$\boldsymbol{\tau}'$ = 32/1 , r' = 1/2 , $\boldsymbol{\tau}$ / n^2 = 1/2

p^2 = 1/8 , min $\boldsymbol{\tau}$ = 8/1 , $\boldsymbol{\tau}$-check is positive? 24/1

max r = 8/1 , r-check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 1, partitions and, 14, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 14
out of total no. of elements equal to 28

dim span idems 7 vs no. of idems 14

"PT1" = {{1, 6, 7, 8}, {2, 3, 4, 5}}

"RG1" = {5, 8}

"RG2" = {4, 8}

"RG3" = {3, 8}

"RG4" = {2, 8}

"RG5" = {5, 7}

"RG6" = {4, 7}

"RG7" = {2, 7}

"RG8" = {5, 6}

"RG9" = {4, 6}

"RG10" = {3, 6}

"RG11" = {2, 6}

"RG12" = {1, 4}

"RG13" = {1, 3}

"RG14" = {1, 2}

$$M_c = \begin{pmatrix} 3 & \frac{-3}{7} & \frac{5}{7} & \frac{5}{7} & -1 & -1 & -1 & -1 \\ \frac{-3}{7} & 3 & -1 & -1 & -1 & \frac{5}{7} & \frac{1}{7} & \frac{-3}{7} \\ \frac{5}{7} & -1 & 3 & -1 & -1 & \frac{1}{7} & -1 & \frac{1}{7} \\ \frac{5}{7} & -1 & -1 & 3 & -1 & \frac{-3}{7} & \frac{1}{7} & \frac{-3}{7} \\ -1 & -1 & -1 & -1 & 3 & \frac{-3}{7} & \frac{5}{7} & \frac{5}{7} \\ -1 & \frac{5}{7} & \frac{1}{7} & \frac{-3}{7} & \frac{-3}{7} & 3 & -1 & -1 \\ -1 & \frac{1}{7} & -1 & \frac{1}{7} & \frac{5}{7} & -1 & 3 & -1 \\ -1 & \frac{-3}{7} & \frac{1}{7} & \frac{-3}{7} & \frac{5}{7} & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_c\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{7} & \frac{5}{21} & \frac{5}{21} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{7} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{5}{21} & \frac{1}{21} & \frac{-1}{7} \\ \frac{5}{21} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{1}{21} & \frac{-1}{3} & \frac{1}{21} \\ \frac{5}{21} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{7} & \frac{1}{21} & \frac{-1}{7} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{7} & \frac{5}{21} & \frac{5}{21} \\ \frac{-1}{3} & \frac{5}{21} & \frac{1}{21} & \frac{-1}{7} & \frac{-1}{7} & 1 & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{1}{21} & \frac{-1}{3} & \frac{1}{21} & \frac{5}{21} & \frac{-1}{3} & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{7} & \frac{1}{21} & \frac{-1}{7} & \frac{5}{21} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_c\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 \\ \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ \frac{-1}{7} & 1 & 1 & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} \\ 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 \\ 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 \\ 1 & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 & 1 \end{pmatrix}$$

$$N_c M_c = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_c N_c = \begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_c

[0., 0., 5.082918009, 2.118627666, 3.084168612, 5.881372334, 2.917081990, 4.915831388]

Eigenvalues N_C

[0., 0., 0., 0., 0., 0., 4., 3.]

Eigenvalues M_C−scaled

[0., 0., 1.694306003, 0.7062092219, 1.028056204, 1.960457444, 0.9723606643, 1.638610464]

Eigenvalues N_C−scaled

[0., 0., 0., 0., 0., 0., 4.571428571, 3.428571429]

NullSpace M_C

{[1, 0, 0, 0, 0, 1, 1, 1], [0, 1, 1, 1, 1, 0, 0, 0]}

NullSpace N_C

{[0, 0, 0, 0, 0, −1, 0, 1], [0, 0, 1, −1, 0, 0, 0, 0], [0, 0, 0, −1, 1, 0, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0], [1, 0, 0, 0, 0, −1, 0, 0], [0, 1, 0, −1, 0, 0, 0, 0]}

Eigenvalues M_0

[0., 8., 5.082918009, 2.118627666, 3.084168612, 5.881372334, 2.917081990, 4.915831388]

Eigenvalues N_0

[4., 4., 0., 0., 0., 0., 0., 0.]

NullSpace M_0

{[−1, 1, 1, 1, 1, −1, −1, −1]}

NullSpace N_0

{[−1, 0, 0, 0, 0, 0, 0, 1], [0, −1, 1, 0, 0, 0, 0, 0], [0, −1, 0, 1, 0, 0, 0, 0], [0, −1, 0, 0, 1, 0, 0, 0], [−1, 0, 0, 0, 0, 1, 0, 0], [−1, 0, 0, 0, 0, 0, 1, 0]}

Eigenvalues M

[−4., 4., 1.082918009, −1.881372335, −0.9158313892, 1.881372335, −1.082918009, 0.9158313890]

Eigenvalues N

[0., 0., 0., 0., 0., 0., 4., −4.]

NullSpace M

{}

NullSpace N

{[0, 0, 0, 0, 0, −1, 0, 1], [0, 1, 0, 0, −1, 0, 0, 0], [0, 0, 1, 0, −1, 0, 0, 0], [0, 0, 0, 1, −1, 0, 0, 0], [1, 0, 0, 0, 0, −1, 0, 0], [0, 0, 0, 0, 0, −1, 1, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

==

{2, 3, 5, 6, 7, 8}

R: [2, 7, 6, 3, 8, 8, 5, 5]
B: [6, 3, 2, 7, 4, 4, 1, 1]

TRACE TWO = 1

det AT = 0

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 & 0 \\ 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ \frac{5}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 3, "vs", 6

Level 2 det = $\frac{-3}{2048}\,(5+3s)\,(101-25s+9s^2-7s^3+2s^4)\,(-1+s)$

RANK of R is 6

R ranking is 3, "vs", 6

RBAR ranking 1, "vs", 2

RANK of B is 6

B ranking is 2, "vs", 6

BBAR ranking 2, "vs", 6

"R CYCLES", 1 + v[5] v[8]
"B CYCLES", (1 + v[2] v[3]) (1 + v[1] v[4] v[6] v[7])

Eigenvalues

R: [0., 0., 0., 0., 0., 0., 1., −1.]

B: [1. I, −1. I, 0., 0., 1., −1., 1., −1.]

NullSpace of R

{[0, 0, 0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0, 0]}

NullSpace of B

{[0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1]}

NullSpace of R^*

{[0, 0, 0, 0, 0, 0, −1, 1], [0, 0, 0, 0, −1, 1, 0, 0]}

NullSpace of B^*

{[0, 0, 0, 0, −1, 1, 0, 0], [0, 0, 0, 0, 0, 0, −1, 1]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} & 1 & \frac{5}{9} & \frac{5}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{2}{3} & 0 & 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{1}{3} & 1 & 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & \frac{1}{3} & \frac{2}{3} & 0 & \frac{4}{9} & \frac{4}{9} & \frac{5}{9} & \frac{5}{9} \\ \frac{5}{9} & \frac{1}{3} & \frac{2}{3} & \frac{4}{9} & 0 & 0 & 1 & 1 \\ \frac{5}{9} & \frac{1}{3} & \frac{2}{3} & \frac{4}{9} & 0 & 0 & 1 & 1 \\ \frac{4}{9} & \frac{2}{3} & \frac{1}{3} & \frac{5}{9} & 1 & 1 & 0 & 0 \\ \frac{4}{9} & \frac{2}{3} & \frac{1}{3} & \frac{5}{9} & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 4, "RANK of M is ", 8

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 2, "Rank mark", 2, "for kernel rank", 2

degree 1: $\frac{1}{8}$ ($v[1] + v[2] + v[3] + v[4] + v[5] + v[6] + v[7] + v[8]$)
degree 2: $\frac{1}{4}$ ($v[1]v[4] + v[2]v[3] + v[5]v[8] + v[6]v[7]$)

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT2" = {{1, 3, 7, 8}, {2, 4, 5, 6}}

"PT3" = {{1, 3, 5, 6}, {2, 4, 7, 8}}

"PT4" = {{1, 2, 5, 6}, {3, 4, 7, 8}}

"RG1" = {5, 8}

"RG2" = {6, 7}

"RG3" = {2, 3}

"RG4" = {1, 4}

$$\pi 2 = [0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0]$$

supp $\pi 2$ = {3, 8, 25, 26}

$$u2 = [6, 3, 9, 5, 5, 4, 4, 9, 3, 3, 3, 6, 6, 6, 6, 6, 3, 3, 4, 4, 5, 5, 0, 9, 9, 9, 9, 0]$$

supp u2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27}

Action of R on ranges, [[1], [1], [2], [3]]
Action of B on ranges, [[4], [4], [3], [2]]

$$\boldsymbol{\beta} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

RPARTS [4, 2, 4, 2]

BPARTS [4, 3, 1, 2]

$$\boldsymbol{\alpha} = \begin{pmatrix} \frac{1}{9} & \frac{4}{9} & \frac{2}{9} & \frac{2}{9} \end{pmatrix}$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[6, 5, 4, 3, 3, 4, 5, 6]

B-BLOCKS,

[7, 8, 2, 1, 3, 4, 6, 5]

with invariant measure, [2, 2, 4, 4, 2, 2, 1, 1]

N by blocks, N – check: true

b_1 = {1, 3, 5, 6}
b_2 = {2, 4, 7, 8}
b_3 = {1, 3, 7, 8}
b_4 = {2, 4, 5, 6}
b_5 = {1, 2, 5, 6}
b_6 = {3, 4, 7, 8}
b_7 = {1, 2, 7, 8}
b_8 = {3, 4, 5, 6}

dim(span of partition vectors), rank(N_0), rank(N): 4, 4, 4

$$\text{Centralizer} = \begin{pmatrix} h[2] & 0 & 0 & h[1] & 0 & 0 & 0 & 0 \\ 0 & h[2] & h[1] & 0 & 0 & 0 & 0 & 0 \\ 0 & h[1] & h[2] & 0 & 0 & 0 & 0 & 0 \\ h[1] & 0 & 0 & h[2] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h[2] & 0 & 0 & h[1] \\ 0 & 0 & 0 & 0 & 0 & h[2] & h[1] & 0 \\ 0 & 0 & 0 & 0 & 0 & h[1] & h[2] & 0 \\ 0 & 0 & 0 & 0 & h[1] & 0 & 0 & h[2] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 20, Shape: 11 ⊕ 9/6

$$\text{CLB} = \begin{pmatrix} 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

Ω_R in Vec(K)? , {{5, 8}}, true

Ω_B in Vec(K)? , {{2, 3}, {1, 4, 6, 7}}, false

$$V = \begin{pmatrix} \frac{-1}{40} & \frac{23}{40} & \frac{7}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-9}{40} \\ \frac{9}{40} & \frac{-7}{40} & \frac{-23}{40} & \frac{1}{40} & \frac{9}{40} & \frac{-7}{40} & \frac{17}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{-23}{40} & \frac{-7}{40} & \frac{9}{40} & \frac{1}{40} & \frac{17}{40} & \frac{-7}{40} & \frac{9}{40} \\ \frac{-9}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{7}{40} & \frac{-17}{40} & \frac{-1}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{-17}{40} & \frac{7}{40} & \frac{-1}{40} & \frac{-9}{40} & \frac{23}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \\ \frac{-17}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} & \frac{23}{40} & \frac{-9}{40} & \frac{-1}{40} & \frac{7}{40} \end{pmatrix}$$

Omega-checks true, true

"I+V is singular"

$\pi_R = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ $u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{3}{16} & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} & 0 & \frac{3}{16} & \frac{3}{16} & 0 \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{1, 2, 7, 8}, {3, 4, 5, 6}}

1, "range", [5, 8], [[8, 8, 5, 5, 5, 5, 8, 8], [5, 5, 8, 8, 8, 8, 5, 5]]

2, "range", [6, 7], [[7, 7, 6, 6, 6, 6, 7, 7], [6, 6, 7, 7, 7, 7, 6, 6]]

3, "range", [2, 3], [[3, 3, 2, 2, 2, 2, 3, 3], [2, 2, 3, 3, 3, 3, 2, 2]]

4, "range", [1, 4], [[4, 4, 1, 1, 1, 1, 4, 4], [1, 1, 4, 4, 4, 4, 1, 1]]

2, "partition", {{1, 3, 7, 8}, {2, 4, 5, 6}}

1, "range", [5, 8], [[8, 5, 8, 5, 5, 5, 8, 8], [5, 8, 5, 8, 8, 8, 5, 5]]

2, "range", [6, 7], [[7, 6, 7, 6, 6, 6, 7, 7], [6, 7, 6, 7, 7, 7, 6, 6]]

3, "range", [2, 3], [[3, 2, 3, 2, 2, 2, 3, 3], [2, 3, 2, 3, 3, 3, 2, 2]]

4, "range", [1, 4], [[4, 1, 4, 1, 1, 1, 4, 4], [1, 4, 1, 4, 4, 4, 1, 1]]

3, "partition", {{1, 3, 5, 6}, {2, 4, 7, 8}}

1, "range", [5, 8], [[8, 5, 8, 5, 8, 8, 5, 5], [5, 8, 5, 8, 5, 5, 8, 8]]

2, "range", [6, 7], [[7, 6, 7, 6, 7, 7, 6, 6], [6, 7, 6, 7, 6, 6, 7, 7]]

3, "range", [2, 3], [[3, 2, 3, 2, 3, 3, 2, 2], [2, 3, 2, 3, 2, 2, 3, 3]]

4, "range", [1, 4], [[4, 1, 4, 1, 4, 4, 1, 1], [1, 4, 1, 4, 1, 1, 4, 4]]

4, "partition", {{1, 2, 5, 6}, {3, 4, 7, 8}}

1, "range", [5, 8], [[8, 8, 5, 5, 8, 8, 5, 5], [5, 5, 8, 8, 5, 5, 8, 8]]

2, "range", [6, 7], [[7, 7, 6, 6, 7, 7, 6, 6], [6, 6, 7, 7, 6, 6, 7, 7]]

3, "range", [2, 3], [[3, 3, 2, 2, 3, 3, 2, 2], [2, 2, 3, 3, 2, 2, 3, 3]]

4, "range", [1, 4], [[4, 4, 1, 1, 4, 4, 1, 1], [1, 1, 4, 4, 1, 1, 4, 4]] "group has", 2, "elements"   Group element 1,1 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$g_1 =$ []

$g_2 =$ [[1, 2]]

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true

$\begin{pmatrix} h[1] & h[2] \end{pmatrix}$

"Basis for Z(G)"

1, "coeff", 1

Z[1] = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2, "coeff", 1

Z[2] = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Check abelian

1, 2, true

EIGS = $\begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5$

$t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [1, 6]}, {6, [1, 7]}, {7, [1, 8]}, {8, [2, 3]}, {9, [2, 4]}, {10, [2, 5]}, {11, [2, 6]}, {12, [2, 7]}, {13, [2, 8]}, {14, [3, 4]}, {15, [3, 5]}, {16, [3, 6]}, {17, [3, 7]}, {18, [3, 8]}, {19, [4, 5]}, {20, [4, 6]}, {21, [4, 7]}, {22, [4, 8]}, {23, [5, 6]}, {24, [5, 7]}, {25, [5, 8]}, {26, [6, 7]}, {27, [6, 8]}, {28, [7, 8]}

KERNEL HIERARCHY

$\pi 2 =$
$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$

{3, 8, 25, 26}

$u2 =$
$\begin{pmatrix} 6 & 3 & 9 & 5 & 5 & 4 & 4 & 9 & 3 & 3 & 3 & 6 & 6 & 6 & 6 & 6 & 3 & 3 & 4 & 4 & 5 & 5 & 0 & 9 & 9 & 9 & 9 & 0 \end{pmatrix}$

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27}

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} & \frac{9}{2} \end{pmatrix}$

picheck $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{4}{9} & 0 & 0 & \frac{5}{9} \\ 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 0 & \frac{5}{9} & 0 & 0 & \frac{4}{9} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{4}{9} & \frac{5}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{5}{9} & \frac{4}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_3 = \begin{pmatrix} 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{4}{9} & 0 & 0 & \frac{5}{9} & 0 & 0 & 0 & 0 \\ \frac{4}{9} & 0 & 0 & \frac{5}{9} & 0 & 0 & 0 & 0 \\ \frac{5}{9} & 0 & 0 & \frac{4}{9} & 0 & 0 & 0 & 0 \\ \frac{5}{9} & 0 & 0 & \frac{4}{9} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_2 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_3 = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$PP_4 = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{1}{4} & \frac{1}{12} & \frac{1}{6} & 0 & \frac{1}{9} & \frac{1}{9} & \frac{5}{36} & \frac{5}{36} \\ \frac{1}{12} & \frac{1}{4} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{6} & 0 & \frac{1}{4} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{12} & \frac{1}{4} & \frac{5}{36} & \frac{5}{36} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{6} & \frac{1}{12} & \frac{5}{36} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{9} & \frac{1}{6} & \frac{1}{12} & \frac{5}{36} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{5}{36} & \frac{1}{12} & \frac{1}{6} & \frac{1}{9} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{5}{36} & \frac{1}{12} & \frac{1}{6} & \frac{1}{9} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad NM = \begin{pmatrix} 4 & \frac{4}{3} & \frac{8}{3} & 0 & \frac{16}{9} & \frac{16}{9} & \frac{20}{9} & \frac{20}{9} \\ \frac{4}{3} & 4 & 0 & \frac{8}{3} & \frac{8}{3} & \frac{8}{3} & \frac{4}{3} & \frac{4}{3} \\ \frac{8}{3} & 0 & 4 & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \\ 0 & \frac{8}{3} & \frac{4}{3} & 4 & \frac{20}{9} & \frac{20}{9} & \frac{16}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{8}{3} & \frac{4}{3} & \frac{20}{9} & 4 & 4 & 0 & 0 \\ \frac{16}{9} & \frac{8}{3} & \frac{4}{3} & \frac{20}{9} & 4 & 4 & 0 & 0 \\ \frac{20}{9} & \frac{4}{3} & \frac{8}{3} & \frac{16}{9} & 0 & 0 & 4 & 4 \\ \frac{20}{9} & \frac{4}{3} & \frac{8}{3} & \frac{16}{9} & 0 & 0 & 4 & 4 \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [-1, 0, 0, -1, 1, 0, 0, 1]$

$$\ker N_c = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -t & s & s & -t & -s & t & t & -s \\ -t & t & t & -t & -s & s & s & -s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{RB checks}$$

$\pi\boldsymbol{\Delta}$ via ker NC $\begin{pmatrix} -1 & 0 & 0 & 1 \end{pmatrix}$

$$\ker M_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -s & -t \\ 0 & 0 & t & s \\ 0 & 0 & -t & -s \\ 0 & 0 & s & t \\ t & s & 0 & 0 \\ t & s & 0 & 0 \\ -t & -s & 0 & 0 \\ -t & -s & 0 & 0 \end{pmatrix} \text{ RB checks}$$

$$\ker M_c = \begin{pmatrix} 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t & -t & 0 & s+t & 0 \\ t & s & 0 & 0 & 0 \\ s & -s & 0 & s+t & 0 \\ s & t & 0 & 0 & 0 \\ 0 & 0 & s & 0 & t \\ 0 & 0 & s & 0 & t \\ s+t & 0 & -s & s+t & -t \\ s+t & 0 & -s & s+t & -t \end{pmatrix} \text{ RB checks}$$

$n\pi x^{\dagger} = \begin{pmatrix} 4 & 0 & 0 & 4 & 0 \end{pmatrix}$

$$RN_0R^* = \begin{pmatrix} 1 & \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{1}{3} & 1 & 0 & \frac{2}{3} & 1 & 1 & 0 & 0 \\ \frac{2}{3} & 0 & 1 & \frac{1}{3} & 0 & 0 & 1 & 1 \\ 0 & \frac{2}{3} & \frac{1}{3} & 1 & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 1 & 0 & \frac{2}{3} & 1 & 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 & \frac{2}{3} & 1 & 1 & 0 & 0 \\ \frac{2}{3} & 0 & 1 & \frac{1}{3} & 0 & 0 & 1 & 1 \\ \frac{2}{3} & 0 & 1 & \frac{1}{3} & 0 & 0 & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & \frac{1}{3} & \frac{2}{3} & 0 & \frac{5}{9} & \frac{5}{9} & \frac{4}{9} & \frac{4}{9} \\ \frac{1}{3} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} & 1 & \frac{4}{9} & \frac{4}{9} & \frac{5}{9} & \frac{5}{9} \\ \frac{5}{9} & \frac{1}{3} & \frac{2}{3} & \frac{4}{9} & 1 & 1 & 0 & 0 \\ \frac{5}{9} & \frac{1}{3} & \frac{2}{3} & \frac{4}{9} & 1 & 1 & 0 & 0 \\ \frac{4}{9} & \frac{2}{3} & \frac{1}{3} & \frac{5}{9} & 0 & 0 & 1 & 1 \\ \frac{4}{9} & \frac{2}{3} & \frac{1}{3} & \frac{5}{9} & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 6, "vs", 2

$$CNM = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 4&0&0&4&0&0&0&0\\ 0&4&4&0&0&0&0&0\\ 0&4&4&0&0&0&0&0\\ 4&0&0&4&0&0&0&0\\ 0&0&0&0&4&0&0&4\\ 0&0&0&0&0&4&4&0\\ 0&0&0&0&0&4&4&0\\ 0&0&0&0&4&0&0&4 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&\frac{1}{3}&\frac{2}{3}&0&\frac{4}{9}&\frac{4}{9}&\frac{5}{9}&\frac{5}{9}\\ \frac{1}{3}&1&0&\frac{2}{3}&\frac{2}{3}&\frac{2}{3}&\frac{1}{3}&\frac{1}{3}\\ \frac{2}{3}&0&1&\frac{1}{3}&\frac{1}{3}&\frac{1}{3}&\frac{2}{3}&\frac{2}{3}\\ 0&\frac{2}{3}&\frac{1}{3}&1&\frac{5}{9}&\frac{5}{9}&\frac{4}{9}&\frac{4}{9}\\ \frac{4}{9}&\frac{2}{3}&\frac{1}{3}&\frac{5}{9}&1&1&0&0\\ \frac{4}{9}&\frac{2}{3}&\frac{1}{3}&\frac{5}{9}&1&1&0&0\\ \frac{5}{9}&\frac{1}{3}&\frac{2}{3}&\frac{4}{9}&0&0&1&1\\ \frac{5}{9}&\frac{1}{3}&\frac{2}{3}&\frac{4}{9}&0&0&1&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 16T + 0\Omega$$

$\Omega\left(\frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\ \frac{1}{8}\right)$

$T\left(\frac{1}{4}\ \frac{1}{4}\ \frac{1}{6}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{4}\ \frac{1}{12}\ \frac{5}{36}\ \frac{5}{36}\ \frac{1}{9}\ \frac{1}{9}\ 0\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{4}\right)$

"IS NM in Vec(K)?", true

$\mathrm{NM}\left(4\ \ 4\ \ \frac{8}{3}\ \frac{16}{9}\ \frac{8}{3}\ \frac{8}{3}\ 4\ \frac{4}{3}\ \frac{20}{9}\ \frac{20}{9}\ \frac{16}{9}\ \frac{16}{9}\ 0\ \frac{8}{3}\ \frac{4}{3}\ 4\right)$

"IS MN in Vec(K)?", true

$\mathrm{MN}\left(4\ \ 4\ \ \frac{8}{3}\ \frac{16}{9}\ \frac{8}{3}\ \frac{8}{3}\ 4\ \frac{4}{3}\ \frac{20}{9}\ \frac{20}{9}\ \frac{16}{9}\ \frac{16}{9}\ 0\ \frac{8}{3}\ \frac{4}{3}\ 4\right)$

$\boldsymbol{\tau}$ = 32/1 , rank = 2 , ratio = 16/1 , n^2 / r = 32/1

$\boldsymbol{\tau}'$ = 32/1 , r' = 1/2 , $\boldsymbol{\tau}$ / n^2 = 1/2

p^2 = 1/8 , min $\boldsymbol{\tau}$ = 8/1 , $\boldsymbol{\tau}$–check is positive? 24/1

max r = 8/1 , r–check is positive? 3/4

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 32\Omega$$

There are, 4, partitions and, 4, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 16
out of total no. of elements equal to 32

dim span idems 12 vs no. of idems 16

"PT1" = {{1, 2, 7, 8}, {3, 4, 5, 6}}

"PT2" = {{1, 3, 7, 8}, {2, 4, 5, 6}}

"PT3" = {{1, 3, 5, 6}, {2, 4, 7, 8}}

"PT4" = {{1, 2, 5, 6}, {3, 4, 7, 8}}

"RG1" = {5, 8}

"RG2" = {6, 7}

"RG3" = {2, 3}

"RG4" = {1, 4}

$$M_c = \begin{pmatrix} 3 & -1 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & 3 & 3 & -1 & -1 & -1 & -1 & -1 \\ -1 & 3 & 3 & -1 & -1 & -1 & -1 & -1 \\ 3 & -1 & -1 & 3 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & -1 & 3 \\ -1 & -1 & -1 & -1 & -1 & 3 & 3 & -1 \\ -1 & -1 & -1 & -1 & -1 & 3 & 3 & -1 \\ -1 & -1 & -1 & -1 & 3 & -1 & -1 & 3 \end{pmatrix} \quad N_c =$$

$$\begin{pmatrix} \frac{7}{8} & \frac{5}{24} & \frac{13}{24} & \frac{-1}{8} & \frac{23}{72} & \frac{23}{72} & \frac{31}{72} & \frac{31}{72} \\ \frac{5}{24} & \frac{7}{8} & \frac{-1}{8} & \frac{13}{24} & \frac{13}{24} & \frac{13}{24} & \frac{5}{24} & \frac{5}{24} \\ \frac{13}{24} & \frac{-1}{8} & \frac{7}{8} & \frac{5}{24} & \frac{5}{24} & \frac{5}{24} & \frac{13}{24} & \frac{13}{24} \\ \frac{-1}{8} & \frac{13}{24} & \frac{5}{24} & \frac{7}{8} & \frac{31}{72} & \frac{31}{72} & \frac{23}{72} & \frac{23}{72} \\ \frac{23}{72} & \frac{13}{24} & \frac{5}{24} & \frac{31}{72} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{23}{72} & \frac{13}{24} & \frac{5}{24} & \frac{31}{72} & \frac{7}{8} & \frac{7}{8} & \frac{-1}{8} & \frac{-1}{8} \\ \frac{31}{72} & \frac{5}{24} & \frac{13}{24} & \frac{23}{72} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \\ \frac{31}{72} & \frac{5}{24} & \frac{13}{24} & \frac{23}{72} & \frac{-1}{8} & \frac{-1}{8} & \frac{7}{8} & \frac{7}{8} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & 1 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & 1 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & 1 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 & \frac{-1}{3} & \frac{-1}{3} & 1 \end{pmatrix} \quad N_C\text{-scaled} =$$

$$\begin{pmatrix} 1 & \frac{5}{21} & \frac{13}{21} & \frac{-1}{7} & \frac{23}{63} & \frac{23}{63} & \frac{31}{63} & \frac{31}{63} \\ \frac{5}{21} & 1 & \frac{-1}{7} & \frac{13}{21} & \frac{13}{21} & \frac{13}{21} & \frac{5}{21} & \frac{5}{21} \\ \frac{13}{21} & \frac{-1}{7} & 1 & \frac{5}{21} & \frac{5}{21} & \frac{5}{21} & \frac{13}{21} & \frac{13}{21} \\ \frac{-1}{7} & \frac{13}{21} & \frac{5}{21} & 1 & \frac{31}{63} & \frac{31}{63} & \frac{23}{63} & \frac{23}{63} \\ \frac{23}{63} & \frac{13}{21} & \frac{5}{21} & \frac{31}{63} & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{23}{63} & \frac{13}{21} & \frac{5}{21} & \frac{31}{63} & 1 & 1 & \frac{-1}{7} & \frac{-1}{7} \\ \frac{31}{63} & \frac{5}{21} & \frac{13}{21} & \frac{23}{63} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 \\ \frac{31}{63} & \frac{5}{21} & \frac{13}{21} & \frac{23}{63} & \frac{-1}{7} & \frac{-1}{7} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Eigenvalues M_C

[8., 8., 8., 0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 0., 0., 0., 3., 2.247353463, 0.6217973830, 1.130849154]

Eigenvalues M_C–scaled

[2.666666667, 2.666666667, 2.666666667, 0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 0., 0., 0., 3.428571429, 2.568403958, 0.7106255807, 1.292399033]

NullSpace M_C

{[0, −1, 1, 0, 0, 0, 0, 0], [1, 1, 0, 0, 1, 1, 0, 0], [0, 0, 0, 0, −1, 0, 0, 1], [1, 1, 0, 0, 1, 0, 1, 0], [−1, 0, 0, 1, 0, 0, 0, 0]}

NullSpace N_C

{[0, 0, 0, 0, 0, 0, −1, 1], [0, −1, −1, 0, 0, 1, 1, 0], [1, −1, −1, 1, 0, 0, 0, 0], [0, −1, −1, 0, 1, 0, 1, 0]}

Eigenvalues M_0

[8., 8., 8., 8., 0., 0., 0., 0.]

Eigenvalues N_0

[0., 0., 0., 0., 4., 2.247353463, 0.6217973830, 1.130849154]

NullSpace M_0

{[0, −1, 1, 0, 0, 0, 0, 0], [−1, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, −1, 0], [0, 0, 0, 0, −1, 0, 0, 1]}

NullSpace N_0

{[0, −1, −1, 0, 1, 0, 1, 0], [0, −1, −1, 0, 1, 0, 0, 1], [1, −1, −1, 1, 0, 0, 0, 0], [0, 0, 0, 0, −1, 1, 0, 0]}

Eigenvalues M

[4., −4., 4., −4., 4., −4., 4., −4.]

Eigenvalues N

[0., 0., 0., 0., 4., −0.6217973825, −2.247353464, −1.130849154]

NullSpace M

{}

NullSpace N

{[0, −1, −1, 0, 1, 0, 0, 1], [0, −1, −1, 0, 0, 1, 0, 1], [0, 0, 0, 0, 0, 0, 1, −1], [1, −1, −1, 1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 6 & 3 & 9 & 5 & 5 & 4 & 4 \\ 6 & 0 & 9 & 3 & 3 & 3 & 6 & 6 \\ 3 & 9 & 0 & 6 & 6 & 6 & 3 & 3 \\ 9 & 3 & 6 & 0 & 4 & 4 & 5 & 5 \\ 5 & 3 & 6 & 4 & 0 & 0 & 9 & 9 \\ 5 & 3 & 6 & 4 & 0 & 0 & 9 & 9 \\ 4 & 6 & 3 & 5 & 9 & 9 & 0 & 0 \\ 4 & 6 & 3 & 5 & 9 & 9 & 0 & 0 \end{pmatrix}$$