

T-Run

[2, 3, 2, 3, 1], [4, 5, 5, 5, 4]

$$\tilde{\pi} = [6, 7, 8, 9, 12]$$

$$\delta = [1, 2, 2, 2, 3]$$

POSSIBLE RANKS

$$1 \times 42$$

$$2 \times 21$$

$$3 \times 14$$

$$6 \times 7$$

BASE DETERMINANT 211/1024, .2060546875

NullSpace of Δ

{1, 2, 3, 4, 5}

Nullspace of A

[[4, 5], {1, 2, 3}]

STRATIFIED CYCLE COVERS

Degree 0

1

Degree 1

0

Degree 2

$$v[5] v[4] + v[2] v[3]$$

Degree 3

$$v[3] v[5] v[4] + v[1] v[2] v[5] + v[1] v[5] v[4]$$

Degree 4

$$2 v[2] v[3] v[5] v[4] + v[1] v[2] v[3] v[5] + v[1] v[3] v[5] v[4]$$

Degree 5

$$2 v[1] v[2] v[3] v[5] v[4]$$

=====

{4, 5}

$$R: [2, 3, 2, 5, 4]$$

$$B: [4, 5, 5, 3, 1]$$

TRACE TWO = 1

$$\det AT = \frac{-1}{8} (-1 + t)^2 (t) (1 + t)$$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ 0 & \frac{1}{6} & 0 & 0 & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 4, "vs", 5

$$\text{Level 2 det} = \frac{3}{8192} (-1688 + 468s + 203s^2 + 25s^3 - 19s^4 + 3s^5) (-1 + s)$$

RANK of R is 4

R ranking is 2, "vs", 4

RBAR ranking 2, "vs", 4

RANK of B is 4

B ranking is 3, "vs", 4

BBAR ranking 3, "vs", 4

"R CYCLES", $(1 + v[5] v[4]) (1 + v[2] v[3])$

"B CYCLES", $1 + v[1] v[3] v[5] v[4]$

Eigenvalues

R: [0., 1., -1., 1., -1.]

B: [0., -1., 1., 1. I, -1. I]

NullSpace of R

{[1, 0, 0, 0, 0]}

NullSpace of B

{[0, 1, 0, 0, 0]}

NullSpace of R^*

{[1, 0, -1, 0, 0]}

NullSpace of B^*

{[0, -1, 1, 0, 0]}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 10 & 4 \\ 0 & 0 & 0 & 4 & 12 \\ 4 & 10 & 4 & 0 & 0 \\ 8 & 4 & 12 & 0 & 0 \end{pmatrix} \quad \text{N} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

"RANK of N is ", 2, "RANK of M is ", 4

"RANK of the KERNEL is ", 2

"IdemSolvability Check", 3 "Trace mark", 3, "Rank mark", 3, "for kernel rank", 2

degree 1: $\frac{3}{14} (6v[1] + 7v[2] + 8v[3] + 9v[4] + 12v[5])$

degree 2: $\frac{2}{21} (2v[1]v[4] + 4v[1]v[5] + 5v[2]v[4] + 2v[2]v[5] + 2v[3]v[4] + 6v[3]v[5])$

Group spectrum $1 + t + t^2$

KERNEL STRUCTURE

"PT1" = {{4, 5}, {1, 2, 3}}

"RG1" = {3, 5}

"RG2" = {3, 4}

"RG3" = {2, 5}

"RG4" = {2, 4}

"RG5" = {1, 5}

"RG6" = {1, 4}

$$\pi_2 = [0, 0, 2, 4, 0, 5, 2, 2, 6, 0]$$

supp $\pi_2 = \{3, 4, 6, 7, 8, 9\}$

$$u_2 = [0, 0, 1, 1, 0, 1, 1, 1, 1, 0]$$

supp $u_2 = \{3, 4, 6, 7, 8, 9\}$

Action of R on ranges, [[4], [3], [2], [1], [4], [3]]

Action of B on ranges, [[5], [1], [5], [1], [6], [2]]

$$\beta = \left(\frac{2}{7} \quad \frac{2}{21} \quad \frac{2}{21} \quad \frac{5}{21} \quad \frac{4}{21} \quad \frac{2}{21} \right)$$

RPARTS [1]

BPARTS [1]

$$\alpha = (1)$$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[1, 2]

B-BLOCKS,

[2, 1]

with invariant measure, [1, 1]

N by blocks, N - check: true

$b_1 = \{4, 5\}$

$b_2 = \{1, 2, 3\}$

dim(span of partition vectors), rank(N_0), rank(N): 2, 2, 2

LIE STRUCTURE

Dimension of Lie algebra: 11, Shape: 3 \oplus 8/6

$$CLB = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Ω_R in Vec(K)? , {{4, 5}, {2, 3}}, true

Ω_B in Vec(K)? , {{1, 3, 4, 5}}, true

$$V = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{-1}{2} & 0 \\ \frac{-1}{7} & \frac{1}{6} & \frac{10}{21} & \frac{-3}{14} & \frac{-2}{7} \\ \frac{-1}{7} & \frac{1}{2} & \frac{1}{7} & \frac{-3}{14} & \frac{-2}{7} \\ \frac{1}{7} & \frac{-1}{6} & \frac{-10}{21} & \frac{3}{14} & \frac{2}{7} \\ \frac{-3}{7} & \frac{-1}{6} & \frac{2}{21} & \frac{5}{14} & \frac{1}{7} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

$$\pi_R = \begin{pmatrix} 0 & \frac{3}{10} & \frac{3}{10} & \frac{1}{5} & \frac{1}{5} \end{pmatrix} \text{ vs } (0 \ 0 \ 0 \ 0 \ 0) \quad u\Omega_R \text{ vs } \Omega(I-V)^{-1}$$

$$\pi_B = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \text{ vs } \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad u\Omega_B \text{ vs } \Omega(I+V)^{-1}$$

LOCAL GROUPS

1, "partition", {{4, 5}, {1, 2, 3}}

1, "range", [3, 5], [[5, 5, 5, 3, 3], [3, 3, 3, 5, 5]]

2, "range", [3, 4], [[4, 4, 4, 3, 3], [3, 3, 3, 4, 4]]

3, "range", [2, 5], [[5, 5, 5, 2, 2], [2, 2, 2, 5, 5]]

4, "range", [2, 4], [[4, 4, 4, 2, 2], [2, 2, 2, 4, 4]]

5, "range", [1, 5], [[5, 5, 5, 1, 1], [1, 1, 1, 5, 5]]

6, "range", [1, 4], [[4, 4, 4, 1, 1], [1, 1, 1, 4, 4]]

"group has", 2, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$g_1 = [[1, 2]]$$

$$g_2 = []$$

linear dimension, 2

"Symmetric?", true

Is Z in Vec(K)? true
(h[2] h[1])

"Basis for Z(G)"

1, "coeff", 1

$$Z[1] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2, "coeff", 1

$$Z[2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$\text{EIGS} = \begin{pmatrix} 1. & 1. \\ 1. & -1. \end{pmatrix}$$

PermChars :=

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$$

Group spectrum: $1 + t + t^2$

Molien Series to order 10: $1 + t + 2t^2 + 2t^3 + 3t^4 + 3t^5 + 4t^6 + 4t^7 + 5t^8 + 5t^9 + 6t^{10}$

n-choose-rank

{1, [1, 2]}, {2, [1, 3]}, {3, [1, 4]}, {4, [1, 5]}, {5, [2, 3]}, {6, [2, 4]}, {7, [2, 5]}, {8, [3, 4]}, {9, [3, 5]}, {10, [4, 5]}

KERNEL HIERARCHY

$$\pi_2 = (0 \ 0 \ 2 \ 4 \ 0 \ 5 \ 2 \ 2 \ 6 \ 0)$$

{3, 4, 6, 7, 8, 9}

$$u_2 = (0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0)$$

{3, 4, 6, 7, 8, 9}

picheck (6 7 8 9 12)

$$\pi = \begin{pmatrix} \frac{1}{7} & \frac{1}{6} & \frac{4}{21} & \frac{3}{14} & \frac{2}{7} \end{pmatrix}$$

$$\pi_1 = (6 \ 7 \ 8 \ 9 \ 12)$$

$$u_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

picheck (6 7 8 9 12)

Column Projections

$$P_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{NO-checks}$$

$$P_3 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_4 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$PP_1 = \begin{pmatrix} \frac{2}{7} & \frac{1}{3} & \frac{8}{21} & 0 & 0 \\ \frac{2}{7} & \frac{1}{3} & \frac{8}{21} & 0 & 0 \\ \frac{2}{7} & \frac{1}{3} & \frac{8}{21} & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{7} & \frac{4}{7} \\ 0 & 0 & 0 & \frac{3}{7} & \frac{4}{7} \end{pmatrix} \quad \text{idem-checks}$$

$$T = \begin{pmatrix} \frac{2}{7} & \frac{1}{3} & \frac{8}{21} & 0 & 0 \\ \frac{2}{7} & \frac{1}{3} & \frac{8}{21} & 0 & 0 \\ \frac{2}{7} & \frac{1}{3} & \frac{8}{21} & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{7} & \frac{4}{7} \\ 0 & 0 & 0 & \frac{3}{7} & \frac{4}{7} \end{pmatrix} \quad NM = \begin{pmatrix} \frac{12}{7} & 2 & \frac{16}{7} & 0 & 0 \\ \frac{12}{7} & 2 & \frac{16}{7} & 0 & 0 \\ \frac{12}{7} & 2 & \frac{16}{7} & 0 & 0 \\ 0 & 0 & 0 & \frac{18}{7} & \frac{24}{7} \\ 0 & 0 & 0 & \frac{18}{7} & \frac{24}{7} \end{pmatrix}$$

CHECKING NULLSPACES

$$\pi\Delta = [-6, 7, -1, 3, -3]$$

$$\ker N_C = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & -t & t \\ -t & 0 & t & -s & s \\ 0 & -s & s & -t & t \end{pmatrix} \text{ RB checks}$$

$$\pi\Delta \text{ via } \ker NC \ (-1 \ 3 \ 7)$$

M0 is invertible. det= 281450/21609

$$\ker M_C = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = (5)$$

$$RN_0R^* = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}, RR^* = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BN_0B^* = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}, BB^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 4, 4, "vs", 2

$$CNM = \begin{pmatrix} 0 & \frac{13}{42} & \frac{13}{21} & \frac{6}{7} & \frac{12}{7} \\ \frac{-13}{42} & 0 & \frac{13}{42} & \frac{4}{7} & \frac{10}{7} \\ \frac{-13}{21} & \frac{-13}{42} & 0 & \frac{2}{7} & \frac{8}{7} \\ \frac{-6}{7} & \frac{-4}{7} & \frac{-2}{7} & 0 & \frac{13}{14} \\ \frac{-12}{7} & \frac{-10}{7} & \frac{-8}{7} & \frac{-13}{14} & 0 \end{pmatrix} \text{ Skew T} = \begin{pmatrix} 0 & \frac{1}{21} & \frac{2}{21} & 0 & 0 \\ \frac{-1}{21} & 0 & \frac{1}{21} & 0 & 0 \\ \frac{-2}{21} & \frac{-1}{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{7} \\ 0 & 0 & 0 & \frac{-1}{7} & 0 \end{pmatrix} \text{ Skew Omega} =$$

$$\begin{pmatrix} 0 & \frac{1}{42} & \frac{1}{21} & \frac{1}{14} & \frac{1}{7} \\ -\frac{1}{42} & 0 & \frac{1}{42} & \frac{1}{21} & \frac{5}{42} \\ -\frac{1}{21} & -\frac{1}{42} & 0 & \frac{1}{42} & \frac{2}{21} \\ -\frac{1}{14} & -\frac{1}{21} & -\frac{1}{42} & 0 & \frac{1}{14} \\ -\frac{1}{7} & -\frac{5}{42} & -\frac{2}{21} & -\frac{1}{14} & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{13}{7} & 0 & 0 & \frac{4}{7} & \frac{8}{7} \\ 0 & \frac{13}{6} & 0 & \frac{10}{7} & \frac{4}{7} \\ 0 & 0 & \frac{52}{21} & \frac{4}{7} & \frac{12}{7} \\ \frac{4}{7} & \frac{10}{7} & \frac{4}{7} & \frac{39}{14} & 0 \\ \frac{8}{7} & \frac{4}{7} & \frac{12}{7} & 0 & \frac{26}{7} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = 6T + 0\Omega$$

$$\Omega \left(\frac{3}{14} \quad \frac{1}{6} \quad \frac{1}{7} \quad \frac{2}{7} \quad \frac{3}{14} \quad \frac{4}{21} \quad \frac{1}{6} \quad \frac{1}{7} \right)$$

$$T \left(\frac{3}{7} \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{8}{21} \quad \frac{1}{3} \quad \frac{2}{7} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(\frac{18}{7} \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{16}{7} \quad 2 \quad \frac{12}{7} \right)$$

"IS MN in Vec(K)?", false

$$MN (3 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 2 \quad 2)$$

$$\tau = 13/1, \text{ rank} = 2, \text{ ratio} = 13/2, n^2/r = 25/2$$

$$\tau' = 12/1, r' = 1/2, \tau/n^2 = 13/25$$

$$p^2 = 187/882, \text{ min } \tau = 4675/882, \tau\text{-check is positive? } 6791/882$$

$$\text{max } r = 882/187, r\text{-check is positive? } 254/441$$

IS N0M0 a combination of T and Omega? , true

$$N_0M_0 = \frac{1}{2}T + 12\Omega$$

There are, 1, partitions and, 6, ranges, with a group size of, 2

KERNEL HAS LINEAR DIMENSION 8

out of total no. of elements equal to 12

dim span idems 4 vs no. of idems 6

"PT1" = {{4, 5}, {1, 2, 3}}

"RG1" = {3, 5}

"RG2" = {3, 4}

"RG3" = {2, 5}

"RG4" = {2, 4}

"RG5" = {1, 5}

"RG6" = {1, 4}

$$M_C = \begin{pmatrix} \frac{66}{49} & \frac{-25}{42} & \frac{-100}{147} & \frac{-19}{98} & \frac{6}{49} \\ \frac{-25}{42} & \frac{53}{36} & \frac{-50}{63} & \frac{15}{28} & \frac{-13}{21} \\ \frac{-100}{147} & \frac{-50}{63} & \frac{692}{441} & \frac{-22}{49} & \frac{52}{147} \\ \frac{-19}{98} & \frac{15}{28} & \frac{-22}{49} & \frac{321}{196} & \frac{-75}{49} \\ \frac{6}{49} & \frac{-13}{21} & \frac{52}{147} & \frac{-75}{49} & \frac{82}{49} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{695}{882} & \frac{695}{882} & \frac{695}{882} & \frac{-187}{882} & \frac{-187}{882} \\ \frac{695}{882} & \frac{695}{882} & \frac{695}{882} & \frac{-187}{882} & \frac{-187}{882} \\ \frac{695}{882} & \frac{695}{882} & \frac{695}{882} & \frac{-187}{882} & \frac{-187}{882} \\ \frac{-187}{882} & \frac{-187}{882} & \frac{-187}{882} & \frac{695}{882} & \frac{695}{882} \\ \frac{-187}{882} & \frac{-187}{882} & \frac{-187}{882} & \frac{695}{882} & \frac{695}{882} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-175}{396} & \frac{-50}{99} & \frac{-19}{132} & \frac{1}{11} \\ \frac{-150}{371} & 1 & \frac{-200}{371} & \frac{135}{371} & \frac{-156}{371} \\ \frac{-75}{173} & \frac{-175}{346} & 1 & \frac{-99}{346} & \frac{39}{173} \\ \frac{-38}{321} & \frac{35}{107} & \frac{-88}{321} & 1 & \frac{-100}{107} \\ \frac{3}{41} & \frac{-91}{246} & \frac{26}{123} & \frac{-75}{82} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & 1 & 1 & \frac{-187}{695} & \frac{-187}{695} \\ 1 & 1 & 1 & \frac{-187}{695} & \frac{-187}{695} \\ 1 & 1 & 1 & \frac{-187}{695} & \frac{-187}{695} \\ \frac{-187}{695} & \frac{-187}{695} & \frac{-187}{695} & 1 & 1 \\ \frac{-187}{695} & \frac{-187}{695} & \frac{-187}{695} & 1 & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{1}{14} & \frac{1}{12} & \frac{2}{21} & \frac{-3}{28} & \frac{-1}{7} \\ \frac{1}{14} & \frac{1}{12} & \frac{2}{21} & \frac{-3}{28} & \frac{-1}{7} \\ \frac{1}{14} & \frac{1}{12} & \frac{2}{21} & \frac{-3}{28} & \frac{-1}{7} \\ \frac{-1}{14} & \frac{-1}{12} & \frac{-2}{21} & \frac{3}{28} & \frac{1}{7} \\ \frac{-1}{14} & \frac{-1}{12} & \frac{-2}{21} & \frac{3}{28} & \frac{1}{7} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & \frac{-1}{14} & \frac{-1}{14} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{-1}{12} & \frac{-1}{12} \\ \frac{2}{21} & \frac{2}{21} & \frac{2}{21} & \frac{-2}{21} & \frac{-2}{21} \\ \frac{-3}{28} & \frac{-3}{28} & \frac{-3}{28} & \frac{3}{28} & \frac{3}{28} \\ \frac{-1}{7} & \frac{-1}{7} & \frac{-1}{7} & \frac{1}{7} & \frac{1}{7} \end{pmatrix} \quad \text{commutator} =$$

$$\begin{pmatrix} 0 & -\frac{1}{84} & -\frac{1}{42} & \frac{1}{28} & \frac{1}{14} \\ \frac{1}{84} & 0 & -\frac{1}{84} & \frac{1}{42} & \frac{5}{84} \\ \frac{1}{42} & \frac{1}{84} & 0 & \frac{1}{84} & \frac{1}{21} \\ -\frac{1}{28} & -\frac{1}{42} & -\frac{1}{84} & 0 & -\frac{1}{28} \\ -\frac{1}{14} & -\frac{5}{84} & -\frac{1}{21} & \frac{1}{28} & 0 \end{pmatrix}$$

Eigenvalues M_c

[0., 0.2079708372, 1.561966029, 2.086724901, 3.842884718]

Eigenvalues N_c

[0., 0., 0., 2.621828106, 1.318081192]

Eigenvalues M_c -scaled

[0., 0.1323256587, 1.025374530, 1.447684396, 2.394615415]

Eigenvalues N_c -scaled

[0., 0., 0., 3.327269625, 1.672730375]

NullSpace M_c

{[1, 1, 1, 1, 1]}

NullSpace N_c

{[0, -1, 1, 0, 0], [0, 0, 0, 1, -1], [1, -1, 0, 0, 0]}

Eigenvalues M_0

[0.1993457542, 1.553302006, 2.077722402, 3.704131635, 5.465498203]

Eigenvalues N_0

[0., 0., 0., 3., 2.]

NullSpace M_0

{}

NullSpace N_0

{[0, 0, 0, -1, 1], [1, 0, -1, 0, 0], [0, 1, -1, 0, 0]}

Eigenvalues M

[0., -2.501079729, 2.501079729, -1.004940951, 1.004940951]

Eigenvalues N

[0., 0., 0., 2.449489743, -2.449489743]

NullSpace M

$\{[-13], \{[-1, 4, 0, 0]\} \}$

NullSpace N

$\{[1, -1, 0, 0, 0], [0, -1, 1, 0, 0], [0, 0, 0, -1, 1]\}$

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$