T-Run

[2, 1, 5, 2, 3, 4], [6, 6, 6, 3, 4, 5]

$\tilde{\pi}$ = [1, 2, 3, 3, 3, 3]
$\boldsymbol{\delta}$ = [1, 2, 2, 2, 2, 3]

POSSIBLE RANKS

1 x 15
3 x 5

BASE DETERMINANT 163959/1048576, .1563634872

NullSpace of Δ

{1, 2, 3, 4, 5, 6}

Nullspace of A

` det(A) = ` 1/32

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[1] v[2] + v[3] v[5]

Degree 3
v[3] v[4] v[6] + v[3] v[4] v[5] + v[2] v[4] v[6] + v[3] v[6] v[5]

Degree 4
v[3] v[4] v[6] v[5] + v[2] v[4] v[6] v[5] + v[1] v[2] v[4] v[6] + v[1] v[2] v[3] v[5]

Degree 5
v[1] v[2] v[3] v[4] v[5] + v[1] v[2] v[4] v[6] v[5] + v[2] v[3] v[4] v[6] v[5] + v[1] v[2] v[3] v[6] v[5] + v[1] v[2] v[3] v[4] v[6]

Degree 6
2 v[1] v[2] v[3] v[4] v[6] v[5]

===

20, [1, -1, 1, -1, -1, 1]

==

{3, 4, 5, 6}

R: [2, 1, 6, 3, 4, 5]
B: [6, 6, 5, 2, 3, 4]

TRACE TWO = 1

det AT = $\frac{5}{32}(1 + 10t^2 + 5t^4)(1 + t)$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{5}{6} \\ \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \end{pmatrix}$$

AT ranking is 2, "vs", 6

Level 2 det = $\frac{-3}{33554432}(-1+s)(5246688 - 600360s - 2730676s^2 - 1031698s^3 + 339539s^4 + 322527s^5 + 17064s^6 - 27908s^7 - 1058s^8 + 1982s^9 + 4s^{10} - 98s^{11} - 9s^{12} + 3s^{13})$

RANK of R is 6

R ranking is 2, "vs", 6

RBAR ranking 2, "vs", 6

RANK of B is 5

B ranking is 1, "vs", 5

BBAR ranking 1, "vs", 5

"R CYCLES", (1 + v[3] v[4] v[6] v[5]) (1 + v[1] v[2])
"B CYCLES", (1 + v[3] v[5]) (1 + v[2] v[4] v[6])

Eigenvalues

R: [1. I, −1. I, 1., −1., 1., −1.]

B: [0., −1., −0.5000000000 + 0.8660254040 I, −0.5000000000 − 0.8660254040 I, 1., 1.]

NullSpace of R

{}

NullSpace of B

{[1, 0, 0, 0, 0, 0]}

NullSpace of R^*

{}

NullSpace of B^*

{[−1, 1, 0, 0, 0, 0]}

FIXED POINTS DIMENSION 1

$$PROTO\text{-}M = \begin{pmatrix} 0 & 0 & 7 & 7 & 7 & 7 \\ 0 & 0 & 14 & 14 & 14 & 14 \\ 7 & 14 & 0 & 21 & 21 & 21 \\ 7 & 14 & 21 & 0 & 21 & 21 \\ 7 & 14 & 21 & 21 & 0 & 21 \\ 7 & 14 & 21 & 21 & 21 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 5

"RANK of the KERNEL is ", 5

"IdemSolvability Check", 3   "Trace mark", 5, "Rank mark", 5, "for kernel rank", 5

degree 1: $\frac{2}{15}$ ($v[1] + 2v[2] + 3v[3] + 3v[4] + 3v[5] + 3v[6]$)

degree 2: $\frac{1}{15}$ ($v[1]v[3] + v[1]v[4] + v[1]v[5] + v[1]v[6] + 2v[2]v[3] + 2v[2]v[4] + 2v[2]v[5] + 2v[2]v[6] + 3v[3]v[4] + 3v[3]v[5] + 3v[3]v[6] + 3v[4]v[5] + 3v[4]v[6] + 3v[6]v[5]$)

degree 3 : $\frac{1}{15}$ ($v[1]v[3]v[4] + v[1]v[3]v[5] + v[1]v[3]v[6] + v[1]v[4]v[5] + v[1]v[4]v[6] + v[1]v[6]v[5] + 2v[2]v[3]v[4] + 2v[2]v[3]v[5] + 2v[2]v[3]v[6] + 2v[2]v[4]v[5] + 2v[2]v[4]v[6] + 2v[2]v[6]v[5] + 3v[3]v[4]v[5] + 3v[3]v[4]v[6] + 3v[3]v[6]v[5] + 3v[4]v[6]v[5]$)

degree 4 : $\frac{2}{15}$ ($v[1]v[3]v[4]v[5] + v[1]v[3]v[4]v[6] + v[1]v[3]v[6]v[5] + v[1]v[4]v[6]v[5] + 2v[2]v[3]v[4]v[5] + 2v[2]v[3]v[4]v[6] + 2v[2]v[3]v[6]v[5] + 2v[2]v[4]v[6]v[5] + 3v[3]v[4]v[6]v[5]$)

degree 5 : $\frac{2}{3}$ ($v[4]$) ($v[5]$) ($v[3]$) ($v[1] + 2v[2]$) ($v[6]$)

Group spectrum $1 + t + t^2 + t^3 + t^4 + t^5$

KERNEL STRUCTURE

"PT1" = {{4}, {1, 2}, {5}, {6}, {3}}

"RG1" = {2, 3, 4, 5, 6}

"RG2" = {1, 3, 4, 5, 6}

$$\pi 5 = [0, 0, 0, 0, 1, 2]$$

supp $\pi 5$ = {5, 6}

$$u5 = [0, 0, 0, 0, 1, 1]$$

supp u5 = {5, 6}

Action of R on ranges, [[2], [1]]
Action of B on ranges, [[1], [1]]

$\boldsymbol{\beta} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = \begin{pmatrix} 1 \end{pmatrix}$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[3, 2, 4, 5, 1]

B-BLOCKS,

[4, 1, 5, 2, 3]

with invariant measure, [1, 1, 1, 1, 1]

N by blocks, N - check: true

b_1 = {4}
b_2 = {1, 2}
b_3 = {5}
b_4 = {6}
b_5 = {3}

dim(span of partition vectors), rank(N_0), rank(N):   5, 5, 5

LIE STRUCTURE

Dimension of Lie algebra: 22, Shape: 15 ⊕ 7/5

$$CLB = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2}, {3, 4, 5, 6}}, true

Ω_B in Vec(K)? , {{3, 5}, {2, 4, 6}}, true

$$V = \begin{pmatrix} \frac{4}{15} & \frac{8}{15} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{3}{5} & \frac{1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{1}{75} & \frac{2}{75} & \frac{-4}{25} & \frac{1}{25} & \frac{-9}{25} & \frac{11}{25} \\ \frac{-23}{75} & \frac{-46}{75} & \frac{17}{25} & \frac{2}{25} & \frac{7}{25} & \frac{-3}{25} \\ \frac{3}{25} & \frac{6}{25} & \frac{-11}{25} & \frac{9}{25} & \frac{-6}{25} & \frac{-1}{25} \\ \frac{-7}{75} & \frac{-14}{75} & \frac{3}{25} & \frac{-7}{25} & \frac{13}{25} & \frac{-2}{25} \end{pmatrix}$$

Omega-checks true, true

"I-V is singular"

"I+V is singular"

$\pi_R = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} 0 & \frac{2}{9} & \frac{1}{6} & \frac{2}{9} & \frac{1}{6} & \frac{2}{9} \end{pmatrix}$ vs $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4}, {1, 2}, {5}, {6}, {3}}

1, "range", [2, 3, 4, 5, 6], [[6, 6, 5, 4, 3, 2], [6, 6, 5, 4, 2, 3], [6, 6, 5, 3, 4, 2], [6, 6, 5, 3, 2, 4], [6, 6, 5, 2, 4, 3], [6, 6, 5, 2, 3, 4], [6, 6, 4, 5, 3, 2], [6, 6, 4, 5, 2, 3], [6, 6, 4, 3, 5, 2], [6, 6, 4, 3, 2, 5], [6, 6, 4, 2, 5, 3], [6, 6, 4, 2, 3, 5], [6, 6, 3, 5, 4, 2], [6, 6, 3, 5, 2, 4], [6, 6, 3, 4, 5, 2], [6, 6, 3, 4, 2, 5], [6, 6, 3, 2, 5, 4], [6, 6, 3, 2, 4, 5], [6, 6, 2, 5, 4, 3], [6, 6, 2, 5, 3, 4], [6, 6, 2, 4, 5, 3], [6, 6, 2, 4, 3, 5], [6, 6, 2, 3, 5, 4], [6, 6, 2, 3, 4, 5], [5, 5, 6, 4, 3, 2], [5, 5, 6, 4, 2, 3], [5, 5, 6, 3, 4, 2], [5, 5, 6, 3, 2, 4], [5, 5, 6, 2, 4, 3], [5, 5, 6, 2, 3, 4], [5, 5, 4, 6, 3, 2], [5, 5, 4, 6, 2, 3], [5, 5, 4, 3, 6, 2], [5, 5, 4, 3, 2, 6], [5, 5, 4, 2, 6, 3], [5, 5, 4, 2, 3, 6], [5, 5, 3, 6, 4, 2], [5, 5, 3, 6, 2, 4], [5, 5, 3, 4, 6, 2], [5, 5, 3, 4, 2, 6], [5, 5, 3, 2, 6, 4], [5, 5, 3, 2, 4, 6], [5, 5, 2, 6, 4, 3], [5, 5, 2, 6, 3, 4], [5, 5, 2, 4, 6, 3], [5, 5, 2, 4, 3, 6], [5, 5, 2, 3, 6, 4], [5, 5, 2, 3, 4, 6], [4, 4, 6, 5, 3, 2], [4, 4, 6, 5, 2, 3], [4, 4, 6, 3, 5, 2], [4, 4, 6, 3, 2, 5], [4, 4, 6, 2, 5, 3], [4, 4, 6, 2, 3, 5], [4, 4, 5, 6, 3, 2], [4, 4, 5, 6, 2, 3], [4, 4, 5, 3, 6, 2], [4, 4, 5, 3, 2, 6], [4, 4, 5, 2, 6, 3], [4, 4, 5, 2, 3, 6], [4, 4, 3, 6, 5, 2], [4, 4, 3, 6, 2, 5], [4, 4, 3, 5, 6, 2], [4, 4, 3, 5, 2, 6], [4, 4, 3, 2, 6, 5], [4, 4, 3, 2, 5, 6], [4, 4, 2, 6, 5, 3], [4, 4, 2, 6, 3, 5], [4, 4, 2, 5, 6, 3], [4, 4, 2, 5, 3, 6], [4, 4, 2, 3, 6, 5], [4, 4, 2, 3, 5, 6], [3, 3, 6, 5, 4, 2], [3, 3, 6, 5, 2, 4], [3, 3, 6, 4, 5, 2], [3, 3, 6, 4, 2, 5], [3, 3, 6, 2, 5, 4], [3, 3, 6, 2, 4, 5], [3, 3, 5, 6, 4, 2], [3, 3, 5, 6, 2, 4], [3, 3, 5, 4, 6, 2], [3, 3, 5, 4, 2, 6], [3, 3, 5, 2, 6, 4], [3, 3, 5, 2, 4, 6], [3, 3, 4, 6, 5, 2], [3, 3, 4, 6, 2, 5], [3, 3, 4, 5, 6, 2], [3, 3, 4, 5, 2, 6], [3, 3, 4, 2, 6, 5], [3, 3, 4, 2, 5, 6], [3, 3, 2, 6, 5, 4], [3, 3, 2, 6, 4, 5], [3, 3, 2, 5, 6, 4], [3, 3, 2, 5, 4, 6], [3, 3, 2, 4, 6, 5], [3, 3, 2, 4, 5, 6], [2, 2, 6, 5, 4, 3], [2, 2, 6, 5, 3, 4], [2, 2, 6, 4, 5, 3], [2, 2, 6, 4, 3, 5], [2, 2, 6, 3, 5, 4], [2, 2, 6, 3, 4, 5], [2, 2, 5, 6, 4, 3], [2, 2, 5, 6, 3, 4], [2, 2, 5, 4, 6, 3], [2, 2, 5, 4, 3, 6], [2, 2, 5, 3, 6, 4], [2, 2, 5, 3, 4, 6], [2, 2, 4, 6, 5, 3], [2, 2, 4, 6, 3, 5], [2, 2, 4, 5, 6, 3], [2, 2, 4, 5, 3, 6], [2, 2, 4, 3, 6, 5], [2, 2, 4, 3, 5, 6], [2, 2, 3, 6, 5, 4], [2, 2, 3, 6, 4, 5], [2, 2, 3, 5, 6, 4], [2, 2, 3, 5, 4, 6], [2, 2, 3, 4, 6, 5], [2, 2, 3, 4, 5, 6]]

2, "range", [1, 3, 4, 5, 6], [[6, 6, 5, 4, 3, 1], [6, 6, 5, 4, 1, 3], [6, 6, 5, 3, 4, 1], [6, 6, 5, 3, 1, 4], [6, 6, 5, 1, 4, 3], [6, 6, 5, 1, 3, 4], [6, 6, 4, 5, 3, 1], [6, 6, 4, 5, 1, 3], [6, 6, 4, 3, 5, 1], [6, 6, 4, 3, 1, 5], [6, 6, 4, 1, 5, 3], [6, 6, 4, 1, 3, 5], [6, 6, 3, 5, 4, 1], [6, 6, 3, 5, 1, 4], [6, 6, 3, 4, 5, 1], [6, 6, 3, 4, 1, 5], [6, 6, 3, 1, 5, 4], [6, 6, 3, 1, 4, 5], [6, 6, 1, 5, 4, 3], [6, 6, 1, 5, 3, 4], [6, 6, 1, 4, 5, 3], [6, 6, 1, 4, 3, 5], [6, 6, 1, 3, 5, 4], [6, 6, 1, 3, 4, 5], [5, 5, 6, 4, 3, 1], [5, 5, 6, 4, 1, 3], [5, 5, 6, 3, 4, 1], [5, 5, 6, 3, 1, 4], [5, 5, 6, 1, 4, 3], [5, 5, 6, 1, 3, 4], [5, 5, 4, 6, 3, 1], [5, 5, 4, 6, 1, 3], [5, 5, 4, 3, 6, 1], [5, 5, 4, 3, 1, 6], [5, 5, 4, 1, 6, 3], [5, 5, 4, 1, 3, 6], [5, 5, 3, 6, 4, 1], [5, 5, 3, 6, 1, 4], [5, 5, 3, 4, 6, 1], [5, 5, 3, 4, 1, 6], [5, 5, 3, 1, 6, 4], [5, 5, 3, 1, 4, 6], [5, 5, 1, 6, 4, 3], [5, 5, 1, 6, 3, 4], [5, 5, 1, 4, 6, 3], [5, 5, 1, 4, 3, 6], [5, 5, 1, 3, 6, 4], [5, 5, 1, 3, 4, 6], [4, 4, 6, 5, 3, 1], [4, 4, 6, 5, 1, 3], [4, 4, 6, 3, 5, 1], [4, 4, 6, 3, 1, 5], [4, 4, 6, 1, 5, 3], [4, 4, 6, 1, 3, 5], [4, 4, 5, 6, 3, 1], [4, 4, 5, 6, 1, 3], [4, 4, 5, 3, 6, 1], [4, 4, 5, 3, 1, 6], [4, 4, 5, 1, 6, 3], [4, 4, 5, 1, 3, 6], [4, 4, 3, 6, 5, 1], [4, 4, 3, 6, 1, 5], [4, 4, 3, 5, 6, 1], [4, 4, 3, 5, 1, 6], [4, 4, 3, 1, 6, 5], [4, 4, 3, 1, 5, 6], [4, 4, 1, 6, 5, 3], [4, 4, 1, 6, 3, 5], [4, 4, 1, 5, 6, 3], [4, 4, 1, 5, 3, 6], [4, 4, 1, 3, 6, 5], [4, 4, 1, 3, 5, 6], [3, 3, 6, 5, 4, 1], [3, 3, 6, 5, 1, 4], [3, 3, 6, 4, 5, 1], [3, 3, 6, 4, 1, 5], [3, 3, 6, 1, 5, 4], [3, 3, 6, 1, 4, 5], [3, 3, 5, 6, 4, 1], [3, 3, 5, 6, 1, 4], [3, 3, 5, 4, 6, 1], [3, 3, 5, 4, 1, 6], [3, 3, 5, 1, 6, 4], [3, 3, 5, 1, 4, 6], [3, 3, 4, 6, 5, 1], [3, 3, 4, 6, 1, 5], [3, 3, 4, 5, 6, 1], [3, 3, 4, 5, 1, 6], [3, 3, 4, 1, 6, 5], [3, 3, 4, 1, 5, 6], [3, 3, 1, 6, 5, 4], [3, 3, 1, 6, 4, 5], [3, 3, 1, 5, 6, 4], [3, 3, 1, 5, 4, 6], [3, 3, 1, 4, 6, 5], [3, 3, 1, 4, 5, 6], [1, 1, 6, 5, 4, 3], [1, 1, 6, 5, 3, 4], [1, 1, 6, 4, 5, 3], [1, 1, 6, 4, 3, 5], [1, 1, 6, 3, 5, 4], [1, 1, 6, 3, 4, 5], [1, 1, 5, 6, 4, 3], [1, 1, 5, 6, 3, 4], [1, 1, 5, 4, 6, 3], [1, 1, 5, 4, 3, 6], [1, 1, 5, 3, 6, 4], [1, 1, 5, 3, 4, 6], [1, 1, 4, 6, 5, 3], [1, 1, 4, 6, 3, 5], [1, 1, 4, 5, 6, 3], [1, 1, 4, 5, 3, 6], [1, 1, 4, 3, 6, 5], [1, 1, 4, 3, 5, 6], [1, 1, 3, 6, 5, 4], [1, 1, 3, 6, 4, 5], [1, 1, 3, 5, 6, 4], [1, 1, 3, 5, 4, 6], [1, 1, 3, 4, 6, 5], [1, 1, 3, 4, 5, 6]]

"group has", 120, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 5], [2, 4]]

$g_2 =$ [[1, 5, 2, 4]]

$g_3 =$ [[1, 5], [2, 4, 3]]

$g_4 =$ [[1, 5, 3, 2, 4]]

$g_5 =$ [[1, 5, 2, 4, 3]]

linear dimension, 17

"Symmetric?", true

Is Z in Vec(K)? true

$(72h[1] + 12h[2] \quad -24h[1] - 6h[2] \quad -48h[1] - 12h[2] \quad 6h[2] \quad 6h[2] \quad -48h[1] - 12h[2] \quad 24h[1] \quad 6h[2] \quad 24h[1] \quad 6h[2] \quad -24h[1]$

"Basis for Z(G)"

1, "coeff", 24

$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2, "coeff", 6

$$Z[2] = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. \\ 4. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1 + t + t^2 + t^3 + t^4 + t^5$

Molien Series to order 10: $1 + t + 2t^2 + 3t^3 + 5t^4 + 7t^5 + 10t^6 + 13t^7 + 18t^8 + 23t^9 + 30t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5]}, {2, [1, 2, 3, 4, 6]}, {3, [1, 2, 3, 5, 6]}, {4, [1, 2, 4, 5, 6]}, {5, [1, 3, 4, 5, 6]}, {6, [2, 3, 4, 5, 6]}

KERNEL HIERARCHY

$\pi 5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$

{5, 6}

$u5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

{5, 6}

picheck $\begin{pmatrix} 1 & 2 & 3 & 3 & 3 & 3 \end{pmatrix}$

$\pi = \begin{pmatrix} \frac{1}{15} & \frac{2}{15} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$

$\pi 4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$

$u4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$

picheck $\begin{pmatrix} 4 & 8 & 12 & 12 & 12 & 12 \end{pmatrix}$

$\pi 3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 4 & 4 & 4 & 4 & 4 & 6 & 6 & 6 & 6 \end{pmatrix}$

$u3 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} & \frac{2}{25} \end{pmatrix}$

picheck $\begin{pmatrix} 12 & 24 & 36 & 36 & 36 & 36 \end{pmatrix}$

$\pi 2 = \begin{pmatrix} 0 & 6 & 6 & 6 & 6 & 12 & 12 & 12 & 12 & 18 & 18 & 18 & 18 & 18 & 18 \end{pmatrix}$

$u2 = \begin{pmatrix} 0 & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} & \frac{6}{125} \end{pmatrix}$

picheck $\begin{pmatrix} 24 & 48 & 72 & 72 & 72 & 72 \end{pmatrix}$

$\pi 1 = \begin{pmatrix} 24 & 48 & 72 & 72 & 72 & 72 \end{pmatrix}$

$u1 = \begin{pmatrix} \frac{24}{625} & \frac{24}{625} & \frac{24}{625} & \frac{24}{625} & \frac{24}{625} & \frac{24}{625} \end{pmatrix}$

picheck $\begin{pmatrix} 24 & 48 & 72 & 72 & 72 & 72 \end{pmatrix}$

Column Projections

$$P_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks} \quad \text{N0-checks}$$

Row Projections

$$\mathrm{PP}_1 = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{idem-checks}$$

$$\mathrm{T} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathrm{NM} = \begin{pmatrix} \frac{28}{15} & \frac{56}{15} & \frac{21}{5} & \frac{21}{5} & \frac{21}{5} & \frac{21}{5} \\ \frac{28}{15} & \frac{56}{15} & \frac{21}{5} & \frac{21}{5} & \frac{21}{5} & \frac{21}{5} \\ \frac{7}{5} & \frac{14}{5} & \frac{28}{5} & \frac{21}{5} & \frac{21}{5} & \frac{21}{5} \\ \frac{7}{5} & \frac{14}{5} & \frac{21}{5} & \frac{28}{5} & \frac{21}{5} & \frac{21}{5} \\ \frac{7}{5} & \frac{14}{5} & \frac{21}{5} & \frac{21}{5} & \frac{28}{5} & \frac{21}{5} \\ \frac{7}{5} & \frac{14}{5} & \frac{21}{5} & \frac{21}{5} & \frac{21}{5} & \frac{28}{5} \end{pmatrix}$$

CHECKING NULLSPACES

$\pi\boldsymbol{\Delta} = [1, -1, 0, 0, 0, 0]$

ker $N_c = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} -s & s & 0 & 0 & 0 & 0 \end{pmatrix}$ RB checks

M0 is invertible. det= 64/15625

$$\ker M_c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} s+t \\ s+t \\ s+t \\ s+t \\ s+t \\ s+t \end{pmatrix} \quad \text{RB checks}$$

$n\pi x^{\dagger} = (6)$

$$R N_0 R^* = \begin{pmatrix} 1&1&0&0&0&0\\ 1&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}, RR^* = \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$$

$$B N_0 B^* = \begin{pmatrix} 1&1&0&0&0&0\\ 1&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}, BB^* = \begin{pmatrix} 1&1&0&0&0&0\\ 1&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 6, 5, "vs", 5

$$CNM = \begin{pmatrix} 0 & \frac{8}{15} & \frac{14}{15} & \frac{14}{15} & \frac{14}{15} & \frac{14}{15}\\ \frac{-8}{15} & 0 & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15}\\ \frac{-14}{15} & \frac{-7}{15} & 0&0&0&0\\ \frac{-14}{15} & \frac{-7}{15} & 0&0&0&0\\ \frac{-14}{15} & \frac{-7}{15} & 0&0&0&0\\ \frac{-14}{15} & \frac{-7}{15} & 0&0&0&0 \end{pmatrix} \quad \text{Skew } T = \begin{pmatrix} 0 & \frac{1}{3} & 0&0&0&0\\ \frac{-1}{3} & 0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & \frac{1}{15} & \frac{2}{15} & \frac{2}{15} & \frac{2}{15} & \frac{2}{15}\\ \frac{-1}{15} & 0 & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15}\\ \frac{-2}{15} & \frac{-1}{15} & 0&0&0&0\\ \frac{-2}{15} & \frac{-1}{15} & 0&0&0&0\\ \frac{-2}{15} & \frac{-1}{15} & 0&0&0&0\\ \frac{-2}{15} & \frac{-1}{15} & 0&0&0&0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} \frac{8}{15} & 0 & \frac{7}{15} & \frac{7}{15} & \frac{7}{15} & \frac{7}{15}\\ 0 & \frac{16}{15} & \frac{14}{15} & \frac{14}{15} & \frac{14}{15} & \frac{14}{15}\\ \frac{7}{15} & \frac{14}{15} & \frac{8}{5} & \frac{7}{5} & \frac{7}{5} & \frac{7}{5}\\ \frac{7}{15} & \frac{14}{15} & \frac{7}{5} & \frac{8}{5} & \frac{7}{5} & \frac{7}{5}\\ \frac{7}{15} & \frac{14}{15} & \frac{7}{5} & \frac{7}{5} & \frac{8}{5} & \frac{7}{5}\\ \frac{7}{15} & \frac{14}{15} & \frac{7}{5} & \frac{7}{5} & \frac{7}{5} & \frac{8}{5} \end{pmatrix} \quad N_0 = \begin{pmatrix} 1&1&0&0&0&0\\ 1&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$NM = \frac{7}{5} T + 21 \Omega$$

$$\Omega \left(\frac{1}{15} \;\; \frac{7}{5} \;\; 1 \;\; \frac{2}{5} \;\; \frac{2}{15} \;\; \frac{1}{15} \;\; 1 \;\; \frac{3}{5} \;\; \frac{1}{5} \;\; \frac{2}{15} \;\; \frac{1}{15} \;\; \frac{3}{5} \;\; \frac{1}{5} \;\; \frac{1}{5} \;\; \frac{2}{15} \;\; \frac{1}{15} \;\; \frac{1}{5} \;\; \frac{1}{5} \;\; \frac{1}{5} \;\; \frac{1}{5} \;\; \frac{2}{15} \;\; \frac{1}{15} \right)$$

$$T \left(0 \;\; 3 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 2 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \frac{2}{3} \;\; \frac{1}{3} \right)$$

"IS NM in Vec(K)?", true

$$NM \left(\frac{7}{5} \;\; \frac{168}{5} \;\; \frac{112}{5} \;\; \frac{42}{5} \;\; \frac{14}{5} \;\; \frac{7}{5} \;\; \frac{119}{5} \;\; 14 \;\; \frac{21}{5} \;\; \frac{14}{5} \;\; \frac{7}{5} \;\; 14 \;\; \frac{21}{5} \;\; \frac{28}{5} \;\; \frac{14}{5} \;\; \frac{7}{5} \;\; \frac{21}{5} \;\; \frac{21}{5} \;\; \frac{21}{5} \;\; \frac{21}{5} \;\; \frac{56}{15} \;\; \frac{28}{15} \right)$$

"IS MN in Vec(K)?", false

MN

$$\left(\frac{105}{58}\ \frac{9513}{290}\ \frac{6181}{290}\ \frac{231}{29}\ \frac{105}{58}\ \frac{105}{58}\ \frac{3346}{145}\ \frac{3871}{290}\ \frac{231}{58}\ \frac{105}{58}\ \frac{105}{58}\ \frac{3871}{290}\ \frac{231}{58}\ \frac{1561}{290}\ \frac{105}{58}\ \frac{105}{58}\ \frac{105}{29}\ \frac{105}{29}\ \frac{105}{29}\ \frac{105}{29}\ \frac{469}{145}\ \frac{469}{145}\right)$$

$$\boldsymbol{\tau} = 8/1 \text{ , rank} = 5 \text{ , ratio} = 8/5 \text{ , } n^2 / r = 36/5$$

$$\boldsymbol{\tau}' = 28/1 \text{ , } r' = 4/5 \text{ , } \boldsymbol{\tau} / n^2 = 2/9$$

$$p^2 = 41/225 \text{ , min } \boldsymbol{\tau} = 164/25 \text{ , } \boldsymbol{\tau}\text{-check is positive? } 36/25$$

$$\text{max r} = 225/41 \text{ , } r\text{-check is positive? } 4/45$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = \frac{1}{5} T + 7\Omega$$

There are, 1, partitions and, 2, ranges, with a group size of, 120

KERNEL HAS LINEAR DIMENSION 22
out of total no. of elements equal to 240

dim span idems 2 vs no. of idems 2

"PT1" = {{4}, {1, 2}, {5}, {6}, {3}}

"RG1" = {2, 3, 4, 5, 6}

"RG2" = {1, 3, 4, 5, 6}

$$M_C = \begin{pmatrix} \frac{28}{75} & \frac{-8}{25} & \frac{-1}{75} & \frac{-1}{75} & \frac{-1}{75} & \frac{-1}{75} \\ \frac{-8}{25} & \frac{32}{75} & \frac{-2}{75} & \frac{-2}{75} & \frac{-2}{75} & \frac{-2}{75} \\ \frac{-1}{75} & \frac{-2}{75} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{-1}{75} & \frac{-2}{75} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{-1}{75} & \frac{-2}{75} & \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} \\ \frac{-1}{75} & \frac{-2}{75} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{184}{225} & \frac{184}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} \\ \frac{184}{225} & \frac{184}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} \\ \frac{-41}{225} & \frac{-41}{225} & \frac{184}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} \\ \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{184}{225} & \frac{-41}{225} & \frac{-41}{225} \\ \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{184}{225} & \frac{-41}{225} \\ \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{-41}{225} & \frac{184}{225} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-6}{7} & \frac{-1}{28} & \frac{-1}{28} & \frac{-1}{28} & \frac{-1}{28} \\ \frac{-3}{4} & 1 & \frac{-1}{16} & \frac{-1}{16} & \frac{-1}{16} & \frac{-1}{16} \\ \frac{-1}{12} & \frac{-1}{6} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-1}{12} & \frac{-1}{6} & \frac{-1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-1}{12} & \frac{-1}{6} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{4} \\ \frac{-1}{12} & \frac{-1}{6} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & 1 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & 1 & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} \\ 1 & 1 & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} \\ \frac{-41}{184} & \frac{-41}{184} & 1 & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} \\ \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & 1 & \frac{-41}{184} & \frac{-41}{184} \\ \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & 1 & \frac{-41}{184} \\ \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & \frac{-41}{184} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} \frac{4}{75} & \frac{8}{75} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{4}{75} & \frac{8}{75} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{-1}{75} & \frac{-2}{75} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{-1}{75} & \frac{-2}{75} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{-1}{75} & \frac{-2}{75} & \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} \\ \frac{-1}{75} & \frac{-2}{75} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} \end{pmatrix} \quad M_C N_C = \begin{pmatrix} \frac{4}{75} & \frac{4}{75} & \frac{-1}{75} & \frac{-1}{75} & \frac{-1}{75} & \frac{-1}{75} \\ \frac{8}{75} & \frac{8}{75} & \frac{-2}{75} & \frac{-2}{75} & \frac{-2}{75} & \frac{-2}{75} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} & \frac{-1}{25} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} & \frac{-1}{25} \\ \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{-1}{25} & \frac{4}{25} \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & \frac{-4}{75} & \frac{2}{75} & \frac{2}{75} & \frac{2}{75} & \frac{2}{75} \\ \frac{4}{75} & 0 & \frac{1}{75} & \frac{1}{75} & \frac{1}{75} & \frac{1}{75} \\ \frac{-2}{75} & \frac{-1}{75} & 0 & 0 & 0 & 0 \\ \frac{-2}{75} & \frac{-1}{75} & 0 & 0 & 0 & 0 \\ \frac{-2}{75} & \frac{-1}{75} & 0 & 0 & 0 & 0 \\ \frac{-2}{75} & \frac{-1}{75} & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0.7217725413, 0.1182274587, 0.2000000000, 0.2000000000, 0.2000000000]

Eigenvalues N_C

[0., 1.808357699, 0.0983089673, 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 1.804745225, 0.4452547747, 1.250000000, 1.250000000, 1.250000000]

Eigenvalues N_C–scaled

[0., 2.211306970, 0.120214770, 1.222826087, 1.222826087, 1.222826087]

NullSpace M_C

{[1, 1, 1, 1, 1, 1]}

NullSpace N_C

{[−1, 1, 0, 0, 0, 0]}

Eigenvalues M_0

[6.576546381, 0.1089619612, 0.7144916588, 0.2000000000, 0.2000000000, 0.2000000000]

Eigenvalues N_0

[0., 2., 1., 1., 1., 1.]

NullSpace M_0

{}

NullSpace N_0

{[1, −1, 0, 0, 0, 0]}

Eigenvalues M

[0., 5.060668092, −0.860668092, −1.400000000, −1.400000000, −1.400000000]

Eigenvalues N

[0., 4.701562118, −1.701562118, −1., −1., −1.]

NullSpace M

{[−2, 1, 0, 0, 0, 0]}

NullSpace N

{[1, −1, 0, 0, 0, 0]}

Harmonic Basis

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$