T-Run

[2, 1, 4, 3, 1], [4, 3, 5, 5, 2]

$\tilde{\pi}$ = [1, 1, 1, 1, 1]
$\boldsymbol{\delta}$ = [2, 2, 2, 2, 2]

POSSIBLE RANKS

1 x 5

BASE DETERMINANT 725/4096, .1770019531

NullSpace of Δ

{1, 2, 3, 4, 5}

Nullspace of A

` det(A) = ` 1/16

STRATIFIED CYCLE COVERS

Degree 0
1

Degree 1
0

Degree 2
v[3] v[4] + v[1] v[2]

Degree 3
v[2] v[3] v[5] + v[1] v[4] v[5]

Degree 4
v[2] v[3] v[4] v[5] + v[1] v[2] v[4] v[5] + v[1] v[2] v[3] v[5] + v[1] v[2] v[3] v[4] + v[1] v[3] v[4] v[5]

Degree 5
2 v[1] v[2] v[3] v[4] v[5]

===

{2, 4}

R: [2, 3, 4, 5, 1]
B: [4, 1, 5, 3, 2]

TRACE TWO = 1

det AT = $\frac{5}{16}\left(1 + 10t^2 + 5t^4\right)$

$$AT = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{5}{6} & 0 \\ \frac{5}{6} & 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & 0 & \frac{1}{6} \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 \end{pmatrix}$$

AT ranking is 1, "vs", 5

Level 2 det = $\frac{1}{65536}(116+92s+21s^2+7s^3+3s^4+s^5)(-1+s)(-100-4s-9s^2+4s^3+s^4)$

RANK of R is 5

R ranking is 1, "vs", 5

RBAR ranking 1, "vs", 5

RANK of B is 5

B ranking is 1, "vs", 5

BBAR ranking 1, "vs", 5

"R CYCLES", 1 + v[1] v[2] v[3] v[4] v[5]
"B CYCLES", 1 + v[1] v[2] v[3] v[4] v[5]

Eigenvalues

R: [1., 0.3090169942 + 0.9510565160 I, −0.8090169942 + 0.5877852520 I, −0.8090169942 − 0.5877852520 I, 0.3090169942 − 0.9510565160 I]

B: [1., 0.3090169942 + 0.9510565160 I, −0.8090169942 + 0.5877852520 I, −0.8090169942 − 0.5877852520 I, 0.3090169942 − 0.9510565160 I]

NullSpace of R

{}

NullSpace of B

{}

NullSpace of R^*

{}

NullSpace of B^*

{}

FIXED POINTS DIMENSION 1

$$\text{PROTO-M} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

"RANK of N is ", 5, "RANK of M is ", 5

"RANK of the KERNEL is ", 5

"IdemSolvability Check", 3 "Trace mark", 5, "Rank mark", 5, "for kernel rank", 5

degree 1: $\frac{1}{5}(v[1]+v[2]+v[3]+v[4]+v[5])$

degree 2: $\frac{1}{10}(v[1]v[2]+v[1]v[3]+v[1]v[4]+v[1]v[5]+v[2]v[3]+v[2]v[4]+v[2]v[5]+v[3]v[4]+v[3]v[5]+v[4]v[5])$

degree 3 : $\frac{1}{10}(v[1]v[2]v[3]+v[1]v[2]v[4]+v[1]v[2]v[5]+v[1]v[3]v[4]+v[1]v[3]v[5]+v[1]v[4]v[5]+v[2]v[3]v[4]+v[2]v[3]v[5]+v[2]v[4]v[5]+v[3]v[4]v[5])$

degree 4 : $\frac{1}{5}(v[1]v[2]v[3]v[4]+v[1]v[2]v[3]v[5]+v[1]v[2]v[4]v[5]+v[1]v[3]v[4]v[5]+v[2]v[3]v[4]v[5])$

degree 5 : $1(v[1])(v[3])(v[5])(v[4])(v[2])$

Group spectrum $1+t+t^2+t^3+t^4+t^5$

KERNEL STRUCTURE

"PT1" = {{4}, {1}, {5}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5}

$$\pi 5 = [1]$$

supp $\pi 5$ = {1}

$$u5 = [1]$$

supp u5 = {1}

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

$\boldsymbol{\beta} = (1)$

RPARTS [1]

BPARTS [1]

$\boldsymbol{\alpha} = (1)$

Action of R and B on the blocks of the partitions:

R-BLOCKS,

[4, 3, 1, 5, 2]

B-BLOCKS,

[2, 5, 4, 1, 3]

with invariant measure, [1, 1, 1, 1, 1]

N by blocks, N - check: true

b_1 = {4}
b_2 = {1}
b_3 = {5}
b_4 = {3}
b_5 = {2}

dim(span of partition vectors), rank(N_0), rank(N): 5, 5, 5

$$\text{Centralizer} = \begin{pmatrix} h[1] & h[2] & h[2] & h[2] & h[2] \\ h[2] & h[1] & h[2] & h[2] & h[2] \\ h[2] & h[2] & h[1] & h[2] & h[2] \\ h[2] & h[2] & h[2] & h[1] & h[2] \\ h[2] & h[2] & h[2] & h[2] & h[1] \end{pmatrix}$$

LIE STRUCTURE

Dimension of Lie algebra: 16, Shape: 15 ⊕ 1/0

$$\text{CLB} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

R and B Cycles. V.

$$\Omega_R = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix} \quad \Omega_B = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

Ω_R in Vec(K)? , {{1, 2, 3, 4, 5}}, true

Ω_B in Vec(K)? , {{1, 2, 3, 4, 5}}, true

$$V = \begin{pmatrix} \frac{2}{15} & \frac{7}{15} & \frac{1}{45} & \frac{-19}{45} & \frac{-1}{5} \\ \frac{-7}{15} & \frac{-2}{15} & \frac{19}{45} & \frac{-1}{45} & \frac{1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{2}{15} & \frac{7}{15} & \frac{-1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{-7}{15} & \frac{-2}{15} & \frac{1}{5} \\ \frac{1}{3} & \frac{-1}{3} & \frac{-1}{9} & \frac{1}{9} & 0 \end{pmatrix}$$

Omega-checks true, true

$\pi_R = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$ $\quad u\Omega_R$ vs $\Omega(I-V)^{-1}$

$\pi_B = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$ vs $\begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$ $\quad u\Omega_B$ vs $\Omega(I+V)^{-1}$

LOCAL GROUPS

1, "partition", {{4}, {1}, {5}, {3}, {2}}

1, "range", [1, 2, 3, 4, 5], [[5, 4, 3, 2, 1], [5, 4, 2, 1, 3], [5, 4, 1, 3, 2], [5, 3, 4, 1, 2], [5, 3, 2, 4, 1], [5, 3, 1, 2, 4], [5, 2, 4, 3, 1], [5, 2, 3, 1, 4], [5, 2, 1, 4, 3], [5, 1, 4, 2, 3], [5, 1, 3, 4, 2], [5, 1, 2, 3, 4], [4, 5, 3, 1, 2], [4, 5, 2, 3, 1], [4, 5, 1, 2, 3], [4, 3, 5, 2, 1], [4, 3, 2, 1, 5], [4, 3, 1, 5, 2], [4, 2, 5, 1, 3], [4, 2, 3, 5, 1], [4, 2, 1, 3, 5], [4, 1, 5, 3, 2], [4, 1, 3, 2, 5], [4, 1, 2, 5, 3], [3, 5, 4, 2, 1], [3, 5, 2, 1, 4], [3, 5, 1, 4, 2], [3, 4, 5, 1, 2], [3, 4, 2, 5, 1], [3, 4, 1, 2, 5], [3, 2, 5, 4, 1], [3, 2, 4, 1, 5], [3, 2, 1, 5, 4], [3, 1, 5, 2, 4], [3, 1, 4, 5, 2], [3, 1, 2, 4, 5], [2, 5, 4, 1, 3], [2, 5, 3, 4, 1], [2, 5, 1, 3, 4], [2, 4, 5, 3, 1], [2, 4, 3, 1, 5], [2, 4, 1, 5, 3], [2, 3, 5, 1, 4], [2, 3, 4, 5, 1], [2, 3, 1, 4, 5], [2, 1, 5, 4, 3], [2, 1, 4, 3, 5], [2, 1, 3, 5, 4], [1, 5, 4, 3, 2], [1, 5, 3, 2, 4], [1, 5, 2, 4, 3], [1, 4, 5, 2, 3], [1, 4, 3, 5, 2], [1, 4, 2, 3, 5], [1, 3, 5, 4, 2], [1, 3, 4, 2, 5], [1, 3, 2, 5, 4], [1, 2, 5, 3, 4], [1, 2, 4, 5, 3], [1, 2, 3, 4, 5]]

"group has", 60, "elements" Group element 1,1 = $\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

$g_1 =$ [[1, 5], [2, 4]]

$g_2 =$ [[1, 5, 3, 2, 4]]

$g_3 =$ [[1, 5, 2, 4, 3]]

$g_4 =$ [[1, 5, 2, 3, 4]]

$g_5 =$ [[1, 5], [2, 3]]

linear dimension, 17

"Symmetric?", true

Is Z in Vec(K)? true

$\left(12h[1]-3h[2] \quad -12h[1] \quad 6h[2] \quad -3h[2] \quad 3h[2] \quad -12h[1]-3h[2] \quad -12h[1]-3h[2] \quad 12h[1]+6h[2] \quad 12h[1]-3h[2] \quad 3h[2] \quad -1\right.$

"Basis for Z(G)"

1, "coeff", 12

$$Z[\,1\,] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2, "coeff", 3

$$Z[\,2\,] = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Check abelian

1, 2, true

$$EIGS = \begin{pmatrix} 1. & 1. & 1. & 1. & 1. \\ 4. & -1. & -1. & -1. & -1. \end{pmatrix}$$

Group spectrum: $1+t+t^2+t^3+t^4+t^5$

Molien Series to order 10: $1+t+2t^2+3t^3+5t^4+7t^5+10t^6+13t^7+18t^8+23t^9+31t^{10}$

n-choose-rank

{1, [1, 2, 3, 4, 5]}

KERNEL HIERARCHY

$\pi 5 = (1)$

{1}

$u5 = (1)$

{1}

picheck $(1 \quad 1 \quad 1 \quad 1 \quad 1)$

$\pi = \left(\frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{5}\right)$

$\pi 4 = (1 \quad 1 \quad 1 \quad 1 \quad 1)$

$u4 = \left(\frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{5}\right)$

picheck $(4 \quad 4 \quad 4 \quad 4 \quad 4)$

$\pi 3 = (2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2)$

$u3 = \left(\frac{2}{25}\ \frac{2}{25}\ \frac{2}{25}\ \frac{2}{25}\ \frac{2}{25}\ \frac{2}{25}\ \frac{2}{25}\ \frac{2}{25}\ \frac{2}{25}\ \frac{2}{25}\right)$

picheck $(12\ \ 12\ \ 12\ \ 12\ \ 12)$

$\pi 2 = (6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6\ \ 6)$

$u2 = \left(\frac{6}{125}\ \frac{6}{125}\ \frac{6}{125}\ \frac{6}{125}\ \frac{6}{125}\ \frac{6}{125}\ \frac{6}{125}\ \frac{6}{125}\ \frac{6}{125}\ \frac{6}{125}\right)$

picheck $(24\ \ 24\ \ 24\ \ 24\ \ 24)$

$\pi 1 = (24\ \ 24\ \ 24\ \ 24\ \ 24)$

$u1 = \left(\frac{24}{625}\ \frac{24}{625}\ \frac{24}{625}\ \frac{24}{625}\ \frac{24}{625}\right)$

picheck $(24\ \ 24\ \ 24\ \ 24\ \ 24)$

Column Projections

$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks N0-checks

Row Projections

$\mathrm{PP}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ idem-checks

$\mathrm{T} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ $\mathrm{NM} = \begin{pmatrix} 4 & 3 & 3 & 3 & 3 \\ 3 & 4 & 3 & 3 & 3 \\ 3 & 3 & 4 & 3 & 3 \\ 3 & 3 & 3 & 4 & 3 \\ 3 & 3 & 3 & 3 & 4 \end{pmatrix}$

CHECKING NULLSPACES

Group case, not scaling MC

$\pi\boldsymbol{\Delta} = [0, 0, 0, 0, 0]$

$\ker N_c = (1\ \ 1\ \ 1\ \ 1\ \ 1)$ $(t+s\ \ t+s\ \ t+s\ \ t+s\ \ t+s)$ RB checks

$\ker M_0 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} -t & s-t & -t & -t \\ 0 & 0 & s & t \\ -s+t & -s & -s & -s \\ s & 0 & t & 0 \\ 0 & t & 0 & s \end{pmatrix}$ RB checks

$$\ker M_c = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & s & t & 0 \\ 0 & s & 0 & 0 & t \\ t & 0 & 0 & s & 0 \\ s & t & 0 & 0 & 0 \\ 0 & 0 & t & 0 & s \end{pmatrix} \text{ RB checks}$$

$$n\pi x^\dagger = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathrm{R}N_0\mathrm{R}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{RR}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{B}N_0\mathrm{B}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{BB}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Do these reconstitute N? , true

Ranks: 5, 5, "vs", 5

$$\mathrm{CNM} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Skew Omega} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \quad N_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM a combination of T and Omega?", true

$$\mathrm{NM} = 1T + 15\Omega$$

$$\Omega \begin{pmatrix} \frac{2}{5} & \frac{1}{2} & \frac{-7}{5} & 1 & 1 & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

$$T \begin{pmatrix} 1 & \frac{1}{2} & -5 & 2 & 2 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

"IS NM in Vec(K)?", true

$$\mathrm{NM} \begin{pmatrix} 7 & 8 & -26 & 17 & 17 & 10 & 3 & 3 & 10 & 3 & 4 & 3 & 3 & 3 & 3 & 3 & 4 \end{pmatrix}$$

"IS MN in Vec(K)?", true

$$\mathrm{MN} \begin{pmatrix} 7 & 8 & -26 & 17 & 17 & 10 & 3 & 3 & 10 & 3 & 4 & 3 & 3 & 3 & 3 & 3 & 4 \end{pmatrix}$$

$$\boldsymbol{\tau} = 5/1 \text{ , rank} = 5 \text{ , ratio} = 1/1 \text{ , } n^2 / r = 5/1$$

$$\boldsymbol{\tau}' = 20/1 \text{ , } r' = 4/5 \text{ , } \boldsymbol{\tau} / n^2 = 1/5$$

$$p^2 = 1/5\ ,\ \min\ \boldsymbol{\tau} = 5/1\ ,\ \boldsymbol{\tau}\text{-check is positive? } 0/1$$

$$\max\ r = 5/1\ ,\ r\text{-check is positive? } 0/1$$

IS N0M0 a combination of T and Omega? , true

$$N_0 M_0 = 0T + 5\Omega$$

There are, 1, partitions and, 1, ranges, with a group size of, 60

KERNEL HAS LINEAR DIMENSION 17
out of total no. of elements equal to 60

dim span idems 1 vs no. of idems 1

"PT1" = {{4}, {1}, {5}, {3}, {2}}

"RG1" = {1, 2, 3, 4, 5}

$$M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C = \begin{pmatrix} \frac{4}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{4}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{4}{5} & \frac{-1}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{4}{5} & \frac{-1}{5} \\ \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{-1}{5} & \frac{4}{5} \end{pmatrix}$$

$$M_C\text{-scaled} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad N_C\text{-scaled} = \begin{pmatrix} 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & 1 & \frac{-1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & 1 \end{pmatrix}$$

$$N_C M_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M_C N_C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{commutator} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigenvalues M_C

[0., 0., 0., 0., 0.]

Eigenvalues N_C

[0., 1., 1., 1., 1.]

Eigenvalues M_C–scaled

[0., 0., 0., 0., 0.]

Eigenvalues N_C–scaled

[0., 1.250000000, 1.250000000, 1.250000000, 1.250000000]

NullSpace M_C

{[0, 0, 0, 1, 0], [1, 0, 0, 0, 0], [0, 1, 0, 0, 0], [0, 0, 0, 0, 1], [0, 0, 1, 0, 0]}

NullSpace N_c

{[1, 1, 1, 1, 1]}

Eigenvalues M_0

[0., 0., 0., 0., 5.]

Eigenvalues N_0

[1., 1., 1., 1., 1.]

NullSpace M_0

{[0, 0, 1, −1, 0], [1, 0, 0, −1, 0], [0, 1, 0, −1, 0], [0, 0, 0, −1, 1]}

NullSpace N_0

{}

Eigenvalues M

[4., −1., −1., −1., −1.]

Eigenvalues N

[4., −1., −1., −1., −1.]

NullSpace M

{}

NullSpace N

{}

Harmonic Basis

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$