RCC-Dets.mw

```
> ##################################################

> R:=R1:B:=B1: ##############################
  FROM RC-RUN FOR PCC COLORING

> ############ START HERE ########################

> Delta:=evalm((R-B)/2):A:=evalm((R+B)/2):
  `R`=matrans(R);`B`=matrans(B);n;

> unassign('e','w','t','k'):wpi:=clearDenoms(evalm(1/w[1]*linsolve(J-transpose(A

> pi:=evalm(wpi/W):omega:=diag(seq(pi[k],k=1..n)):"PI-CHECK",iszero(symm

> Omega:=stackmatrix(seq(pi,k=1..n)):`pi`=evalm(pi);

> nsd:=nullspace(Delta):
      print(`NullSpace  of Delta `,nullity(Delta));
      print(seq(readVec(nsd[k]),k=1..nullity(Delta)));
          nsk:=nullspace(Delta):
      print(`Nullspace of A`,nullity(A));
      NSA:=nullspace(A):
      if(det(A)=0) then
          for ix to nops(NSA) do
      nx:=NSA[ix];
      P:={};Q:={};
          for aa to n do
              if(nx[aa]=1) then P:={op(P),aa} fi;
              if(nx[aa]=-1) then Q:={op(Q),aa} fi;
          od;
          print([P,Q]);
      od;
      else
          print(` det(A) = `);
          fracfm(det(A))
```

```
      fi;
```

```
> if(nullity(A)=nullity(Delta)) then "CC" else "NOT CC" fi;
```

$$R = [3, 3, 1, 1, 7, 7, 5, 5]$$

$$B = [6, 8, 8, 6, 2, 4, 4, 2]$$

$$8$$

$$wpi := [1, 1, 1, 1, 1, 1, 1, 1]$$

$$W := 8$$

"PI-CHECK", *true*

$$\pi = \left[\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8} \right]$$

NullSpace of Delta, 2

{2, 4, 5, 7}, {1, 3, 6, 8}

Nullspace of A, 2

[{6, 8}, {1, 3}]

[{5, 7}, {2, 4}]

"CC"

```
> unassign('f'):f:=vector(n);ff:=diag(seq(f[k],k=1..n)):PA:=multiply(omega,A)
```

$$f := array(1 .. 8, [])$$

true

CYCLE DECOMPOSITION HERE

```
> ccls:=goCycles(PA):
```

```
> AP:=evalm(add(ccls[k][1]/nops(ccls[k][2])*cyc(ccls[k][2]),k=1..nops(ccls))
  A",iszero(AP-omega&*A);

> cvx:=seq(ccls[k][1],k=1..nops(ccls)):

> vcx:=clearDenoms(evalm(Vector([cvx])));if(`+`(cvx)=1) then
  print("convexchex") else print("ERROR!")
  fi;WCYC:=add(vcx[k],k=1..nops(ccls));

> print(`
  `);print("==========================================
  `);

> "R=",matrans(R),"B=",matrans(B);print("Values,Cycles",ccls);print("Friedman
  weights",wpi,cat(` W = `,add(wpi[k],k=1..n),` with WCYC = `,WCYC));
```

"finding cycles"

"found cycle of length", 2

"found cycle of length", 3

"found cycle of length", 3

"found cycle of length", 3

"found cycle of length", 3

"found cycle of length", 2

"Reconstituting A", *true*

$vcx := [3, 3, 2, 2, 3, 3]$

"convexchex"

$WCYC := 16$

"==========

"R=", [3, 3, 1, 1, 7, 7, 5, 5], "B=", [6, 8, 8, 6, 2, 4, 4, 2]

$$\text{"Values,Cycles"}, \left\{ \left[\frac{3}{16}, [1, 6, 4] \right], \left[\frac{3}{16}, [2, 3, 8] \right], \left[\frac{1}{8}, [5, 7] \right], \left[\frac{1}{8}, [1, 3] \right], \left[\frac{3}{16}, [4, 6, 7] \right], \left[\frac{3}{16}, [2, 8, 5] \right] \right\}$$

"Friedman weights", [1, 1, 1, 1, 1, 1, 1, 1], $W = 8$ *with* $WCYC = 16$

SAVING R, B, and CYCLE DATA

```
> outfile:=cat("/home/ph/maple/CYCLES",matrans(R)[],".m");save(R,B,ccls,out
```

outfile := "/home/ph/maple/CYCLES33117755.m"

READING CYCLE DATA. FIRST RENAME FILE TO CYCLESn.m WHERE n=# of VERTICES

```
> #outfile:=cat("/home/ph/maple/CYCLES",n,".m");read (outfile);ccls;

> ##################################################

> seq(ccls[k][1],k=1..nops(ccls)):if(1<>`+`(%)) then "Error. Recheck
  calculations."
  fi;Digits:=2:print(seq([k,evalf(ccls[k][1])],k=1..nops(ccls)));Digits:=10:
```

[1, 0.12], [2, 0.19], [3, 0.19], [4, 0.19], [5, 0.19], [6, 0.12]

```
> for i to nops(ccls) do
  prob[i]:=add(pi[ccls[i][2][j]],j=1..nops(ccls[i][2]));od:mx:=max(seq(prob[i
  j to nops(ccls) do if(prob[j]=mx) then MX:=[op(MX),j];print("max prob cycle'

> mx:=min(seq(prob[i],i=1..nops(ccls))):"min",mx;for j to nops(ccls) do
  if(prob[j]=mx) then print("min prob cycle",j,ccls[j]) fi
  ;od;Digits:=2:print(seq([k,evalf(prob[k])],k=1..nops(ccls)));Digits:=10:
```

$$\text{"max"}, \frac{3}{8}$$

$$\text{"max prob cycle"}, 2, \left[\frac{3}{16}, [2, 8, 5]\right]$$

$$\text{"max prob cycle"}, 3, \left[\frac{3}{16}, [2, 3, 8]\right]$$

$$\text{"max prob cycle"}, 4, \left[\frac{3}{16}, [1, 6, 4]\right]$$

$$\text{"max prob cycle"}, 5, \left[\frac{3}{16}, [4, 6, 7]\right]$$

$$\text{"min"}, \frac{1}{4}$$

$$\text{"min prob cycle"}, 1, \left[\frac{1}{8}, [1, 3]\right]$$

$$\text{"min prob cycle"}, 6, \left[\frac{1}{8}, [5, 7]\right]$$

$$[1, 0.25], [2, 0.38], [3, 0.38], [4, 0.38], [5, 0.38], [6, 0.25]$$

Now color the maximum likelihood cycle Red.

```
> j:=MX[1]:print(ccls[j]):CYMAX:=evalm(cyc(ccls[j][2])):rr:=readrows(CYMA
  k to n do if(rr[k]<>[]) then
  Delta:=rowreplace(Delta,k,evalm(2*Delta[k,op(rr[k])]*row(Delta,k))) fi; od:
```

$$\left[\frac{3}{16}, [2, 8, 5]\right]$$

```
> j:=1:print(ccls[j]):CY1:=evalm(cyc(ccls[j][2])):rr:=readrows(CY1):for k to
  n do if(rr[k]<>[]) then
  Delta:=rowreplace(Delta,k,evalm(2*Delta[k,op(rr[k])]*row(Delta,k))) fi;
  od:

> #iszero(multiply(CYMAX,CY1));
```

$$\left[\frac{1}{8}, [1, 3]\right]$$

```
> #evalm(PA):mx:=1:
```

```
        #for i to nd do
            #for j to nd do
                #if(PA[i,j]>0 and PA[i,j]<mx) then
                #       mx:=PA[i,j];
                #       ix:=i;
                ##        jx:=j;
                #fi
            #od:       od:

> #mx;

> #ccx:=[11,1,9,13]:for k to nops(ccx) do k1:=k+1: if(k=nops(ccx))then
  k1:=1 fi; vl:=Delta[ccx[k],ccx[k1]]:if(vl<>1) then
  Delta:=rowreplace(Delta,ccx[k],evalm(2*vl*row(Delta,ccx[k]))) fi; od:

> R:=evalm(1/2*(A+Delta)):B:=evalm(1/2*(A-Delta)):

    RECOLORING STARTS HERE

> #colrs:={{1,2,25,30,4,32,6, 11, 13, 17, 24, 28, 34,36}};

>

> colrs:={{3,2,6,7}};

> unassign('COLR','e','f'):print("wpi",wpi);print("pi",pi);

> ee:=vector(n,1):KOL:=1:COLR:=colrs[KOL];for i to nops(COLR) do
  ee[convert(COLR[i],decimal,hex)]:=-1
  od:print(map(convert,COLR,decimal,hex));

> phi:=diag(seq(ee[q],q=1..n)):Detta:=evalm(phi&*Delta):R:=evalm(A+Detta

> unassign('t'):print();

> for t in [1,-1] do print();print();if(t=1) then print("R") else print("B")
  fi;X:=evalm(A+t*Detta):print(matrans(X));print(map(convert,matrans(X),dec
```

```
> if(tailchk(X)) then print("tailcheck")
  fi;rk:=rank(X):rca:=op(readcycles(abel(X)));rcc:=`union`(rca);print();print(

> KK:=pipow(pi,X);

> print();print(rank(KK),"vs",rk);print();print();

> #if(rank(KK)=rk) then

> YZ:=evalm(X^rk):rx:=rank(YZ);KXK:=submatrix(KK,rk-rx+1..rk,[op(rcc)]):

> print("with cycle rank",rx) ;print("DET",det(KXK));

> vv:=Vector(row(KXK,1));

> FT:=FourierTransform(vv);print("Fourier Transform"):for i to rowdim(KXK)
  do print(i,FT[i]) od;

> #fi;

> for j to rank(X) do
  Y:=evalm(X^j);KK:=pipow(pi,Y):;print(readrows(Y),rank(KK),"vs",rank(Y));

> if(iszero(Y-Y^2)) then print(j,"th power is idempotent, checking next
  one");break fi;od;

> Y:=evalm(X&*Y):KK:=pipow(pi,Y):print(KK);print(readrows(Y),rank(KK),"vs'

> od:

> dot(YY[1],YY[-1]):
```

$$colrs := \{\{2, 3, 6, 7\}\}$$

"wpi", [1, 1, 1, 1, 1, 1, 1, 1]

$$\text{"pi"}, \left[\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}\right]$$

$$COLR := \{2, 3, 6, 7\}$$

$$\{2, 3, 6, 7\}$$

$$\text{"R"}$$

$$[3, 8, 8, 1, 7, 4, 4, 5]$$

$$[3, 8, 8, 1, 7, 4, 4, 5]$$

$$\text{"Cycles"}, \{1, 3, 4, 5, 7, 8\}$$

$$3, \text{"vs"}, 6$$

$$\begin{bmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \end{bmatrix}$$

$$\text{"with cycle rank"}, 6$$

$$\text{"DET"}, 0$$

$$\text{"Fourier Transform"}$$

$$1, 0.4082482904638630\,18 + 0.\,I$$

$$2, 0. + 0.\,I$$

3, -0.0510310363079828772 + 0.0883883476483184327 *I*

4, 0. + 0. *I*

5, -0.0510310363079828772 - 0.0883883476483184327 *I*

6, 0. + 0. *I*

[[3], [8], [8], [1], [7], [4], [4], [5]], 3, "vs", 6

[[8], [5], [5], [3], [4], [1], [1], [7]], 3, "vs", 6

[[5], [7], [7], [8], [1], [3], [3], [4]], 1, "vs", 6

[[7], [4], [4], [5], [3], [8], [8], [1]], 3, "vs", 6

[[4], [1], [1], [7], [8], [5], [5], [3]], 3, "vs", 6

[[1], [3], [3], [4], [5], [7], [7], [8]], 1, "vs", 6

6, "th power is idempotent, checking next one"

$$\begin{bmatrix} \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & 0 & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & 0 & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & 0 & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & 0 & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{4} & \frac{1}{8} \end{bmatrix}$$

[[3], [8], [8], [1], [7], [4], [4], [5]], 3, "vs", 6

"B"

[*6, 3, 1, 6, 2, 7, 5, 2*]

[6, 3, 1, 6, 2, 7, 5, 2]

"Cycles", {1, 2, 3, 5, 6, 7}

3, "vs", 6

$$\begin{bmatrix} \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

"with cycle rank", 6

"DET", 0

"Fourier Transform"

1, 0.4082482904638630 18 + 0. *I*

2, 0. + 0. *I*

3, -0.05103103630798287 72 - 0.08838834764831843 27 *I*

4, 0. + 0. *I*

5, -0.05103103630798287 72 + 0.08838834764831843 27 *I*

6, 0. + 0. *I*

[[6], [3], [1], [6], [2], [7], [5], [2]], 3, "vs", 6

[[7], [1], [6], [7], [3], [5], [2], [3]], 3, "vs", 6

[[5], [6], [7], [5], [1], [2], [3], [1]], 1, "vs", 6

[[2], [7], [5], [2], [6], [3], [1], [6]], 3, "vs", 6

[[3], [5], [2], [3], [7], [1], [6], [7]], 3, "vs", 6

[[1], [2], [3], [1], [5], [6], [7], [5]], 1, "vs", 6

6, "th power is idempotent, checking next one"

$$\begin{bmatrix} \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & 0 \end{bmatrix}$$

[[6], [3], [1], [6], [2], [7], [5], [2]], 3, "vs", 6

```
> q:=[1,2,4,3];Q:=matrix(rx,rx,(i,j)->if(j=q[i]) then 1 else 0 fi);

> rx;jj:=IdentityMatrix(rx):

> KPK:=multiply(KXK,Q);

> vv:=Vector(row(KPK,1));

> FT:=FourierTransform(vv):print("Fourier Transform"):for i to rowdim(KXK)
  do print(i,FT[i]) od;
```

$$q := [1, 2, 4, 3]$$

$$Q := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$4$$

$$KPK := \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{8} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{8} & \frac{3}{8} & \frac{1}{4} \end{bmatrix}$$

$$vv := \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{3}{8} \right]$$

"Fourier Transform"

1, 0.500000000000000000 + 0. *I*

2, 0.062500000000000000 + 0.062500000000000000 *I*

3, -0.125000000000000000 + 0. *I*

4, 0.062500000000000000 - 0.062500000000000000 *I*

{}

> dot(R,B);