

New Graph

[5, 5, 1, 5, 2], [3, 4, 5, 2, 1]

$$\pi = [2, 2, 1, 1, 3]$$

POSSIBLE RANKS

1 x 9
3 x 3

BASE DETERMINANT 351/4096, .8569335938e-1

NullSpace of Δ

{1, 2, 3, 4, 5}

Nullspace of A

$$\det(A) = -1/16$$

1 . Coloring, {}

R: [5, 5, 1, 5, 2]

B: [3, 4, 5, 2, 1]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 5

Omega Rank for R : cycles: {{2, 5}} order: 2

[See Matrix](#)

$$[y_1, y_2, 0, 0, y_3]$$

Omega Rank for B : cycles: {{1, 3, 5}, {2, 4}}

[See Matrix](#)

$$[y_4, y_3, y_2, y_1, -y_4 + 2y_3 - y_2 + 2y_1]$$

$$p' = -1 - s + s^3 + s^4$$

2 . Coloring, {2}

$$\Omega p(\Delta)=0: \quad p = s + 4s^3 \quad p' = s + 4s^3$$

R: [5, 4, 1, 5, 2]

B: [3, 5, 5, 2, 1]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 4	3 vs 5	3 vs 5	2 vs 4	2 vs 4

Omega Rank for R : cycles: {{2, 4, 5}} order: 3

[See Matrix](#)

$$[y_2 - y_1, y_2, 0, y_1, y_2]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3

[See Matrix](#)

$$[y_2, y_1, y_2 - y_1, 0, y_2]$$

$$p' = s^2 - s^3 \quad p = s^2 - s^4$$

[See 3-level graph](#)

	M	N
	0 1 1 0 2	0 1 1 0 1
	1 0 0 1 2	1 0 0 1 1
	[1 0 0 0 1]	[1 0 0 1 1]
	0 1 0 0 1	0 1 1 0 1
	2 2 1 1 0	1 1 1 1 0

$$\tau = 9, r' = 2/3$$

R: [5, 4, 1, 5, 2]

B: [3, 5, 5, 2, 1]

Ranges

Action of R on ranges, [[3], [1], [3]]

Action of B on ranges, [[2], [2], [1]]

Cycles: R, {{2, 4, 5}}, B, {{1, 3, 5}}

$$\beta(\{1, 2, 5\}) = 1/3$$

$$\beta(\{1, 3, 5\}) = 1/3$$

$$\beta(\{2, 4, 5\}) = 1/3$$

Partitions

$$\alpha(\{\{5\}, \{1, 4\}, \{2, 3\}\}) = 1/1$$

$$b1 = \{5\}, b2 = \{1, 4\}, b3 = \{2, 3\}$$

Action of R and B on the blocks of the partitions: = **[2, 3, 1]** **[3, 1, 2]**
with invariant measure [1, 1, 1]

N by blocks, check: true . [See partition graph.](#)

[See level-3 partition graph.](#)

Right Group	
Coloring	{2}
Rank	3
R,B	[5, 4, 1, 5, 2], [3, 5, 5, 2, 1]
Π_2	[1, 1, 0, 2, 0, 1, 2, 0, 1, 1]
u_2	[1, 1, 0, 1, 0, 1, 1, 1, 1, 1] (dim 1)
wpp	[2, 2, 2, 2, 1]
Π_3	[0, 0, 1, 0, 1, 0, 0, 0, 1, 0]
u_3	[0, 0, 1, 0, 1, 0, 0, 0, 1, 1]

3 . Coloring, {3}

$$\Omega_p(\Delta)=0: \quad p = s + 2s^2 - 4s^3 - 8s^4$$

R: [5, 5, 5, 5, 2]

B: [3, 4, 1, 2, 1]

[See graph](#)

[See pair graph](#)

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
3 vs 4	4 vs 5	4 vs 5	2 vs 2	2 vs 4

Omega Rank for R : cycles: {{2, 5}} order: 2

[See Matrix](#)

$$[0, y_1, 0, 0, y_2]$$

Omega Rank for B : cycles: {{2, 4}, {1, 3}} order: 2

[See Matrix](#)

$$[2 y_1, y_2, 2 y_2, y_1, 0]$$

$$p' = s - s^3 \quad p = s - s^3$$

4 . Coloring, {4}

R: [5, 5, 1, 2, 2]

B: [3, 4, 5, 5, 1]

[See graph](#)

[See pair graph](#)

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R : cycles: {{2, 5}} order: 2

[See Matrix](#)

$$[y_2, y_3, 0, 0, y_1]$$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3

[See Matrix](#)

$$[y_4, 0, y_3, y_2, y_1]$$

5 . Coloring, {5}

R: [5, 5, 1, 5, 1]

B: [3, 4, 5, 2, 2]

[See graph](#)

[See pair graph](#)

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B

4 vs 4	5 vs 5	5 vs 5	2 vs 2	4 vs 4
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Omega Rank for R : cycles: $\{\{1, 5\}\}$ order: 2

[See Matrix](#)

$$[y_1, 0, 0, 0, y_2]$$

Omega Rank for B : cycles: $\{\{2, 4\}\}$ order: 4

[See Matrix](#)

$$[0, y_1, y_2, y_3, y_4]$$

6 . Coloring, $\{2, 3\}$

R: [5, 4, 5, 5, 2]

B: [3, 5, 1, 2, 1]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R : cycles: $\{\{2, 4, 5\}\}$ order: 3

[See Matrix](#)

$$[0, y_3, 0, y_1, y_2]$$

Omega Rank for B : cycles: $\{\{1, 3\}\}$ order: 4

[See Matrix](#)

$$[y_1, y_2, y_3, 0, y_4]$$

7 . Coloring, $\{2, 4\}$

R: [5, 4, 1, 2, 2]

B: [3, 5, 5, 5, 1]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	3 vs 3

Omega Rank for R : cycles: $\{\{2, 4\}\}$ order: 4

[See Matrix](#)

$$[y_1, y_2, 0, y_3, y_4]$$

Omega Rank for B : cycles: $\{\{1, 3, 5\}\}$ order: 3

[See Matrix](#)

$$[y_1, 0, y_2, 0, y_3]$$

8 . Coloring, $\{2, 5\}$

R: [5, 4, 1, 5, 1]

B: [3, 5, 5, 2, 2]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	3 vs 3

Omega Rank for R : cycles: $\{\{1, 5\}\}$ order: 2

[See Matrix](#)

$$[y_3, 0, 0, y_2, y_1]$$

Omega Rank for B : cycles: $\{\{2, 5\}\}$ order: 2

[See Matrix](#)

$$[0, y_3, y_1, 0, y_2]$$

9 . Coloring, $\{3, 4\}$

R: [5, 5, 5, 2, 2]

B: [3, 4, 1, 5, 1]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 2	4 vs 4

Omega Rank for R : cycles: $\{\{2, 5\}\}$ order: 2

[See Matrix](#)

$$[0, y_1, 0, 0, y_2]$$

Omega Rank for B : cycles: $\{\{1, 3\}\}$ order: 4

[See Matrix](#)

$$[y_4, 0, y_1, y_2, y_3]$$

10 . Coloring, $\{3, 5\}$

$$\Omega_p(\Delta)=0: \quad p = s + 2s^2 - 4s^3 - 8s^4$$

R: [5, 5, 5, 5, 1]

B: [3, 4, 1, 2, 2]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	4 vs 5	4 vs 5	2 vs 2	2 vs 4

Omega Rank for R : cycles: $\{\{1, 5\}\}$ order: 2

[See Matrix](#)

$$[y_1, 0, 0, 0, y_2]$$

Omega Rank for B : cycles: $\{\{1, 3\}, \{2, 4\}\}$ order: 2

[See Matrix](#)

$$[y_2, 2y_1, y_1, 2y_2, 0]$$

$$p = -s + s^3 \quad p' = s - s^3$$

11 . Coloring, $\{4, 5\}$

R: [5, 5, 1, 2, 1]

B: [3, 4, 5, 5, 2]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R : cycles: $\{\{1, 5\}\}$ order: 2

[See Matrix](#)

$$[y_2, y_3, 0, 0, y_1]$$

Omega Rank for B : cycles: $\{\{2, 4, 5\}\}$ order: 3

[See Matrix](#)

$$[0, y_3, y_4, y_2, y_1]$$

12 . Coloring, {2, 3, 4}

R: [5, 4, 5, 2, 2]

B: [3, 5, 1, 5, 1]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	3 vs 3

Omega Rank for R : cycles: $\{\{2, 4\}\}$ order: 2

[See Matrix](#)

$$[0, y_3, 0, y_2, y_1]$$

Omega Rank for B : cycles: $\{\{1, 3\}\}$ order: 2

[See Matrix](#)

$$[y_1, 0, y_2, 0, y_3]$$

13 . Coloring, {2, 3, 5}

R: [5, 4, 5, 5, 1]

B: [3, 5, 1, 2, 2]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	2 vs 4

Omega Rank for R : cycles: {{1, 5}} order: 2

[See Matrix](#)

$$[y_3, 0, 0, y_1, y_2]$$

Omega Rank for B : cycles: {{1, 3}, {2, 5}} order: 2

[See Matrix](#)

$$[y_1, 2 y_2, y_2, 0, 2 y_1]$$

$$p' = -s + s^3 \quad p = -s + s^3$$

14 . Coloring, {2, 4, 5}

R: [5, 4, 1, 2, 1]

B: [3, 5, 5, 5, 2]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 3

Omega Rank for R : cycles: {{1, 5}, {2, 4}} order: 2

[See Matrix](#)

$$[2 y_2, y_1, 0, y_2, 2 y_1]$$

$$p = s - s^3 \quad p' = s - s^3$$

Omega Rank for B : cycles: {{2, 5}} order: 2

[See Matrix](#)

$$[0, y_1, y_2, 0, y_3]$$

15 . Coloring, {3, 4, 5}

R: [5, 5, 5, 2, 1]

B: [3, 4, 1, 5, 2]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 5

Omega Rank for R : cycles: {{1, 5}} order: 2

[See Matrix](#)

$$[y_1, y_2, 0, 0, y_3]$$

Omega Rank for B : cycles: {{1, 3}, {2, 4, 5}}

[See Matrix](#)

$$[y_1, 2y_1 + 2y_2 - y_4 - y_3, y_2, y_4, y_3]$$

$$p' = -1 - s + s^3 + s^4$$

16 . Coloring, {2, 3, 4, 5}

$$\Omega p(\Delta)=0: \quad p = s - 4s^3 \quad p' = s - 4s^3$$

R: [5, 4, 5, 2, 1]

B: [3, 5, 1, 5, 2]

[See graph](#)

[See pair graph](#)

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 4	3 vs 5	3 vs 5	2 vs 4	2 vs 4

Omega Rank for R : cycles: {{1, 5}, {2, 4}} order: 2

[See Matrix](#)

$$[y_2, y_1, 0, y_2 - y_1, y_2]$$

$$p = -s + s^3 \quad p' = -s + s^3$$

Omega Rank for B : cycles: {{1, 3}, {2, 5}} order: 2

[See Matrix](#)

$$[y_2 - y_1, y_2, y_1, 0, y_2]$$

$$p = s - s^3 \quad p' = -s + s^3$$

[See 3-level graph](#)

	M	N
	0 1 0 1 2	0 1 0 1 1
	1 0 1 0 2	1 0 1 0 1
	[0 1 0 0 1]	[0 1 0 1 1]
	1 0 0 0 1	1 0 1 0 1
	2 2 1 1 0	1 1 1 1 0

$$\tau = 9, r' = 2/3$$

R: [5, 4, 5, 2, 1]

B: [3, 5, 1, 5, 2]

Ranges

Action of R on ranges, [[2], [1], [2]]

Action of B on ranges, [[3], [3], [1]]

Cycles: R, {{1, 5}, {2, 4}}, B, {{1, 3}, {2, 5}}

$$\beta(\{1, 2, 5\}) = 1/3$$

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$$\beta(\{2, 3, 5\}) = 1/3$$

Partitions

$$\alpha(\{1, 3, 5\}, \{2, 4\}) = 1/1$$

$$b1 = \{1, 3\}, b2 = \{5\}, b3 = \{2, 4\}$$

Action of R and B on the blocks of the partitions: = [2, 1, 3] [1, 3, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . [See partition graph.](#)

[See level-3 partition graph.](#)

Right Group	
Coloring	{2, 3, 4, 5}

Rank	3
R,B	[5, 4, 5, 2, 1], [3, 5, 1, 5, 2]
Π_2	[1, 0, 1, 2, 1, 0, 2, 0, 1, 1]
u_2	[1, 0, 1, 1, 1, 0, 1, 1, 1, 1] (dim 1)
wpp	[2, 2, 2, 2, 1]
Π_3	[0, 0, 1, 0, 0, 1, 0, 1, 0, 0]
u_3	[0, 0, 1, 0, 0, 1, 0, 1, 0, 1]

SUMMARY	
Graph Type	NOT CC
$\nu(A)$	0
$\nu(\Delta)$	1
Π	[2, 2, 1, 1, 3]
Dbly Stoch	false

SANDWICH		Total 0
No .	Coloring	Rank

RT GROUPS		Total 2	
No .	Coloring	Rank	Solv
1	{2}	3	Not Solvable
2	{2, 3, 4, 5}	3	Not Solvable

Δ -RANK'D	SC'D !RK'D	τ -RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
12	0	12, 12	13, 9	2	16	16