New Graph

```
[3, 3, 1, 1, 7, 7, 5, 5], [6, 8, 8, 6, 2, 4, 4, 2]
```

π = [1, 1, 1, 1, 1, 1, 1, 1]

POSSIBLE RANKS

1 x 8
2 x 4

BASE DETERMINANT 4236243/134217728, .3156246990e-1

NullSpace of Δ

{1, 3, 6, 8}, {2, 4, 5, 7}

Nullspace of A

[{2, 4},{5, 7}] , [{1, 3},{6, 8}]

1 . Coloring, {}

$\Omega p(\Delta)=0$: $p' = s^3$ $p' = s^5$ $p' = s^4$ $p = s^2$ $p' = s^2$

R: [3, 3, 1, 1, 7, 7, 5, 5]
B: [6, 8, 8, 6, 2, 4, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 *vs* 6	1 *vs* 6	1 *vs* 6	1 *vs* 4	1 *vs* 4

Omega Rank for R : cycles: {{5, 7}, {1, 3}} order: 2

See Matrix

$[y_1, 0, y_1, 0, y_1, 0, y_1, 0]$

$p = -s + s^4$ $p = -s + s^2$ $p = -s + s^3$

Omega Rank for B : cycles: {{4, 6}, {2, 8}} order: 2

See Matrix

$[0, y_1, 0, y_1, 0, y_1, 0, y_1]$

$p = -s + s^4$ $p = -s + s^2$ $p = -s + s^3$

See 4-level graph

$$M \quad \backslash; \quad N$$

$$\begin{bmatrix} 0&0&2&0&2&0&2&0\\ 0&0&0&2&0&2&0&2\\ 2&0&0&0&2&0&2&0\\ 0&2&0&0&0&2&0&2\\ 2&0&2&0&0&0&2&0\\ 0&2&0&2&0&0&0&2\\ 2&0&2&0&2&0&0&0\\ 0&2&0&2&0&2&0&0 \end{bmatrix} \begin{bmatrix} 0&1&2&1&2&2&2&2\\ 1&0&1&2&2&2&2&2\\ 2&1&0&1&2&2&2&2\\ 1&2&1&0&2&2&2&2\\ 2&2&2&2&0&1&2&1\\ 2&2&2&2&1&0&1&2\\ 2&2&2&2&2&1&0&1\\ 2&2&2&2&1&2&1&0 \end{bmatrix}$$

τ= 16 , r'= 3/4

R: [3, 3, 1, 1, 7, 7, 5, 5]
B: [6, 8, 8, 6, 2, 4, 4, 2]

Ranges

Action of R on ranges, [[1], [1]]
Action of B on ranges, [[2], [2]]

Cycles: R , {{5, 7}, {1, 3}}, B , {{4, 6}, {2, 8}}

β({1, 3, 5, 7}) = 1/2
β({2, 4, 6, 8}) = 1/2

Partitions

Action of R on partitions, [[1], [1]]
Action of B on partitions, [[2], [2]]

α([{1, 2}, {7, 8}, {5, 6}, {3, 4}]) = 1/2
α([{6, 7}, {5, 8}, {2, 3}, {1, 4}]) = 1/2

b1 = {1, 2} , b2 = {6, 7} , b3 = {5, 8} , b4 = {7, 8} , b5 = {5, 6} , b6 = {2, 3} , b7 = {3, 4} , b8 = {1, 4}

Action of R and B on the blocks of the partitions: = [7, 5, 4, 5, 4, 1, 1, 7] [3, 8, 6, 6, 8, 3, 2, 2]
with invariant measure [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . See partition graph.

See level-4 partition graph.

Sandwich	
Coloring	{}

Rank	4
R,B	[3, 3, 1, 1, 7, 7, 5, 5], [6, 8, 8, 6, 2, 4, 4, 2]
π_2	[0, 2, 0, 2, 0, 2, 0, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 0, 2, 0, 2, 0, 2, 0, 0, 2, 0]
u_2	[1, 2, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 2, 2, 2, 1, 2, 1, 1, 2, 1] (dim 3)
wpp	[2, 2, 2, 2, 2, 2, 2, 2]
π_4	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
u_4	[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 2, 1, 1, 2, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 2, 1, 1, 2, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0]

2 . Coloring, {2}

R: [3, 8, 1, 1, 7, 7, 5, 5]
B: [6, 3, 8, 6, 2, 4, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{5, 7}, {1, 3}} order: 2

See Matrix

$$[-4\,y_3 + 3\,y_2, 0, y_3, 0, y_2, 0, -5\,y_3 + 4\,y_2 - y_1, y_1]$$

$p = -s^2 + s^4$ $p' = -s^2 + s^4$

Omega Rank for B : cycles: {{4, 6}, {2, 3, 8}} order: 6

See Matrix

$$[0, y_1, -y_1 + 2\,y_3 - y_2, y_3, 0, y_3, 0, y_2]$$

$p' = s - s^4$ $p = -s + s^4$

3 . Coloring, {3}

R: [3, 3, 8, 1, 7, 7, 5, 5]
B: [6, 8, 1, 6, 2, 4, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$[y_1, 0, y_1 - y_2 + y_4 + y_3, 0, y_2, 0, y_4, y_3]$$

$p = - s^4 + s^5$

Omega Rank for B : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$[y_2, 3 y_1 - 4 y_3, 0, -y_2 + 4 y_1 - 5 y_3, 0, y_1, 0, y_3]$$

$p' = - s^2 + s^4$ $p = - s^2 + s^4$

4 . Coloring, {4}

R: [3, 3, 1, 6, 7, 7, 5, 5]
B: [6, 8, 8, 1, 2, 4, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{5, 7}, {1, 3}} order: 2

See Matrix

$$[y_1, 0, -4 y_1 + 3 y_2, 0, -5 y_1 - y_3 + 4 y_2, y_3, y_2, 0]$$

$p = - s^2 + s^4$ $p' = s^2 - s^4$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

$$[y_2, y_3, 0, -y_2 + 2 y_3 - y_1, 0, y_1, 0, y_3]$$

$p = - s + s^4$ $p' = s - s^4$

5 . Coloring, {5}

R: [3, 3, 1, 1, 2, 7, 5, 5]
B: [6, 8, 8, 6, 7, 4, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$[-y_1 + y_2 + y_3 - y_4, y_1, y_2, 0, y_3, 0, y_4, 0]$$

$p = s^4 - s^5$

Omega Rank for B : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$[0, y_1, 0, y_2, 0, y_3, -5 y_1 + 4 y_2 - y_3, -4 y_1 + 3 y_2]$$

$p' = - s^2 + s^4$ $p = - s^2 + s^4$

6 . Coloring, {6}

R: [3, 3, 1, 1, 7, 4, 5, 5]
B: [6, 8, 8, 6, 2, 7, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{1, 3}, {5, 7}} order: 2

See Matrix

$$[y_2, 0, 4 y_2 - y_1 - 5 y_3, y_1, 3 y_2 - 4 y_3, 0, y_3, 0]$$

$p = s^2 - s^4 \quad p' = s^2 - s^4$

Omega Rank for B : cycles: {{2, 8}, {4, 6, 7}} order: 6

See Matrix

$$[0, y_3, 0, y_2, 0, y_1, 2\, y_3 - y_2 - y_1, y_3]$$

$p = - s + s^4 \quad p' = s - s^4$

7 . Coloring, {7}

R: [3, 3, 1, 1, 7, 7, 4, 5]
B: [6, 8, 8, 6, 2, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$[y_1 + y_2 + y_3 - y_4, 0, y_1, y_2, y_3, 0, y_4, 0]$$

$p = - s^4 + s^5$

Omega Rank for B : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$[0, y_1, 0, y_2, 4\, y_1 - 5\, y_2 - y_3, 3\, y_1 - 4\, y_2, 0, y_3]$$

$p = - s^2 + s^4 \quad p' = - s^2 + s^4$

8 . Coloring, {8}

R: [3, 3, 1, 1, 7, 7, 5, 2]
B: [6, 8, 8, 6, 2, 4, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{1, 3}, {5, 7}} order: 2

See Matrix

$$[-y_1 + 4 y_2 - 5 y_3, y_1, y_2, 0, y_3, 0, 3 y_2 - 4 y_3, 0]$$

$p = - s^2 + s^4$ $p' = - s^2 + s^4$

Omega Rank for B : cycles: {{4, 6}, {2, 5, 8}} order: 6

See Matrix

$$[0, y_2, 0, y_3, y_1, y_3, 0, -y_2 - y_1 + 2 y_3]$$

$p = s - s^4$ $p' = - s + s^4$

9 . Coloring, {2, 3}

R: [3, 8, 8, 1, 7, 7, 5, 5]
B: [6, 3, 1, 6, 2, 4, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	6 *vs* 6	6 *vs* 6	5 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$[y_1, 0, y_2, 0, y_3, 0, y_4, y_5]$$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

$$[y_5, y_1, y_2, y_3, 0, y_4, 0, 0]$$

10 . Coloring, {2, 4}

$\Omega p(\Delta)=0$: $p' = s + 4s^5$ $p = s + 4s^5$ $p' = s^2 - 2s^4 + 4s^5$ $p' = s^3 - 2s^4 + 2s^5$

R: [3, 8, 1, 6, 7, 7, 5, 5]
B: [6, 3, 8, 1, 2, 4, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 6	3 *vs* 7	3 *vs* 7	2 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{1, 3}, {5, 7}} order: 2

See Matrix

$$[y_1, 0, y_1, 0, 3\,y_1 - y_2, y_2, 3\,y_1 - y_2, y_2]$$

$p = -s^2 + s^3$ $p = -s^2 + s^4$ $p = -s^2 + s^5$ $p = -s^2 + s^6$

Omega Rank for B : cycles: {{1, 4, 6}, {2, 3, 8}} order: 3

See Matrix

$$[y_1, y_2, y_1, y_2, 0, y_3, 0, y_3]$$

$p = -s + s^4$ $p' = -s^2 + s^5$ $p' = -s + s^4$

M \; N

$$\begin{bmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&1&2&1&1&1&1&1 \\ 1&0&1&2&1&1&1&1 \\ 2&1&0&1&1&1&1&1 \\ 1&2&1&0&1&1&1&1 \\ 1&1&1&1&0&1&2&1 \\ 1&1&1&1&1&0&1&2 \\ 1&1&1&1&2&1&0&1 \\ 1&1&1&1&1&2&1&0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 8, 1, 6, 7, 7, 5, 5]
B: [6, 3, 8, 1, 2, 4, 4, 2]

Ranges

Action of R on ranges, [[1], [4], [3], [3]]
Action of B on ranges, [[4], [1], [2], [2]]

Cycles: R , {{1, 3}, {5, 7}}, B , {{1, 4, 6}, {2, 3, 8}}

β({1, 3}) = 1/4
β({2, 4}) = 1/4

β({5, 7}) = 1/4
β({6, 8}) = 1/4

Partitions

Action of R on partitions, [[7], [4], [3], [5], [5], [4], [3], [7]]
Action of B on partitions, [[2], [8], [8], [2], [1], [6], [6], [1]]

α([{1, 2, 5, 8}, {3, 4, 6, 7}]) = 1/8
α([{3, 4, 5, 8}, {1, 2, 6, 7}]) = 1/8
α([{3, 4, 7, 8}, {1, 2, 5, 6}]) = 1/8
α([{1, 2, 7, 8}, {3, 4, 5, 6}]) = 1/8
α([{1, 4, 7, 8}, {2, 3, 5, 6}]) = 1/8
α([{2, 3, 5, 8}, {1, 4, 6, 7}]) = 1/8
α([{2, 3, 7, 8}, {1, 4, 5, 6}]) = 1/8
α([{1, 4, 5, 8}, {2, 3, 6, 7}]) = 1/8

b1 = {3, 4, 7, 8} , b2 = {1, 2, 5, 6} , b3 = {1, 4, 7, 8} , b4 = {2, 3, 5, 6} , b5 = {3, 4, 5, 8} , b6 = {1, 2, 6, 7} , b7 = {1, 2, 7, 8} , b8 = {3, 4, 5, 6} , b9 = {2, 3, 5, 8} , b10 = {1, 4, 6, 7} , b11 = {1, 4, 5, 8} , b12 = {2, 3, 6, 7} , b13 = {2, 3, 7, 8} , b14 = {1, 4, 5, 6} , b15 = {1, 2, 5, 8} , b16 = {3, 4, 6, 7}

Action of R and B on the blocks of the partitions: = [2, 1, 4, 3, 7, 8, 4, 3, 7, 8, D, E, 2, 1, D, E] [C, B, 10, F, C, B, 5, 6, 9, A, 10, F, 9, A, 5, 6]
with invariant measure [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{2, 4}
Rank	2
R,B	[3, 8, 1, 6, 7, 7, 5, 5], [6, 3, 8, 1, 2, 4, 4, 2]
π_2	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

11 . Coloring, {2, 5}

R: [3, 8, 1, 1, 2, 7, 5, 5]
B: [6, 3, 8, 6, 7, 4, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	5 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{2, 5, 8}, {1, 3}} order: 6

See Matrix

$$[3\,y_1, 5\,y_1 + 5\,y_2 - 3\,y_3 - 3\,y_4 - 3\,y_5, 3\,y_2, 0, 3\,y_3, 0, 3\,y_4, 3\,y_5]$$

$p = -s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{4, 6}, {2, 3, 8}} order: 6

See Matrix

$$[0, y_3, y_3, 5\,y_3 - y_1 - y_2, 0, y_1, y_2, y_3]$$

$p = s^2 - s^6$ $p' = s^3 - s^5$ $p' = s^2 - s^4$

12 . Coloring, {2, 6}

$\Omega p(\Delta)=0$: $p = s + 4s^4$ $p' = s + 4s^4$ $p' = s^2 + 4s^5$

R: [3, 8, 1, 1, 7, 4, 5, 5]
B: [6, 3, 8, 6, 2, 7, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 6	4 *vs* 8	4 *vs* 8	2 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{1, 3}, {5, 7}} order: 2

See Matrix

$$[y_2, 0, y_2 - y_1, y_1, y_2, 0, y_2 - y_1, y_1]$$

$p = -s^2 + s^3$ $p = -s^2 + s^4$ $p = -s^2 + s^5$ $p = -s^2 + s^6$

Omega Rank for B : cycles: {{4, 6, 7}, {2, 3, 8}} order: 3

See Matrix

$$[0, y_3, y_1, y_2, 0, y_3, y_1, y_2]$$

$p = s - s^4$ $p' = s - s^4$ $p' = s^2 - s^5$

M \; N

$$\begin{bmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&2&2&1&4&2&2&3\\ 2&0&3&2&2&4&1&2\\ 2&3&0&1&2&1&4&3\\ 1&2&1&0&3&2&3&4\\ 4&2&2&3&0&2&2&1\\ 2&4&1&2&2&0&3&2\\ 2&1&4&3&2&3&0&1\\ 3&2&3&4&1&2&1&0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 8, 1, 1, 7, 4, 5, 5]
B: [6, 3, 8, 6, 2, 7, 4, 2]

Ranges

Action of R on ranges, [[3], [4], [1], [1]]
Action of B on ranges, [[2], [3], [4], [2]]

Cycles: R , {{1, 3}, {5, 7}}, B , {{4, 6, 7}, {2, 3, 8}}

β({1, 5}) = 1/4
β({2, 6}) = 1/4
β({3, 7}) = 1/4
β({4, 8}) = 1/4

Partitions

Action of R on partitions, [[3], [2], [2], [4], [4]]
Action of B on partitions, [[5], [4], [1], [1], [4]]

α([{3, 5, 6, 8}, {1, 2, 4, 7}]) = 1/4
α([{2, 3, 4, 5}, {1, 6, 7, 8}]) = 1/8
α([{1, 2, 7, 8}, {3, 4, 5, 6}]) = 1/8
α([{1, 3, 4, 6}, {2, 5, 7, 8}]) = 3/8
α([{1, 2, 3, 4}, {5, 6, 7, 8}]) = 1/8

b1 = {3, 5, 6, 8} , b2 = {1, 2, 4, 7} , b3 = {2, 3, 4, 5} , b4 = {1, 6, 7, 8} , b5 = {1, 2, 7, 8} , b6 = {3, 4, 5, 6} , b7 = {1, 3, 4, 6} , b8 = {2, 5, 7, 8} , b9 = {1, 2, 3, 4} , b10 = {5, 6, 7, 8}

Action of R and B on the blocks of the partitions: = [5, 6, 4, 3, 3, 4, 7, 8, 7, 8] [9, A, 8, 7, 1, 2, 2, 1, 8, 7]
with invariant measure [2, 2, 1, 1, 1, 1, 3, 3, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{2, 6}

Rank	2
R,B	[3, 8, 1, 1, 7, 4, 5, 5], [6, 3, 8, 6, 2, 7, 4, 2]
$\mathbf{\pi_2}$	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
$\mathbf{u_2}$	[2, 2, 1, 4, 2, 2, 3, 3, 2, 2, 4, 1, 2, 1, 2, 1, 4, 3, 3, 2, 3, 4, 2, 2, 1, 3, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

13 . Coloring, {2, 7}

R: [3, 8, 1, 1, 7, 7, 4, 5]
B: [6, 3, 8, 6, 2, 4, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	6 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{1, 3}} order: 6

See Matrix

$$[y_4, 0, y_5, y_6, y_1, 0, y_2, y_3]$$

Omega Rank for B : cycles: {{4, 6}, {2, 3, 8}} order: 6

See Matrix

$$[0, -3\ y_3 + 5\ y_4 - 3\ y_2 + 5\ y_1 - 3\ y_5, 3\ y_3, 3\ y_4, 3\ y_2, 3\ y_1, 0, 3\ y_5]$$

$p = s^2 + s^3 - s^5 - s^6$

14 . Coloring, {2, 8}

$\Omega p(\Delta)=0$: $p = s + 4s^4 + 8s^5 - 16s^6$ $p' = s - 4s^3 - 4s^4 + 8s^5$ $p' = s^2 + 2s^3 - 4s^5$

R: [3, 8, 1, 1, 7, 7, 5, 2]
B: [6, 3, 8, 6, 2, 4, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 6	4 *vs* 8	4 *vs* 8	2 *vs* 6	1 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {1, 3}, {5, 7}} order: 2

See Matrix

$$[3\, y_1 - y_2, y_1, y_2, 0, y_2, 0, 3\, y_1 - y_2, y_1]$$

$p' = s - s^5$ $p' = s^3 - s^5$ $p = - s + s^5$ $p = - s + s^3$

Omega Rank for B : cycles: {{4, 6}, {2, 3, 5, 8}} order: 4

See Matrix

$$[0, y_1, y_1, 2\, y_1, y_1, 2\, y_1, 0, y_1]$$

$p' = s - s^4$ $p' = s^2 - s^4$ $p' = s^3 - s^4$ $p' = - s^4 + s^5$ $p = s - s^5$

M \; N

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 7 & 6 & 3 & 5 & 8 & 11 & 4 \\ 7 & 0 & 5 & 6 & 6 & 5 & 4 & 11 \\ 6 & 5 & 0 & 3 & 11 & 8 & 5 & 6 \\ 3 & 6 & 3 & 0 & 8 & 11 & 8 & 5 \\ 5 & 6 & 11 & 8 & 0 & 3 & 6 & 5 \\ 8 & 5 & 8 & 11 & 3 & 0 & 3 & 6 \\ 11 & 4 & 5 & 8 & 6 & 3 & 0 & 7 \\ 4 & 11 & 6 & 5 & 5 & 6 & 7 & 0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 8, 1, 1, 7, 7, 5, 2]
B: [6, 3, 8, 6, 2, 4, 4, 5]

Ranges

Action of R on ranges, [[3], [2], [1], [1]]
Action of B on ranges, [[4], [3], [2], [4]]

Cycles: R , {{2, 8}, {1, 3}, {5, 7}}, B , {{4, 6}, {2, 3, 5, 8}}

β({1, 7}) = 1/4
β({2, 8}) = 1/4
β({3, 5}) = 1/4
β({4, 6}) = 1/4

Partitions

Action of R on partitions, [[1], [4], [3], [2], [1], [3]]
Action of B on partitions, [[6], [6], [2], [2], [4], [5]]

α([{1, 2, 5, 6}, {3, 4, 7, 8}]) = 1/11
α([{2, 5, 6, 7}, {1, 3, 4, 8}]) = 3/11
α([{1, 5, 6, 8}, {2, 3, 4, 7}]) = 2/11
α([{1, 2, 3, 4}, {5, 6, 7, 8}]) = 2/11
α([{1, 2, 4, 5}, {3, 6, 7, 8}]) = 1/11
α([{1, 4, 5, 8}, {2, 3, 6, 7}]) = 2/11

b1 = {1, 5, 6, 8} , b2 = {2, 3, 4, 7} , b3 = {1, 2, 4, 5} , b4 = {3, 6, 7, 8} , b5 = {1, 4, 5, 8} , b6 = {2, 3, 6, 7} , b7 = {1, 2, 3, 4} , b8 = {5, 6, 7, 8} , b9 = {1, 2, 5, 6} , b10 = {2, 5, 6, 7} , b11 = {3, 4, 7, 8} , b12 = {1, 3, 4, 8}

Action of R and B on the blocks of the partitions: = [2, 1, B, 9, 2, 1, C, A, B, 8, 9, 7] [C, A, 8, 7, 4, 3, A, C, 5, 5, 6, 6] with invariant measure [2, 2, 1, 1, 2, 2, 2, 2, 1, 3, 1, 3]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{2, 8}
Rank	2
R,B	[3, 8, 1, 1, 7, 7, 5, 2], [6, 3, 8, 6, 2, 4, 4, 5]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]
u_2	[7, 6, 3, 5, 8, 11, 4, 5, 6, 6, 5, 4, 11, 3, 11, 8, 5, 6, 8, 11, 8, 5, 3, 6, 5, 3, 6, 7] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

15 . Coloring, {3, 4}

R: [3, 3, 8, 6, 7, 7, 5, 5]
B: [6, 8, 1, 1, 2, 4, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	6 *vs* 6	6 *vs* 6	4 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$[0, 0, 2\, y_2, 0, y_4, y_2, y_3, y_1]$$

$p = - s^3 + s^5$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

$$[3\, y_2, 3\, y_1, 0, -3\, y_2 + 5\, y_1 - 3\, y_3 + 5\, y_4, 0, 3\, y_3, 0, 3\, y_4]$$

$p = - s - s^2 + s^4 + s^5$

16 . Coloring, {3, 5}

$\Omega p(\Delta)=0$: $p = s - 4s^4 - 8s^5$ $p' = s - 4s^4 - 8s^5$ $p' = s^2 + 2s^3 + 4s^4 + 4s^5$

R: [3, 3, 8, 1, 2, 7, 5, 5]
B: [6, 8, 1, 6, 7, 4, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 6	4 *vs* 8	4 *vs* 8	2 *vs* 6	2 *vs* 6

Omega Rank for R : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

$$[y_1, y_2, y_1 + y_2, 0, y_1 + y_2, 0, y_1, y_2]$$

$p = - s^2 + s^3$ $p = - s^2 + s^4$ $p = - s^2 + s^5$ $p = - s^2 + s^6$

Omega Rank for B : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$[y_1, y_2, 0, -y_1 + 3\, y_2, 0, -y_1 + 3\, y_2, y_1, y_2]$$

$p' = s^3 - s^4$ $p' = - s^4 + s^5$ $p = s^2 - s^5$ $p' = s^2 - s^4$

M \; N

$$\begin{bmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&2&4&2&3&5&7&5\\ 2&0&4&4&3&3&5&7\\ 4&4&0&5&7&2&3&3\\ 2&4&5&0&2&7&5&3\\ 3&3&7&2&0&5&4&4\\ 5&3&2&7&5&0&2&4\\ 7&5&3&5&4&2&0&2\\ 5&7&3&3&4&4&2&0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 3, 8, 1, 2, 7, 5, 5]
B: [6, 8, 1, 6, 7, 4, 4, 2]

Ranges

Action of R on ranges, [[3], [3], [2], [1]]
Action of B on ranges, [[4], [2], [1], [4]]

Cycles: R , {{2, 3, 5, 8}}, B , {{2, 8}, {4, 6}}

β({1, 7}) = 1/4
β({2, 8}) = 1/4
β({3, 5}) = 1/4
β({4, 6}) = 1/4

Partitions

Action of R on partitions, [[5], [4], [1], [5], [4], [2]]
Action of B on partitions, [[5], [3], [6], [3], [5], [6]]

α([{1, 2, 3, 4}, {5, 6, 7, 8}]) = 1/14
α([{1, 2, 5, 6}, {3, 4, 7, 8}]) = 1/14
α([{2, 5, 6, 7}, {1, 3, 4, 8}]) = 1/7
α([{1, 2, 3, 6}, {4, 5, 7, 8}]) = 3/14
α([{1, 2, 4, 5}, {3, 6, 7, 8}]) = 5/14
α([{1, 4, 5, 8}, {2, 3, 6, 7}]) = 1/7

b1 = {1, 2, 3, 6} , b2 = {4, 5, 7, 8} , b3 = {1, 2, 4, 5} , b4 = {3, 6, 7, 8} , b5 = {1, 4, 5, 8} , b6 = {2, 3, 6, 7} , b7 = {1, 2, 3, 4} , b8 = {5, 6, 7, 8} , b9 = {1, 2, 5, 6} , b10 = {2, 5, 6, 7} , b11 = {3, 4, 7, 8} , b12 = {1, 3, 4, 8}

Action of R and B on the blocks of the partitions: = [3, 4, 2, 1, B, 9, 3, 4, 2, 8, 1, 7] [C, A, 4, 3, 6, 5, 4, 3, C, 5, A, 6]
with invariant measure [3, 3, 5, 5, 2, 2, 1, 1, 1, 2, 1, 2]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{3, 5}
Rank	2
R,B	[3, 3, 8, 1, 2, 7, 5, 5], [6, 8, 1, 6, 7, 4, 4, 2]

π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]
u_2	[2, 4, 2, 3, 5, 7, 5, 4, 4, 3, 3, 5, 7, 5, 7, 2, 3, 3, 2, 7, 5, 3, 5, 4, 4, 2, 4, 2] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

17 . Coloring, {3, 6}

R: [3, 3, 8, 1, 7, 4, 5, 5]
B: [6, 8, 1, 6, 2, 7, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	6 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{5, 7}} order: 6

See Matrix

$$[y_6, 0, y_3, y_4, y_5, 0, y_1, y_2]$$

Omega Rank for B : cycles: {{2, 8}, {4, 6, 7}} order: 6

See Matrix

$$[5 y_1 - 3 y_5 - 3 y_4 - 3 y_2 + 5 y_3, 3 y_1, 0, 3 y_5, 0, 3 y_4, 3 y_2, 3 y_3]$$

$p = - s^{2} - s^{3} + s^{5} + s^{6}$

18 . Coloring, {3, 7}

$\Omega p(\Delta)=0$: $p = s - 4s^{4}$ $p' = s - 4s^{4}$ $p' = s^{2} - 4s^{5}$

R: [3, 3, 8, 1, 7, 7, 4, 5]
B: [6, 8, 1, 6, 2, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 6	4 *vs* 8	4 *vs* 8	3 *vs* 6	2 *vs* 6

Omega Rank for R : cycles: {{1, 3, 4, 5, 7, 8}} order: 6

See Matrix

$$[y_1, 0, y_2, y_3, y_1, 0, y_2, y_3]$$

$p' = s^2 - s^5$ $p' = - s + s^4$ $p = - s + s^4$

Omega Rank for B : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$[y_2, y_1, 0, -y_2 + y_1, y_2, y_1, 0, -y_2 + y_1]$$

$p = - s^2 + s^6$ $p = - s^2 + s^3$ $p = - s^2 + s^4$ $p = - s^2 + s^5$

M \; N

$$\begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&1&2&1&4&3&2&3 \\ 1&0&2&2&3&4&2&2 \\ 2&2&0&3&2&2&4&1 \\ 1&2&3&0&3&2&1&4 \\ 4&3&2&3&0&1&2&1 \\ 3&4&2&2&1&0&2&2 \\ 2&2&4&1&2&2&0&3 \\ 3&2&1&4&1&2&3&0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 3, 8, 1, 7, 7, 4, 5]
B: [6, 8, 1, 6, 2, 4, 5, 2]

Ranges

Action of R on ranges, [[3], [3], [4], [1]]
Action of B on ranges, [[2], [4], [1], [2]]

Cycles: R , {{1, 3, 4, 5, 7, 8}}, B , {{2, 8}, {4, 6}}

β({1, 5}) = 1/4
β({2, 6}) = 1/4
β({3, 7}) = 1/4
β({4, 8}) = 1/4

Partitions

Action of R on partitions, [[2], [4], [5], [5], [2]]
Action of B on partitions, [[3], [1], [3], [5], [5]]

α([{1, 4, 6, 7}, {2, 3, 5, 8}]) = 1/8
α([{1, 2, 3, 8}, {4, 5, 6, 7}]) = 1/4
α([{1, 3, 4, 6}, {2, 5, 7, 8}]) = 1/8
α([{1, 2, 3, 4}, {5, 6, 7, 8}]) = 1/8

α([{1, 2, 4, 7}, {3, 5, 6, 8}]) = 3/8

b1 = {1, 2, 3, 8} , b2 = {4, 5, 6, 7} , b3 = {1, 2, 4, 7} , b4 = {3, 5, 6, 8} , b5 = {1, 3, 4, 6} , b6 = {1, 4, 6, 7} , b7 = {2, 5, 7, 8} , b8 = {2, 3, 5, 8} , b9 = {1, 2, 3, 4} , b10 = {5, 6, 7, 8}

Action of R and B on the blocks of the partitions: = [9, A, 2, 1, 3, 2, 4, 1, 3, 4] [8, 6, 4, 3, 5, 5, 7, 7, 4, 3]
with invariant measure [2, 2, 3, 3, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{3, 7}
Rank	2
R,B	[3, 3, 8, 1, 7, 7, 4, 5], [6, 8, 1, 6, 2, 4, 5, 2]
π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[1, 2, 1, 4, 3, 2, 3, 2, 2, 3, 4, 2, 2, 3, 2, 2, 4, 1, 3, 2, 1, 4, 1, 2, 1, 2, 2, 3] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

19 . Coloring, {3, 8}

R: [3, 3, 8, 1, 7, 7, 5, 2]
B: [6, 8, 1, 6, 2, 4, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	5 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{2, 3, 8}, {5, 7}} order: 6

See Matrix

$$[-3 y_1 - 3 y_2 + 5 y_3 + 5 y_4 - 3 y_5, 3 y_1, 3 y_2, 0, 3 y_3, 0, 3 y_4, 3 y_5]$$

$p = - s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{4, 6}, {2, 5, 8}} order: 6

See Matrix

$$[5\ y_3 - y_1 - y_2, y_3, 0, y_1, y_3, y_2, 0, y_3]$$

$p = - s^2 + s^4$ $p' = - s^2 + s^4$ $p = - s^2 + s^6$

20 . Coloring, {4, 5}

R: [3, 3, 1, 6, 2, 7, 5, 5]
B: [6, 8, 8, 1, 7, 4, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	6 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{1, 3}} order: 6

See Matrix

$$[y_2, y_1, y_5, 0, y_6, y_4, y_3, 0]$$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

$$[5\ y_1 - 3\ y_2 - 3\ y_3 - 3\ y_4 + 5\ y_5, 3\ y_1, 0, 3\ y_2, 0, 3\ y_3, 3\ y_4, 3\ y_5]$$

$p = - s^2 - s^3 + s^5 + s^6$

21 . Coloring, {4, 6}

$\Omega p(\Delta)=0$: $p = s + 4s^4 + 8s^5 - 16s^6$ $p' = s - 4s^3 - 4s^4 + 8s^5$ $p' = s^2 + 2s^3 - 4s^5$

R: [3, 3, 1, 6, 7, 4, 5, 5]
B: [6, 8, 8, 1, 2, 7, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 6	4 *vs* 8	4 *vs* 8	2 *vs* 6	1 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {1, 3}, {5, 7}} order: 2

See Matrix

$$[y_2, 0, -y_2 + 3\,y_1, y_1, -y_2 + 3\,y_1, y_1, y_2, 0]$$

$p = -s + s^5 \quad p' = s^3 - s^5 \quad p' = s - s^5 \quad p = -s + s^3$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6, 7}} order: 4

See Matrix

$$[y_1, 2\,y_1, 0, y_1, 0, y_1, y_1, 2\,y_1]$$

$p = s - s^6 \quad p' = s^2 - s^5 \quad p' = s - s^5 \quad p' = s^3 - s^5 \quad p' = s^4 - s^5$

M \; N

0	0	0	0	0	0	1	0		0	3	6	5	5	6	11	8
0	0	0	0	0	0	0	1		3	0	3	6	8	5	8	11
0	0	0	0	1	0	0	0		6	3	0	7	11	4	5	8
0	0	0	0	0	1	0	0		5	6	7	0	4	11	6	5
0	0	1	0	0	0	0	0		5	8	11	4	0	7	6	3
0	0	0	1	0	0	0	0		6	5	4	11	7	0	5	6
1	0	0	0	0	0	0	0		11	8	5	6	6	5	0	3
0	1	0	0	0	0	0	0		8	11	8	5	3	6	3	0

τ= 32 , r'= 1/2

R: [3, 3, 1, 6, 7, 4, 5, 5]
B: [6, 8, 8, 1, 2, 7, 4, 2]

Ranges

Action of R on ranges, [[3], [3], [1], [4]]
Action of B on ranges, [[4], [2], [2], [1]]

Cycles: R , {{4, 6}, {1, 3}, {5, 7}}, B , {{2, 8}, {1, 4, 6, 7}}

β({1, 7}) = 1/4
β({2, 8}) = 1/4
β({3, 5}) = 1/4
β({4, 6}) = 1/4

Partitions

Action of R on partitions, [[4], [5], [6], [4], [2], [6]]
Action of B on partitions, [[5], [3], [1], [3], [2], [2]]

α([{2, 3, 4, 7}, {1, 5, 6, 8}]) = 1/11
α([{1, 2, 3, 6}, {4, 5, 7, 8}]) = 3/11
α([{1, 4, 5, 8}, {2, 3, 6, 7}]) = 2/11
α([{3, 4, 7, 8}, {1, 2, 5, 6}]) = 1/11
α([{1, 2, 3, 4}, {5, 6, 7, 8}]) = 2/11
α([{1, 2, 4, 5}, {3, 6, 7, 8}]) = 2/11

b1 = {1, 2, 3, 6} , b2 = {2, 3, 4, 7} , b3 = {1, 5, 6, 8} , b4 = {4, 5, 7, 8} , b5 = {1, 4, 5, 8} , b6 = {2, 3, 6, 7} , b7 = {3, 4, 7, 8} , b8 = {1, 2, 5, 6} , b9 = {1, 2, 4, 5} , b10 = {1, 2, 3, 4} , b11 = {3, 6, 7, 8} , b12 = {5, 6, 7, 8}

Action of R and B on the blocks of the partitions: = [A, 8, 7, C, B, 9, 8, 7, B, 1, 9, 4] [5, C, A, 6, 2, 3, 6, 5, 4, 4, 1, 1] with invariant measure [3, 1, 1, 3, 2, 2, 1, 1, 2, 2, 2, 2]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{4, 6}
Rank	2
R,B	[3, 3, 1, 6, 7, 4, 5, 5], [6, 8, 8, 1, 2, 7, 4, 2]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]
u_2	[3, 6, 5, 5, 6, 11, 8, 3, 6, 8, 5, 8, 11, 7, 11, 4, 5, 8, 4, 11, 6, 5, 7, 6, 3, 5, 6, 3] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

22 . Coloring, {4, 7}

R: [3, 3, 1, 6, 7, 7, 4, 5]
B: [6, 8, 8, 1, 2, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	5 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{1, 3}, {4, 6, 7}} order: 6

See Matrix

$$[3\ y_4,\ 0,\ 3\ y_3,\ 3\ y_5,\ 5\ y_4 + 5\ y_3 - 3\ y_5 - 3\ y_1 - 3\ y_2,\ 3\ y_1,\ 3\ y_2,\ 0]$$

$p = - s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

$$[y_2,\ y_3,\ 0,\ y_2,\ y_1,\ y_2,\ 0,\ 5\ y_2 - y_3 - y_1]$$

$p' = s^3 - s^5$ $p = s^2 - s^6$ $p' = s^2 - s^4$

23 . Coloring, {4, 8}

$\Omega p(\Delta)=0$: $p' = s^2 + 4s^5$ $p = s + 4s^4$ $p' = s + 4s^4$

R: [3, 3, 1, 6, 7, 7, 5, 2]
B: [6, 8, 8, 1, 2, 4, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 6	4 *vs* 8	4 *vs* 8	2 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{1, 3}, {5, 7}} order: 2

See Matrix

$$[y_2, -y_2 + y_1, y_1, 0, y_2, -y_2 + y_1, y_1, 0]$$

$p = s^2 - s^4$ $p' = s^2 - s^3$ $p' = - s^3 + s^4$ $p' = - s^3 + s^5$

Omega Rank for B : cycles: {{2, 5, 8}, {1, 4, 6}} order: 3

See Matrix

$$[y_1, y_2, 0, y_3, y_1, y_2, 0, y_3]$$

$p' = - s^2 + s^5$ $p' = - s + s^4$ $p = s - s^4$

M \; N

$$\begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&1&2&3&4&3&2&1 \\ 1&0&1&2&3&4&3&2 \\ 2&1&0&2&2&3&4&2 \\ 3&2&2&0&1&2&2&4 \\ 4&3&2&1&0&1&2&3 \\ 3&4&3&2&1&0&1&2 \\ 2&3&4&2&2&1&0&2 \\ 1&2&2&4&3&2&2&0 \end{bmatrix}$$

$\tau = 32$, $r' = 1/2$

R: [3, 3, 1, 6, 7, 7, 5, 2]
B: [6, 8, 8, 1, 2, 4, 4, 5]

Ranges

Action of R on ranges, [[3], [3], [1], [2]]
Action of B on ranges, [[2], [4], [4], [1]]

Cycles: R , {{1, 3}, {5, 7}}, B , {{2, 5, 8}, {1, 4, 6}}

β({1, 5}) = 1/4
β({2, 6}) = 1/4
β({3, 7}) = 1/4
β({4, 8}) = 1/4

Partitions

Action of R on partitions, [[1], [3], [1], [4], [4]]
Action of B on partitions, [[4], [5], [2], [2], [4]]

α([{1, 2, 4, 7}, {3, 5, 6, 8}]) = 1/8
α([{1, 6, 7, 8}, {2, 3, 4, 5}]) = 1/4
α([{1, 2, 7, 8}, {3, 4, 5, 6}]) = 1/8
α([{1, 2, 3, 8}, {4, 5, 6, 7}]) = 3/8
α([{1, 2, 3, 4}, {5, 6, 7, 8}]) = 1/8

b1 = {1, 2, 4, 7} , b2 = {3, 5, 6, 8} , b3 = {1, 6, 7, 8} , b4 = {2, 3, 4, 5} , b5 = {1, 2, 7, 8} , b6 = {3, 4, 5, 6} , b7 = {1, 2, 3, 8} , b8 = {4, 5, 6, 7} , b9 = {1, 2, 3, 4} , b10 = {5, 6, 7, 8}

Action of R and B on the blocks of the partitions: = [2, 1, 6, 5, 2, 1, 7, 8, 7, 8] [8, 7, 9, A, 4, 3, 4, 3, 8, 7]
with invariant measure [1, 1, 2, 2, 1, 1, 3, 3, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{4, 8}
Rank	2
R,B	[3, 3, 1, 6, 7, 7, 5, 2], [6, 8, 8, 1, 2, 4, 4, 5]
π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[1, 2, 3, 4, 3, 2, 1, 1, 2, 3, 4, 3, 2, 2, 2, 3, 4, 2, 1, 2, 2, 4, 1, 2, 3, 1, 2, 2] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

24 . Coloring, {5, 6}

R: [3, 3, 1, 1, 2, 4, 5, 5]
B: [6, 8, 8, 6, 7, 7, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	6 *vs* 6	6 *vs* 6	4 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$[y_4, y_3, y_1, y_2, 2\ y_2, 0, 0, 0]$$

$p = - s^3 + s^5$

Omega Rank for B : cycles: {{2, 8}, {4, 6, 7}} order: 6

See Matrix

$$[0, 3\ y_1, 0, 5\ y_1 - 3\ y_4 - 3\ y_3 + 5\ y_2, 0, 3\ y_4, 3\ y_3, 3\ y_2]$$

$p = - s - s^2 + s^4 + s^5$

25 . Coloring, {5, 7}

Ωp(Δ)=0: $p' = s^3 - 2s^5$ $p = s - 4s^5$ $p' = s - 4s^5$ $p' = s^2 - 2s^4$

R: [3, 3, 1, 1, 2, 7, 4, 5]
B: [6, 8, 8, 6, 7, 4, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 6	3 *vs* 7	3 *vs* 7	3 *vs* 6	2 *vs* 6

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$[y_2, y_1, y_2, y_1, y_3, 0, y_3, 0]$$

$p' = s^4 - s^5$ $p' = s^3 - s^5$ $p = s^3 - s^6$

Omega Rank for B : cycles: {{2, 8}, {4, 6}, {5, 7}} order: 2

See Matrix

$$[0, 3\ y_1 - y_2, 0, 3\ y_1 - y_2, y_1, y_2, y_1, y_2]$$

$p' = -s + s^3$ $p' = -s + s^5$ $p = -s + s^3$ $p = -s + s^5$

M \; N

$$\begin{bmatrix} 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0 \end{bmatrix} \begin{bmatrix} 0&1&2&1&1&1&1&1\\ 1&0&1&2&1&1&1&1\\ 2&1&0&1&1&1&1&1\\ 1&2&1&0&1&1&1&1\\ 1&1&1&1&0&1&2&1\\ 1&1&1&1&1&0&1&2\\ 1&1&1&1&2&1&0&1\\ 1&1&1&1&1&2&1&0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 3, 1, 1, 2, 7, 4, 5]
B: [6, 8, 8, 6, 7, 4, 5, 2]

Ranges

Action of R on ranges, [[1], [1], [2], [3]]
Action of B on ranges, [[4], [4], [3], [2]]

Cycles: R , {{1, 3}}, B , {{2, 8}, {4, 6}, {5, 7}}

β({1, 3}) = 1/4
β({2, 4}) = 1/4
β({5, 7}) = 1/4
β({6, 8}) = 1/4

Partitions

Action of R on partitions, [[3], [5], [7], [1], [3], [1], [7], [5]]
Action of B on partitions, [[8], [6], [4], [4], [6], [2], [2], [8]]

α([{3, 4, 7, 8}, {1, 2, 5, 6}]) = 1/8
α([{1, 4, 6, 7}, {2, 3, 5, 8}]) = 1/8
α([{3, 4, 5, 8}, {1, 2, 6, 7}]) = 1/8
α([{1, 4, 5, 8}, {2, 3, 6, 7}]) = 1/8
α([{3, 4, 6, 7}, {1, 2, 5, 8}]) = 1/8
α([{1, 4, 5, 6}, {2, 3, 7, 8}]) = 1/8
α([{1, 2, 7, 8}, {3, 4, 5, 6}]) = 1/8
α([{1, 4, 7, 8}, {2, 3, 5, 6}]) = 1/8

b1 = {1, 4, 7, 8} , b2 = {2, 3, 5, 6} , b3 = {1, 4, 5, 8} , b4 = {1, 4, 5, 6} , b5 = {2, 3, 6, 7} , b6 = {2, 3, 7, 8} , b7 = {3, 4, 7, 8} , b8 = {1, 2, 7, 8} , b9 = {3, 4, 5, 6} , b10 = {1, 2, 5, 6} , b11 = {3, 4, 5, 8} , b12 = {1, 2, 6, 7} , b13 = {3, 4, 6, 7} , b14 = {1, 4, 6, 7} , b15 = {2, 3, 5, 8} , b16 = {1, 2, 5, 8}

Action of R and B on the blocks of the partitions: = [D, 10, 7, 7, A, A, C, 9, 8, B, 8, 9, C, D, 10, B] [2, 1, 5, E, 3, F, 2, F, E, 1, 5, 3, 4, 4, 6, 6]
with invariant measure [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{5, 7}
Rank	2
R,B	[3, 3, 1, 1, 2, 7, 4, 5], [6, 8, 8, 6, 7, 4, 5, 2]
π_2	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

26 . Coloring, {5, 8}

R: [3, 3, 1, 1, 2, 7, 5, 2]
B: [6, 8, 8, 6, 7, 4, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	6 *vs* 6	6 *vs* 6	5 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$[y_1, y_4, y_3, 0, y_2, 0, y_5, 0]$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

$[0, 0, 0, y_4, y_5, y_2, y_1, y_3]$

27 . Coloring, {6, 7}

R: [3, 3, 1, 1, 7, 4, 4, 5]
B: [6, 8, 8, 6, 2, 7, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	6 *vs* 6	6 *vs* 6	5 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$[y_1, 0, y_5, y_2, y_3, 0, y_4, 0]$

Omega Rank for B : cycles: {{2, 8}} order: 4

See Matrix

$[0, y_1, 0, 0, y_4, y_3, y_2, y_5]$

28 . Coloring, {6, 8}

Ωp(Δ)=0: $p' = s + 4s^5$ $p' = s^2 - 2s^4 + 4s^5$ $p = s + 4s^5$ $p' = s^3 - 2s^4 + 2s^5$

R: [3, 3, 1, 1, 7, 4, 5, 2]
B: [6, 8, 8, 6, 2, 7, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 6	3 *vs* 7	3 *vs* 7	2 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{1, 3}, {5, 7}} order: 2

See Matrix

$[-y_2 + 3 y_1, y_2, -y_2 + 3 y_1, y_2, y_1, 0, y_1, 0]$

$p' = s^2 - s^4$ $p' = s^3 - s^4$ $p = s^2 - s^5$ $p' = - s^4 + s^5$

Omega Rank for B : cycles: {{4, 6, 7}, {2, 5, 8}} order: 3

See Matrix

$[0, y_1, 0, y_1, y_2, y_3, y_2, y_3]$

$p = - s + s^4$ $p' = - s + s^4$ $p' = - s^2 + s^5$

M \; N

$$\begin{bmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&1&2&1&1&1&1&1 \\ 1&0&1&2&1&1&1&1 \\ 2&1&0&1&1&1&1&1 \\ 1&2&1&0&1&1&1&1 \\ 1&1&1&1&0&1&2&1 \\ 1&1&1&1&1&0&1&2 \\ 1&1&1&1&2&1&0&1 \\ 1&1&1&1&1&2&1&0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 3, 1, 1, 7, 4, 5, 2]
B: [6, 8, 8, 6, 2, 7, 4, 5]

Ranges

Action of R on ranges, [[1], [1], [3], [2]]
Action of B on ranges, [[4], [4], [2], [3]]

Cycles: R , {{1, 3}, {5, 7}}, B , {{4, 6, 7}, {2, 5, 8}}

β({1, 3}) = 1/4
β({2, 4}) = 1/4
β({5, 7}) = 1/4
β({6, 8}) = 1/4

Partitions

Action of R on partitions, [[6], [5], [6], [4], [4], [7], [7], [5]]
Action of B on partitions, [[8], [2], [1], [1], [8], [2], [3], [3]]

α([{1, 4, 5, 6}, {2, 3, 7, 8}]) = 1/8
α([{1, 4, 6, 7}, {2, 3, 5, 8}]) = 1/8
α([{1, 4, 5, 8}, {2, 3, 6, 7}]) = 1/8
α([{3, 4, 5, 8}, {1, 2, 6, 7}]) = 1/8
α([{1, 2, 7, 8}, {3, 4, 5, 6}]) = 1/8
α([{3, 4, 6, 7}, {1, 2, 5, 8}]) = 1/8
α([{3, 4, 7, 8}, {1, 2, 5, 6}]) = 1/8
α([{1, 4, 7, 8}, {2, 3, 5, 6}]) = 1/8

b1 = {1, 4, 7, 8} , b2 = {2, 3, 5, 6} , b3 = {1, 4, 5, 8} , b4 = {1, 4, 5, 6} , b5 = {2, 3, 6, 7} , b6 = {2, 3, 7, 8} , b7 = {3, 4, 7, 8} , b8 = {1, 2, 7, 8} , b9 = {3, 4, 5, 6} , b10 = {1, 2, 5, 6} , b11 = {3, 4, 5, 8} , b12 = {1, 2, 6, 7} , b13 = {3, 4, 6, 7} , b14 = {1, 4, 6, 7} , b15 = {2, 3, 5, 8} , b16 = {1, 2, 5, 8}

Action of R and B on the blocks of the partitions: = [9, 8, D, D, 10, 10, A, B, C, 7, C, B, A, 9, 8, 7] [5, 3, 6, 1, 4, 2, 5, 2, 1, 3, 6, 4, E, E, F, F]
with invariant measure [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{6, 8}

Rank	2
R,B	[3, 3, 1, 1, 7, 4, 5, 2], [6, 8, 8, 6, 2, 7, 4, 5]
π_2	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

29 . Coloring, {7, 8}

R: [3, 3, 1, 1, 7, 7, 4, 2]
B: [6, 8, 8, 6, 2, 4, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	6 *vs* 6	6 *vs* 6	4 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$[y_1, y_2, y_3, y_4, 0, 0, 2\, y_2, 0]$$

$p = - s^3 + s^5$

Omega Rank for B : cycles: {{4, 6}, {2, 5, 8}} order: 6

See Matrix

$$[0, 3\, y_2, 0, 3\, y_3, -3\, y_2 + 5\, y_3 + 5\, y_1 - 3\, y_4, 3\, y_1, 0, 3\, y_4]$$

$p = - s - s^2 + s^4 + s^5$

30 . Coloring, {2, 3, 4}

R: [3, 8, 8, 6, 7, 7, 5, 5]
B: [6, 3, 1, 1, 2, 4, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{5, 7}} order: 4

See Matrix

$$[0, 0, y_3, 0, y_2, y_3, 2\,y_3 + y_2 - y_1, y_1]$$

$p = s^3 - s^5 \quad p' = s^3 - s^4$

Omega Rank for B : cycles: {{1, 4, 6}} order: 3

See Matrix

$$[y_5, y_4, y_3, y_2, 0, y_1, 0, 0]$$

31 . Coloring, {2, 3, 5}

R: [3, 8, 8, 1, 2, 7, 5, 5]
B: [6, 3, 1, 6, 7, 4, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	5 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{2, 5, 8}} order: 3

See Matrix

$$[y_4, y_1, y_2, 0, y_3, 0, y_4, y_5]$$

$p = - s^3 + s^6$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

$$[y_1 - y_2 + y_3, y_4, y_1, y_2, 0, y_3, y_4, 0]$$

$p = s^4 - s^6 \quad p' = s^4 - s^5$

32 . Coloring, {2, 3, 6}

R: [3, 8, 8, 1, 7, 4, 5, 5]
B: [6, 3, 1, 6, 2, 7, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	5 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{5, 7}} order: 6

See Matrix

$$[y_5, 0, y_3, y_4, y_2, 0, y_1, -y_5 + y_3 + y_4 + y_2 - y_1]$$

$p = s^5 - s^6$

Omega Rank for B : cycles: {{4, 6, 7}} order: 6

See Matrix

$$[y_1, y_2, y_3, y_4, 0, y_6, y_5, 0]$$

33 . Coloring, {2, 3, 7}

R: [3, 8, 8, 1, 7, 7, 4, 5]
B: [6, 3, 1, 6, 2, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	6 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{1, 3, 4, 5, 7, 8}} order: 6

See Matrix

$$[y_1, 0, y_2, y_3, y_4, 0, y_5, y_6]$$

Omega Rank for B : cycles: {{4, 6}} order: 6

See Matrix

$$[y_3, y_2, y_3 + y_2 + y_1 - y_4 - y_5, y_1, y_4, y_5, 0, 0]$$

$p = - s^{5} + s^{6}$

34 . Coloring, {2, 3, 8}

R: [3, 8, 8, 1, 7, 7, 5, 2]
B: [6, 3, 1, 6, 2, 4, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {5, 7}} order: 4

See Matrix

$$[4 y_1 + 4 y_2 - 5 y_3 - y_4, y_1, y_2, 0, y_3, 0, 3 y_1 + 3 y_2 - 4 y_3, y_4]$$

$p = - s^{3} + s^{5}$ $p' = - s^{3} + s^{5}$

Omega Rank for B : cycles: {{4, 6}} order: 6

See Matrix

$$[-y_1 + y_4 - y_3 + y_2 + y_5, y_1, y_4, y_3, y_2, y_5, 0, 0]$$

$p = - s^{5} + s^{6}$

35 . Coloring, {2, 4, 5}

R: [3, 8, 1, 6, 2, 7, 5, 5]
B: [6, 3, 8, 1, 7, 4, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	3 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{1, 3}, {2, 5, 8}} order: 6

See Matrix

$$[y_1, 2 y_1 - y_2, y_1, 0, 2 y_1, y_2, 2 y_1 - y_3, y_3]$$

$p = - s^{3} + s^{5}$ $p = - s^{3} + s^{6}$ $p = - s^{3} + s^{7}$ $p = - s^{3} + s^{4}$

Omega Rank for B : cycles: {{1, 4, 6}, {2, 3, 8}}   order: 3

See Matrix

$$[5\, y_4 - y_1 - y_2 - y_3, y_4, y_4, y_1, 0, y_2, y_3, y_4]$$

$p = - s^{2} + s^{5}$ $p' = - s^{3} + s^{6}$ $p' = - s^{2} + s^{5}$

36 . Coloring, {2, 4, 6}

R: [3, 8, 1, 6, 7, 4, 5, 5]
B: [6, 3, 8, 1, 2, 7, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 7	3 *vs* 7

Omega Rank for R : cycles: {{4, 6}, {1, 3}, {5, 7}}   order: 2

See Matrix

$$[y_1, 0, y_1, y_1, 2\, y_1, y_1, 2\, y_1 - y_2, y_2]$$

$p = s^{2} - s^{4}$ $p' = s^{2} - s^{3}$ $p' = - s^{3} + s^{4}$ $p' = - s^{3} + s^{5}$ $p' = - s^{3} + s^{6}$

Omega Rank for B : cycles: {{2, 3, 8}, {1, 4, 6, 7}}

See Matrix

$$[y_2, y_1, 4\, y_2 - y_1 - y_3, y_2, 0, y_2, y_2, y_3]$$

$p = - s + s^{7}$ $p = - s + s^{4}$ $p' = s - s^{4}$ $p' = s^{2} - s^{5}$

37 . Coloring, {2, 4, 7}

R: [3, 8, 1, 6, 7, 7, 4, 5]
B: [6, 3, 8, 1, 2, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	3 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{4, 6, 7}, {1, 3}} order: 6

See Matrix

$$[y_1, 0, y_1, 2\,y_1 - y_3, 2\,y_1 - y_2, y_2, 2\,y_1, y_3]$$

$p = -s^3 + s^4$ $p = -s^3 + s^5$ $p = -s^3 + s^6$ $p = -s^3 + s^7$

Omega Rank for B : cycles: {{2, 3, 8}, {1, 4, 6}} order: 3

See Matrix

$$[y_3, 5\,y_3 - y_1 - y_2 - y_4, y_1, y_3, y_2, y_3, 0, y_4]$$

$p' = s^2 - s^5$ $p' = s^3 - s^6$ $p = s^2 - s^5$

38 . Coloring, {2, 4, 8}

R: [3, 8, 1, 6, 7, 7, 5, 2]
B: [6, 3, 8, 1, 2, 4, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 7	3 *vs* 7

Omega Rank for R : cycles: {{2, 8}, {1, 3}, {5, 7}} order: 2

See Matrix

$$[y_2, y_2, y_2, 0, y_1, 2\,y_2 - y_1, 2\,y_2, y_2]$$

$p' = -s^2 + s^6$ $p' = -s^2 + s^5$ $p' = -s^2 + s^4$ $p' = -s^2 + s^3$ $p = s^2 - s^3$

Omega Rank for B : cycles: {{1, 4, 6}, {2, 3, 5, 8}}

See Matrix

$$[y_3, y_2, y_2, y_1, y_2, -y_3 + 4\,y_2 - y_1, 0, y_2]$$

$p = -s + s^4$ $p' = -s + s^4$ $p' = -s^2 + s^5$ $p = -s + s^7$

39 . Coloring, {2, 5, 6}

R: [3, 8, 1, 1, 2, 4, 5, 5]
B: [6, 3, 8, 6, 7, 7, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{1, 3}, {2, 5, 8}} order: 6

See Matrix

$$[y_1 + y_3, 2\,y_1 + 2\,y_3 - y_2 - y_4, y_1, y_3, y_2, 0, 0, y_4]$$

$p = -s^2 + s^5$ $p' = -s^2 + s^5$

Omega Rank for B : cycles: {{4, 6, 7}, {2, 3, 8}} order: 3

See Matrix

$$[0, y_2, y_2, y_3, 0, 5\,y_2 - y_3 - y_1, y_1, y_2]$$

$p' = s^2 - s^5$ $p = -s + s^4$ $p' = -s + s^4$

40 . Coloring, {2, 5, 7}

R: [3, 8, 1, 1, 2, 7, 4, 5]
B: [6, 3, 8, 6, 7, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	4 *vs* 7	2 *vs* 7

Omega Rank for R : cycles: {{1, 3}, {2, 5, 8}} order: 6

See Matrix

$$[5\,y_4 - y_1 - y_2 - y_3, y_4, y_1, y_2, y_4, 0, y_3, y_4]$$

$p = s^3 - s^5$ $p' = -s^3 + s^5$ $p' = -s^4 + s^6$

Omega Rank for B : cycles: {{4, 6}, {5, 7}, {2, 3, 8}} order: 6

See Matrix

$$[0, y_2, y_2, y_1, y_2, 3\,y_2 - y_1, y_2, y_2]$$

$p' = -s^4 + s^6$ $p' = -s^3 + s^5$ $p' = s - s^3$ $p' = s^2 - s^4$ $p = s - s^5$

41 . Coloring, {2, 5, 8}

R: [3, 8, 1, 1, 2, 7, 5, 2]
B: [6, 3, 8, 6, 7, 4, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {1, 3}} order: 4

See Matrix

$$[y_1, y_2, -4\,y_1 + 3\,y_2 + 3\,y_4, 0, -5\,y_1 + 4\,y_2 + 4\,y_4 - y_3, 0, y_4, y_3]$$

$p = -s^3 + s^5$ $p' = -s^3 + s^5$

Omega Rank for B : cycles: {{4, 6}} order: 6

See Matrix

$$[0, 0, -y_1 - y_2 + y_3 + y_4 + y_5, y_1, y_2, y_3, y_4, y_5]$$

$p = -s^5 + s^6$

42 . Coloring, {2, 6, 7}

R: [3, 8, 1, 1, 7, 4, 4, 5]
B: [6, 3, 8, 6, 2, 7, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	5 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{1, 3}} order: 6

See Matrix

$$[y_3, 0, y_2, y_1, y_3 - y_2 - y_1 + y_5 + y_4, 0, y_5, y_4]$$

$p = s^5 - s^6$

Omega Rank for B : cycles: {{2, 3, 8}} order: 6

See Matrix

$$[0, y_6, y_5, 0, y_2, y_3, y_1, y_4]$$

43 . Coloring, {2, 6, 8}

R: [3, 8, 1, 1, 7, 4, 5, 2]
B: [6, 3, 8, 6, 2, 7, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 7	3 *vs* 7

Omega Rank for R : cycles: {{2, 8}, {1, 3}, {5, 7}} order: 2

See Matrix

$$[2 y_2, y_2, y_1, 2 y_2 - y_1, y_2, 0, y_2, y_2]$$

$p' = s^3 - s^4$ $p = s^2 - s^5$ $p' = - s^4 + s^6$ $p' = - s^4 + s^5$ $p' = s^2 - s^4$

Omega Rank for B : cycles: {{4, 6, 7}, {2, 3, 5, 8}}

See Matrix

$$[0, y_3, y_3, y_2, y_3, 4 y_3 - y_2 - y_1, y_1, y_3]$$

$p' = s^2 - s^5$ $p' = s^3 - s^6$ $p = s - s^7$ $p' = s - s^4$

44 . Coloring, {2, 7, 8}

R: [3, 8, 1, 1, 7, 7, 4, 2]
B: [6, 3, 8, 6, 2, 4, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {1, 3}} order: 4

See Matrix

$$[6 y_2 - y_1 - y_3 - y_4, y_2, y_1, y_3, 0, 0, y_4, y_2]$$

$p = - s^3 + s^5$ $p' = - s^3 + s^5$

Omega Rank for B : cycles: {{4, 6}, {2, 3, 5, 8}} order: 4

See Matrix

$$[0, y_1, y_4, y_3, 4 y_1 - y_4 - 5 y_3 + 4 y_2, 3 y_1 - 4 y_3 + 3 y_2, 0, y_2]$$

$p' = - s + s^5$ $p = - s + s^5$

45 . Coloring, {3, 4, 5}

R: [3, 3, 8, 6, 2, 7, 5, 5]
B: [6, 8, 1, 1, 7, 4, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	6 *vs* 6	2 *vs* 6

Omega Rank for R : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

$$[0, y_2, y_1, 0, y_6, y_5, y_4, y_3]$$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

$$[2 y_2, y_2, 0, 2 y_2, 0, 2 y_2 - y_1, y_1, y_2]$$

$p = - s^2 + s^3$ $p = - s^2 + s^4$ $p = - s^2 + s^5$ $p = - s^2 + s^6$

46 . Coloring, {3, 4, 6}

R: [3, 3, 8, 6, 7, 4, 5, 5]
B: [6, 8, 1, 1, 2, 7, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {5, 7}} order: 4

See Matrix

$$[0, 0, 6\, y_2 - y_1 - y_3 - y_4, y_2, y_1, y_2, y_3, y_4]$$

$p = s^3 - s^5$ $p' = - s^3 + s^5$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6, 7}} order: 4

See Matrix

$$[y_2, y_1, 0, 4\, y_2 - 5\, y_1 - y_3 + 4\, y_4, 0, y_3, y_4, 3\, y_2 - 4\, y_1 + 3\, y_4]$$

$p' = - s + s^5$ $p = - s + s^5$

47 . Coloring, {3, 4, 7}

R: [3, 3, 8, 6, 7, 7, 4, 5]
B: [6, 8, 1, 1, 2, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	6 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{4, 6, 7}} order: 6

See Matrix

$$[0, 0, y_1, y_3, y_4, y_2, y_5, y_6]$$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}} order: 6

See Matrix

$$[2\, y_3 + 2\, y_4 - y_2 - y_1,\, y_3 + y_4,\, 0,\, y_2,\, y_3,\, y_1,\, 0,\, y_4]$$

$p = - s^{2} + s^{5}$ $p' = - s^{2} + s^{5}$

48 . Coloring, {3, 4, 8}

R: [3, 3, 8, 6, 7, 7, 5, 2]
B: [6, 8, 1, 1, 2, 4, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{2, 3, 8}, {5, 7}} order: 6

See Matrix

$$[0,\, -y_1 + 2\, y_2 - y_4,\, y_1,\, 0,\, y_3,\, -y_3 + y_2,\, y_2,\, y_4]$$

$p' = - s^{2} + s^{5}$ $p = - s^{2} + s^{5}$

Omega Rank for B : cycles: {{2, 5, 8}, {1, 4, 6}} order: 3

See Matrix

$$[-y_2 - y_1 + 5\, y_3,\, y_3,\, 0,\, y_2,\, y_3,\, y_1,\, 0,\, y_3]$$

$p = - s + s^{4}$ $p' = - s^{2} + s^{5}$ $p' = - s + s^{4}$

49 . Coloring, {3, 5, 6}

R: [3, 3, 8, 1, 2, 4, 5, 5]
B: [6, 8, 1, 6, 7, 7, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	6 *vs* 6	2 *vs* 6

Omega Rank for R : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

$$[y_1, y_4, y_2, y_3, y_5, 0, 0, y_6]$$

Omega Rank for B : cycles: {{2, 8}, {4, 6, 7}} order: 6

See Matrix

$$[2\ y_2 - y_1, y_2, 0, y_1, 0, 2\ y_2, 2\ y_2, y_2]$$

$p = -s^2 + s^3$ $p = -s^2 + s^4$ $p = -s^2 + s^5$ $p = -s^2 + s^6$

50 . Coloring, {3, 5, 7}

R: [3, 3, 8, 1, 2, 7, 4, 5]
B: [6, 8, 1, 6, 7, 4, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	4 *vs* 7	2 *vs* 7

Omega Rank for R : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

$$[-y_2 + y_3 + y_4, y_2, y_3 + y_4, y_3, y_4, 0, y_3 + y_4 - y_1, y_1]$$

$p = -s^4 + s^7$ $p = -s^4 + s^5$ $p = -s^4 + s^6$

Omega Rank for B : cycles: {{2, 8}, {4, 6}, {5, 7}} order: 2

See Matrix

$$[y_1, y_2, 0, -y_1 + 2\ y_2, y_2, 2\ y_2, y_2, y_2]$$

$p' = -s^2 + s^3$ $p' = -s^2 + s^4$ $p' = -s^2 + s^5$ $p' = -s^2 + s^6$ $p = s^2 - s^3$

51 . Coloring, {3, 5, 8}

R: [3, 3, 8, 1, 2, 7, 5, 2]
B: [6, 8, 1, 6, 7, 4, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	5 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{2, 3, 8}} order: 3

See Matrix

$$[y_4, y_3, y_1, 0, y_2, 0, y_4, y_5]$$

$p = - s^3 + s^6$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

$$[y_1, 0, 0, -y_3 + y_4 + y_2, y_3, y_4, y_2, y_1]$$

$p = - s^4 + s^6$ $p = - s^4 + s^5$

52 . Coloring, {3, 6, 7}

R: [3, 3, 8, 1, 7, 4, 4, 5]
B: [6, 8, 1, 6, 2, 7, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	6 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{1, 3, 4, 5, 7, 8}} order: 6

See Matrix

$$[y_1, 0, y_4, y_2, y_3, 0, y_5, y_6]$$

Omega Rank for B : cycles: {{2, 8}} order: 6

See Matrix

$$[-y_1 + y_2 + y_3 - y_4 + y_5, y_1, 0, 0, y_2, y_3, y_4, y_5]$$

$p = - s^5 + s^6$

53 . Coloring, {3, 6, 8}

R: [3, 3, 8, 1, 7, 4, 5, 2]
B: [6, 8, 1, 6, 2, 7, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	3 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{2, 3, 8}, {5, 7}} order: 6

See Matrix

$$[-y_1 + 2 y_3, y_1, 2 y_3, 2 y_3 - y_2, y_3, 0, y_3, y_2]$$

$p = - s^3 + s^4$ $p = - s^3 + s^5$ $p = - s^3 + s^7$ $p = - s^3 + s^6$

Omega Rank for B : cycles: {{4, 6, 7}, {2, 5, 8}} order: 3

See Matrix

$$[-y_1 + 5 y_4 - y_2 - y_3, y_4, 0, y_1, y_4, y_2, y_3, y_4]$$

$p = s^2 - s^5$ $p' = - s^2 + s^5$ $p' = - s^3 + s^6$

54 . Coloring, {3, 7, 8}

R: [3, 3, 8, 1, 7, 7, 4, 2]
B: [6, 8, 1, 6, 2, 4, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	6 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{2, 3, 8}} order: 6

See Matrix

$$[y_1, y_2, y_5, y_3, 0, 0, y_6, y_4]$$

Omega Rank for B : cycles: {{4, 6}, {2, 5, 8}} order: 6

See Matrix

$$[-y_3 + y_1, 2 y_1 - y_2 - y_4, 0, y_3, y_2, y_1, 0, y_4]$$

$p = - s^2 + s^5$ $p' = - s^2 + s^5$

55 . Coloring, {4, 5, 6}

R: [3, 3, 1, 6, 2, 4, 5, 5]
B: [6, 8, 8, 1, 7, 7, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {1, 3}} order: 4

See Matrix

$$[y_4, y_3, y_2, y_1, -y_4 - y_3 - y_2 + 6 y_1, y_1, 0, 0]$$

$p' = - s^3 + s^5$ $p = - s^3 + s^5$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6, 7}} order: 4

See Matrix

$$[-5 y_1 + 4 y_2 + 4 y_3 - y_4, y_1, 0, y_2, 0, y_3, y_4, -4 y_1 + 3 y_2 + 3 y_3]$$

$p = - s + s^5$ $p' = - s + s^5$

56 . Coloring, {4, 5, 7}

R: [3, 3, 1, 6, 2, 7, 4, 5]
B: [6, 8, 8, 1, 7, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	4 *vs* 7	2 *vs* 7

Omega Rank for R : cycles: {{1, 3}, {4, 6, 7}} order: 6

See Matrix

$$[y_1, y_2, y_3, y_4, -y_1 - y_2 - y_3 + 5\, y_4, y_4, y_4, 0]$$

$p = -s^3 + s^5$ $p' = -s^3 + s^5$ $p = -s^3 + s^7$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}, {5, 7}} order: 6

See Matrix

$$[y_2, 3\, y_2 - y_1, 0, y_2, y_2, y_2, y_2, y_1]$$

$p = -s + s^5$ $p = -s + s^7$ $p = -s + s^3$ $p' = -s + s^5$ $p' = -s + s^3$

57 . Coloring, {4, 5, 8}

R: [3, 3, 1, 6, 2, 7, 5, 2]
B: [6, 8, 8, 1, 7, 4, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	5 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{1, 3}} order: 6

See Matrix

$$[y_1, -y_1 + y_2 + y_3 + y_4 - y_5, y_2, 0, y_3, y_4, y_5, 0]$$

$p = -s^5 + s^6$

Omega Rank for B : cycles: {{1, 4, 6}} order: 6

See Matrix

$$[y_3, 0, 0, y_2, y_1, y_4, y_5, y_6]$$

58 . Coloring, {4, 6, 7}

R: [3, 3, 1, 6, 7, 4, 4, 5]
B: [6, 8, 8, 1, 2, 7, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {1, 3}} order: 4

See Matrix

$$[y_1, 0, -4 y_1 + 3 y_3 + 3 y_4, -5 y_1 - y_2 + 4 y_3 + 4 y_4, y_2, y_3, y_4, 0]$$

$p = - s^3 + s^5$ $p' = - s^3 + s^5$

Omega Rank for B : cycles: {{2, 8}} order: 6

See Matrix

$$[-y_1 + y_2 + y_3 - y_4 + y_5, y_1, 0, 0, y_2, y_3, y_4, y_5]$$

$p = - s^5 + s^6$

59 . Coloring, {4, 6, 8}

R: [3, 3, 1, 6, 7, 4, 5, 2]
B: [6, 8, 8, 1, 2, 7, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 7	3 *vs* 7

Omega Rank for R : cycles: {{4, 6}, {1, 3}, {5, 7}} order: 2

See Matrix

$$[y_1, -y_1 + 2 y_2, 2 y_2, y_2, y_2, y_2, y_2, 0]$$

$p' = - s^2 + s^3$ $p' = - s^2 + s^4$ $p' = - s^2 + s^5$ $p' = - s^2 + s^6$ $p = s^2 - s^3$

Omega Rank for B : cycles: {{2, 5, 8}, {1, 4, 6, 7}}

See Matrix

$$[y_2, 4 y_2 - y_1 - y_3, 0, y_2, y_1, y_2, y_2, y_3]$$

$p = - s + s^4$ $p' = - s + s^4$ $p' = - s^2 + s^5$ $p = - s + s^7$

60 . Coloring, {4, 7, 8}

R: [3, 3, 1, 6, 7, 7, 4, 2]
B: [6, 8, 8, 1, 2, 4, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{1, 3}, {4, 6, 7}} order: 6

See Matrix

$$[y_1, y_2, y_1 + y_2, 2 y_1 + 2 y_2 - y_3 - y_4, 0, y_3, y_4, 0]$$

$p = - s^2 + s^5$ $p' = - s^2 + s^5$

Omega Rank for B : cycles: {{2, 5, 8}, {1, 4, 6}} order: 3

See Matrix

$$[y_2, 5 y_2 - y_1 - y_3, 0, y_2, y_1, y_2, 0, y_3]$$

$p' = s^2 - s^5$ $p = - s + s^4$ $p' = - s + s^4$

61 . Coloring, {5, 6, 7}

R: [3, 3, 1, 1, 2, 4, 4, 5]
B: [6, 8, 8, 6, 7, 7, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$[y_4, y_3, y_2, 2 y_1, y_1, 0, 0, 0]$$

$p = - s^3 + s^5$

Omega Rank for B : cycles: {{2, 8}, {5, 7}} order: 2

See Matrix

$$[0, y_1, 0, 0, -5 y_1 - y_2 + 4 y_3, y_2, y_3, -4 y_1 + 3 y_3]$$

$p' = s^2 - s^4$ $p = s^2 - s^4$

62 . Coloring, {5, 6, 8}

R: [3, 3, 1, 1, 2, 4, 5, 2]
B: [6, 8, 8, 6, 7, 7, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$$[-y_1 + y_2 + 2 y_3, y_1, y_2, y_3, y_3, 0, 0, 0]$$

$p = - s^3 + s^4$ $p = - s^3 + s^5$

Omega Rank for B : cycles: {{4, 6, 7}} order: 3

See Matrix

$$[0, 0, 0, y_4, y_5, y_1, y_2, y_3]$$

63 . Coloring, {5, 7, 8}

R: [3, 3, 1, 1, 2, 7, 4, 2]
B: [6, 8, 8, 6, 7, 4, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$[y_3, 2\,y_4, y_1, y_2, 0, 0, y_4, 0]$

$p = -s^3 + s^5$

Omega Rank for B : cycles: {{4, 6}, {5, 7}} order: 2

See Matrix

$[0, 0, 0, y_1, y_2, -4\,y_1 + 3\,y_2, -5\,y_1 + 4\,y_2 - y_3, y_3]$

$p = -s^2 + s^4$ $p' = -s^2 + s^4$

64 . Coloring, {6, 7, 8}

R: [3, 3, 1, 1, 7, 4, 4, 2]
B: [6, 8, 8, 6, 2, 7, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{1, 3}} order: 4

See Matrix

$[y_1, y_2, y_3, y_1 + 2\,y_2 - y_3, 0, 0, y_2, 0]$

$p = -s^3 + s^5$ $p = -s^3 + s^4$

Omega Rank for B : cycles: {{2, 5, 8}} order: 3

See Matrix

$[0, y_1, 0, 0, y_2, y_3, y_4, y_5]$

65 . Coloring, {2, 3, 4, 5}

R: [3, 8, 8, 6, 2, 7, 5, 5]
B: [6, 3, 1, 1, 7, 4, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{2, 5, 8}} order: 3

See Matrix

$$[0, y_1, y_5, 0, y_2, y_5, y_3, y_4]$$

$p = s^3 - s^6$

Omega Rank for B : cycles: {{1, 4, 6}} order: 3

See Matrix

$$[y_3, y_4, y_2, y_1, 0, y_5, y_4, 0]$$

$p = s^3 - s^6$

66 . Coloring, {2, 3, 4, 6}

R: [3, 8, 8, 6, 7, 4, 5, 5]
B: [6, 3, 1, 1, 2, 7, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	3 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {5, 7}} order: 4

See Matrix

$$[0, 0, y_3, y_2, -y_3 + 3 y_2, y_2, y_1, 3 y_2 - y_1]$$

$p' = s^4 - s^5$ $p = s^3 - s^6$ $p' = s^3 - s^5$

Omega Rank for B : cycles: {{1, 4, 6, 7}} order: 4

See Matrix

$$[y_1, y_2, y_3, y_4, 0, y_5, y_6, 0]$$

67 . Coloring, {2, 3, 4, 7}

R: [3, 8, 8, 6, 7, 7, 4, 5]
B: [6, 3, 1, 1, 2, 4, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	6 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{4, 6, 7}} order: 6

See Matrix

$$[0, 0, y_5, y_6, y_2, y_3, y_4, y_1]$$

Omega Rank for B : cycles: {{1, 4, 6}} order: 6

See Matrix

$$[y_3, y_2, y_1, y_6, y_4, y_5, 0, 0]$$

68 . Coloring, {2, 3, 4, 8}

R: [3, 8, 8, 6, 7, 7, 5, 2]
B: [6, 3, 1, 1, 2, 4, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {5, 7}} order: 2

See Matrix

$$[0, -y_1 + y_2, y_1, 0, -y_1 + y_2, y_1, y_2, y_2]$$

$p = s^2 - s^5$ $p' = s^2 - s^4$ $p' = - s^4 + s^5$ $p' = s^3 - s^4$

Omega Rank for B : cycles: {{1, 4, 6}} order: 6

See Matrix

$$[y_1, y_2, y_3, y_4, y_5, y_6, 0, 0]$$

69 . Coloring, {2, 3, 5, 6}

$\Omega p(\Delta)=0$: $p = s^2$ $p' = s^3$ $p' = s^2$ $p' = s^4$ $p' = s^5$

R: [3, 8, 8, 1, 2, 4, 5, 5]
B: [6, 3, 1, 6, 7, 7, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 *vs* 6	6 *vs* 6	6 *vs* 6	6 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{2, 5, 8}} order: 6

See Matrix

$[y_4, y_5, y_1, y_2, y_3, 0, 0, y_6]$

Omega Rank for B : cycles: {{4, 6, 7}} order: 6

See Matrix

$[y_2, y_1, y_5, y_6, 0, y_3, y_4, 0]$

70 . Coloring, {2, 3, 5, 7}

$\Omega p(\Delta)=0$: $p = s - 2s^3 + 8s^6$

R: [3, 8, 8, 1, 2, 7, 4, 5]
B: [6, 3, 1, 6, 7, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 *vs* 6	7 *vs* 7	7 *vs* 7	7 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{2, 5, 8}} order: 6

See Matrix

$[y_4, y_2, y_3, y_1, y_7, 0, y_6, y_5]$

Omega Rank for B : cycles: {{4, 6}, {5, 7}} order: 4

See Matrix

$$[-y_1 - y_4 + 3\ y_3, y_1, -y_2 + 3\ y_3, y_4, y_3, y_2, y_3, 0]$$

$p' = s^4 - s^6$ $p' = s^5 - s^6$ $p = s^4 - s^7$

71 . Coloring, {2, 3, 5, 8}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^6$ $p' = s^3 + 2s^5$ $p' = s^2 + 2s^4$

R: [3, 8, 8, 1, 2, 7, 5, 2]
B: [6, 3, 1, 6, 7, 4, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 6	3 *vs* 6	3 *vs* 6	3 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{2, 8}} order: 4

See Matrix

$$[y_2, y_3, y_1, 0, y_1, 0, y_2, y_3]$$

$p = - s^3 + s^6$ $p = - s^3 + s^4$ $p = - s^3 + s^5$

Omega Rank for B : cycles: {{4, 6}} order: 4

See Matrix

$$[y_3, 0, y_1, y_2, y_1, y_2, y_3, 0]$$

$p = s^3 - s^6$ $p' = s^3 - s^5$ $p' = s^4 - s^5$

M \; N

$$\begin{bmatrix} 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&5&6&3&3&6&9&4 \\ 5&0&3&4&6&5&4&9 \\ 6&3&0&5&9&4&3&6 \\ 3&4&5&0&4&9&6&5 \\ 3&6&9&4&0&5&6&3 \\ 6&5&4&9&5&0&3&4 \\ 9&4&3&6&6&3&0&5 \\ 4&9&6&5&3&4&5&0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 8, 8, 1, 2, 7, 5, 2]
B: [6, 3, 1, 6, 7, 4, 4, 5]

Ranges

Action of R on ranges, [[3], [2], [2], [1]]
Action of B on ranges, [[4], [3], [1], [4]]

Cycles: R , {{2, 8}}, B , {{4, 6}}

β({1, 7}) = 1/4
β({2, 8}) = 1/4
β({3, 5}) = 1/4
β({4, 6}) = 1/4

Partitions

Action of R on partitions, [[6], [5], [3], [3], [4], [4], [5], [6]]
Action of B on partitions, [[6], [2], [8], [2], [6], [4], [8], [4]]

α([{2, 4, 5, 7}, {1, 3, 6, 8}]) = 0/1
α([{1, 2, 4, 5}, {3, 6, 7, 8}]) = 2/9
α([{2, 3, 4, 7}, {1, 5, 6, 8}]) = 2/9
α([{1, 4, 5, 8}, {2, 3, 6, 7}]) = 2/9
α([{1, 2, 3, 6}, {4, 5, 7, 8}]) = 1/9
α([{1, 2, 3, 4}, {5, 6, 7, 8}]) = 1/9
α([{3, 4, 7, 8}, {1, 2, 5, 6}]) = 0/1
α([{1, 3, 4, 8}, {2, 5, 6, 7}]) = 1/9

b1 = {1, 2, 3, 6} , b2 = {2, 4, 5, 7} , b3 = {1, 3, 6, 8} , b4 = {1, 2, 4, 5} , b5 = {3, 6, 7, 8} , b6 = {2, 3, 4, 7} , b7 = {1, 5, 6, 8} , b8 = {1, 4, 5, 8} , b9 = {2, 3, 6, 7} , b10 = {1, 2, 3, 4} , b11 = {5, 6, 7, 8} , b12 = {3, 4, 7, 8} , b13 = {4, 5, 7, 8} , b14 = {1, 2, 5, 6} , b15 = {1, 3, 4, 8} , b16 = {2, 5, 6, 7}

Action of R and B on the blocks of the partitions: = [8, B, A, D, 1, 7, 6, 6, 7, 8, 9, 1, 9, D, A, B] [A, B, A, 5, 4, 10, F, 5, 4, 9, 8, 10, B, F, 9, 8]
with invariant measure [1, 0, 0, 2, 2, 2, 2, 2, 2, 1, 1, 0, 1, 0, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{2, 3, 5, 8}
Rank	2
R,B	[3, 8, 8, 1, 2, 7, 5, 2], [6, 3, 1, 6, 7, 4, 4, 5]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]
u_2	[5, 6, 3, 3, 6, 9, 4, 3, 4, 6, 5, 4, 9, 5, 9, 4, 3, 6, 4, 9, 6, 5, 5, 6, 3, 3, 4, 5] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

72 . Coloring, {2, 3, 6, 7}

$\Omega p(\Delta)=0$: $p' = s^2 + 2s^4$ $p' = s^3 + 2s^5$ $p = s^2 - 4s^6$

R: [3, 8, 8, 1, 7, 4, 4, 5]
B: [6, 3, 1, 6, 2, 7, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 6	3 *vs* 6	3 *vs* 6	3 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{1, 3, 4, 5, 7, 8}} order: 6

See Matrix

$[y_1, 0, y_2, y_3, y_1, 0, y_2, y_3]$

$p' = - s + s^4$ $p' = - s^2 + s^5$ $p = - s + s^4$

Omega Rank for B : cycles: {{1, 2, 3, 5, 6, 7}} order: 6

See Matrix

$[y_2, y_3, y_1, 0, y_2, y_3, y_1, 0]$

$p = - s + s^4$ $p' = - s + s^4$ $p' = - s^2 + s^5$

M \; N

$$\begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&3&2&1&5&2&3&4 \\ 3&0&1&4&2&5&4&1 \\ 2&1&0&3&3&4&5&2 \\ 1&4&3&0&4&1&2&5 \\ 5&2&3&4&0&3&2&1 \\ 2&5&4&1&3&0&1&4 \\ 3&4&5&2&2&1&0&3 \\ 4&1&2&5&1&4&3&0 \end{bmatrix}$$

$\tau = 32$, $r' = 1/2$

R: [3, 8, 8, 1, 7, 4, 4, 5]
B: [6, 3, 1, 6, 2, 7, 5, 2]

Ranges

Action of R on ranges, [[3], [4], [4], [1]]
Action of B on ranges, [[2], [3], [1], [2]]

Cycles: R , {{1, 3, 4, 5, 7, 8}}, B , {{1, 2, 3, 5, 6, 7}}

β({1, 5}) = 1/4
β({2, 6}) = 1/4
β({3, 7}) = 1/4
β({4, 8}) = 1/4

Partitions

Action of R on partitions, [[3], [1], [2], [2]]
Action of B on partitions, [[2], [4], [2], [3]]

α([{4, 5, 6, 7}, {1, 2, 3, 8}]) = 1/5
α([{1, 4, 6, 7}, {2, 3, 5, 8}]) = 2/5
α([{1, 2, 3, 4}, {5, 6, 7, 8}]) = 1/5
α([{2, 5, 7, 8}, {1, 3, 4, 6}]) = 1/5

b1 = {1, 4, 6, 7} , b2 = {1, 2, 3, 4} , b3 = {5, 6, 7, 8} , b4 = {4, 5, 6, 7} , b5 = {1, 2, 3, 8} , b6 = {2, 3, 5, 8} , b7 = {2, 5, 7, 8} , b8 = {1, 3, 4, 6}

Action of R and B on the blocks of the partitions: = [4, 1, 6, 3, 2, 5, 6, 1] [8, 6, 1, 1, 6, 7, 3, 2]
with invariant measure [2, 1, 1, 1, 1, 2, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{2, 3, 6, 7}
Rank	2
R,B	[3, 8, 8, 1, 7, 4, 4, 5], [6, 3, 1, 6, 2, 7, 5, 2]
π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[3, 2, 1, 5, 2, 3, 4, 1, 4, 2, 5, 4, 1, 3, 3, 4, 5, 2, 4, 1, 2, 5, 3, 2, 1, 1, 4, 3] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

73 . Coloring, {2, 3, 6, 8}

Ωp(Δ)=0: $p = s - 2s^3 - 8s^6$

R: [3, 8, 8, 1, 7, 4, 5, 2]
B: [6, 3, 1, 6, 2, 7, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 7	7 *vs* 7

Omega Rank for R : cycles: {{2, 8}, {5, 7}} order: 4

See Matrix

$$[3\,y_3 - y_4, -y_1 - y_2 + 3\,y_3, y_1, y_2, y_3, 0, y_3, y_4]$$

$p = -s^4 + s^5$ $p = -s^4 + s^6$ $p = -s^4 + s^7$

Omega Rank for B : cycles: {{4, 6, 7}} order: 6

See Matrix

$$[y_1, y_2, y_7, y_4, y_5, y_6, y_3, 0]$$

74 . Coloring, {2, 3, 7, 8}

Ωp(Δ)=0: $p = s^4$ $p' = s^4$ $p' = s^5$

R: [3, 8, 8, 1, 7, 7, 4, 2]
B: [6, 3, 1, 6, 2, 4, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 6	6 *vs* 6	6 *vs* 6	6 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{2, 8}} order: 6

See Matrix

$$[y_3, y_2, y_1, y_6, 0, 0, y_5, y_4]$$

Omega Rank for B : cycles: {{4, 6}} order: 6

See Matrix

$$[y_6, y_5, y_4, y_3, y_2, y_1, 0, 0]$$

75 . Coloring, {2, 4, 5, 6}

$\Omega p(\Delta)=0$: $p = s^2 + s^3 - 4s^6$

R: [3, 8, 1, 6, 2, 4, 5, 5]
B: [6, 3, 8, 1, 7, 7, 4, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{4, 6}, {1, 3}, {2, 5, 8}} order: 6

See Matrix

$$[y_2, 4y_2 - y_1 - y_3, y_2, y_2, y_1, y_2, 0, y_3]$$

$p' = s^3 - s^6$ $p' = s^2 - s^5$ $p' = s - s^4$ $p = s - s^7$

Omega Rank for B : cycles: {{1, 4, 6, 7}, {2, 3, 8}}

See Matrix

$$[5y_3 - y_1 - y_2 - y_4, y_3, y_3, y_1, 0, y_2, y_4, y_3]$$

$p = -s + s^5$ $p' = -s + s^5$ $p' = -s^2 + s^6$

76 . Coloring, {2, 4, 5, 7}

$\Omega p(\Delta)=0$: $p' = s^5$ $p = s$ $p' = s$ $p' = s^2$ $p' = s^4$ $p' = s^3$

R: [3, 8, 1, 6, 2, 7, 4, 5]
B: [6, 3, 8, 1, 7, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
0 *vs* 6	1 *vs* 8	1 *vs* 8	1 *vs* 8	1 *vs* 8

Omega Rank for R : cycles: {{1, 3}, {4, 6, 7}, {2, 5, 8}} order: 6

See Matrix

$$[y_1, y_1, y_1, y_1, y_1, y_1, y_1, y_1]$$

$p' = -s + s^4$ $p' = -s + s^7$ $p' = -s + s^6$ $p' = -s + s^5$ $p' = -s + s^3$ $p' = -s + s^2$ $p' = 1 - s$

Omega Rank for B : cycles: {{1, 4, 6}, {5, 7}, {2, 3, 8}} order: 6

See Matrix

$$[y_1, y_1, y_1, y_1, y_1, y_1, y_1, y_1]$$

$p' = -1 + s \quad p' = -1 + s^2 \quad p' = -1 + s^3 \quad p' = -1 + s^4 \quad p' = -1 + s^5 \quad p' = -1 + s^6 \quad p' = -1 + s^7$

See 8-level graph

M \; N

$$\begin{bmatrix} 0&1&1&1&1&1&1&1\\ 1&0&1&1&1&1&1&1\\ 1&1&0&1&1&1&1&1\\ 1&1&1&0&1&1&1&1\\ 1&1&1&1&0&1&1&1\\ 1&1&1&1&1&0&1&1\\ 1&1&1&1&1&1&0&1\\ 1&1&1&1&1&1&1&0 \end{bmatrix} \quad \begin{bmatrix} 0&1&1&1&1&1&1&1\\ 1&0&1&1&1&1&1&1\\ 1&1&0&1&1&1&1&1\\ 1&1&1&0&1&1&1&1\\ 1&1&1&1&0&1&1&1\\ 1&1&1&1&1&0&1&1\\ 1&1&1&1&1&1&0&1\\ 1&1&1&1&1&1&1&0 \end{bmatrix}$$

τ= 8 , r'= 7/8

R: [3, 8, 1, 6, 2, 7, 4, 5]
B: [6, 3, 8, 1, 7, 4, 5, 2]

Ranges

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

Cycles: R , {{1, 3}, {4, 6, 7}, {2, 5, 8}}, B , {{1, 4, 6}, {5, 7}, {2, 3, 8}}

β({1, 2, 3, 4, 5, 6, 7, 8}) = 1/1

Partitions
α([{8}, {1}, {3}, {4}, {2}, {5}, {6}, {7}]) = 1/1

b1 = {8} , b2 = {1} , b3 = {3} , b4 = {4} , b5 = {2} , b6 = {5} , b7 = {6} , b8 = {7}

Action of R and B on the blocks of the partitions: = [5, 3, 2, 8, 6, 1, 4, 7] [3, 4, 5, 7, 1, 8, 2, 6]
with invariant measure [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . See partition graph.

See level-8 partition graph.

Right Group	
Coloring	{2, 4, 5, 7}

Rank	8
R,B	[3, 8, 1, 6, 2, 7, 4, 5], [6, 3, 8, 1, 7, 4, 5, 2]
π_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
u_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] (dim 2)
wpp	[1, 1, 1, 1, 1, 1, 1, 1]
π_8	[1]
u_8	[1]

77 . Coloring, {2, 4, 5, 8}

$\Omega p(\Delta)=0$: $p = s - 2s^3 - 8s^6$

R: [3, 8, 1, 6, 2, 7, 5, 2]
B: [6, 3, 8, 1, 7, 4, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
5 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 7	7 *vs* 7

Omega Rank for R : cycles: {{2, 8}, {1, 3}} order: 4

See Matrix

$[y_3, y_2, y_3, 0, y_4, 3\,y_3 - y_4 - y_1, 3\,y_3 - y_2, y_1]$

$p = -s^4 + s^6$ $p = -s^4 + s^7$ $p = -s^4 + s^5$

Omega Rank for B : cycles: {{1, 4, 6}} order: 6

See Matrix

$[y_1, 0, y_2, y_3, y_6, y_4, y_5, y_7]$

78 . Coloring, {2, 4, 6, 7}

$\Omega p(\Delta)=0$: $p = s - 2s^3 - 8s^6$

R: [3, 8, 1, 6, 7, 4, 4, 5]
B: [6, 3, 8, 1, 2, 7, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 7	7 *vs* 7

Omega Rank for R : cycles: {{4, 6}, {1, 3}} order: 4

See Matrix

$$[y_4, 0, y_4, y_3, 3\,y_4 - y_3, y_2, 3\,y_4 - y_2 - y_1, y_1]$$

$p' = -s^5 + s^6$ $p = s^4 - s^6$ $p' = s^4 - s^5$

Omega Rank for B : cycles: {{2, 3, 8}} order: 6

See Matrix

$$[y_1, y_2, y_4, 0, y_3, y_5, y_6, y_7]$$

79 . Coloring, {2, 4, 6, 8}

Ωp(Δ)=0: $p' = s^3$ $p' = s^4$ $p' = s^5$ $p' = s^2$ $p' = s$ $p = s$

R: [3, 8, 1, 6, 7, 4, 5, 2]
B: [6, 3, 8, 1, 2, 7, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
0 *vs* 6	1 *vs* 8	1 *vs* 8	1 *vs* 8	1 *vs* 8

Omega Rank for R : cycles: {{2, 8}, {4, 6}, {1, 3}, {5, 7}} order: 2

See Matrix

$$[y_1, y_1, y_1, y_1, y_1, y_1, y_1, y_1]$$

$p' = -s + s^7$ $p' = -s + s^6$ $p' = -s + s^5$ $p' = -s + s^3$ $p' = -s + s^4$ $p' = -s + s^2$ $p' = 1 - s$

Omega Rank for B : cycles: {{1, 4, 6, 7}, {2, 3, 5, 8}} order: 4

See Matrix

$[y_1, y_1, y_1, y_1, y_1, y_1, y_1, y_1]$

$p' = -1 + s^4$ $p' = -1 + s^5$ $p' = -1 + s^6$ $p' = -1 + s$ $p' = -1 + s^2$ $p' = -1 + s^3$ $p' = -1 + s^7$

See 8-level graph

M \; N

$$\begin{bmatrix} 0&1&1&1&1&1&1&1\\ 1&0&1&1&1&1&1&1\\ 1&1&0&1&1&1&1&1\\ 1&1&1&0&1&1&1&1\\ 1&1&1&1&0&1&1&1\\ 1&1&1&1&1&0&1&1\\ 1&1&1&1&1&1&0&1\\ 1&1&1&1&1&1&1&0 \end{bmatrix} \quad \begin{bmatrix} 0&1&1&1&1&1&1&1\\ 1&0&1&1&1&1&1&1\\ 1&1&0&1&1&1&1&1\\ 1&1&1&0&1&1&1&1\\ 1&1&1&1&0&1&1&1\\ 1&1&1&1&1&0&1&1\\ 1&1&1&1&1&1&0&1\\ 1&1&1&1&1&1&1&0 \end{bmatrix}$$

τ= 8 , r'= 7/8

R: [3, 8, 1, 6, 7, 4, 5, 2]
B: [6, 3, 8, 1, 2, 7, 4, 5]

Ranges

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

Cycles: R , {{2, 8}, {4, 6}, {1, 3}, {5, 7}}, B , {{1, 4, 6, 7}, {2, 3, 5, 8}}

β({1, 2, 3, 4, 5, 6, 7, 8}) = 1/1

Partitions
α([{8}, {1}, {3}, {4}, {2}, {5}, {6}, {7}]) = 1/1

b1 = {8} , b2 = {1} , b3 = {3} , b4 = {4} , b5 = {2} , b6 = {5} , b7 = {6} , b8 = {7}

Action of R and B on the blocks of the partitions: = [5, 3, 2, 7, 1, 8, 4, 6] [3, 4, 5, 8, 6, 1, 2, 7]
with invariant measure [1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . See partition graph.

See level-8 partition graph.

Right Group	
Coloring	{2, 4, 6, 8}
Rank	8

R,B	[3, 8, 1, 6, 7, 4, 5, 2], [6, 3, 8, 1, 2, 7, 4, 5]
π_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
u_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] (dim 4)
wpp	[1, 1, 1, 1, 1, 1, 1, 1]
π_8	[1]
u_8	[1]

80 . Coloring, {2, 4, 7, 8}

$\Omega p(\Delta)=0$: $p = s^2 + s^3 - 4s^6$

R: [3, 8, 1, 6, 7, 7, 4, 2]
B: [6, 3, 8, 1, 2, 4, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{2, 8}, {4, 6, 7}, {1, 3}} order: 6

See Matrix

$$[y_3, y_3, y_3, 4\,y_3 - y_2 - y_1, 0, y_2, y_1, y_3]$$

$p = -s + s^4$ $p' = -s + s^4$ $p' = -s^2 + s^5$ $p = -s + s^7$

Omega Rank for B : cycles: {{1, 4, 6}, {2, 3, 5, 8}}

See Matrix

$$[y_3, y_4, y_2, y_3, y_1, y_3, 0, 5\,y_3 - y_4 - y_2 - y_1]$$

$p = -s + s^5$ $p' = -s + s^5$ $p' = -s^2 + s^6$

81 . Coloring, {2, 5, 6, 7}

R: [3, 8, 1, 1, 2, 4, 4, 5]
B: [6, 3, 8, 6, 7, 7, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	3 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{1, 3}, {2, 5, 8}} order: 6

See Matrix

$$[y_2, y_3, -y_2 + 5\, y_3 - y_1, y_1, y_3, 0, 0, y_3]$$

$p' = - s^{2} + s^{4}$ $p = - s^{2} + s^{6}$ $p = - s^{2} + s^{4}$

Omega Rank for B : cycles: {{2, 3, 8}, {5, 7}} order: 6

See Matrix

$$[0, y_3, y_3, 0, y_1, y_2, 5\, y_3 - y_1 - y_2, y_3]$$

$p = s^{2} - s^{4}$ $p' = - s^{3} + s^{5}$ $p' = s^{2} - s^{4}$

82 . Coloring, {2, 5, 6, 8}

R: [3, 8, 1, 1, 2, 4, 5, 2]
B: [6, 3, 8, 6, 7, 7, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {1, 3}} order: 2

See Matrix

$$[y_2, y_2, y_1, y_2 - y_1, y_2 - y_1, 0, 0, y_1]$$

$p' = - s^{2} + s^{5}$ $p' = - s^{2} + s^{4}$ $p' = - s^{2} + s^{3}$ $p = s^{2} - s^{3}$

Omega Rank for B : cycles: {{4, 6, 7}} order: 6

See Matrix

$$[0, 0, y_6, y_5, y_4, y_3, y_2, y_1]$$

83 . Coloring, {2, 5, 7, 8}

R: [3, 8, 1, 1, 2, 7, 4, 2]
B: [6, 3, 8, 6, 7, 4, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	4 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {1, 3}} order: 4

See Matrix

$$[y_1, y_2, 4\,y_1 - 5\,y_2 - y_3 + 4\,y_4, y_3, 0, 0, y_4, 3\,y_1 - 4\,y_2 + 3\,y_4]$$

$p = s^3 - s^5$ $p' = s^3 - s^5$

Omega Rank for B : cycles: {{4, 6}, {5, 7}} order: 4

See Matrix

$$[0, 0, -5\,y_1 - y_3 + 4\,y_2 + 4\,y_4, y_1, y_3, -4\,y_1 + 3\,y_2 + 3\,y_4, y_2, y_4]$$

$p = s^3 - s^5$ $p' = s^3 - s^5$

84 . Coloring, {2, 6, 7, 8}

R: [3, 8, 1, 1, 7, 4, 4, 2]
B: [6, 3, 8, 6, 2, 7, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	3 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {1, 3}} order: 4

See Matrix

$$[3\,y_3 - y_2, y_3, 3\,y_3 - y_1, y_1, 0, 0, y_2, y_3]$$

$p = -s^3 + s^4$ $p = -s^3 + s^6$ $p = -s^3 + s^5$

Omega Rank for B : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

$$[0, y_4, y_1, 0, y_2, y_3, y_5, y_6]$$

85 . Coloring, {3, 4, 5, 6}

$\Omega p(\Delta)=0$: $p' = s^3 - 2s^5$ $p = s^2 - 4s^6$ $p' = s^2 - 2s^4$

R: [3, 3, 8, 6, 2, 4, 5, 5]
B: [6, 8, 1, 1, 7, 7, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 6	3 *vs* 6	3 *vs* 6	2 *vs* 6	2 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {2, 3, 5, 8}} order: 4

See Matrix

$$[0, y_2, -y_2 + 3\, y_1, y_1, -y_2 + 3\, y_1, y_1, 0, y_2]$$

$p' = - s + s^3$ $p = - s + s^5$ $p = - s + s^3$ $p' = - s + s^5$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6, 7}} order: 4

See Matrix

$$[y_2, y_1, 0, -y_2 + 3\, y_1, 0, -y_2 + 3\, y_1, y_2, y_1]$$

$p = s - s^3$ $p' = - s + s^3$ $p' = - s^2 + s^4$ $p' = - s + s^5$

M \; N

$$\begin{bmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&3&4&5&5&4&9&6\\ 3&0&5&6&4&3&6&9\\ 4&5&0&3&9&6&5&4\\ 5&6&3&0&6&9&4&3\\ 5&4&9&6&0&3&4&5\\ 4&3&6&9&3&0&5&6\\ 9&6&5&4&4&5&0&3\\ 6&9&4&3&5&6&3&0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 3, 8, 6, 2, 4, 5, 5]
B: [6, 8, 1, 1, 7, 7, 4, 2]

Ranges

Action of R on ranges, [[3], [3], [2], [4]]
Action of B on ranges, [[4], [2], [1], [1]]

Cycles: R , {{4, 6}, {2, 3, 5, 8}}, B , {{2, 8}, {1, 4, 6, 7}}

β({1, 7}) = 1/4
β({2, 8}) = 1/4
β({3, 5}) = 1/4
β({4, 6}) = 1/4

Partitions

Action of R on partitions, [[4], [6], [5], [3], [6], [4]]
Action of B on partitions, [[2], [5], [6], [1], [6], [1]]

α([{1, 3, 4, 8}, {2, 5, 6, 7}]) = 2/9
α([{2, 3, 4, 7}, {1, 5, 6, 8}]) = 1/9
α([{1, 2, 4, 5}, {3, 6, 7, 8}]) = 1/9
α([{1, 2, 3, 6}, {4, 5, 7, 8}]) = 2/9
α([{1, 2, 3, 4}, {5, 6, 7, 8}]) = 1/9
α([{3, 4, 7, 8}, {1, 2, 5, 6}]) = 2/9

b1 = {1, 2, 3, 6} , b2 = {1, 3, 4, 8} , b3 = {2, 5, 6, 7} , b4 = {2, 3, 4, 7} , b5 = {1, 5, 6, 8} , b6 = {4, 5, 7, 8} , b7 = {1, 2, 4, 5} , b8 = {3, 6, 7, 8} , b9 = {1, 2, 3, 4} , b10 = {5, 6, 7, 8} , b11 = {3, 4, 7, 8} , b12 = {1, 2, 5, 6}

Action of R and B on the blocks of the partitions: = [7, 1, 6, C, B, 8, A, 9, C, B, 1, 6] [2, 4, 5, A, 9, 3, B, C, B, C, 3, 2] with invariant measure [2, 2, 2, 1, 1, 2, 1, 1, 1, 1, 2, 2]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{3, 4, 5, 6}
Rank	2
R,B	[3, 3, 8, 6, 2, 4, 5, 5], [6, 8, 1, 1, 7, 7, 4, 2]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]
u_2	[3, 4, 5, 5, 4, 9, 6, 5, 6, 4, 3, 6, 9, 3, 9, 6, 5, 4, 6, 9, 4, 3, 3, 4, 5, 5, 6, 3] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

86 . Coloring, {3, 4, 5, 7}

$\Omega p(\Delta)=0$: $p = - s^{2} + s^{3} + 4s^{6}$

R: [3, 3, 8, 6, 2, 7, 4, 5]
B: [6, 8, 1, 1, 7, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 7	3 *vs* 7

Omega Rank for R : cycles: {{4, 6, 7}, {2, 3, 5, 8}}

See Matrix

$$[0, -y_2 + 5 y_4 - y_3 - y_1, y_2, y_4, y_3, y_4, y_4, y_1]$$

$p = - s + s^{5}$ $p' = - s + s^{5}$ $p' = - s^{2} + s^{6}$

Omega Rank for B : cycles: {{2, 8}, {1, 4, 6}, {5, 7}} order: 6

See Matrix

$$[y_1, y_3, 0, y_2, y_3, -y_1 - y_2 + 4 y_3, y_3, y_3]$$

$p = s - s^{4}$ $p' = - s + s^{4}$ $p' = - s^{2} + s^{5}$ $p' = - s^{3} + s^{6}$

87 . Coloring, {3, 4, 5, 8}

$\Omega p(\Delta)=0$: $p' = s^{2}$ $p' = s^{3}$ $p' = s^{4}$ $p = s^{2}$ $p' = s^{5}$

R: [3, 3, 8, 6, 2, 7, 5, 2]
B: [6, 8, 1, 1, 7, 4, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 *vs* 6	6 *vs* 6	6 *vs* 6	6 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{2, 3, 8}} order: 6

See Matrix

$$[0, y_1, y_2, 0, y_3, y_4, y_5, y_6]$$

Omega Rank for B : cycles: {{1, 4, 6}} order: 6

See Matrix

$[y_2, 0, 0, y_3, y_1, y_6, y_4, y_5]$

88 . Coloring, {3, 4, 6, 7}

$\Omega p(\Delta)=0$: $p = s^4$ $p' = s^4$ $p' = s^5$

R: [3, 3, 8, 6, 7, 4, 4, 5]
B: [6, 8, 1, 1, 2, 7, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 6	6 *vs* 6	6 *vs* 6	6 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{4, 6}} order: 6

See Matrix

$[0, 0, y_2, y_1, y_4, y_5, y_6, y_3]$

Omega Rank for B : cycles: {{2, 8}} order: 6

See Matrix

$[y_2, y_1, 0, 0, y_3, y_6, y_5, y_4]$

89 . Coloring, {3, 4, 6, 8}

$\Omega p(\Delta)=0$: $p = s^2 + s^3 - 4s^6$

R: [3, 3, 8, 6, 7, 4, 5, 2]
B: [6, 8, 1, 1, 2, 7, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{4, 6}, {2, 3, 8}, {5, 7}} order: 6

See Matrix

$$[0, y_1, y_2, y_3, y_3, y_3, y_3, -y_1 - y_2 + 4 y_3]$$

$p = s - s^4$ $p' = - s + s^4$ $p' = - s^2 + s^5$ $p' = - s^3 + s^6$

Omega Rank for B : cycles: {{1, 4, 6, 7}, {2, 5, 8}}

See Matrix

$$[5 y_3 - y_4 - y_1 - y_2, y_3, 0, y_4, y_3, y_1, y_2, y_3]$$

$p' = - s + s^5$ $p' = - s^2 + s^6$ $p = - s + s^5$

90 . Coloring, {3, 4, 7, 8}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^6$ $p' = s^3 + 2s^5$ $p' = s^2 + 2s^4$

R: [3, 3, 8, 6, 7, 7, 4, 2]
B: [6, 8, 1, 1, 2, 4, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 6	3 *vs* 6	3 *vs* 6	3 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{4, 6, 7}, {2, 3, 8}} order: 3

See Matrix

$$[0, y_1, y_3, y_2, 0, y_1, y_3, y_2]$$

$p' = s^2 - s^5$ $p' = s - s^4$ $p = s - s^4$

Omega Rank for B : cycles: {{1, 4, 6}, {2, 5, 8}} order: 3

See Matrix

$$[y_1, y_2, 0, y_3, y_1, y_2, 0, y_3]$$

$p' = s - s^4$ $p' = s^2 - s^5$ $p = - s + s^4$

M \ ; N

$$\begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&1&4&3&5&4&1&2 \\ 1&0&3&2&4&5&2&3 \\ 4&3&0&1&1&2&5&4 \\ 3&2&1&0&2&3&4&5 \\ 5&4&1&2&0&1&4&3 \\ 4&5&2&3&1&0&3&2 \\ 1&2&5&4&4&3&0&1 \\ 2&3&4&5&3&2&1&0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 3, 8, 6, 7, 7, 4, 2]
B: [6, 8, 1, 1, 2, 4, 5, 5]

Ranges

Action of R on ranges, [[3], [3], [4], [2]]
Action of B on ranges, [[2], [4], [1], [1]]

Cycles: R , {{4, 6, 7}, {2, 3, 8}}, B , {{1, 4, 6}, {2, 5, 8}}

β({1, 5}) = 1/4
β({2, 6}) = 1/4
β({3, 7}) = 1/4
β({4, 8}) = 1/4

Partitions

Action of R on partitions, [[2], [3], [1], [2]]
Action of B on partitions, [[2], [4], [2], [1]]

α([{1, 2, 3, 4}, {5, 6, 7, 8}]) = 1/5
α([{3, 4, 5, 6}, {1, 2, 7, 8}]) = 2/5
α([{1, 2, 4, 7}, {3, 5, 6, 8}]) = 1/5
α([{1, 6, 7, 8}, {2, 3, 4, 5}]) = 1/5

b1 = {1, 6, 7, 8} , b2 = {2, 3, 4, 5} , b3 = {3, 4, 5, 6} , b4 = {1, 2, 3, 4} , b5 = {5, 6, 7, 8} , b6 = {1, 2, 7, 8} , b7 = {1, 2, 4, 7} , b8 = {3, 5, 6, 8}

Action of R and B on the blocks of the partitions: = [3, 6, 7, 6, 3, 8, 5, 4] [4, 5, 1, 3, 6, 2, 3, 6]
with invariant measure [1, 1, 2, 1, 1, 2, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{3, 4, 7, 8}
Rank	2
R,B	[3, 3, 8, 6, 7, 7, 4, 2], [6, 8, 1, 1, 2, 4, 5, 5]

π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[1, 4, 3, 5, 4, 1, 2, 3, 2, 4, 5, 2, 3, 1, 1, 2, 5, 4, 2, 3, 4, 5, 1, 4, 3, 3, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

91 . Coloring, {3, 5, 6, 7}

R: [3, 3, 8, 1, 2, 4, 4, 5]
B: [6, 8, 1, 6, 7, 7, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	6 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

$$[y_4, y_3, y_2, y_1, y_6, 0, 0, y_5]$$

Omega Rank for B : cycles: {{2, 8}, {5, 7}} order: 4

See Matrix

$$[y_3, y_2, 0, 0, 3\ y_2 - y_1, y_1, -y_3 + 3\ y_2, y_2]$$

$p = - s^3 + s^4$ $p = - s^3 + s^5$ $p = - s^3 + s^6$

92 . Coloring, {3, 5, 6, 8}

R: [3, 3, 8, 1, 2, 4, 5, 2]
B: [6, 8, 1, 6, 7, 7, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	5 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{2, 3, 8}} order: 3

See Matrix

$$[y_1, y_3, y_2, y_5, y_5, 0, 0, y_4]$$

$p = s^3 - s^6$

Omega Rank for B : cycles: {{4, 6, 7}} order: 3

See Matrix

$$[y_4, 0, 0, y_2, y_1, y_5, y_3, y_4]$$

$p = - s^3 + s^6$

93 . Coloring, {3, 5, 7, 8}

R: [3, 3, 8, 1, 2, 7, 4, 2]
B: [6, 8, 1, 6, 7, 4, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	6 *vs* 6	2 *vs* 6

Omega Rank for R : cycles: {{2, 3, 8}} order: 6

See Matrix

$$[y_1, y_2, y_3, y_4, 0, 0, y_5, y_6]$$

Omega Rank for B : cycles: {{4, 6}, {5, 7}} order: 2

See Matrix

$$[y_2, 0, 0, y_1 - y_2, y_1, y_1, y_1 - y_2, y_2]$$

$p' = s^3 - s^5$ $p' = s^4 - s^5$ $p' = s^2 - s^5$ $p = s^2 - s^6$

94 . Coloring, {3, 6, 7, 8}

R: [3, 3, 8, 1, 7, 4, 4, 2]
B: [6, 8, 1, 6, 2, 7, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	6 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{2, 3, 8}} order: 6

See Matrix

$$[y_1, y_2, y_3, y_4, 0, 0, y_5, y_6]$$

Omega Rank for B : cycles: {{2, 5, 8}} order: 6

See Matrix

$$[y_6, y_5, 0, 0, y_4, y_3, y_2, y_1]$$

95 . Coloring, {4, 5, 6, 7}

R: [3, 3, 1, 6, 2, 4, 4, 5]
B: [6, 8, 8, 1, 7, 7, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	4 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {1, 3}} order: 4

See Matrix

$$[y_2, y_1, y_4, 3\, y_2 + 3\, y_1 - 4\, y_3, 4\, y_2 + 4\, y_1 - y_4 - 5\, y_3, y_3, 0, 0]$$

$p' = s^3 - s^5$ $p = - s^3 + s^5$

Omega Rank for B : cycles: {{2, 8}, {5, 7}} order: 4

See Matrix

$$[-5\, y_1 + 4\, y_3 + 4\, y_2 - y_4, y_1, 0, 0, y_3, y_2, y_4, -4\, y_1 + 3\, y_3 + 3\, y_2]$$

$p = s^3 - s^5$ $p' = - s^3 + s^5$

96 . Coloring, {4, 5, 6, 8}

R: [3, 3, 1, 6, 2, 4, 5, 2]

B: [6, 8, 8, 1, 7, 7, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	3 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {1, 3}} order: 4

See Matrix

$$[y_3, -y_3 + 3\, y_1, y_2, y_1, -y_2 + 3\, y_1, y_1, 0, 0]$$

$p' = -s^3 + s^4$ $p' = -s^3 + s^5$ $p = s^3 - s^4$

Omega Rank for B : cycles: {{1, 4, 6, 7}} order: 4

See Matrix

$$[y_1, 0, 0, y_4, y_5, y_6, y_2, y_3]$$

97 . Coloring, {4, 5, 7, 8}

R: [3, 3, 1, 6, 2, 7, 4, 2]
B: [6, 8, 8, 1, 7, 4, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	3 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{1, 3}, {4, 6, 7}} order: 6

See Matrix

$$[-y_1 - y_2 + 5\, y_3, y_1, y_2, y_3, 0, y_3, y_3, 0]$$

$p = s^2 - s^6$ $p' = s^3 - s^5$ $p' = s^2 - s^4$

Omega Rank for B : cycles: {{1, 4, 6}, {5, 7}} order: 6

See Matrix

$$[y_1, 0, 0, y_1, 5\, y_1 - y_2 - y_3, y_1, y_2, y_3]$$

$p = - s^{2} + s^{4}$ $p' = - s^{2} + s^{4}$ $p = - s^{2} + s^{6}$

98 . Coloring, {4, 6, 7, 8}

R: [3, 3, 1, 6, 7, 4, 4, 2]
B: [6, 8, 8, 1, 2, 7, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {1, 3}} order: 2

See Matrix

$[y_2, -y_2 + y_1, y_1, y_1, 0, y_2, -y_2 + y_1, 0]$

$p' = s^{3} - s^{5}$ $p' = s^{4} - s^{5}$ $p = s^{2} - s^{6}$ $p' = s^{2} - s^{5}$

Omega Rank for B : cycles: {{2, 5, 8}} order: 6

See Matrix

$[y_3, y_2, 0, 0, y_1, y_5, y_6, y_4]$

99 . Coloring, {5, 6, 7, 8}

Ωp(Δ)=0: $p = s^{3}$ $p' = s^{3}$ $p' = s^{4}$ $p' = s^{5}$

R: [3, 3, 1, 1, 2, 4, 4, 2]
B: [6, 8, 8, 6, 7, 7, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 6	2 *vs* 6	2 *vs* 6	2 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{1, 3}} order: 2

See Matrix

$[y_2, y_1, y_2, y_1, 0, 0, 0, 0]$

$p = - s^2 + s^4 \quad p = - s^2 + s^3$

Omega Rank for B : cycles: {{5, 7}} order: 2

See Matrix

$[0, 0, 0, 0, y_2, y_1, y_2, y_1]$

$p' = s^2 - s^3 \quad p = s^2 - s^4$

$$M \qquad \backslash; \quad N$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 \end{bmatrix}$$

$\tau = 32$, r'= 1/2

R: [3, 3, 1, 1, 2, 4, 4, 2]
B: [6, 8, 8, 6, 7, 7, 5, 5]

Ranges

Action of R on ranges, [[1], [1], [2], [2]]
Action of B on ranges, [[4], [4], [3], [3]]

Cycles: R , {{1, 3}}, B , {{5, 7}}

β({1, 3}) = 1/4
β({2, 4}) = 1/4
β({5, 7}) = 1/4
β({6, 8}) = 1/4

Partitions

Action of R on partitions, [[8], [8], [6], [8], [6], [6], [8], [6]]
Action of B on partitions, [[2], [7], [7], [2], [7], [2], [7], [2]]

α([{1, 4, 5, 8}, {2, 3, 6, 7}]) = 0/1
α([{1, 4, 5, 6}, {2, 3, 7, 8}]) = 1/4
α([{1, 2, 5, 6}, {3, 4, 7, 8}]) = 0/1
α([{1, 4, 6, 7}, {2, 3, 5, 8}]) = 0/1
α([{3, 4, 5, 6}, {1, 2, 7, 8}]) = 0/1
α([{1, 2, 6, 7}, {3, 4, 5, 8}]) = 1/4
α([{2, 3, 5, 6}, {1, 4, 7, 8}]) = 1/4
α([{3, 4, 6, 7}, {1, 2, 5, 8}]) = 1/4

b1 = {1, 4, 5, 8} , b2 = {3, 4, 5, 6} , b3 = {1, 2, 6, 7} , b4 = {1, 4, 6, 7} , b5 = {1, 2, 5, 6} , b6 = {3, 4, 5, 8} , b7 = {2, 3, 5, 6} , b8 = {1, 4, 7, 8} , b9 = {3, 4, 6, 7} , b10 = {1, 2, 5, 8} , b11 = {2, 3, 6, 7} , b12 = {1, 2, 7, 8} , b13 = {3, 4, 7, 8} , b14 = {2, 3, 5, 8} , b15 = {1, 4, 5, 6} , b16 = {2, 3, 7, 8}

Action of R and B on the blocks of the partitions: = [9, 3, 6, 9, 6, 3, A, 9, 3, 6, A, 6, 3, A, 9, A] [10, 8, F, F, 8, 10, 8, 7, F, 10, F, 7, 7, 10, 8, 7]
with invariant measure [0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{5, 6, 7, 8}
Rank	2
R,B	[3, 3, 1, 1, 2, 4, 4, 2], [6, 8, 8, 6, 7, 7, 5, 5]
π_2	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

100 . Coloring, {2, 3, 4, 5, 6}

R: [3, 8, 8, 6, 2, 4, 5, 5]
B: [6, 3, 1, 1, 7, 7, 4, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	2 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{2, 5, 8}, {4, 6}} order: 6

See Matrix

$$[0, y_2, -y_2 + 2 y_1, y_1, 2 y_1, y_1, 0, 2 y_1]$$

$p = s^2 - s^3 \quad p' = - s^2 + s^3 \quad p' = - s^2 + s^4 \quad p' = - s^2 + s^5$

Omega Rank for B : cycles: {{1, 4, 6, 7}} order: 4

See Matrix

$$[y_3, y_4, y_1, y_2, 0, y_6, y_5, 0]$$

101 . Coloring, {2, 3, 4, 5, 7}

R: [3, 8, 8, 6, 2, 7, 4, 5]
B: [6, 3, 1, 1, 7, 4, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	4 *vs* 7	3 *vs* 7

Omega Rank for R : cycles: {{2, 5, 8}, {4, 6, 7}} order: 3

See Matrix

$$[0, -y_1 + 5 y_3 - y_2 - y_4, y_1, y_3, y_2, y_3, y_3, y_4]$$

$p' = s^2 - s^5$ $p = - s^2 + s^5$ $p' = - s^3 + s^6$

Omega Rank for B : cycles: {{1, 4, 6}, {5, 7}} order: 6

See Matrix

$$[2 y_2, -y_1 + 2 y_2, -y_3 + 2 y_2, y_3, y_2, y_1, y_2, 0]$$

$p = - s^3 + s^4$ $p = - s^3 + s^6$ $p = - s^3 + s^7$ $p = - s^3 + s^5$

102 . Coloring, {2, 3, 4, 5, 8}

R: [3, 8, 8, 6, 2, 7, 5, 2]
B: [6, 3, 1, 1, 7, 4, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{2, 8}} order: 4

See Matrix

$$[0, y_1 - y_3 + y_4, y_2, 0, y_1, y_2, y_3, y_4]$$

$p = - s^4 + s^5$ $p = - s^4 + s^6$

Omega Rank for B : cycles: {{1, 4, 6}} order: 3

See Matrix

$$[y_1, 0, y_5, y_4, y_5, y_2, y_3, 0]$$

$p = - s^3 + s^6$

103 . Coloring, {2, 3, 4, 6, 7}

R: [3, 8, 8, 6, 7, 4, 4, 5]
B: [6, 3, 1, 1, 2, 7, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	5 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{4, 6}} order: 6

See Matrix

$$[0, 0, y_3, y_4, y_5, y_3 + y_4 + y_5 - y_1 - y_2, y_1, y_2]$$

$p = s^5 - s^6$

Omega Rank for B : cycles: {{1, 2, 3, 5, 6, 7}} order: 6

See Matrix

$$[y_4, y_3, y_2, 0, y_1, y_5, y_6, 0]$$

104 . Coloring, {2, 3, 4, 6, 8}

R: [3, 8, 8, 6, 7, 4, 5, 2]
B: [6, 3, 1, 1, 2, 7, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{2, 8}, {4, 6}, {5, 7}} order: 2

See Matrix

$$[0, -y_1 + 2 y_2, y_1, y_2, y_2, y_2, y_2, 2 y_2]$$

$p = - s^2 + s^4$ $p = - s^2 + s^3$ $p = - s^2 + s^5$ $p = - s^2 + s^6$ $p = - s^2 + s^7$

Omega Rank for B : cycles: {{1, 4, 6, 7}} order: 4

See Matrix

$$[y_2 + y_3, y_2 + y_3 - y_4, -y_1 + y_2 + y_3, y_1, y_2, y_3, y_4, 0]$$

$p = s^4 - s^7$ $p' = s^4 - s^6$ $p' = s^5 - s^6$

105 . Coloring, {2, 3, 4, 7, 8}

R: [3, 8, 8, 6, 7, 7, 4, 2]
B: [6, 3, 1, 1, 2, 4, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {4, 6, 7}} order: 6

See Matrix

$$[0, -y_1 + y_4, y_1, 2 y_4 - y_2 - y_3, 0, y_2, y_3, y_4]$$

$p = s^2 - s^5$ $p' = - s^2 + s^5$

Omega Rank for B : cycles: {{1, 4, 6}} order: 6

See Matrix

$$[y_1, y_2, y_3, y_5, y_6, y_4, 0, 0]$$

106 . Coloring, {2, 3, 5, 6, 7}

R: [3, 8, 8, 1, 2, 4, 4, 5]
B: [6, 3, 1, 6, 7, 7, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	6 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{2, 5, 8}} order: 6

See Matrix

$$[y_5, y_6, y_4, y_3, y_2, 0, 0, y_1]$$

Omega Rank for B : cycles: {{5, 7}} order: 6

See Matrix

$$[y_2, y_1, y_2 + y_1 - y_5 - y_4 + y_3, 0, y_5, y_4, y_3, 0]$$

$p = s^5 - s^6$

107 . Coloring, {2, 3, 5, 6, 8}

R: [3, 8, 8, 1, 2, 4, 5, 2]
B: [6, 3, 1, 6, 7, 7, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{2, 8}} order: 4

See Matrix

$$[y_1 + y_2 - y_4, y_1, y_2, y_3, y_3, 0, 0, y_4]$$

$p' = - s^4 + s^5$ $p = s^4 - s^5$

Omega Rank for B : cycles: {{4, 6, 7}} order: 3

See Matrix

$$[y_1, 0, y_2, y_3, y_2, y_5, y_4, 0]$$

$p = - s^3 + s^6$

108 . Coloring, {2, 3, 5, 7, 8}

R: [3, 8, 8, 1, 2, 7, 4, 2]
B: [6, 3, 1, 6, 7, 4, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	5 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{2, 8}} order: 6

See Matrix

$$[y_2 + y_1 + y_3 - y_5 - y_4, y_2, y_1, y_3, 0, 0, y_5, y_4]$$

$p = s^5 - s^6$

Omega Rank for B : cycles: {{4, 6}, {5, 7}} order: 4

See Matrix

$$[y_2, 0, 4 y_2 + 4 y_1 - y_3 - 5 y_4, y_1, 3 y_2 + 3 y_1 - 4 y_4, y_3, y_4, 0]$$

$p = - s^3 + s^5$ $p' = - s^3 + s^5$

109 . Coloring, {2, 3, 6, 7, 8}

R: [3, 8, 8, 1, 7, 4, 4, 2]
B: [6, 3, 1, 6, 2, 7, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	5 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{2, 8}} order: 6

See Matrix

$$[y_2, y_3, y_1, y_2 - y_3 - y_1 + y_5 + y_4, 0, 0, y_5, y_4]$$

$p = - s^5 + s^6$

Omega Rank for B : cycles: {{1, 2, 3, 5, 6, 7}} order: 6

See Matrix

$[y_1, y_2, y_6, 0, y_5, y_4, y_3, 0]$

110 . Coloring, {2, 4, 5, 6, 7}

R: [3, 8, 1, 6, 2, 4, 4, 5]
B: [6, 3, 8, 1, 7, 7, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{4, 6}, {1, 3}, {2, 5, 8}} order: 6

See Matrix

$[y_2, y_2, y_2, 3\ y_2 - y_1, y_2, y_1, 0, y_2]$

$p = s - s^3$ $p' = - s + s^5$ $p' = - s^2 + s^6$ $p' = - s + s^3$ $p' = - s^2 + s^4$

Omega Rank for B : cycles: {{2, 3, 8}, {5, 7}} order: 6

See Matrix

$[5\ y_4 - y_2 - y_3 - y_1, y_4, y_4, 0, y_2, y_3, y_1, y_4]$

$p = - s^3 + s^5$ $p' = - s^3 + s^5$ $p = - s^3 + s^7$

111 . Coloring, {2, 4, 5, 6, 8}

R: [3, 8, 1, 6, 2, 4, 5, 2]
B: [6, 3, 8, 1, 7, 7, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{2, 8}, {4, 6}, {1, 3}} order: 2

See Matrix

$$[y_1, 2\,y_1, y_1, y_1, 2\,y_1 - y_2, y_1, 0, y_2]$$

$p' = s^{2} - s^{3}$ $p = s^{2} - s^{4}$ $p' = - s^{3} + s^{4}$ $p' = - s^{3} + s^{6}$ $p' = - s^{3} + s^{5}$

Omega Rank for B : cycles: {{1, 4, 6, 7}} order: 4

See Matrix

$$[y_3 + y_2 - y_4, 0, y_3, y_2, y_1, y_3 + y_2 - y_1, y_3 + y_2, y_4]$$

$p = - s^{4} + s^{6}$ $p = - s^{4} + s^{7}$ $p = - s^{4} + s^{5}$

112 . Coloring, {2, 4, 5, 7, 8}

R: [3, 8, 1, 6, 2, 7, 4, 2]
B: [6, 3, 8, 1, 7, 4, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{2, 8}, {1, 3}, {4, 6, 7}} order: 6

See Matrix

$$[y_2, 3\,y_2 - y_1, y_2, y_2, 0, y_2, y_2, y_1]$$

$p' = s^{2} - s^{4}$ $p' = - s + s^{3}$ $p' = - s^{4} + s^{6}$ $p' = - s + s^{5}$ $p = s - s^{5}$

Omega Rank for B : cycles: {{1, 4, 6}, {5, 7}} order: 6

See Matrix

$$[y_3, 0, 5\,y_3 - y_1 - y_2 - y_4, y_3, y_1, y_3, y_2, y_4]$$

$p = s^{3} - s^{7}$ $p' = s^{3} - s^{5}$ $p' = s^{4} - s^{6}$

113 . Coloring, {2, 4, 6, 7, 8}

R: [3, 8, 1, 6, 7, 4, 4, 2]
B: [6, 3, 8, 1, 2, 7, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	2 *vs* 7	4 *vs* 7

Omega Rank for R : cycles: {{2, 8}, {4, 6}, {1, 3}} order: 2

See Matrix

$$[y_2, y_2, y_2, 2\,y_2, 0, y_1, 2\,y_2 - y_1, y_2]$$

$p' = -s^3 + s^6$ $p' = -s^3 + s^5$ $p = s^2 - s^4$ $p' = -s^3 + s^4$ $p' = s^2 - s^3$

Omega Rank for B : cycles: {{2, 3, 5, 8}} order: 4

See Matrix

$$[y_1, y_2 + y_4 - y_1, y_3, 0, y_2 + y_4, y_2 - y_3 + y_4, y_2, y_4]$$

$p = -s^4 + s^7$ $p = -s^4 + s^6$ $p = -s^4 + s^5$

114 . Coloring, {2, 5, 6, 7, 8}

R: [3, 8, 1, 1, 2, 4, 4, 2]
B: [6, 3, 8, 6, 7, 7, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{2, 8}, {1, 3}} order: 2

See Matrix

$$[-5\,y_2 + 4\,y_3 + 4\,y_1, y_2, y_3, y_1, 0, 0, 0, -4\,y_2 + 3\,y_3 + 3\,y_1]$$

$p = -s^2 + s^4$ $p' = -s^2 + s^4$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$$[0, 0, y_1, 0, y_2, 2\,y_1, y_3, y_4]$$

$p = s^3 - s^5$

115 . Coloring, {3, 4, 5, 6, 7}

R: [3, 3, 8, 6, 2, 4, 4, 5]
B: [6, 8, 1, 1, 7, 7, 5, 2]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {2, 3, 5, 8}} order: 4

See Matrix

$$[0, y_4, y_3, y_1, y_2, 3\, y_3 - 4\, y_1 + 3\, y_2, 0, -y_4 + 4\, y_3 - 5\, y_1 + 4\, y_2]$$

$p = s - s^5$ $p' = s - s^5$

Omega Rank for B : cycles: {{2, 8}, {5, 7}} order: 4

See Matrix

$$[y_3, y_4, 0, 0, -y_3 + 6\, y_4 - y_1 - y_2, y_1, y_2, y_4]$$

$p = - s^3 + s^5$ $p' = - s^3 + s^5$

116 . Coloring, {3, 4, 5, 6, 8}

R: [3, 3, 8, 6, 2, 4, 5, 2]
B: [6, 8, 1, 1, 7, 7, 4, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	2 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {2, 3, 8}} order: 6

See Matrix

$$[0, 2\, y_1, 2\, y_1, y_1, 2\, y_1 - y_2, y_1, 0, y_2]$$

$p = - s^2 + s^3$ $p = - s^2 + s^4$ $p = - s^2 + s^5$ $p = - s^2 + s^6$

Omega Rank for B : cycles: {{1, 4, 6, 7}} order: 4

See Matrix

$$[y_1, 0, 0, y_2, y_3, y_4, y_5, y_6]$$

117 . Coloring, {3, 4, 5, 7, 8}

R: [3, 3, 8, 6, 2, 7, 4, 2]
B: [6, 8, 1, 1, 7, 4, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 6	4 *vs* 6

Omega Rank for R : cycles: {{4, 6, 7}, {2, 3, 8}} order: 3

See Matrix

$$[0, y_1, -y_1 + 5 y_3 - y_2, y_3, 0, y_3, y_3, y_2]$$

$p' = - s + s^4$ $p' = - s^2 + s^5$ $p = - s + s^4$

Omega Rank for B : cycles: {{5, 7}, {1, 4, 6}} order: 6

See Matrix

$$[y_4, 0, 0, y_3, y_2, -y_4 - y_3 + 2 y_2, y_2 - y_1, y_1]$$

$p = - s^2 + s^5$ $p' = - s^2 + s^5$

118 . Coloring, {3, 4, 6, 7, 8}

R: [3, 3, 8, 6, 7, 4, 4, 2]
B: [6, 8, 1, 1, 2, 7, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	4 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {2, 3, 8}} order: 6

See Matrix

$$[0, y_1, -y_1 + 2 y_4 + 2 y_3 - y_2, y_4 + y_3, 0, y_4, y_3, y_2]$$

$p' = s^2 - s^5 \quad p = s^2 - s^5$

Omega Rank for B : cycles: {{2, 5, 8}} order: 6

See Matrix

$$[y_1, y_2, 0, 0, y_4, y_5, y_6, y_3]$$

119 . Coloring, {3, 5, 6, 7, 8}

R: [3, 3, 8, 1, 2, 4, 4, 2]
B: [6, 8, 1, 6, 7, 7, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	5 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{2, 3, 8}} order: 3

See Matrix

$$[y_1, y_2, y_3, y_4, 0, 0, 0, y_5]$$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$$[y_1, 0, 0, 0, y_2, y_3, -2 y_1 + y_2 + y_3, y_1]$$

$p = - s^3 + s^4 \quad p = - s^3 + s^5$

120 . Coloring, {4, 5, 6, 7, 8}

R: [3, 3, 1, 6, 2, 4, 4, 2]
B: [6, 8, 8, 1, 7, 7, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{4, 6}, {1, 3}} order: 2

See Matrix

$$[-y_1 + 4 y_2 - 5 y_3, y_1, y_2, 3 y_2 - 4 y_3, 0, y_3, 0, 0]$$

$p' = s^2 - s^4$ $p = - s^2 + s^4$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$$[y_4, 0, 0, 0, y_3, y_2, y_1, 2 y_4]$$

$p = - s^3 + s^5$

121 . Coloring, {2, 3, 4, 5, 6, 7}

R: [3, 8, 8, 6, 2, 4, 4, 5]
B: [6, 3, 1, 1, 7, 7, 5, 2]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	5 *vs* 6	6 *vs* 6

Omega Rank for R : cycles: {{4, 6}, {2, 5, 8}} order: 6

See Matrix

$$[0, -3 y_1 + 5 y_2 - 3 y_5 + 5 y_3 - 3 y_4, 3 y_1, 3 y_2, 3 y_5, 3 y_3, 0, 3 y_4]$$

$p = - s^2 - s^3 + s^5 + s^6$

Omega Rank for B : cycles: {{5, 7}} order: 6

See Matrix

$$[y_4, y_1, y_3, 0, y_2, y_5, y_6, 0]$$

122 . Coloring, {2, 3, 4, 5, 6, 8}

$\Omega p(\Delta)=0$: $p' = s + 4s^4 - 8s^5$ $p' = s^2 - 2s^3 + 4s^4 - 4s^5$ $p = s + 4s^4 - 8s^5$

R: [3, 8, 8, 6, 2, 4, 5, 2]
B: [6, 3, 1, 1, 7, 7, 4, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 6	4 *vs* 8	4 *vs* 8	2 *vs* 6	2 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$[0, y_2, 3\,y_1 - y_2, y_1, 3\,y_1 - y_2, y_1, 0, y_2]$$

$p = s^2 - s^6$ $p' = s^2 - s^5$ $p' = s^4 - s^5$ $p' = s^3 - s^5$

Omega Rank for B : cycles: {{1, 4, 6, 7}} order: 4

See Matrix

$$[y_2, 0, y_2 - y_1, y_1, y_2 - y_1, y_1, y_2, 0]$$

$p = -s^2 + s^3$ $p = -s^2 + s^4$ $p = -s^2 + s^6$ $p = -s^2 + s^5$

M \; N

$$\begin{bmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0 \end{bmatrix} \begin{bmatrix} 0&5&4&4&3&3&7&2\\ 5&0&2&4&5&3&2&7\\ 4&2&0&2&7&5&3&5\\ 4&4&2&0&5&7&3&3\\ 3&5&7&5&0&2&4&2\\ 3&3&5&7&2&0&4&4\\ 7&2&3&3&4&4&0&5\\ 2&7&5&3&2&4&5&0 \end{bmatrix}$$

$\tau = 32$, $r' = 1/2$

R: [3, 8, 8, 6, 2, 4, 5, 2]
B: [6, 3, 1, 1, 7, 7, 4, 5]

Ranges

Action of R on ranges, [[3], [2], [2], [4]]
Action of B on ranges, [[4], [3], [1], [1]]

Cycles: R , {{2, 8}, {4, 6}}, B , {{1, 4, 6, 7}}

β({1, 7}) = 1/4
β({2, 8}) = 1/4
β({3, 5}) = 1/4
β({4, 6}) = 1/4

Partitions

Action of R on partitions, [[4], [6], [6], [4], [5], [5]]
Action of B on partitions, [[4], [3], [4], [3], [2], [1]]

α([{5, 6, 7, 8}, {1, 2, 3, 4}]) = 1/14
α([{1, 2, 5, 6}, {3, 4, 7, 8}]) = 1/14
α([{1, 3, 4, 8}, {2, 5, 6, 7}]) = 3/14
α([{2, 3, 4, 7}, {1, 5, 6, 8}]) = 5/14
α([{1, 4, 5, 8}, {2, 3, 6, 7}]) = 1/7
α([{4, 5, 7, 8}, {1, 2, 3, 6}]) = 1/7

b1 = {4, 5, 7, 8} , b2 = {1, 4, 5, 8} , b3 = {1, 2, 3, 6} , b4 = {1, 2, 5, 6} , b5 = {2, 3, 6, 7} , b6 = {1, 3, 4, 8} , b7 = {2, 5, 6, 7} , b8 = {5, 6, 7, 8} , b9 = {2, 3, 4, 7} , b10 = {3, 4, 7, 8} , b11 = {1, 5, 6, 8} , b12 = {1, 2, 3, 4}

Action of R and B on the blocks of the partitions: = [5, 5, 2, 1, 2, 3, 1, 9, B, 3, 9, B] [8, A, C, 6, 4, 9, B, B, 7, 7, 6, 9] with invariant measure [2, 2, 2, 1, 2, 3, 3, 1, 5, 1, 5, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{2, 3, 4, 5, 6, 8}
Rank	2
R,B	[3, 8, 8, 6, 2, 4, 5, 2], [6, 3, 1, 1, 7, 7, 4, 5]
π_2	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0]
u_2	[5, 4, 4, 3, 3, 7, 2, 2, 4, 5, 3, 2, 7, 2, 7, 5, 3, 5, 5, 7, 3, 3, 2, 4, 2, 4, 4, 5] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

123 . Coloring, {2, 3, 4, 5, 7, 8}

R: [3, 8, 8, 6, 2, 7, 4, 2]
B: [6, 3, 1, 1, 7, 4, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	8 *vs* 8	8 *vs* 8	3 *vs* 6	5 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {4, 6, 7}} order: 6

See Matrix

$$[0, y_1, -y_1 + 5 y_2 - y_3, y_2, 0, y_2, y_2, y_3]$$

$p = - s^2 + s^4$ $p' = - s^2 + s^4$ $p = - s^2 + s^6$

Omega Rank for B : cycles: {{1, 4, 6}, {5, 7}} order: 6

See Matrix

$$[-3 y_1 - 3 y_2 + 5 y_5 - 3 y_3 + 5 y_4, 0, 3 y_1, 3 y_2, 3 y_5, 3 y_3, 3 y_4, 0]$$

$p = - s^2 - s^3 + s^5 + s^6$

124 . Coloring, {2, 3, 4, 6, 7, 8}

$\Omega p(\Delta)=0$: $p = s + 4s^4$ $p' = s + 4s^4$ $p' = s^2 + 4s^5$

R: [3, 8, 8, 6, 7, 4, 4, 2]
B: [6, 3, 1, 1, 2, 7, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 6	4 *vs* 8	4 *vs* 8	2 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$[0, -y_1 + y_2, y_1, y_2, 0, -y_1 + y_2, y_1, y_2]$$

$p' = - s^2 + s^5$ $p = s^2 - s^3$ $p' = - s^2 + s^3$ $p' = - s^2 + s^4$

Omega Rank for B : cycles: {{1, 2, 3, 5, 6, 7}} order: 6

See Matrix

$$[y_1, y_2, y_3, 0, y_1, y_2, y_3, 0]$$

$p = - s + s^4$ $p' = - s + s^4$ $p' = - s^2 + s^5$

M \; N

$$\begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \end{bmatrix} \quad \begin{bmatrix} 0&3&2&2&4&1&2&2 \\ 3&0&1&2&1&4&3&2 \\ 2&1&0&1&2&3&4&3 \\ 2&2&1&0&2&2&3&4 \\ 4&1&2&2&0&3&2&2 \\ 1&4&3&2&3&0&1&2 \\ 2&3&4&3&2&1&0&1 \\ 2&2&3&4&2&2&1&0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 8, 8, 6, 7, 4, 4, 2]
B: [6, 3, 1, 1, 2, 7, 5, 5]

Ranges

Action of R on ranges, [[3], [4], [4], [2]]
Action of B on ranges, [[2], [3], [1], [1]]

Cycles: R , {{2, 8}, {4, 6}}, B , {{1, 2, 3, 5, 6, 7}}

β({1, 5}) = 1/4
β({2, 6}) = 1/4
β({3, 7}) = 1/4
β({4, 8}) = 1/4

Partitions

Action of R on partitions, [[3], [5], [4], [4], [5]]
Action of B on partitions, [[2], [5], [1], [5], [1]]

α([{2, 5, 7, 8}, {1, 3, 4, 6}]) = 1/4
α([{5, 6, 7, 8}, {1, 2, 3, 4}]) = 1/8
α([{1, 4, 6, 7}, {2, 3, 5, 8}]) = 1/8
α([{1, 2, 3, 8}, {4, 5, 6, 7}]) = 1/8
α([{2, 3, 4, 5}, {1, 6, 7, 8}]) = 3/8

b1 = {2, 5, 7, 8} , b2 = {1, 3, 4, 6} , b3 = {1, 4, 6, 7} , b4 = {1, 2, 3, 8} , b5 = {4, 5, 6, 7} , b6 = {5, 6, 7, 8} , b7 = {2, 3, 4, 5} , b8 = {1, 6, 7, 8} , b9 = {2, 3, 5, 8} , b10 = {1, 2, 3, 4}

Action of R and B on the blocks of the partitions: = [9, 3, 5, 4, 5, 7, 8, 7, 4, 8] [6, A, 2, 7, 8, 8, 1, 2, 1, 7]
with invariant measure [2, 2, 1, 1, 1, 1, 3, 3, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{2, 3, 4, 6, 7, 8}
Rank	2
R,B	[3, 8, 8, 6, 7, 4, 4, 2], [6, 3, 1, 1, 2, 7, 5, 5]

π_2	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]
u_2	[3, 2, 2, 4, 1, 2, 2, 1, 2, 1, 4, 3, 2, 1, 2, 3, 4, 3, 2, 2, 3, 4, 3, 2, 2, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

125 . Coloring, {2, 3, 5, 6, 7, 8}

R: [3, 8, 8, 1, 2, 4, 4, 2]
B: [6, 3, 1, 6, 7, 7, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	6 *vs* 6	6 *vs* 6	5 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{2, 8}} order: 4

See Matrix

$[y_1, y_2, y_3, y_4, 0, 0, 0, y_5]$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$[y_1, 0, y_5, 0, y_4, y_2, y_3, 0]$

126 . Coloring, {2, 4, 5, 6, 7, 8}

$\Omega p(\Delta)=0$: $p' = s - 4s^5$ $p = s - 4s^5$ $p' = s^2 - 2s^4$ $p' = s^3 - 2s^5$

R: [3, 8, 1, 6, 2, 4, 4, 2]
B: [6, 3, 8, 1, 7, 7, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 6	3 *vs* 7	3 *vs* 7	2 *vs* 6	3 *vs* 6

Omega Rank for R : cycles: {{2, 8}, {4, 6}, {1, 3}} order: 2

See Matrix

$$[y_1, 3\ y_1 - y_2, y_1, 3\ y_1 - y_2, 0, y_2, 0, y_2]$$

$p' = - s^2 + s^4$ $p' = - s + s^5$ $p' = - s + s^3$ $p = s - s^3$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$$[y_1, 0, y_1, 0, y_3, y_2, y_3, y_2]$$

$p' = s^4 - s^5$ $p = s^3 - s^6$ $p' = s^3 - s^5$

M \; N

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 \end{bmatrix}$$

τ= 32 , r'= 1/2

R: [3, 8, 1, 6, 2, 4, 4, 2]
B: [6, 3, 8, 1, 7, 7, 5, 5]

Ranges

Action of R on ranges, [[1], [4], [2], [2]]
Action of B on ranges, [[4], [1], [3], [3]]

Cycles: R , {{2, 8}, {4, 6}, {1, 3}}, B , {{5, 7}}

β({1, 3}) = 1/4
β({2, 4}) = 1/4
β({5, 7}) = 1/4
β({6, 8}) = 1/4

Partitions

Action of R on partitions, [[7], [1], [3], [6], [7], [6], [1], [3]]
Action of B on partitions, [[8], [4], [2], [5], [5], [8], [2], [4]]

α([{3, 4, 6, 7}, {1, 2, 5, 8}]) = 1/8
α([{1, 4, 5, 6}, {2, 3, 7, 8}]) = 1/8
α([{1, 2, 6, 7}, {3, 4, 5, 8}]) = 1/8
α([{2, 3, 5, 6}, {1, 4, 7, 8}]) = 1/8
α([{1, 2, 7, 8}, {3, 4, 5, 6}]) = 1/8
α([{1, 4, 5, 8}, {2, 3, 6, 7}]) = 1/8

α([{1, 4, 6, 7}, {2, 3, 5, 8}]) = 1/8
α([{1, 2, 5, 6}, {3, 4, 7, 8}]) = 1/8

b1 = {3, 4, 6, 7} , b2 = {1, 2, 5, 8} , b3 = {1, 2, 7, 8} , b4 = {1, 4, 5, 6} , b5 = {3, 4, 5, 6} , b6 = {2, 3, 7, 8} , b7 = {1, 4, 5, 8} , b8 = {1, 4, 6, 7} , b9 = {2, 3, 6, 7} , b10 = {1, 2, 6, 7} , b11 = {3, 4, 5, 8} , b12 = {2, 3, 5, 6} , b13 = {1, 4, 7, 8} , b14 = {1, 2, 5, 6} , b15 = {3, 4, 7, 8} , b16 = {2, 3, 5, 8}

Action of R and B on the blocks of the partitions: = [8, 10, 10, 1, 8, 2, 9, 1, 7, B, A, 7, 9, B, A, 2] [E, F, 5, D, 3, C, F, 4, E, 4, 6, 3, 5, D, C, 6]
with invariant measure [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{2, 4, 5, 6, 7, 8}
Rank	2
R,B	[3, 8, 1, 6, 2, 4, 4, 2], [6, 3, 8, 1, 7, 7, 5, 5]
π_2	[0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0]
u_2	[1, 2, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 1] (dim 1)
wpp	[4, 4, 4, 4, 4, 4, 4, 4]

127 . Coloring, {3, 4, 5, 6, 7, 8}

R: [3, 3, 8, 6, 2, 4, 4, 2]
B: [6, 8, 1, 1, 7, 7, 5, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
6 *vs* 6	6 *vs* 6	6 *vs* 6	4 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{4, 6}, {2, 3, 8}} order: 6

See Matrix

$$[0, -3 y_1 + 5 y_3 + 5 y_2 - 3 y_4, 3 y_1, 3 y_3, 0, 3 y_2, 0, 3 y_4]$$

$p = - s - s^2 + s^4 + s^5$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$$[2\ y_4,\ 0,\ 0,\ 0,\ y_1,\ y_2,\ y_3,\ y_4]$$

$p = -s^3 + s^5$

128 . Coloring, {2, 3, 4, 5, 6, 7, 8}

R: [3, 8, 8, 6, 2, 4, 4, 2]
B: [6, 3, 1, 1, 7, 7, 5, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
6 *vs* 6	7 *vs* 7	7 *vs* 7	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{2, 8}, {4, 6}} order: 2

See Matrix

$$[0,\ y_1,\ -y_1 - 5\ y_3 + 4\ y_2,\ -4\ y_3 + 3\ y_2,\ 0,\ y_3,\ 0,\ y_2]$$

$p' = s^2 - s^4$ $p = -s^2 + s^4$

Omega Rank for B : cycles: {{5, 7}} order: 4

See Matrix

$$[y_1 + y_2 + y_3 - y_4,\ 0,\ y_1,\ 0,\ y_2,\ y_3,\ y_4,\ 0]$$

$p = s^4 - s^5$

SUMMARY	
Graph Type	CC
ν(A)	2
ν(Δ)	2
π	[1, 1, 1, 1, 1, 1, 1, 1]
Dbly Stoch	true

SANDWICH		Total 18
No .	**Coloring**	**Rank**
1	{2, 6}	2
2	{3, 7}	2
3	{3, 4, 5, 6}	2
4	{3, 4, 7, 8}	2
5	{}	4
6	{2, 8}	2
7	{2, 3, 6, 7}	2
8	{2, 3, 4, 5, 6, 8}	2
9	{3, 5}	2
10	{2, 3, 4, 6, 7, 8}	2
11	{4, 8}	2
12	{5, 6, 7, 8}	2
13	{2, 3, 5, 8}	2
14	{2, 4}	2
15	{6, 8}	2
16	{5, 7}	2
17	{2, 4, 5, 6, 7, 8}	2
18	{4, 6}	2

RT GROUPS		Total 2	
No .	**Coloring**	**Rank**	**Solv**
1	{2, 4, 5, 7}	8	["group", Not Solvable]
2	{2, 4, 6, 8}	8	["group", Not Solvable]

CC Colorings		Total 2
No .	**Coloring**	**Sandwich,Rank**

1	{}	true, 4
2	{5, 6, 7, 8}	true, 2

Δ-RANK'D	**SC'D !RK'D**	**τ-RANK'D**	**R/B RANK'D**	**NOT SYNC'D**	**Total Runs**	2^{n-1}
96	0	108 , 108	24 , 32	20	128	128
