New Graph

```
[2, 4, 4, 2, 6, 5], [3, 6, 5, 3, 1, 4]
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π = [1, 2, 2, 3, 2, 2]

POSSIBLE RANKS

1 x 12
2 x 6
3 x 4

BASE DETERMINANT 231/2048, .1127929688

NullSpace of Δ

{2, 3}, {1, 4, 5, 6}

Nullspace of A

[{3},{2}] , [{5, 6},{1, 4}]

1 . Coloring, {}

$\Omega p(\Delta)=0$: $p = s^3 + 2s^4$

R: [2, 4, 4, 2, 6, 5]
B: [3, 6, 5, 3, 1, 4]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	3 *vs* 5	3 *vs* 5	1 *vs* 4	3 *vs* 5

Omega Rank for R : cycles: {{5, 6}, {2, 4}} order: 2

See Matrix

$[0, 2 y_1, 0, 2 y_1, y_1, y_1]$

$p = s - s^4$ $p' = s - s^3$ $p' = s^2 - s^3$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3

See Matrix

$[y_3 - y_2, 0, y_3, y_2, y_1, y_3 - y_1]$

$p = - s^{3} + s^{5}$ $p = - s^{3} + s^{4}$

See 3-level graph

M N

$$
\begin{bmatrix} 0 & 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 & 2 & 2 \\ 2 & 0 & 0 & 2 & 2 & 2 \\ 0 & 4 & 2 & 0 & 3 & 3 \\ 1 & 2 & 2 & 3 & 0 & 0 \\ 1 & 2 & 2 & 3 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}
$$

τ= 12 , r'= 2/3

R: [2, 4, 4, 2, 6, 5]
B: [3, 6, 5, 3, 1, 4]

Ranges

Action of R on ranges, [[4], [3], [4], [3], [4], [3]]
Action of B on ranges, [[1], [5], [2], [6], [1], [5]]

Cycles: R , {{5, 6}, {2, 4}}, B , {{1, 3, 5}}

β({1, 3, 5}) = 1/8
β({1, 3, 6}) = 1/8
β({2, 4, 5}) = 1/4
β({2, 4, 6}) = 1/4
β({3, 4, 5}) = 1/8
β({3, 4, 6}) = 1/8

Partitions
α([{5, 6}, {1, 4}, {2, 3}]) = 1/1

b1 = {5, 6} , b2 = {1, 4} , b3 = {2, 3}

Action of R and B on the blocks of the partitions: = [1, 3, 2] [3, 1, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . See partition graph.

See level-3 partition graph.

Right Group	
Coloring	{}
Rank	3
R,B	[2, 4, 4, 2, 6, 5], [3, 6, 5, 3, 1, 4]

π_2	[0, 2, 0, 1, 1, 0, 4, 2, 2, 2, 2, 2, 3, 3, 0]
u_2	[1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 2, 2, 0, 1, 1, 0, 0]
u_3	[0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]

2 . Coloring, {2}

R: [2, 6, 4, 2, 6, 5]
B: [3, 4, 5, 3, 1, 4]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	4 *vs* 5	5 *vs* 5	3 *vs* 4	4 *vs* 4

Omega Rank for R : cycles: {{5, 6}} order: 4

See Matrix

$$[0, y_2, 0, y_2 + y_1 - y_3, y_1, y_3]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3

See Matrix

$$[y_1, 0, y_4, y_2, y_3, 0]$$

3 . Coloring, {3}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$ $p' = s^2 + 2s^3$

R: [2, 4, 5, 2, 6, 5]
B: [3, 6, 4, 3, 1, 4]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{5, 6}, {2, 4}} order: 2

See Matrix

$$[0, y_1, 0, y_2, y_1, y_2]$$

$p = s - s^3 \quad p' = s - s^3$

Omega Rank for B : cycles: {{3, 4}} order: 2

See Matrix

$$[y_1, 0, y_2, y_2, 0, y_1]$$

$p' = - s^2 + s^3 \quad p = s^2 - s^3$

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

τ= 18 , r'= 1/2

R: [2, 4, 5, 2, 6, 5]
B: [3, 6, 4, 3, 1, 4]

Ranges

Action of R on ranges, [[2], [4], [2], [2]]
Action of B on ranges, [[3], [1], [3], [3]]

Cycles: R , {{5, 6}, {2, 4}}, B , {{3, 4}}

β({1, 6}) = 1/6
β({2, 5}) = 1/3
β({3, 4}) = 1/3
β({4, 6}) = 1/6

Partitions
α([{3, 5, 6}, {1, 2, 4}]) = 1/1

b1 = {3, 5, 6} , b2 = {1, 2, 4}

Action of R and B on the blocks of the partitions: = [1, 2] [2, 1]
with invariant measure [1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Right Group	
Coloring	{3}
Rank	2
R,B	[2, 4, 5, 2, 6, 5], [3, 6, 4, 3, 1, 4]
π_2	[0, 0, 0, 0, 1, 0, 0, 2, 0, 2, 0, 0, 0, 1, 0]
u_2	[0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

4 . Coloring, {4}

$\Omega p(\Delta)=0$: $p = s + 2s^3 + 4s^4$

R: [2, 4, 4, 3, 6, 5]
B: [3, 6, 5, 2, 1, 4]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 4	4 *vs* 6	4 *vs* 6	2 *vs* 5	3 *vs* 6

Omega Rank for R : cycles: {{5, 6}, {3, 4}} order: 2

See Matrix

$$[0, -y_1 + 2 y_2, y_1, 2 y_2, y_2, y_2]$$

$p = s^2 - s^5$ $p' = s^3 - s^4$ $p' = s^2 - s^4$

Omega Rank for B : cycles: {{1, 3, 5}, {2, 4, 6}} order: 3

See Matrix

$$[4 y_3 - 5 y_1 + 4 y_2, 3 y_3 - 4 y_1 + 4 y_2, y_3, y_1, y_2, 4 y_3 - 4 y_1 + 3 y_2]$$

$p' = s^2 - s^5$ $p' = s - s^4$ $p' = 1 - s^3$

See 3-level graph

	M						N				
0	4	1	0	2	3	0	1	1	0	1	1
4	0	0	6	6	4	1	0	0	1	1	1
1	0	0	9	4	6	1	0	0	1	1	1
0	6	9	0	8	7	0	1	1	0	1	1
2	6	4	8	0	0	1	1	1	1	0	0
3	4	6	7	0	0	1	1	1	1	0	0

τ= 12 , r'= 2/3

R: [2, 4, 4, 3, 6, 5]
B: [3, 6, 5, 2, 1, 4]

Ranges

Action of R on ranges, [[5], [4], [4], [7], [6], [7], [6]]
Action of B on ranges, [[3], [7], [6], [2], [5], [1], [4]]

Cycles: R , {{5, 6}, {3, 4}}, B , {{1, 3, 5}, {2, 4, 6}}

β({1, 2, 5}) = 1/10
β({1, 2, 6}) = 1/10
β({1, 3, 6}) = 1/20
β({2, 4, 5}) = 1/5
β({2, 4, 6}) = 1/10
β({3, 4, 5}) = 1/5
β({3, 4, 6}) = 1/4

Partitions
α([{5, 6}, {1, 4}, {2, 3}]) = 1/1

b1 = {5, 6} , b2 = {1, 4} , b3 = {2, 3}

Action of R and B on the blocks of the partitions: = [1, 3, 2] [3, 1, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . See partition graph.

See level-3 partition graph.

Right Group	
Coloring	{4}
Rank	3
R,B	[2, 4, 4, 3, 6, 5], [3, 6, 5, 2, 1, 4]
π_2	[4, 1, 0, 2, 3, 0, 6, 6, 4, 9, 4, 6, 8, 7, 0]

u_2	[1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 2, 2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 4, 2, 0, 4, 5, 0, 0]
u_3	[0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]

5 . Coloring, {5}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$ $p' = s^2 - 2s^3$

R: [2, 4, 4, 2, 1, 5]
B: [3, 6, 5, 3, 6, 4]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	3 *vs* 5	3 *vs* 5	3 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{2, 4}} order: 4

See Matrix

$$[y_1, y_1 + y_2 - y_3, 0, y_2, y_3, 0]$$

$p = s^3 - s^4$

Omega Rank for B : cycles: {{3, 4, 5, 6}} order: 4

See Matrix

$$[0, 0, y_2, y_1, y_1, y_2]$$

$p = s - s^3$ $p' = s - s^3$

M N

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 3 & 2 & 0 & 3 & 1 \\ 3 & 0 & 1 & 3 & 0 & 2 \\ 2 & 1 & 0 & 2 & 1 & 3 \\ 0 & 3 & 2 & 0 & 3 & 1 \\ 3 & 0 & 1 & 3 & 0 & 2 \\ 1 & 2 & 3 & 1 & 2 & 0 \end{bmatrix}$$

τ= 18 , r'= 1/2

R: [2, 4, 4, 2, 1, 5]
B: [3, 6, 5, 3, 6, 4]

Ranges

Action of R on ranges, [[2], [2], [4], [1]]
Action of B on ranges, [[3], [3], [4], [3]]

Cycles: R , {{2, 4}}, B , {{3, 4, 5, 6}}

β({1, 2}) = 1/6
β({2, 4}) = 1/6
β({3, 6}) = 1/3
β({4, 5}) = 1/3

Partitions

Action of R on partitions, [[2], [2]]
Action of B on partitions, [[2], [1]]

α([{2, 5, 6}, {1, 3, 4}]) = 1/3
α([{2, 3, 5}, {1, 4, 6}]) = 2/3

b1 = {2, 5, 6} , b2 = {1, 3, 4} , b3 = {2, 3, 5} , b4 = {1, 4, 6}

Action of R and B on the blocks of the partitions: = [4, 3, 4, 3] [3, 4, 2, 1]
with invariant measure [1, 1, 2, 2]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{5}
Rank	2
R,B	[2, 4, 4, 2, 1, 5], [3, 6, 5, 3, 6, 4]
π_2	[1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 2, 0, 0]
u_2	[3, 2, 0, 3, 1, 1, 3, 0, 2, 2, 1, 3, 3, 1, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

6 . Coloring, {6}

R: [2, 4, 4, 2, 6, 4]
B: [3, 6, 5, 3, 1, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	3 *vs* 4	4 *vs* 4	2 *vs* 3	4 *vs* 4

Omega Rank for R : cycles: {{2, 4}} order: 2

See Matrix

$$[0, y_1 - y_2, 0, y_1, 0, y_2]$$

$p = - s^2 + s^3$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3

See Matrix

$$[y_1, 0, y_4, 0, y_3, y_2]$$

7 . Coloring, {2, 3}

R: [2, 6, 5, 2, 6, 5]
B: [3, 4, 4, 3, 1, 4]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 4	4 *vs* 4	3 *vs* 3	3 *vs* 3

Omega Rank for R : cycles: {{5, 6}} order: 2

See Matrix

$$[0, y_2, 0, 0, y_1, y_3]$$

Omega Rank for B : cycles: {{3, 4}} order: 2

See Matrix

$$[y_3, 0, y_1, y_2, 0, 0]$$

8 . Coloring, {2, 4}

R: [2, 6, 4, 3, 6, 5]
B: [3, 4, 5, 2, 1, 4]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{3, 4}, {5, 6}} order: 2

See Matrix

$$[0, -7 y_3 - y_2 + 6 y_1, y_3, -6 y_3 + 5 y_1, y_2, y_1]$$

$p = - s^2 + s^4$ $p' = s^2 - s^4$

Omega Rank for B : cycles: {{2, 4}, {1, 3, 5}} order: 6

See Matrix

$$[5 y_2, 7 y_2 + 7 y_1 - 5 y_3 + 7 y_4, 5 y_1, 5 y_3, 5 y_4, 0]$$

$p = s + s^2 - s^4 - s^5$

9 . Coloring, {2, 5}

R: [2, 6, 4, 2, 1, 5]
B: [3, 4, 5, 3, 6, 4]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 5	3 *vs* 4

Omega Rank for R : cycles: {{1, 2, 5, 6}} order: 4

See Matrix

$$[y_1, y_1 + y_2 - y_3 + y_4, 0, y_2, y_3, y_4]$$

$p = - s^2 + s^3 - s^4 + s^5$

Omega Rank for B : cycles: {{3, 4, 5, 6}} order: 4

See Matrix

$$[0, 0, y_1 + y_2 - y_3, y_1, y_2, y_3]$$

$p = s - s^2 + s^3 - s^4$

10 . Coloring, {2, 6}

$\Omega p(\Delta)=0$: $p = s^3$ $p' = s^3$

R: [2, 6, 4, 2, 6, 4]
B: [3, 4, 5, 3, 1, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	2 *vs* 4	2 *vs* 4	1 *vs* 3	2 *vs* 4

Omega Rank for R : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[0, y_1, 0, y_1, 0, y_1]$$

$p = - s + s^2$ $p = - s + s^3$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3

See Matrix

$$[y_2 - y_1, 0, y_2, y_1, y_2, 0]$$

$p = - s^2 + s^4$ $p = - s^2 + s^3$

See 3-level graph

M N

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 \\ 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 & 1 & 2 \\ 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

τ= 12 , r'= 2/3

R: [2, 6, 4, 2, 6, 4]
B: [3, 4, 5, 3, 1, 5]

Ranges

Action of R on ranges, [[2], [2], [2]]
Action of B on ranges, [[1], [3], [1]]

Cycles: R , {{2, 4, 6}}, B , {{1, 3, 5}}

β({1, 3, 5}) = 1/4
β({2, 4, 6}) = 1/2
β({3, 4, 5}) = 1/4

Partitions
α([{2, 5}, {3, 6}, {1, 4}]) = 1/1

b1 = {2, 5} , b2 = {3, 6} , b3 = {1, 4}

Action of R and B on the blocks of the partitions: = [3, 1, 2] [2, 3, 1]
with invariant measure [1, 1, 1]

N by blocks, check: true . See partition graph.

See level-3 partition graph.

Right Group	
Coloring	{2, 6}
Rank	3
R,B	[2, 6, 4, 2, 6, 4], [3, 4, 5, 3, 1, 5]
π_2	[0, 1, 0, 1, 0, 0, 2, 0, 2, 1, 2, 0, 1, 2, 0]
u_2	[1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0]
u_3	[1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1]

11 . Coloring, {3, 4}

R: [2, 4, 5, 3, 6, 5]
B: [3, 6, 4, 2, 1, 4]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{5, 6}} order: 4

See Matrix

$$[0, y_4, y_3, y_1, y_2, -y_4 - y_3 + y_1 + y_2]$$

$p = - s^4 + s^5$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[y_1, y_3, y_4, y_5, 0, y_2]$$

12 . Coloring, {3, 5}

R: [2, 4, 5, 2, 1, 5]
B: [3, 6, 4, 3, 6, 4]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 4	4 *vs* 4	4 *vs* 4	3 *vs* 3

Omega Rank for R : cycles: {{2, 4}} order: 4

See Matrix

$$[y_1, y_2, 0, y_3, y_4, 0]$$

Omega Rank for B : cycles: {{3, 4}} order: 2

See Matrix

$$[0, 0, y_3, y_2, 0, y_1]$$

13 . Coloring, {3, 6}

R: [2, 4, 5, 2, 6, 4]
B: [3, 6, 4, 3, 1, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 4	4 *vs* 5

Omega Rank for R : cycles: {{2, 4}} order: 4

See Matrix

$$[0, y_1 + y_2 - y_3, 0, y_1, y_2, y_3]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{3, 4}} order: 4

See Matrix

$$[y_1 - y_4 + y_3 - y_2, 0, y_1, y_4, y_3, y_2]$$

$p = s^4 - s^5$

14 . Coloring, {4, 5}

R: [2, 4, 4, 3, 1, 5]
B: [3, 6, 5, 2, 6, 4]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{3, 4}} order: 4

See Matrix

$$[y_3, y_3 - y_1 + y_2 - y_4, y_1, y_2, y_4, 0]$$

$p = - s^4 + s^5$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[0, y_5, y_4, y_3, y_2, y_1]$$

15 . Coloring, {4, 6}

R: [2, 4, 4, 3, 6, 4]
B: [3, 6, 5, 2, 1, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	2 *vs* 4	5 *vs* 5

Omega Rank for R : cycles: {{3, 4}} order: 2

See Matrix

$$[0, y_2, y_1, 3\, y_2 + y_1, 0, 2\, y_2]$$

$p = - s^2 + s^3$ $p = - s^2 + s^4$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3

See Matrix

$$[y_2, y_1, y_5, 0, y_3, y_4]$$

16 . Coloring, {5, 6}

R: [2, 4, 4, 2, 1, 4]
B: [3, 6, 5, 3, 6, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	4 *vs* 4	4 *vs* 4	3 *vs* 3	3 *vs* 3

Omega Rank for R : cycles: {{2, 4}} order: 2

See Matrix

$$[y_1, y_3, 0, y_2, 0, 0]$$

Omega Rank for B : cycles: {{5, 6}} order: 2

See Matrix

$$[0, 0, y_1, 0, y_3, y_2]$$

17 . Coloring, {2, 3, 4}

$\Omega p(\Delta)=0$: $p = s - 2s^3 - 4s^4$

R: [2, 6, 5, 3, 6, 5]
B: [3, 4, 4, 2, 1, 4]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 4	4 *vs* 4

Omega Rank for R : cycles: {{5, 6}} order: 2

See Matrix

$$[0, y_3, 3\,y_3, 0, y_1, y_2]$$

$p = -s^2 + s^4$

Omega Rank for B : cycles: {{2, 4}} order: 4

See Matrix

$$[y_2, y_1, y_3, y_4, 0, 0]$$

18 . Coloring, {2, 3, 5}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$ $p' = s^2 + 2s^3$

R: [2, 6, 5, 2, 1, 5]
B: [3, 4, 4, 3, 6, 4]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 3

Omega Rank for R : cycles: {{1, 2, 5, 6}} order: 4

See Matrix

$$[y_2, y_1, 0, 0, y_1, y_2]$$

$p = -s + s^3 \quad p' = -s + s^3$

Omega Rank for B : cycles: {{3, 4}} order: 2

See Matrix

$$[0, 0, y_2, y_1, 0, -y_2 + y_1]$$

$p = -s^2 + s^3$

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 2 & 3 & 0 & 1 & 3 \\ 2 & 0 & 1 & 2 & 3 & 1 \\ 3 & 1 & 0 & 3 & 2 & 0 \\ 0 & 2 & 3 & 0 & 1 & 3 \\ 1 & 3 & 2 & 1 & 0 & 2 \\ 3 & 1 & 0 & 3 & 2 & 0 \end{bmatrix}$$

τ= 18 , r'= 1/2

R: [2, 6, 5, 2, 1, 5]
B: [3, 4, 4, 3, 6, 4]

Ranges

Action of R on ranges, [[2], [1], [2], [2]]
Action of B on ranges, [[3], [4], [3], [3]]

Cycles: R , {{1, 2, 5, 6}}, B , {{3, 4}}

β({1, 6}) = 1/6
β({2, 5}) = 1/3
β({3, 4}) = 1/3
β({4, 6}) = 1/6

Partitions

Action of R on partitions, [[2], [1]]
Action of B on partitions, [[2], [2]]

α([{1, 2, 4}, {3, 5, 6}]) = 1/3
α([{2, 3, 6}, {1, 4, 5}]) = 2/3

b1 = {1, 2, 4} , b2 = {3, 5, 6} , b3 = {2, 3, 6} , b4 = {1, 4, 5}

Action of R and B on the blocks of the partitions: = [4, 3, 1, 2] [3, 4, 4, 3]
with invariant measure [1, 1, 2, 2]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Sandwich	
Coloring	{2, 3, 5}
Rank	2
R,B	[2, 6, 5, 2, 1, 5], [3, 4, 4, 3, 6, 4]
π_2	[0, 0, 0, 0, 1, 0, 0, 2, 0, 2, 0, 0, 0, 1, 0]
u_2	[2, 3, 0, 1, 3, 1, 2, 3, 1, 3, 2, 0, 1, 3, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

19 . Coloring, {2, 3, 6}

R: [2, 6, 5, 2, 6, 4]
B: [3, 4, 4, 3, 1, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	4 *vs* 5	4 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[0, y_4, 0, y_3, y_1, y_2]$$

Omega Rank for B : cycles: {{3, 4}} order: 4

See Matrix

$$[y_1 - y_2 + y_3, 0, y_1, y_2, y_3, 0]$$

$p = - s^{3} + s^{4}$

20 . Coloring, {2, 4, 5}

R: [2, 6, 4, 3, 1, 5]
B: [3, 4, 5, 2, 6, 4]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 6	4 *vs* 5

Omega Rank for R : cycles: {{1, 2, 5, 6}, {3, 4}} order: 4

See Matrix

$$[y_2, 6 y_2 - 7 y_1 - y_3 + 6 y_4, y_1, 5 y_2 - 6 y_1 + 5 y_4, y_3, y_4]$$

$p' = - 1 + s^{4}$ $p' = - s + s^{5}$

Omega Rank for B : cycles: {{2, 4}} order: 4

See Matrix

$$[0, y_4, y_3, y_2, y_1, -y_4 - y_3 + y_2 + y_1]$$

$p = s^{4} - s^{5}$

21 . Coloring, {2, 4, 6}

R: [2, 6, 4, 3, 6, 4]
B: [3, 4, 5, 2, 1, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	4 *vs* 5

Omega Rank for R : cycles: {{3, 4}} order: 4

See Matrix

$$[0, y_1, y_4, y_3, 0, y_2]$$

Omega Rank for B : cycles: {{1, 3, 5}, {2, 4}} order: 6

See Matrix

$$[7 y_1 - 5 y_4 + 7 y_3 - 5 y_2, 5 y_1, 5 y_4, 5 y_3, 5 y_2, 0]$$

$p = - s - s^2 + s^4 + s^5$

22 . Coloring, {2, 5, 6}

R: [2, 6, 4, 2, 1, 4]
B: [3, 4, 5, 3, 6, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	4 *vs* 4	3 *vs* 4	4 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[y_1, y_2, 0, y_3, 0, y_4]$$

Omega Rank for B : cycles: {{5, 6}} order: 4

See Matrix

$$[0, 0, y_1 + y_2 - y_3, y_1, y_2, y_3]$$

$p = - s^3 + s^4$

23 . Coloring, {3, 4, 5}

R: [2, 4, 5, 3, 1, 5]
B: [3, 6, 4, 2, 6, 4]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	5 *vs* 5	4 *vs* 4

Omega Rank for R : cycles: {{1, 2, 3, 4, 5}} order: 5

See Matrix

$[y_3, y_4, y_5, y_2, y_1, 0]$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3

See Matrix

$[0, y_1, y_2, y_3, 0, y_4]$

24 . Coloring, {3, 4, 6}

R: [2, 4, 5, 3, 6, 4]
B: [3, 6, 4, 2, 1, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 5	5 *vs* 6

Omega Rank for R : cycles: {{3, 4, 5, 6}} order: 4

See Matrix

$[0, y_2, y_3, y_2 + y_3 - y_1 + y_4, y_1, y_4]$

$p = - s^2 + s^3 - s^4 + s^5$

Omega Rank for B : cycles: {{1, 2, 3, 4, 5, 6}} order: 6

See Matrix

$[y_2, y_1, y_2 - y_1 + y_3 - y_4 + y_5, y_3, y_4, y_5]$

$p' = 1 - s + s^2 - s^3 + s^4 - s^5$

25 . Coloring, {3, 5, 6}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$ $p' = s^2 - 2s^3$

R: [2, 4, 5, 2, 1, 4]
B: [3, 6, 4, 3, 6, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 4	3 *vs* 5	3 *vs* 5	3 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{2, 4}} order: 4

See Matrix

$$[y_2, y_1 + y_2 - y_3, 0, y_1, y_3, 0]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{5, 6}, {3, 4}} order: 2

See Matrix

$$[0, 0, y_2, y_1, y_1, y_2]$$

$p = s - s^3$ $p' = s - s^3$

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

τ= 18 , r'= 1/2

R: [2, 4, 5, 2, 1, 4]
B: [3, 6, 4, 3, 6, 5]

Ranges

Action of R on ranges, [[2], [2], [4], [1]]
Action of B on ranges, [[3], [3], [4], [3]]

Cycles: R , {{2, 4}}, B , {{5, 6}, {3, 4}}

β({1, 2}) = 1/6
β({2, 4}) = 1/6
β({3, 6}) = 1/3
β({4, 5}) = 1/3

Partitions
α([{2, 5, 6}, {1, 3, 4}]) = 1/1

b1 = {2, 5, 6} , b2 = {1, 3, 4}

Action of R and B on the blocks of the partitions: = [2, 1] [1, 2]
with invariant measure [1, 1]

N by blocks, check: true . See partition graph.

See level-2 partition graph.

Right Group	
Coloring	{3, 5, 6}
Rank	2
R,B	[2, 4, 5, 2, 1, 4], [3, 6, 4, 3, 6, 5]
π_2	[1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 2, 0, 0]
u_2	[1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

26 . Coloring, {4, 5, 6}

R: [2, 4, 4, 3, 1, 4]
B: [3, 6, 5, 2, 6, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{3, 4}} order: 4

See Matrix

$$[y_1, y_2, y_3, y_4, 0, 0]$$

Omega Rank for B : cycles: {{5, 6}} order: 2

See Matrix

$$[0, 3\,y_1, y_1, 0, y_2, y_3]$$

$$p = s^2 - s^4$$

27 . Coloring, {2, 3, 4, 5}

R: [2, 6, 5, 3, 1, 5]
B: [3, 4, 4, 2, 6, 4]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	5 *vs* 5	2 *vs* 4

Omega Rank for R : cycles: {{1, 2, 5, 6}} order: 4

See Matrix

$$[y_3, y_4, y_5, 0, y_1, y_2]$$

Omega Rank for B : cycles: {{2, 4}} order: 2

See Matrix

$$[0, -3\, y_2 + y_1, y_2, y_1, 0, 2\, y_2]$$

$p = - s^2 + s^3$ $p = - s^2 + s^4$

28 . Coloring, {2, 3, 4, 6}

R: [2, 6, 5, 3, 6, 4]
B: [3, 4, 4, 2, 1, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{3, 4, 5, 6}} order: 4

See Matrix

$$[0, y_1, y_2, y_3, y_4, y_5]$$

Omega Rank for B : cycles: {{2, 4}} order: 4

See Matrix

$$[y_1 + y_2 - y_3 + y_4, y_1, y_2, y_3, y_4, 0]$$

$p = - s^4 + s^5$

29 . Coloring, {2, 3, 5, 6}

$\Omega p(\Delta)=0$: $p = s^3 - 2s^4$

R: [2, 6, 5, 2, 1, 4]
B: [3, 4, 4, 3, 6, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	3 *vs* 5	3 *vs* 5	3 *vs* 5	1 *vs* 4

Omega Rank for R : cycles: {{2, 4, 6}} order: 3

See Matrix

$$[y_2 + y_3 - y_1, y_2 + y_3, 0, y_1, y_2, y_3]$$

$p = - s^3 + s^5$ $p = - s^3 + s^4$

Omega Rank for B : cycles: {{5, 6}, {3, 4}} order: 2

See Matrix

$$[0, 0, 2 y_1, 2 y_1, y_1, y_1]$$

$p' = - s + s^2$ $p = s - s^2$ $p' = - s + s^3$

See 3-level graph

M N

$$\begin{bmatrix} 0 & 2 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 4 & 2 & 2 \\ 0 & 2 & 4 & 0 & 3 & 3 \\ 1 & 2 & 2 & 3 & 0 & 0 \\ 1 & 2 & 2 & 3 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$\tau= 12$, $r'= 2/3$

R: [2, 6, 5, 2, 1, 4]
B: [3, 4, 4, 3, 6, 5]

Ranges

Action of R on ranges, [[2], [4], [2], [4], [1], [3]]
Action of B on ranges, [[6], [5], [6], [5], [6], [5]]

Cycles: R , {{2, 4, 6}}, B , {{5, 6}, {3, 4}}

β({1, 2, 5}) = 1/8
β({1, 2, 6}) = 1/8
β({2, 4, 5}) = 1/8
β({2, 4, 6}) = 1/8
β({3, 4, 5}) = 1/4
β({3, 4, 6}) = 1/4

Partitions
α([{5, 6}, {1, 4}, {2, 3}]) = 1/1

b1 = {5, 6} , b2 = {1, 4} , b3 = {2, 3}

Action of R and B on the blocks of the partitions: = [3, 1, 2] [1, 3, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . See partition graph.

See level-3 partition graph.

Right Group	
Coloring	{2, 3, 5, 6}
Rank	3
R,B	[2, 6, 5, 2, 1, 4], [3, 4, 4, 3, 6, 5]
π_2	[2, 0, 0, 1, 1, 0, 2, 2, 2, 4, 2, 2, 3, 3, 0]
u_2	[1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 2, 2, 0, 0]
u_3	[0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]

30 . Coloring, {2, 4, 5, 6}

R: [2, 6, 4, 3, 1, 4]
B: [3, 4, 5, 2, 6, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	5 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{3, 4}} order: 4

See Matrix

$$[y_4, y_1, y_2, y_3, 0, y_5]$$

Omega Rank for B : cycles: {{5, 6}, {2, 4}} order: 2

See Matrix

$$[0, 5 y_2 - 6 y_1 + 5 y_3, y_2, y_1, 6 y_2 - 7 y_1 + 6 y_3, y_3]$$

$p = - s^2 + s^4$ $p' = - s^2 + s^4$

31 . Coloring, {3, 4, 5, 6}

R: [2, 4, 5, 3, 1, 4]
B: [3, 6, 4, 2, 6, 5]

See graph

See pair graph

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{1, 2, 3, 4, 5}} order: 5

See Matrix

$$[y_3, y_2, y_1, y_5, y_4, 0]$$

Omega Rank for B : cycles: {{5, 6}} order: 4

See Matrix

$$[0, -y_1 + y_3 - y_4 + y_2, y_1, y_3, y_4, y_2]$$

$p = s^4 - s^5$

32 . Coloring, {2, 3, 4, 5, 6}

$\Omega p(\Delta)=0$: $p = s - 2s^3 + 4s^4$

R: [2, 6, 5, 3, 1, 4]
B: [3, 4, 4, 2, 6, 5]

See graph

See pair graph

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	4 *vs* 6	4 *vs* 6	4 *vs* 6	2 *vs* 5

Omega Rank for R : cycles: {{1, 2, 3, 4, 5, 6}} order: 6

See Matrix

$$[y_2 + y_3 - y_4, y_1, -y_1 + y_2 + y_3, y_4, y_2, y_3]$$

$p' = s - s^2 + s^4 - s^5$ $p' = 1 - s^2 + s^3 - s^5$

Omega Rank for B : cycles: {{5, 6}, {2, 4}} order: 2

See Matrix

$$[0, y_1, -y_1 + 2\, y_2, 2\, y_2, y_2, y_2]$$

$p = s^2 - s^4$ $p' = s^2 - s^3$ $p' = - s^3 + s^4$

See 3-level graph

$$M = \begin{bmatrix} 0 & 2 & 7 & 0 & 4 & 5 \\ 2 & 0 & 0 & 16 & 10 & 8 \\ 7 & 0 & 0 & 11 & 8 & 10 \\ 0 & 16 & 11 & 0 & 14 & 13 \\ 4 & 10 & 8 & 14 & 0 & 0 \\ 5 & 8 & 10 & 13 & 0 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$\tau = 12$, $r' = 2/3$

R: [2, 6, 5, 3, 1, 4]
B: [3, 4, 4, 2, 6, 5]

Ranges

Action of R on ranges, [[2], [6], [1], [5], [4], [8], [3], [7]]
Action of B on ranges, [[8], [7], [8], [7], [6], [5], [6], [5]]

Cycles: R , {{1, 2, 3, 4, 5, 6}}, B , {{5, 6}, {2, 4}}

β({1, 2, 5}) = 1/27
β({1, 2, 6}) = 1/54
β({1, 3, 5}) = 2/27
β({1, 3, 6}) = 13/108
β({2, 4, 5}) = 13/54
β({2, 4, 6}) = 11/54
β({3, 4, 5}) = 4/27
β({3, 4, 6}) = 17/108

Partitions
α([{5, 6}, {1, 4}, {2, 3}]) = 1/1

b1 = {5, 6} , b2 = {1, 4} , b3 = {2, 3}

Action of R and B on the blocks of the partitions: = [3, 1, 2] [1, 3, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . See partition graph.

See level-3 partition graph.

Right Group	
Coloring	{2, 3, 4, 5, 6}
Rank	3
R,B	[2, 6, 5, 3, 1, 4], [3, 4, 4, 2, 6, 5]
π_2	[2, 7, 0, 4, 5, 0, 16, 10, 8, 11, 8, 10, 14, 13, 0]
u_2	[1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 4, 2, 0, 8, 13, 0, 0, 0, 0, 0, 0, 26, 22, 0, 16, 17, 0, 0]
u_3	[0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]

SUMMARY	
Graph Type	CC
$\nu(A)$	2
$\nu(\Delta)$	2
π	[1, 2, 2, 3, 2, 2]

Dbly Stoch	false

SANDWICH		Total 2
No .	**Coloring**	**Rank**
1	{5}	2
2	{2, 3, 5}	2

RT GROUPS		Total 7	
No .	**Coloring**	**Rank**	**Solv**
1	{3}	2	Solvable
2	{}	3	Not Solvable
3	{2, 3, 4, 5, 6}	3	Not Solvable
4	{2, 6}	3	Solvable
5	{2, 3, 5, 6}	3	Not Solvable
6	{3, 5, 6}	2	Solvable
7	{4}	3	Not Solvable

Δ-RANK'D	**SC'D !RK'D**	**τ-RANK'D**	**R/B RANK'D**	**NOT SYNC'D**	**Total Runs**	2^{n-1}
22	0	21 , 21	12 , 10	9	32	32
