

New Graph

[3, 5, 6, 1, 2, 1], [5, 1, 2, 6, 3, 4]

$$\pi = [3, 3, 3, 1, 3, 2]$$

POSSIBLE RANKS

$$\begin{array}{l} 1 \times 15 \\ 3 \times 5 \end{array}$$

BASE DETERMINANT 163959/1048576, .1563634872

NullSpace of Δ

{1, 2, 3, 4, 5, 6}

Range of Δ : $[-\lambda_1 - \lambda_5 - \lambda_3 - \lambda_4 - \lambda_2, \lambda_1, \lambda_5, \lambda_3, \lambda_4, \lambda_2]$

1 . Coloring, {}

R: [3, 5, 6, 1, 2, 1] **B**: [5, 1, 2, 6, 3, 4]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A + \tau \Delta$:

$$[3^4 (-3 + \tau)^4, 3^4 (-3 + \tau)^4, 3^4 (-3 + \tau)^4, 3^4 (-1 + \tau)^4, 3^4 (-3 + \tau)^4, -6^4]$$

For $\tau=1/2$, [-5, -5, -5, -1, -5, -4] . FixedPtCheck, [5, 5, 5, 1, 5, 4]

$$\det(A + \tau \Delta) = 1^4 (1 + 10\tau^2 + 5\tau^4)^4 (-1 + \tau)^4$$

Delta Range : $[y_4, y_5, y_3, y_1, y_2, -y_4 - y_5 - y_3 - y_1 - y_2]$

$$[3, 3, 3, 1, 3, 2]$$

$$+ \quad \backslash ; \quad - \quad \backslash ; \quad \Delta$$

\$ [[3, 3, 3, 0, 3, 3], [6, 6, 6, 1, 6, 5], [12, 12, 12, 3, 12, 9], [24, 24, 24, 7, 24, 17], [48, 48, 48, 15, 48, 33]]\$ \$ [[3, 3, 3, 2, 3, 1], [6, 6, 6, 3, 6, 3], [12, 12, 12, 5, 12, 7], [24, 24, 24, 9, 24, 15], [48, 48, 48, 17, 48, 31]]\$ \$ [[0, 0, 0, -1, 0, 1], [0, 0, 0, -1, 0, 1], [0, 0, 0, -1, 0, 1], [0, 0, 0, -1, 0, 1], [0, 0, 0, -1, 0, 1]]\$

$$[0, 0, 0, -y_1, 0, y_1]$$

$p' = s^3 - 2s^4$ $p = s - 16s^5$
 S+ \; S- \; NM
 \$ [[1, 9, 2, 1, 4, 0], [3, 0, 9, 0, 1, 4], [1, 3, 1, 3, 7, 2], [5, 4, 1, 1, 4, 2], [7, 1, 4, 0, 1, 4], [5, 4, 1, 2, 4, 1]]
 \$ \$ [[3, 0, 6, 0, 2, 6], [3, 6, 0, 1, 7, 0], [8, 3, 3, 0, 0, 3], [3, 1, 6, 1, 3, 3], [0, 7, 2, 3, 5, 0], [3, 1, 6, 2, 3, 2]]\$ \$ [[12, 9, 9, 3, 9, 6], [9, 12, 9, 3, 9, 6], [9, 9, 12, 3, 9, 6], [9, 9, 9, 4, 9, 8], [9, 9, 9, 3, 12, 6], [9, 9, 9, 4, 9, 8]]\$
 CmmCk true, true, true
 $p' = s - 8s^4$ $p' = s^2 - 4s^4$

Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
1 vs 5	2 vs 6	2 vs 6	1 vs 5	2 vs 6

Omega Rank for R : cycles: {{1, 3, 6}, {2, 5}}, net cycles: 2 . order: 6

\$ [[3, 3, 3, 0, 3, 3], [3, 3, 3, 0, 3, 3], [3, 3, 3, 0, 3, 3], [3, 3, 3, 0, 3, 3], [3, 3, 3, 0, 3, 3]]\$

$$[y_1, y_1, y_1, 0, y_1, y_1]$$

$$p = -s + s^2 \quad p = -s + s^3 \quad p = -s + s^5 \quad p = -s + s^4$$

Omega Rank for B : cycles: {{1, 2, 3, 5}, {4, 6}}, net cycles: 2 . order: 4

\$ [[3, 3, 3, 2, 3, 1], [3, 3, 3, 1, 3, 2], [3, 3, 3, 2, 3, 1], [3, 3, 3, 1, 3, 2], [3, 3, 3, 2, 3, 1], [3, 3, 3, 1, 3, 2]]\$

$$[y_1, y_1, y_1, y_1 - y_2, y_1, y_2]$$

$$p' = s - s^5 \quad p' = 1 - s^4 \quad p' = s^3 - s^5 \quad p' = s^2 - s^4$$

Â« NOT SYNC'D Â»

Nullspace of $\{\Omega\Delta^i\}$:

$$[x_1, x_2, x_3, x_4, -16x_1 - 8x_2 - 4x_3 - 2x_4]$$

$$\text{For } A+2\Delta: [-y_1 - y_2 - y_3 - y_4, y_1, y_2, y_4, y_3, y_4]$$

$$\text{For } A-2\Delta: [-y_1 - y_2 - y_3 - y_4, y_1, y_2, y_4, y_3, y_4]$$

Range of $\{\Omega\Delta^i\}$: $[0, 0, 0, -\mu_1, 0, \mu_1]$

rank of M is 5 , rank of N is 5

M N

\$ [[0, 3, 3, 1, 3, 2] , [3, 0, 3, 1, 3, 2] , [3, 3, 0, 1, 3, 2] , [1, 1, 1, 0, 1, 0] , [3, 3, 3, 1, 0, 2] , [2, 2, 2, 0, 2, 0]]
 \$ \$ [[0, 1, 1, 1, 1, 1] , [1, 0, 1, 1, 1, 1] , [1, 1, 0, 1, 1, 1] , [1, 1, 1, 0, 1, 0] , [1, 1, 1, 1, 0, 1] , [1, 1, 1, 0, 1, 0]] \$

Check is $\Omega\Delta N$ zero? *true*, $\pi\Delta = [0, 0, 0, -1, 0, 1]$

ker M, $[0, 0, 0, -2\lambda_1, 0, \lambda_1]$

Range M, $[x_1, x_2, x_3, x_4, x_5, 2x_4]$

$\tau = 8$, $r' = 4/5$

Ranges

Action of R on ranges, $[[2], [2]]$

Action of B on ranges, $[[2], [1]]$

$\beta(\{1, 2, 3, 4, 5\}) = 1/3$

$\beta(\{1, 2, 3, 5, 6\}) = 2/3$

ker N, $[0, 0, 0, -\mu_1, 0, \mu_1]$

Range of N

$[y_1, y_2, y_3, y_5, y_4, y_5]$

Partitions

$\alpha(\{\{1\}, \{5\}, \{3\}, \{4, 6\}, \{2\}\}) = 1/1$

$b1 = \{1\}$, $b2 = \{5\}$, $b3 = \{3\}$, $b4 = \{4, 6\}$, $b5 = \{2\}$

Action of R and B on the blocks of the partitions: \$ [[0, 0, 0, 1, 1] , [1, 0, 0, 0, 1] , [1, 1, 0, 0, 0] , [0, 0, 1, 1, 0] , [0, 1, 1, 0, 0]] \$ = \$ [[0, 0, 0, 1, 0] , [0, 0, 0, 0, 1] , [1, 0, 0, 0, 0] , [0, 0, 1, 0, 0] , [0, 1, 0, 0, 0]] \$ + \$ [[0, 0, 0, 0, 1] , [1, 0, 0, 0, 0] , [0, 1, 0, 0, 0] , [0, 0, 0, 1, 0] , [0, 0, 1, 0, 0]] \$
 $['4'$, $'5'$, $'1'$, $'3'$, $'2'$, $['5'$, $'1'$, $'2'$, $'4'$, $'3'$] with invariant measure [1, 1, 1, 1, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-5 partition graph.

'

Right Group	
Coloring	{ }
Rank	5
R,B	[3, 5, 6, 1, 2, 1], [5, 1, 2, 6, 3, 4]
π_2	[3, 3, 1, 3, 2, 3, 1, 3, 2, 1, 3, 2, 1, 0, 2]
u_2	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1] (dim 1)
wpp	[1, 1, 1, 2, 1, 2]
π_5	[1, 0, 2, 0, 0, 0]
u_5	[1, 0, 1, 0, 0, 0]

2. Coloring, {2}

R: [3, 1, 6, 1, 2, 1] **B:** [5, 5, 2, 6, 3, 4]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A+\tau\Delta$:

‘ [‘ 15‘ (‘ 1 + τ ‘)‘‘ (‘ 5 - 2 τ + τ^2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -15‘ (‘ 5 + 2 τ + τ^2 ‘)‘‘ (‘ - 1 + τ ‘)‘‘ (‘ - 3 + τ ‘)‘ , 15‘ (‘ 5 - τ + 3 τ^2 + τ^3 ‘)‘‘ (‘ - 3 + τ ‘)‘ , 15‘ (‘ 5 - τ + 3 τ^2 + τ^3 ‘)‘‘ (‘ - 1 + τ ‘)‘ , 15‘ (‘ - 1 + τ ‘)‘‘ (‘ - 5 + τ^2 ‘)‘‘ (‘ - 3 + τ ‘)‘ , -30‘ (‘ 5 - τ + 3 τ^2 + τ^3 ‘)‘ ‘]‘

For $\tau=1/2$, [-255, -125, -215, -43, -95, -172] . FixedPtCheck, [255, 125, 215, 43, 95, 172]

$\det(A + \tau \Delta) = 1 \cdot (1 + 3\tau^2) \cdot (-1 + \tau)^2 \cdot (1 + \tau)$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	4 vs 5

bi =

$\$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 3/4, 0, 0, 0, 1/4], [1/4, 0, 0, 0, 0, 3/4], [0, 1/4, 3/4, 0, 0, 0], [1/4, 0, 0, 3/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$

$\$ [[312/355, 369/355, -596/2485, -528/2485, -15296/7455, 4864/7455], [82/355, 1117/1065,$

10972/7455, 13456/7455, -5952/2485, -15616/7455], [72/355, 249/355, 1772/7455, -9104/7455, -4672/2485, 15104/7455], [-68/355, -531/355, -44668/7455, -18224/7455, -5696/7455, 81664/7455], [-43/355, 752/1065, 30136/22365, 4096/7455, 7424/7455, -76288/22365], [-423/355, -1596/355, -3056/2485, -1184/7455, 62464/7455, -3072/2485]] \$ x \$ [[3/2, 3, 3, 3/2, 9/2, 3/2], [3/2, 27/8, 15/4, 9/8, 27/8, 15/8], [51/32, 117/32, 93/32, 45/32, 117/32, 57/32], [219/128, 99/32, 201/64, 171/128, 63/16, 57/32], [795/512, 855/256, 1731/512, 171/128, 1845/512, 915/512], [3309/2048, 3519/1024, 3165/1024, 2745/2048, 7515/2048, 3783/2048]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 0 . order: 3

$$[y_2, y_3, y_4, 0, 0, y_1]$$

R = \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 2/15, 2/15, -1/5], [0, 2/15, -1/5, 2/15], [0, -1/5, 2/15, 2/15], [0, 2/15, -1/5, 2/15], [1/3, -1/5, 2/15, -1/5], [0, 2/15, -1/5, 2/15]] \$ x \$ [[6, 3, 3, 0, 0, 3], [6, 0, 6, 0, 0, 3], [3, 0, 6, 0, 0, 6], [6, 0, 3, 0, 0, 6]] \$

Omega Rank for B : cycles: {{2, 3, 5}, {4, 6}}, net cycles: 2 . order: 6

$$$ [[0, 3, 3, 2, 6, 1], [0, 3, 6, 1, 3, 2], [0, 6, 3, 2, 3, 1], [0, 3, 3, 1, 6, 2], [0, 3, 6, 2, 3, 1]] $$$

$$[0, -y_1 + 4 y_2 - y_3 + 4 y_4, y_1, y_2, y_3, y_4]$$

$$p = -s - s^2 + s^4 + s^5$$

Â» SYNC'D 783/4096, 0.1911621094

3. Coloring, {3}

R: [3, 5, 2, 1, 2, 1] **B:** [5, 1, 6, 6, 3, 4]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

' ['15' (' - 5 + τ² ')'' (' 1 + τ ')'' (' - 1 + τ ')'' (' - 3 + τ ')', 15' (' 5 - 2τ + τ² ')'' (' 1 + τ ')''² ' (' - 3 + τ ')', -15' (' 5 + 2τ + τ² ')'' (' 1 + τ ')'' (' - 1 + τ ')'' (' - 3 + τ ')', 15' (' 5 + 2τ + τ² ')'' (' - 1 + τ ')''³ , 15' (' 5 - τ + 3τ² + τ³ ')'' (' 1 + τ ')'' (' - 3 + τ ')', -30' (' 5 + 2τ + τ² ')'' (' - 1 + τ ')''² ']'

For $\tau=1/2$, $[-57, -153, -75, -5, -129, -20]$. FixedPtCheck, $[57, 153, 75, 5, 129, 20]$

$$\det(A + \tau \Delta) = 1' (' 1 + \tau ') (' - 1 + \tau ')^2 (' 1 + 3\tau^2 ')'$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	5 vs 5

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 1/4, 0, 0, 0, 3/4], [1/4, 0, 0, 0, 0, 3/4], [0, 1/4, 3/4, 0, 0, 0], \\ & [1/4, 0, 0, 3/4, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], \\ & [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[-971/550, 115/66, 2578/825, 2072/825, 4064/275, -16768/825], [849/550, -151/110, -634/275, -216/275, -6368/825, 2944/275], \\ & [-1641/550, 39/22, 6038/825, 3112/825, 17632/825, -8576/275], [-801/550, 97/330, 3598/825, 2456/2475, 3744/275, -43904/2475], \\ & [732/275, -79/55, -1604/275, -1296/275, -5696/275, 24832/825], [562/275, -199/165, -4652/825, -4144/2475, -5056/275, 61696/2475]] \\ & \$ x \$ [[3, 3/2, 3, 3/2, 3, 3], [9/4, 3/2, 3, 9/4, 21/8, 27/8], [81/32, 45/32, 81/32, 81/32, 33/16, 63/16], \\ & [171/64, 147/128, 279/128, 189/64, 9/4, 243/64], [1305/512, 567/512, 603/256, 729/256, 1173/512, 1971/512], \\ & [2565/1024, 2379/2048, 603/256, 5913/2048, 2241/1024, 999/256]] \$ \end{aligned}$$

Check x AllOnes: $[1, 1, 1, 1, 1, 1]$

Omega Rank for R : cycles: $\{\{2, 5\}\}$, net cycles: 0 . order: 4

$$[y_4, y_1, y_2, 0, y_3, 0]$$

$$\begin{aligned} R = & \$ [[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, \\ & 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, \\ & 0, 0, 0]] \$ = \$ [[0, 1/3, -2/15, -2/15], [0, 0, -2/15, 1/5], [0, 0, 1/5, -2/15], [1/3, -1/3, -2/15, 1/5], [0, 0, \\ & 1/5, -2/15], [1/3, -1/3, -2/15, 1/5]] \$ x \$ [[3, 6, 3, 0, 3, 0], [0, 6, 3, 0, 6, 0], [0, 9, 0, 0, 6, 0], [0, 6, 0, 0, \\ & 9, 0]] \$ \end{aligned}$$

Omega Rank for B : cycles: $\{\{4, 6\}\}$, net cycles: 0 . order: 4

$$[y_1, 0, y_2, y_3, y_4, y_5]$$

$$\begin{aligned} B = & \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0, \\ & 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, \\ & 0, 0, 1]] \$ = \$ [[0, 1/3, -1/3, 1/5, -2/15], [1/3, -1/3, 0, -2/15, 1/5], [0, 0, 0, 8/15, -7/15], [0, 0, 0, 8/15, \\ & -7/15], [0, 0, 1/3, -7/15, 1/5], [0, 0, 0, -7/15, 8/15]] \$ x \$ [[3, 0, 3, 2, 3, 4], [0, 0, 3, 4, 3, 5], [0, 0, 3, 5, \\ & 0, 7], [0, 0, 0, 7, 0, 8], [0, 0, 0, 8, 0, 7]] \$ \end{aligned}$$

Â» SYNC'D 25/256 , 0.09765625000

4 . Coloring, {4}

R: [3, 5, 6, 6, 2, 1] **B:** [5, 1, 2, 1, 3, 4]

' See graph

' ' See pair graph

Ω for $A+\tau\Delta$:

$$['15' ('3 + \tau^2')', 15' ('3 + \tau^2')', 15' ('3 + \tau^2')', -15' ('1 + \tau')' (' - 1 + \tau')', 15' ('3 + \tau^2')', 30' ('1 + \tau')']'$$

For $\tau=1/2$, [13, 13, 13, 3, 13, 12] . FixedPtCheck, [13, 13, 13, 3, 13, 12]

$$\det(A + \tau \Delta) = 1' ('1 + 3\tau^2')' ('1 + \tau')' (' - 1 + \tau')'^2$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	3 vs 5	5 vs 5

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 3/4, 0, 0, 0, 1/4], [3/4, 0, 0, 0, 0, 1/4], [0, 1/4, 3/4, 0, 0, 0], \\ & [1/4, 0, 0, 3/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], \\ & [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[-3697/11390, 29111/102510, 15866/10251, -20312/17085, 38432/51255, -51328/51255], \\ & [193/2278, -21799/102510, -105362/51255, -16072/17085, -5536/51255, 169088/51255], [-545/2278, \\ & 29051/102510, 65098/51255, 18568/17085, 122144/51255, -241792/51255], [1573/11390, \\ & 114281/102510, -11266/10251, 8248/17085, -43168/51255, 13952/51255], [922/5695, -26957/51255, \\ & -27956/51255, 19088/17085, -139072/51255, 26368/10251], [5164/5695, -15407/51255, 2300/10251, \\ & -6032/17085, -2368/51255, -18688/51255]] \$ \times \$ [[7/2, 3, 3, 3/2, 3, 1], [29/8, 3, 25/8, 3/4, 27/8, 9/8], \\ & [99/32, 51/16, 55/16, 27/32, 111/32, 31/32], [209/64, 441/128, 27/8, 93/128, 399/128, 137/128], \\ & [1739/512, 1695/512, 1615/512, 411/512, 1695/512, 525/512], [6843/2048, 1635/512, 853/256, \\ & 1575/2048, 27/8, 1013/1024]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 5}, {1, 3, 6}}, net cycles: 2 . order: 6

$$\$ [[2, 3, 3, 0, 3, 4], [4, 3, 2, 0, 3, 3], [3, 3, 4, 0, 3, 2], [2, 3, 3, 0, 3, 4], [4, 3, 2, 0, 3, 3]] \$$$

$$[3 y_2 - y_1 - y_3, y_2, y_1, 0, y_2, y_3]$$

$$p' = s - s^4 \quad p = -s + s^4$$

Omega Rank for B : cycles: {{1, 2, 3, 5}}, net cycles: 0 . order: 4

$$[y_5, y_4, y_3, y_2, y_1, 0]$$

$$B = \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, -2/15, 7/15, -1/3, 1/15], [0, 7/15, -1/3, 1/15, -2/15], [0, -1/3, 1/15, -2/15, 7/15], [0, 7/15, -1/3, 1/15, -2/15], [0, 1/15, -2/15, 7/15, -1/3], [1/2, -1/3, 1/15, -2/15, -1/30]] \$ \times \$ [[4, 3, 3, 2, 3, 0], [5, 3, 3, 0, 4, 0], [3, 3, 4, 0, 5, 0], [3, 4, 5, 0, 3, 0], [4, 5, 3, 0, 3, 0]] \$$$

Â» SYNC'D 85/4096 , 0.02075195312

5 . Coloring, {5}

R: [3, 5, 6, 1, 3, 1] **B:** [5, 1, 2, 6, 2, 4]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$['15' ('5 - \tau + 3\tau^2 + \tau^3') ' (' - 3 + \tau ') ', 15' (' - 5 + \tau^2 ') ' (' - 1 + \tau ') ' (' - 3 + \tau ') ', 15' (' 1 + \tau ') ' (' 5 - 2\tau + \tau^2 ') ' (' - 3 + \tau ') ', 15' (' 1 + \tau ') ' (' - 1 + \tau ') ' (' 5 - 2\tau + \tau^2 ') ', -15' (' 5 + 2\tau + \tau^2 ') ' (' - 1 + \tau ') ' (' - 3 + \tau ') ', -30' (' 1 + \tau ') ' (' 5 - 2\tau + \tau^2 ') ']'$$

For τ=1/2, [-215, -95, -255, -51, -125, -204] . FixedPtCheck, [215, 95, 255, 51, 125, 204]

$$\det(A + \tau \Delta) = 1' (' 1 + 3\tau^2 ') ' (' 1 + \tau ') ' (' - 1 + \tau ')'^2$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	4 vs 5

bi =

$$\$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 3/4, 0, 0, 0, 1/4], [1/4, 0, 0, 0, 0, 3/4], [0, 3/4, 1/4, 0, 0, 0], [1/4, 0, 0, 3/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$$\$ [[1791/14605, 556/14605, -232/14605, -39776/43815, -26624/43815, 20992/14605], [-341/2921, 2028/14605, -42752/131445, -2656/8763, 29312/43815, 512/131445], [2199/2921, 4568/14605, -22248/14605, -2240/2921, -51968/43815, 108544/43815], [5101/14605, 25966/14605, -49736/43815, -16736/43815, -46848/14605, 38912/14605], [-5387/14605, -25694/43815, 5288/8763, 62752/43815, 9088/14605, -14336/8763], [-3789/14605, -10864/14605, 107744/43815, 44224/43815, 34432/14605, -208384/43815]] \$ \times \$ [[3, 9/2, 3/2, 3/2, 3, 3/2], [33/8, 27/8, 3/2, 9/8, 27/8, 3/2], [51/16, 117/32, 15/8, 9/8, 63/16, 39/32], [213/64, 279/64, 57/32, 117/128, 423/128, 21/16], [1959/512, 1953/512, 849/512, 63/64, 459/128, 579/512], [3471/1024, 8055/2048, 3795/2048, 1737/2048, 3915/1024, 2361/2048]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 0 . order: 3

$$[y_1, 0, y_2, 0, y_3, y_4]$$

$$R = \begin{bmatrix} [0, 0, 1, 0, 0, 0] \\ [0, 0, 0, 0, 1, 0] \\ [0, 0, 0, 0, 0, 1] \\ [1, 0, 0, 0, 0, 0] \\ [0, 0, 1, 0, 0, 0] \\ [1, 0, 0, 0, 0, 0] \end{bmatrix} \times \begin{bmatrix} [1, 0, 0, 0, 0, 0] \\ [0, 0, 0, 0, 0, 0] \\ [0, 0, 1, 0, 0, 0] \\ [0, 0, 0, 0, 0, 0] \\ [0, 0, 0, 0, 1, 0] \\ [0, 0, 0, 0, 0, 1] \end{bmatrix} = \begin{bmatrix} [0, 2/15, -1/5, 2/15] \\ [1/3, -1/5, 2/15, -1/5] \\ [0, 2/15, 2/15, -1/5] \\ [0, -1/5, 2/15, 2/15] \\ [0, 2/15, -1/5, 2/15] \\ [0, -1/5, 2/15, 2/15] \end{bmatrix} \times \begin{bmatrix} [3, 0, 6, 0, 3, 3] \\ [3, 0, 6, 0, 0, 6] \\ [6, 0, 3, 0, 0, 6] \\ [6, 0, 6, 0, 0, 3] \end{bmatrix}$$

Omega Rank for B : cycles: {{4, 6}, {1, 2, 5}}, net cycles: 2 . order: 6

$$\begin{bmatrix} [3, 6, 0, 2, 3, 1] \\ [6, 3, 0, 1, 3, 2] \\ [3, 3, 0, 2, 6, 1] \\ [3, 6, 0, 1, 3, 2] \\ [6, 3, 0, 2, 3, 1] \end{bmatrix}$$

$$[-y_1 + 4y_2 - y_3 + 4y_4, y_1, 0, y_2, y_3, y_4]$$

$$p = -s - s^2 + s^4 + s^5$$

Â» SYNC'D 819/4096, 0.1999511719

6. Coloring, {6}

R: [3, 5, 6, 1, 2, 4] **B:** [5, 1, 2, 6, 3, 1]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

$$[15(3 + \tau^2), 15(3 + \tau^2), 15(3 + \tau^2), 15(1 + \tau)^2, 15(3 + \tau^2), 30(1 + \tau)]$$

For τ=1/2, [13, 13, 13, 9, 13, 12] . FixedPtCheck, [13, 13, 13, 9, 13, 12]

$$\det(A + \tau \Delta) = (1 + \tau)^2 (1 + 10\tau^2 + 5\tau^4)$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 6	5 vs 5

bi =

$\$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 3/4, 0, 0, 0, 1/4], [1/4, 0, 0, 0, 0, 3/4], [0, 1/4, 3/4, 0, 0, 0], [3/4, 0, 0, 1/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$

$\$ [[-4/153, 23/45, -352/255, -32/255, -1216/765, 2048/765], [296/765, -41/45, -244/255, -96/85, 1024/765, 1024/765], [64/765, 11/45, -424/255, 592/255, -1984/765, 256/153], [2683/765, -32/9, -2392/255, -32/255, -7744/765, 15104/765], [-184/765, -17/45, 692/255, -16/17, 1216/765, -2048/765], [-1193/765, 116/45, 1688/255, -32/255, 5312/765, -11008/765]] \$ \times \$ [[4, 3, 3, 1/2, 3, 3/2], [7/2, 3, 13/4, 3/8, 15/4, 9/8], [51/16, 27/8, 59/16, 9/32, 27/8, 35/32], [219/64, 231/64, 213/64, 35/128, 207/64, 145/128], [29/8, 423/128, 105/32, 145/512, 111/32, 531/512], [3407/1024, 213/64, 449/128, 531/2048, 1815/512, 2115/2048]] \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 4, 6}, {2, 5}}, net cycles: 2 . order: 4

$\$ [[1, 3, 3, 2, 3, 3], [2, 3, 1, 3, 3, 3], [3, 3, 2, 3, 3, 1], [3, 3, 3, 1, 3, 2], [1, 3, 3, 2, 3, 3], [2, 3, 1, 3, 3, 3]] \$$

$[3 y_3 - y_1 - y_2 - y_4, y_3, y_1, y_2, y_3, y_4]$

$$p' = -s + s^5 \quad p' = -1 + s^4$$

Omega Rank for B : cycles: {{1, 2, 3, 5}}, net cycles: 0 . order: 4

$[y_1, y_2, y_3, 0, y_4, y_5]$

$B = \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 7/15, -2/15, 1/15, -1/3], [0, -2/15, 1/15, -1/3, 7/15], [0, 1/15, -1/3, 7/15, -2/15], [1, 1/15, -1/3, 7/15, -17/15], [0, -1/3, 7/15, -2/15, 1/15], [0, -2/15, 1/15, -1/3, 7/15]] \$ \times \$ [[5, 3, 3, 0, 3, 1], [4, 3, 3, 0, 5, 0], [3, 3, 5, 0, 4, 0], [3, 5, 4, 0, 3, 0], [5, 4, 3, 0, 3, 0]] \$$

Â» SYNC'D 15/512 , 0.02929687500

7. Coloring, {2, 3}

R: [3, 1, 2, 1, 2, 1] **B:** [5, 5, 6, 6, 3, 4]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

$['15' (' - 5 - \tau - 3\tau^2 + \tau^3 ')'' (' 1 + \tau ')'' (' - 3 + \tau ')', -15' (' 1 + \tau ')'^2 (' 5 - 2\tau + \tau^2 ')'' (' - 3 + \tau ')', -15' (' 1 + \tau ')'' (' 5 - \tau + 3\tau^2 + \tau^3 ')'' (' - 3 + \tau ')', 15' (' - 1 + \tau ')'^2 (' 5 - \tau + 3\tau^2 + \tau^3 ')', 15' (' 5 + 2\tau + \tau^2 ')'' (' - 1 + \tau ')'' (' 1 + \tau ')'' (' - 3 + \tau ')', -30' (' - 1 + \tau ')'' (' 5 - \tau + 3\tau^2$

$$+ \tau^3 \text{ ' ' ' ' }$$

For $\tau=1/2$, [735, 765, 645, 43, 375, 172] . FixedPtCheck, [735, 765, 645, 43, 375, 172]

$$\det(A + \tau \Delta) = 1 \text{ ' (' - 1 + } \tau \text{ ') ' }^3 \text{ ' (' 1 + } \tau \text{ ') ' }^2$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	3 vs 3	4 vs 4

bi =

$$\$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 1/4, 0, 0, 0, 3/4], [1/4, 0, 0, 0, 0, 3/4], [0, 1/4, 3/4, 0, 0, 0], [1/4, 0, 0, 3/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$$\$ [[12703/34475, -6562/34475, -4808/8865, -125632/103425, -5248/103425, 526336/310275], [739/4925, -2168/14775, 1064/2955, 25184/14775, 2176/14775, -31744/14775], [-1277/34475, 258/34475, 3232/8865, -101312/103425, -6656/34475, 280576/310275], [-2053/14775, -3214/44325, 1352/8865, 7744/14775, 34048/44325, -51712/44325], [-4891/103425, 40274/103425, -5752/26595, 52384/310275, 4864/34475, -343552/930825], [-8461/103425, -16588/310275, -224/8865, 22688/103425, -139904/310275, 142336/310275]] \$ \times \$ [[3/2, 3/2, 3, 3/2, 9/2, 3], [3/2, 15/8, 15/4, 9/4, 9/4, 27/8], [15/8, 3/2, 33/16, 81/32, 81/32, 9/2], [273/128, 147/128, 303/128, 27/8, 81/32, 441/128], [255/128, 627/512, 1245/512, 1323/512, 315/128, 2205/512], [4155/2048, 2505/2048, 75/32, 6615/2048, 4941/2048, 963/256]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 1 . order: 3

$$[y_1, y_2, y_3, 0, 0, 0]$$

$$R = \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0]] \$ = \$ [[-1/5, 2/15, 2/15], [2/15, 2/15, -1/5], [2/15, -1/5, 2/15], [2/15, 2/15, -1/5], [2/15, -1/5, 2/15], [2/15, 2/15, -1/5]] \$ \times \$ [[6, 6, 3, 0, 0, 0], [6, 3, 6, 0, 0, 0], [3, 6, 6, 0, 0, 0]] \$$$

Omega Rank for B : cycles: {{4, 6}}, net cycles: 0 . order: 4

$$[0, 0, y_4, y_1, y_2, y_3]$$

$$B = \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0]] \$ \times \$ [[0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[1/6, -1/12, -1/30, 1/60], [1/6, -1/12, -1/30, 1/60], [0, 0, 2/15, -1/15], [0, 0, 2/15, -1/15], [0, 1/6, -1/15, -1/30], [0, 0, -1/15, 2/15]] \$ \times \$ [[0, 0, 3, 2, 6, 4], [0, 0, 6, 4, 0, 5], [0, 0, 0, 5, 0, 10], [0, 0, 0, 10, 0, 5]] \$$$

Â» SYNC'D 19/64 , 0.2968750000

8. Coloring, {2, 4}

R: [3, 1, 6, 6, 2, 1] **B:** [5, 5, 2, 1, 3, 4]

' See graph

' ' See pair graph

'

Ω for $A+\tau\Delta$:

' ['15' ('1 + τ ')'' ('3 + τ^2 ')'' ('5 - 2 τ + τ^2 ')', -15' ('-1 + τ ')'' ('3 + τ^2 ')'' ('5 + 2 τ + τ^2 ')', 15' ('5 - τ + 3 τ^2 + τ^3 ')'' ('3 + τ^2 ')', -15' ('-1 + τ ')'' ('5 - τ + 3 τ^2 + τ^3 ')'' ('1 + τ ')', 15' ('-1 + τ ')'' ('-5 + τ^2 ')'' ('3 + τ^2 ')', 30' ('5 - τ + 3 τ^2 + τ^3 ')'' ('1 + τ ')'']'

For $\tau=1/2$, [663, 325, 559, 129, 247, 516] . FixedPtCheck, [663, 325, 559, 129, 247, 516]

$$\det(A + \tau \Delta) = 1' ('-1 + \tau ')'^3 ('1 + \tau ')'^2$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	5 vs 5

bi =

\$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 3/4, 0, 0, 0, 1/4], [3/4, 0, 0, 0, 0, 1/4], [0, 1/4, 3/4, 0, 0, 0], [1/4, 0, 0, 3/4, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =
 \$ [[-103/815, -12928/7335, 22856/7335, 26816/4401, 264832/22005, -424448/22005], [-673/815, -7798/7335, 41096/7335, 48512/4401, 116992/22005, -439808/22005], [-1/5, 44/45, 32/45, -64/27, -896/135, 1024/135], [-923/815, 14012/7335, 55616/7335, 44576/4401, -134528/22005, -270848/22005], [607/815, 242/7335, -5056/815, -38368/4401, 15232/22005, 297472/22005], [1367/815, 12962/7335, -63304/7335, -62080/4401, -309248/22005, 735232/22005]] \$ x \$ [[2, 3, 3, 3/2, 9/2, 1], [17/8, 27/8, 31/8, 3/4, 15/4, 9/8], [27/16, 123/32, 107/32, 27/32, 33/8, 37/32], [241/128, 453/128, 225/64, 111/128, 531/128, 67/64], [115/64, 1881/512, 917/256, 201/256, 1041/256, 561/512], [57/32, 237/64, 3583/1024, 1683/2048, 8403/2048, 559/512]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 0 . order: 3

$$[y_1, y_4, y_2, 0, 0, y_3]$$

R = \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 1/45, 17/90, -13/90], [0, 17/90, -13/90, 1/45], [0, -13/90, 1/45, 17/90], [0, -13/90,

$$1/45, 17/90], [1/3, -13/90, 1/45, -13/90], [0, 17/90, -13/90, 1/45]] \$ x \$ [[5, 3, 3, 0, 0, 4], [7, 0, 5, 0, 0, 3], [3, 0, 7, 0, 0, 5], [5, 0, 3, 0, 0, 7]] \$$$

Omega Rank for B : cycles: {{2, 3, 5}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, y_4, y_5, 0]$$

$$B = \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 0]] \$ = \$ [[0, 0, 1/45, 16/45, -14/45], [0, 0, 1/45, 16/45, -14/45], [0, 0, 16/45, -14/45, 1/45], [0, 1/2, 16/45, -14/45, -43/90], [0, 0, -14/45, 1/45, 16/45], [1/2, -1/4, -14/45, -43/90, 109/180]] \$ x \$ [[1, 3, 3, 2, 6, 0], [2, 3, 6, 0, 4, 0], [0, 6, 4, 0, 5, 0], [0, 4, 5, 0, 6, 0], [0, 5, 6, 0, 4, 0]] \$$$

Â» SYNC'D 245/1024 , 0.2392578125

9 . Coloring, {2, 5}

R: [3, 1, 6, 1, 3, 1] **B:** [5, 5, 2, 6, 2, 4]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

$$['15' ('1 + \tau')'' ('-3 + \tau')', -15' ('-1 + \tau')'' ('-3 + \tau')', 15' ('1 + \tau')'' ('-3 + \tau')', 15' ('-1 + \tau')'' ('1 + \tau')', -15' ('-1 + \tau')'' ('-3 + \tau')', -30' ('1 + \tau')'']'$$

For τ=1/2, [-15, -5, -15, -3, -5, -12] . FixedPtCheck, [15, 5, 15, 3, 5, 12]

$$\det(A + \tau \Delta) = 1' ('-1 + \tau')' ^3 ' ('1 + \tau')' ^2$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	3 vs 3	2 vs 4

bi =

$$\$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 3/4, 0, 0, 0, 1/4], [1/4, 0, 0, 0, 0, 3/4], [0, 3/4, 1/4, 0, 0, 0], [1/4, 0, 0, 3/4, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$$\$ [[42/95, 51/95, 220/57, -1936/285, -2752/285, 3328/285], [86/285, 593/285, 1004/513, -2576/285, -5056/855, 27392/2565], [32/95, 103/285, -572/171, 208/95, 1088/285, -2816/855], [42/95, 913/285, 68/57, -1936/285, -2752/285, 3328/285], [-149/285, -2056/855, -1120/513, 928/95, 2048/285, -30208/2565], [-53/95, -234/95, -176/171, 2624/285, 3328/285, -14336/855]] \$ x \$ [[3/2, 9/2, 3/2, 3/2, 9/2, 3/2], [15/8, 9/2, 3/2, 9/8, 9/2, 3/2], [57/32, 9/2, 51/32, 9/8, 153/32, 39/32], [219/128, 153/32, 105/64, 117/128, 603/128, 159/128], [111/64, 2439/512, 411/256, 477/512, 2493/512, 561/512], [3477/2048,$$

9945/2048, 3381/2048, 1683/2048, 9981/2048, 2253/2048] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 1 . order: 3

$$[y_3, 0, y_2, 0, 0, y_1]$$

$$R = \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[2/15, 2/15, -1/5], [2/15, -1/5, 2/15], [-1/5, 2/15, 2/15], [2/15, -1/5, 2/15], [2/15, 2/15, -1/5], [2/15, -1/5, 2/15]] \$ x \$ [[6, 0, 6, 0, 0, 3], [3, 0, 6, 0, 0, 6], [6, 0, 3, 0, 0, 6]] \$$$

Omega Rank for B : cycles: {{2, 5}, {4, 6}}, net cycles: 2 . order: 2

$$\$ [[0, 6, 0, 2, 6, 1], [0, 6, 0, 1, 6, 2], [0, 6, 0, 2, 6, 1], [0, 6, 0, 1, 6, 2]] \$$$

$$[0, 2y_1 + 2y_2, 0, y_1, 2y_1 + 2y_2, y_2]$$

$$p = s - s^3 \quad p' = s - s^3$$

Â» SYNC'D 15/512, 0.02929687500

10 . Coloring, {2, 6}

R: [3, 1, 6, 1, 2, 4] **B:** [5, 5, 2, 6, 3, 1]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

$$['15' ('1 + \tau')' ('3 + \tau^2')' ('5 - 2\tau + \tau^2')', -15' ('-1 + \tau')' ('3 + \tau^2')' ('5 + 2\tau + \tau^2')', 15' ('5 - \tau + 3\tau^2 + \tau^3')' ('3 + \tau^2')', 15' ('1 + \tau')'^2 ' ('5 - \tau + 3\tau^2 + \tau^3')', 15' ('-1 + \tau')' ('-5 + \tau^2')' ('3 + \tau^2')', 30' ('1 + \tau')' ('5 - \tau + 3\tau^2 + \tau^3')']'$$

For τ=1/2, [663, 325, 559, 387, 247, 516] . FixedPtCheck, [663, 325, 559, 387, 247, 516]

$$\det(A + \tau \Delta) = 1' ('-1 + \tau')' ('1 + \tau')'^2 ' ('1 + 3\tau^2')$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	5 vs 5	5 vs 5

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 3/4, 0, 0, 0, 1/4], [1/4, 0, 0, 0, 0, 3/4], [0, 1/4, 3/4, 0, 0, 0], \\ & [3/4, 0, 0, 1/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], \\ & [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[-50978/21145, 6185/4229, 704276/63435, 94576/12687, 420416/63435, -1533184/63435], \\ & [7012/21145, -10951/12687, -78964/63435, -848/12687, 82496/63435, 38656/63435], [-7998/21145, \\ & 317/4229, 164636/63435, 13840/12687, -28864/63435, -181504/63435], [159262/21145, -25447/4229, \\ & -1725244/63435, -287504/12687, -556864/63435, 3627776/63435], [49921/63435, -524/4229, \\ & -298504/63435, -46784/12687, -126464/63435, 621056/63435], [-16073/21145, 9232/4229, \\ & 125456/63435, 52576/12687, -242944/63435, -231424/63435]] \$ \times \$ [[5/2, 3, 3, 1/2, 9/2, 3/2], [2, 27/8, \\ & 4, 3/8, 33/8, 9/8], [57/32, 129/32, 115/32, 9/32, 129/32, 41/32], [261/128, 237/64, 111/32, 41/128, \\ & 279/64, 71/64], [941/512, 945/256, 1935/512, 71/256, 2205/512, 567/512], [3733/2048, 4005/1024, \\ & 1889/512, 567/2048, 8493/2048, 2361/2048]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 4, 6}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, y_4, 0, y_5]$$

$$\begin{aligned} R = & \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], \\ & [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[0, -2/15, 7/15, -1/3, 1/15], [0, 7/15, -1/3, 1/15, -2/15], [0, 1/15, -2/15, 7/15, -1/3], [0, 7/15, -1/3, 1/15, -2/15], \\ & [1/3, -1/3, 1/15, -2/15, 2/15], [0, -1/3, 1/15, -2/15, 7/15]] \$ \times \$ [[4, 3, 3, 2, 0, 3], [5, 0, 4, 3, 0, 3], [3, 0, 5, 3, 0, 4], [3, 0, 3, 4, 0, 5], [4, 0, 3, 5, 0, 3]] \$ \end{aligned}$$

Omega Rank for B : cycles: {{2, 3, 5}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, 0, y_4, y_5]$$

$$\begin{aligned} B = & \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], \\ & [0, 0, 0, 0, 0, 1]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[0, 0, -14/45, 16/45, 1/45], [0, 0, -14/45, 16/45, 1/45], [0, 0, 16/45, 1/45, -14/45], [1, -2, 1/45, -59/45, 106/45], \\ & [0, 0, 1/45, -14/45, 16/45], [0, 1, 16/45, 1/45, -59/45]] \$ \times \$ [[2, 3, 3, 0, 6, 1], [1, 3, 6, 0, 5, 0], [0, 6, 5, 0, 4, 0], [0, 5, 4, 0, 6, 0], [0, 4, 6, 0, 5, 0]] \$ \end{aligned}$$

Â» SYNC'D 357/4096 , 0.08715820312

11 . Coloring, {3, 4}

R: [3, 5, 2, 6, 2, 1] **B:** [5, 1, 6, 1, 3, 4]

‘ See graph

‘ ‘ See pair graph

Ω for $A+\tau\Delta$:

$$\begin{aligned} & ['15' ('-1 + \tau')'' ('-5 + \tau^2')'' ('3 + \tau^2')', 15' ('1 + \tau')'' ('3 + \tau^2')'' ('5 - 2\tau + \tau^2')', \\ & , -15' ('5 + 2\tau + \tau^2')'' ('-1 + \tau')'' ('3 + \tau^2')', -15' ('5 + 2\tau + \tau^2')'' ('-1 + \tau')'^3, 15' ('5 - \\ & \tau + 3\tau^2 + \tau^3')'' ('3 + \tau^2')', 30' ('5 + 2\tau + \tau^2')'' ('-1 + \tau')'^2]' \end{aligned}$$

For $\tau=1/2$, [247, 663, 325, 25, 559, 100] . FixedPtCheck, [247, 663, 325, 25, 559, 100]

$$\det(A + \tau \Delta) = 1' ('1 + \tau')'^2 ('-1 + \tau')'^3$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	5 vs 5	5 vs 5

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 1/4, 0, 0, 0, 3/4], [3/4, 0, 0, 0, 0, 1/4], [0, 1/4, 3/4, 0, 0, \\ & 0], [1/4, 0, 0, 3/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, \\ & 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \end{aligned}$$

$$\begin{aligned} & \$ [[12747/3084545, 2046052/3084545, -1001440/616909, 611872/3084545, -3480832/9253635, \\ & 11107328/9253635], [1920087/3084545, -2486768/3084545, -197912/616909, 708992/3084545, \\ & 5237888/9253635, -2079232/9253635], [-495063/3084545, -171578/3084545, 1408/616909, \\ & 3857152/3084545, -32163712/9253635, 23187968/9253635], [490187/3084545, -5468628/3084545, \\ & -240520/616909, -1156448/3084545, -2217472/9253635, 24846848/9253635], [34857/3084545, \\ & 1014262/3084545, 845968/616909, -6573568/3084545, 16871168/9253635, -12370432/9253635], \\ & [-911763/3084545, 2131362/3084545, 648224/616909, 2671552/3084545, 21411968/9253635, \\ & -42191872/9253635]] \$ \times \$ [[7/2, 3/2, 3, 3/2, 3, 5/2], [23/8, 3/2, 25/8, 15/8, 3, 21/8], [51/16, 49/32, \\ & 95/32, 63/32, 81/32, 45/16], [213/64, 11/8, 345/128, 135/64, 355/128, 87/32], [843/256, 175/128, \\ & 1491/512, 261/128, 727/256, 1305/512], [6537/2048, 2945/2048, 189/64, 3915/2048, 2879/1024, \\ & 5517/2048]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 5}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, 0, y_4, y_5]$$

$$\begin{aligned} & R = \$ [[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, \\ & 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, \\ & 0, 0, 1]] \$ = \$ [[0, 0, 1, 8/15, -22/15], [0, 0, 0, 8/15, -7/15], [0, 0, 0, -7/15, 8/15], [1, -2, 1, 38/15, \\ & -37/15], [0, 0, 0, -7/15, 8/15], [0, 1, -2, -22/15, 38/15]] \$ \times \$ [[2, 6, 3, 0, 3, 1], [1, 6, 2, 0, 6, 0], [0, 8, 1, \\ & 0, 6, 0], [0, 7, 0, 0, 8, 0], [0, 8, 0, 0, 7, 0]] \$ \end{aligned}$$

Omega Rank for B : cycles: {{1, 3, 4, 5, 6}}, net cycles: 1 . order: 5

$$[y_1, 0, y_2, y_3, y_4, y_5]$$

$$B = \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[16/75, 31/75, -29/75, -14/75, 1/75], [31/75, -29/75, -14/75, 1/75, 16/75], [-14/75, 1/75, 16/75, 31/75, -29/75], [31/75, -29/75, -14/75, 1/75, 16/75], [1/75, 16/75, 31/75, -29/75, -14/75], [-29/75, -14/75, 1/75, 16/75, 31/75]] \$ \times \$ [[4, 0, 3, 2, 3, 3], [2, 0, 3, 3, 4, 3], [3, 0, 4, 3, 2, 3], [3, 0, 2, 3, 3, 4], [3, 0, 3, 4, 3, 2]] \$$$

Â» SYNC'D 515/8192 , 0.06286621094

12 . Coloring, {3, 5}

R: [3, 5, 2, 1, 3, 1] **B:** [5, 1, 6, 6, 2, 4]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$\begin{aligned} & \text{' ['15' ('5 + 2τ + τ^2')'' ('-1 + τ')'' ('-3 + τ')', 15' ('-5 - τ - 3τ^2 + τ^3')'' ('-3 + τ')', } \\ & -15' ('1 + τ')'' ('5 - 2τ + τ^2')'' ('-3 + τ')', 15' ('5 - 2τ + τ^2')'' ('-1 + τ')'^2, -15' ('5 - τ + } \\ & 3τ^2 + τ^3')'' ('-3 + τ')', -30' ('5 - 2τ + τ^2')'' ('-1 + τ')'] \end{aligned}$$

For τ=1/2, [125, 245, 255, 17, 215, 68] . FixedPtCheck, [125, 245, 255, 17, 215, 68]

$$\det(A + \tau \Delta) = 1' ('-1 + \tau')'^3 ('1 + \tau')'^2$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	2 vs 5

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 1/4, 0, 0, 0, 3/4], [1/4, 0, 0, 0, 0, 3/4], [0, 3/4, 1/4, 0, 0, } \\ & 0], [1/4, 0, 0, 3/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, } \\ & 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[144/845, 2053/2535, 17164/7605, -1392/845, -8384/2535, 13568/7605], [14/845, 5563/2535, } \\ & 9884/7605, -2096/2535, -64/2535, -19712/7605], [444/845, -1669/845, -7052/2535, -912/845, 5696/2535, } \\ & 7936/2535], [-488/2535, -13201/7605, -50468/22815, -6128/7605, 25408/7605, 38144/22815], } \\ & [-201/845, 236/845, 1696/845, 11744/2535, -2944/2535, -4608/845], [-293/2535, -8326/7605, } \\ & -69968/22815, -9248/7605, 12928/7605, 88064/22815]] \$ \times \$ [[3, 3, 3/2, 3/2, 3, 3], [27/8, 21/8, 3/2, } \\ & 9/4, 3, 9/4], [99/32, 21/8, 51/32, 27/16, 51/16, 45/16], [99/32, 357/128, 201/128, 135/64, 381/128, } \\ & 315/128], [207/64, 21/8, 777/512, 945/512, 1545/512, 1413/512], [3195/1024, 1353/512, 3201/2048, } \\ & 4239/2048, 789/256, 2583/1024]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 5}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, 0, y_4, 0]$$

$$R = \$ [[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0]] \$ = \$ [[0, 2/15, -1/5, 2/15], [0, -1/5, 2/15, 2/15], [0, 2/15, 2/15, -1/5], [1/3, -1/5, 2/15, -1/5], [0, 2/15, -1/5, 2/15], [1/3, -1/5, 2/15, -1/5]] \$ x \$ [[3, 3, 6, 0, 3, 0], [0, 6, 6, 0, 3, 0], [0, 6, 3, 0, 6, 0], [0, 3, 6, 0, 6, 0]] \$$$

Omega Rank for B : cycles: {{1, 2, 5}, {4, 6}}, net cycles: 2 . order: 6

$$\$ [[3, 3, 0, 2, 3, 4], [3, 3, 0, 4, 3, 2], [3, 3, 0, 2, 3, 4], [3, 3, 0, 4, 3, 2], [3, 3, 0, 2, 3, 4]] \$$$

$$[y_1, y_1, 0, y_2, y_1, 2y_1 - y_2]$$

$$p = -s + s^3 \quad p' = -s + s^3 \quad p = -s + s^5$$

Â» SYNC'D 135/2048 , 0.06591796875

13 . Coloring, {3, 6}

R: [3, 5, 2, 1, 2, 4] **B:** [5, 1, 6, 6, 3, 1]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

$$['15' ('-5 + \tau^2') ('3 + \tau^2') ('-1 + \tau')', 15' ('3 + \tau^2') ('5 - 2\tau + \tau^2') ('1 + \tau')', -15' ('3 + \tau^2') ('5 + 2\tau + \tau^2') ('-1 + \tau')', 15' ('5 + 2\tau + \tau^2') ('-1 + \tau')^2 ('1 + \tau')', 15' ('5 - \tau + 3\tau^2 + \tau^3') ('3 + \tau^2')', 30' ('5 + 2\tau + \tau^2') ('-1 + \tau')^2]'$$

For τ=1/2, [247, 663, 325, 75, 559, 100] . FixedPtCheck, [247, 663, 325, 75, 559, 100]

$$\det(A + \tau \Delta) = 1' ('1 + 3\tau^2') ('-1 + \tau') ('1 + \tau')^2$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	5 vs 5	4 vs 4

bi =

$\$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 1/4, 0, 0, 0, 3/4], [1/4, 0, 0, 0, 0, 3/4], [0, 1/4, 3/4, 0, 0, 0], [3/4, 0, 0, 1/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$

$\$ [[-59201/194690, 160387/194690, 604342/97345, -371976/97345, 2317216/292035, -3146624/292035], [144079/194690, 109507/194690, -350578/97345, 154744/97345, -1134944/292035, 1361536/292035], [-137351/194690, -13823/194690, 1035242/97345, -339096/97345, 3589216/292035, -5431424/292035], [-59291/194690, -581803/194690, 438922/97345, -361176/97345, 2462176/292035, -1714304/292035], [51572/97345, -289/97345, -828148/97345, 394384/97345, -3733184/292035, 4900096/292035], [25492/97345, -46169/97345, -910748/97345, 423504/97345, -2788544/292035, 4331776/292035]] \$ \times \$ [[4, 3/2, 3, 1/2, 3, 3], [7/2, 3/2, 13/4, 3/4, 27/8, 21/8], [105/32, 53/32, 109/32, 21/32, 3, 3], [117/32, 205/128, 393/128, 3/4, 23/8, 195/64], [1881/512, 761/512, 393/128, 195/256, 1609/512, 1467/512], [3537/1024, 3181/2048, 1677/512, 1467/2048, 1601/512, 2943/1024]] \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 5}}, net cycles: 0 . order: 4

$[y_1, y_5, y_2, y_3, y_4, 0]$

$R = \$ [[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0]] \$ = \$ [[0, 0, 1/2, -7/15, 1/30], [0, 0, 0, -7/15, 8/15], [0, 0, 0, 8/15, -7/15], [0, 1/2, -1/4, 1/30, -13/60], [0, 0, 0, 8/15, -7/15], [1/2, -1/4, -5/8, -13/60, 79/120]] \$ \times \$ [[1, 6, 3, 2, 3, 0], [2, 6, 1, 0, 6, 0], [0, 7, 2, 0, 6, 0], [0, 8, 0, 0, 7, 0], [0, 7, 0, 0, 8, 0]] \$$

Omega Rank for B : cycles: {{1, 3, 5, 6}}, net cycles: 1 . order: 4

$[y_1, 0, y_4, 0, y_2, y_3]$

$B = \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[-1/3, 7/15, -2/15, 1/15], [7/15, -2/15, 1/15, -1/3], [-2/15, 1/15, -1/3, 7/15], [-2/15, 1/15, -1/3, 7/15], [1/15, -1/3, 7/15, -2/15], [7/15, -2/15, 1/15, -1/3]] \$ \times \$ [[5, 0, 3, 0, 3, 4], [4, 0, 3, 0, 5, 3], [3, 0, 5, 0, 4, 3], [3, 0, 4, 0, 3, 5]] \$$

Â» SYNC'D 111/1024 , 0.1083984375

14 . Coloring, {4, 5}

R: [3, 5, 6, 6, 3, 1] **B:** [5, 1, 2, 1, 2, 4]

‘ See graph

‘ ‘ See pair graph

Ω for $A+\tau\Delta$:

$$\left[\begin{matrix} 15(\tau^3 + 3\tau^2 + \tau) & (5 - \tau + 3\tau^2 + \tau^3) & 15(-1 + \tau) & (-5 + \tau^2) & (3 + \tau^2) & 15 \\ (\tau + 1) & (3 + \tau^2) & (5 - 2\tau + \tau^2) & -15(-1 + \tau) & (\tau + 1)^2 & (5 - 2\tau + \tau^2) \\ (-1 + \tau) & (3 + \tau^2) & (5 + 2\tau + \tau^2) & 30(\tau + 1)^2 & (5 - 2\tau + \tau^2) & \end{matrix} \right]$$

For $\tau=1/2$, [559, 247, 663, 153, 325, 612] . FixedPtCheck, [559, 247, 663, 153, 325, 612]

$$\det(A + \tau \Delta) = 1^3 (-1 + \tau)^3 (\tau + 1)^2$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	4 vs 4

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 3/4, 0, 0, 0, 1/4], [3/4, 0, 0, 0, 0, 1/4], [0, 3/4, 1/4, 0, 0, 0], \\ & [1/4, 0, 0, 3/4, 0, 0]] \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], \\ & [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[-10098/30785, -10301/18471, 19004/6157, 40112/55413, 554944/277065, -1346816/277065], \\ & [2916/30785, 10983/30785, -158308/92355, -224912/277065, -158144/277065, 150272/55413], \\ & [-48684/30785, -253651/92355, 467164/30785, 1922608/277065, 3094336/277065, -1600768/55413], \\ & [-36178/30785, -10253/18471, 54252/6157, 241616/55413, 1520704/277065, -4672256/277065], \\ & [21489/30785, 98522/92355, -605984/92355, -622976/277065, -287488/55413, 3407872/277065], \\ & [85047/30785, 57232/18471, -119352/6157, -503392/55413, -3840896/277065, 10123264/277065]] \$ \times \\ & \$ [[7/2, 9/2, 3/2, 3/2, 3, 1], [19/4, 27/8, 13/8, 3/4, 15/4, 3/4], [105/32, 129/32, 17/8, 9/16, 141/32, 19/32], \\ & [115/32, 627/128, 123/64, 57/128, 111/32, 43/64], [1069/256, 1035/256, 113/64, 129/256, 2007/512, 303/512], \\ & [7287/2048, 8733/2048, 4145/2048, 909/2048, 2121/512, 581/1024]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 0 . order: 3

$$[y_1, 0, y_2, 0, y_3, y_4]$$

$$\begin{aligned} R = & \$ [[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], \\ & [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[0, 1/45, -14/45, 16/45], [1/3, -14/45, 16/45, -14/45], [0, 16/45, 1/45, -14/45], [0, 16/45, 1/45, -14/45], \\ & [0, 1/45, -14/45, 16/45], [0, -14/45, 16/45, 1/45]] \$ \times \$ [[2, 0, 6, 0, 3, 4], [4, 0, 5, 0, 0, 6], \\ & [6, 0, 4, 0, 0, 5], [5, 0, 6, 0, 0, 4]] \$ \end{aligned}$$

Omega Rank for B : cycles: {{1, 2, 5}}, net cycles: 0 . order: 3

$$[y_1, y_2, 0, y_3, y_4, 0]$$

$$\begin{aligned} B = & \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], \\ & [0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0]] \$ \times \\ & \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0]] \$ \end{aligned}$$

$0, 0, 0]] \$ = \$ [[0, -8/315, 52/315, -23/315], [0, 52/315, -23/315, -8/315], [0, -23/315, -8/315, 52/315], [0, 52/315, -23/315, -8/315], [0, -23/315, -8/315, 52/315], [1/2, -23/315, -8/315, -211/630]] \$ \times \$ [[4, 6, 0, 2, 3, 0], [8, 3, 0, 0, 4, 0], [3, 4, 0, 0, 8, 0], [4, 8, 0, 0, 3, 0]] \$$

Â» SYNC'D 179/512 , 0.3496093750

15 . Coloring, {4, 6}

R: [3, 5, 6, 6, 2, 4] **B:** [5, 1, 2, 1, 3, 1]

' See graph

' ' See pair graph

Ω for $A+\tau\Delta$:

' ['15' ('3 + τ')' ('-1 + τ')', 15' ('3 + τ')' ('-1 + τ')', 15' ('3 + τ')' ('-1 + τ')', -15' ('1 + τ')' ², 15' ('3 + τ')' ('-1 + τ')', -30' ('1 + τ')']'

For $\tau=1/2, [-7, -7, -7, -9, -7, -12]$. FixedPtCheck, [7, 7, 7, 9, 7, 12]

$\det(A + \tau \Delta) = 1' ('1 + 3\tau^2')$ ('-1 + τ')' ('1 + τ')' ²

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	5 vs 6	3 vs 5	4 vs 4

Omega Rank for R : cycles: {{2, 5}, {4, 6}}, net cycles: 1 . order: 2

$\$ [[0, 3, 3, 2, 3, 4], [0, 3, 0, 4, 3, 5], [0, 3, 0, 5, 3, 4], [0, 3, 0, 4, 3, 5], [0, 3, 0, 5, 3, 4]] \$$

$[0, y_2, y_3, 3y_2 - y_3 - y_1, y_2, y_1]$

$p = -s^2 + s^4$ $p' = -s^2 + s^4$

Omega Rank for B : cycles: {{1, 2, 3, 5}}, net cycles: 1 . order: 4

$[y_1, y_2, y_3, 0, y_4, 0]$

$B = \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0]] \$ = \$ [[-1/15, 4/15, -1/15, -1/15], [4/15, -1/15, -1/15, -1/15], [-1/15, -1/15, -1/15, 4/15], [4/15, -1/15, -1/15, -1/15], [-1/15, -1/15, 4/15, -1/15], [4/15, -1/15, -1/15, -1/15]] \$ \times \$ [[6, 3, 3, 0, 3, 0], [3, 3, 3, 0, 6, 0], [3, 3, 6, 0, 3, 0], [3, 6, 3, 0, 3, 0]] \$$

Â» SYNC'D 75/2048 , 0.03662109375

16 . Coloring, {5, 6}

R: [3, 5, 6, 1, 3, 4] **B:** [5, 1, 2, 6, 2, 1]

' See graph

' ' See pair graph

,

Ω for $A+\tau\Delta$:

' ['15' ('5 - τ + 3 τ^2 + τ^3)' ('3 + τ^2)' , 15' ('-5 + τ^2)' ('3 + τ^2)' ('-1 + τ)' , 15' ('1 + τ)' ('3 + τ^2)' ('5 - 2 τ + τ^2)' , 15' ('1 + τ)'^3 ('5 - 2 τ + τ^2)' , -15' ('3 + τ^2)' ('5 + 2 τ + τ^2)' ('-1 + τ)' , 30' ('1 + τ)'^2 ('5 - 2 τ + τ^2)']'

For $\tau=1/2$, [559, 247, 663, 459, 325, 612] . FixedPtCheck, [559, 247, 663, 459, 325, 612]

$\det(A + \tau \Delta) = 1' ('1 + 3\tau^2$)' ('1 + τ)'^2 ('-1 + τ)'

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	5 vs 5	4 vs 4

bi =

\$ [[0, 0, 1/4, 0, 3/4, 0] , [3/4, 0, 0, 0, 1/4, 0] , [0, 3/4, 0, 0, 0, 1/4] , [1/4, 0, 0, 0, 0, 3/4] , [0, 3/4, 1/4, 0, 0, 0] , [3/4, 0, 0, 1/4, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ =

\$ [[42829/293815, -17724/293815, -416776/881445, -184352/176289, 512/881445, 1321472/881445] , [-191701/881445, 1536/58763, 628736/881445, 461792/881445, 52608/58763, -1652224/881445] , [213793/293815, -45376/58763, -1493464/881445, -1027648/881445, 38656/176289, 2425856/881445] , [-285161/293815, -363014/293815, 9080504/881445, 723232/176289, 4627712/881445, -15321088/881445] , [-83309/293815, 186554/881445, 123928/176289, 943136/881445, -632704/881445, -269312/293815] , [125369/293815, 443616/293815, -3547456/881445, -198272/176289, -2839168/881445, 5729792/881445]] \$ x \$ [[4, 9/2, 3/2, 1/2, 3, 3/2] , [37/8, 27/8, 7/4, 3/8, 33/8, 3/4] , [51/16, 141/32, 35/16, 3/16, 69/16, 23/32] , [249/64, 39/8, 15/8, 23/128, 447/128, 11/16] , [2159/512, 2061/512, 945/512, 11/64, 1059/256, 309/512] , [3599/1024, 9189/2048, 4277/2048, 309/2048, 4269/1024, 1209/2048]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 4, 6}}, net cycles: 0 . order: 4

[$y_1, 0, y_2, y_5, y_3, y_4$]

$$R = \$ [[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, -52/255, 38/255, -67/255, 98/255], [1/3, 38/255, -67/255, 98/255, -137/255], [0, 98/255, -52/255, 38/255, -67/255], [0, 38/255, -67/255, 98/255, -52/255], [0, -52/255, 38/255, -67/255, 98/255], [0, -67/255, 98/255, -52/255, 38/255]] \$ \times \$ [[1, 0, 6, 2, 3, 3], [2, 0, 4, 3, 0, 6], [3, 0, 2, 6, 0, 4], [6, 0, 3, 4, 0, 2], [4, 0, 6, 2, 0, 3]] \$$$

Omega Rank for B : cycles: {{1, 2, 5}}, net cycles: 0 . order: 3

$$[y_1, y_2, 0, 0, y_3, y_4]$$

$$B = \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 1/45, 17/90, -13/90], [0, 17/90, -13/90, 1/45], [0, -13/90, 1/45, 17/90], [1, -13/90, 1/45, -73/90], [0, -13/90, 1/45, 17/90], [0, 17/90, -13/90, 1/45]] \$ \times \$ [[5, 6, 0, 0, 3, 1], [7, 3, 0, 0, 5, 0], [3, 5, 0, 0, 7, 0], [5, 7, 0, 0, 3, 0]] \$$$

Â» SYNC'D 645/2048 , 0.3149414062

17 . Coloring, {2, 3, 4}

R: [3, 1, 2, 6, 2, 1] **B:** [5, 5, 6, 1, 3, 4]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

$$['-15' (' - 5 - \tau - 3\tau^2 + \tau^3 ')'' (' 3 + \tau^2 ')', 15' (' 3 + \tau^2 ')'' (' 5 - 2\tau + \tau^2 ')'' (' 1 + \tau ')', 15' (' 5 - \tau + 3\tau^2 + \tau^3 ')'' (' 3 + \tau^2 ')', 15' (' 5 - \tau + 3\tau^2 + \tau^3 ')'' (' - 1 + \tau ')'^2, -15' (' - 1 + \tau ')'' (' 5 + 2\tau + \tau^2 ')'' (' 3 + \tau^2 ')', -30' (' 5 - \tau + 3\tau^2 + \tau^3 ')'' (' - 1 + \tau ')'']'$$

For τ=1/2, [637, 663, 559, 43, 325, 172] . FixedPtCheck, [637, 663, 559, 43, 325, 172]

$$\det(A + \tau \Delta) = 1' (' 1 + \tau ')'^2 (' - 1 + \tau ')'^3$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	5 vs 5

bi =

$$\$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 1/4, 0, 0, 0, 3/4], [3/4, 0, 0, 0, 0, 1/4], [0, 1/4, 3/4, 0, 0, 0], [1/4, 0, 0, 3/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[377012/3515805, -10499917/31642245, -16694476/31642245, -9100336/18985347,$$

2341312/94926735, 120892672/94926735] , [1858832/3515805, -1266577/31642245, 5050844/31642245, 10987760/18985347, 19185472/94926735, -129337088/94926735] , [-600448/3515805, -310537/31642245, 16749284/31642245, -11243152/18985347, -24075968/94926735, 53516032/94926735] , [-140008/3515805, -3839617/31642245, 3592004/31642245, 6103088/18985347, 93422272/94926735, -113086208/94926735] , [42737/3515805, 14859458/31642245, -8249296/31642245, -1548352/18985347, 7802752/94926735, -14716928/94926735] , [-689293/3515805, -2253832/31642245, 2919464/31642245, 13304576/18985347, -54591488/94926735, 11011072/94926735]] \$ x \$ [[2, 3/2, 3, 3/2, 9/2, 5/2] , [17/8, 15/8, 31/8, 15/8, 21/8, 21/8] , [81/32, 13/8, 5/2, 63/32, 3, 27/8] , [349/128, 11/8, 369/128, 81/32, 399/128, 303/128] , [1451/512, 3/2, 773/256, 909/512, 1575/512, 1431/512] , [2463/1024, 3121/2048, 193/64, 4293/2048, 6657/2048, 5547/2048]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, 0, 0, y_4]$$

R = \$ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 1/45, 17/90, -13/90] , [0, 17/90, -13/90, 1/45] , [0, -13/90, 1/45, 17/90] , [1, -13/90, 1/45, -73/90] , [0, -13/90, 1/45, 17/90] , [0, 17/90, -13/90, 1/45]] \$ x \$ [[5, 6, 3, 0, 0, 1] , [7, 3, 5, 0, 0, 0] , [3, 5, 7, 0, 0, 0] , [5, 7, 3, 0, 0, 0]] \$

Omega Rank for B : cycles: {{1, 3, 4, 5, 6}}, net cycles: 1 . order: 5

$$[y_1, 0, y_2, y_3, y_4, y_5]$$

B = \$ [[0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ = \$ [[841/4575, -449/4575, -239/4575, 46/4575, 106/4575] , [841/4575, -449/4575, -239/4575, 46/4575, 106/4575] , [46/4575, 106/4575, 841/4575, -449/4575, -239/4575] , [-449/4575, -239/4575, 46/4575, 106/4575, 841/4575] , [106/4575, 841/4575, -449/4575, -239/4575, 46/4575] , [-239/4575, 46/4575, 106/4575, 841/4575, -449/4575]] \$ x \$ [[1, 0, 3, 2, 6, 3] , [2, 0, 6, 3, 1, 3] , [3, 0, 1, 3, 2, 6] , [3, 0, 2, 6, 3, 1] , [6, 0, 3, 1, 3, 2]] \$

Â» SYNC'D 1215/4096 , 0.2966308594

18 . Coloring, {2, 3, 5}

R: [3, 1, 2, 1, 3, 1] **B:** [5, 5, 6, 6, 2, 4]

‘ See graph

‘ ‘ See pair graph

Ω for $A+\tau\Delta$:

$$\begin{aligned} & [-15^{\binom{1}{1} + \binom{1}{1}} \binom{5}{-2\tau + \tau^2}^{\binom{1}{1}} \binom{-3 + \tau}{\tau}^{\binom{1}{1}}, 15^{\binom{-5 - \tau - 3\tau^2 + \tau^3}{1}} \binom{-3 + \tau}{\tau}^{\binom{1}{1}}, 15^{\binom{1}{1} + \binom{1}{1}} \binom{-5 + \tau^2}{\tau^2}^{\binom{1}{1}} \binom{-3 + \tau}{\tau}^{\binom{1}{1}}, -15^{\binom{-1 + \tau}{1}} \binom{-5 + \tau^2}{\tau^2}^{\binom{1}{1}}, 15^{\binom{5 + 2\tau + \tau^2}{1}} \binom{-1 + \tau}{\tau}^{\binom{1}{1}} \binom{-3 + \tau}{\tau}^{\binom{1}{1}}, 30^{\binom{-1 + \tau}{1}} \binom{-5 + \tau^2}{\tau^2}^{\binom{1}{1}}] \end{aligned}$$

For $\tau=1/2$, [255, 245, 285, 19, 125, 76] . FixedPtCheck, [255, 245, 285, 19, 125, 76]

$$\det(A + \tau \Delta) = 1^{\binom{1}{1} + \binom{1}{1}} \binom{-1 + \tau}{\tau}^{\binom{1}{1}} \binom{-1 + \tau}{\tau}^{\binom{1}{1}}$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	3 vs 3	2 vs 4

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 1/4, 0, 0, 0, 3/4], [1/4, 0, 0, 0, 0, 3/4], [0, 3/4, 1/4, 0, 0, 0], \\ & [1/4, 0, 0, 3/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], \\ & [0, 0, 0, 0, 0, 1]] \$ = \end{aligned}$$

$$\begin{aligned} & \$ [[13/225, -374/225, -2536/2025, 32/225, 512/675, 4096/2025], [-67/225, -334/225, -6176/2025, \\ & -1088/225, 1792/675, 14336/2025], [133/225, 466/225, 5624/2025, 512/225, -1408/675, -11264/2025], \\ & [109/675, 1168/675, 29552/6075, 3776/675, -6784/2025, -54272/6075], [-181/675, -622/675, \\ & -22928/6075, -1184/675, 7936/2025, 17408/6075], [199/675, 1438/675, 33512/6075, 2336/675, \\ & -12544/2025, -31232/6075]] \$ \times \$ [[3/2, 3, 3/2, 3/2, 9/2, 3], [15/8, 15/4, 3/2, 9/4, 27/8, 9/4], [33/16, \\ & 93/32, 21/16, 27/16, 135/32, 45/16], [237/128, 447/128, 201/128, 135/64, 477/128, 9/4], [1005/512, \\ & 51/16, 357/256, 27/16, 513/128, 1413/512], [3909/2048, 3435/1024, 3057/2048, 4239/2048, 7911/2048, \\ & 2367/1024]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 1 . order: 3

$$[y_1, y_3, y_2, 0, 0, 0]$$

$$\begin{aligned} & R = \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], \\ & [0, 0, 0, 1, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], \\ & [0, 0, 0, 0, 0, 0]] \$ = \$ [[2/15, 2/15, -1/5], [2/15, -1/5, 2/15], [-1/5, 2/15, 2/15], [2/15, -1/5, 2/15], [2/15, 2/15, \\ & -1/5], [2/15, -1/5, 2/15]] \$ \times \$ [[6, 3, 6, 0, 0, 0], [3, 6, 6, 0, 0, 0], [6, 6, 3, 0, 0, 0]] \$ \end{aligned}$$

Omega Rank for B : cycles: {{2, 5}, {4, 6}}, net cycles: 2 . order: 2

$$\begin{aligned} & \$ [[0, 3, 0, 2, 6, 4], [0, 6, 0, 4, 3, 2], [0, 3, 0, 2, 6, 4], [0, 6, 0, 4, 3, 2]] \$ \end{aligned}$$

$$[0, 3 y_1, 0, 2 y_1, 3 y_2, 2 y_2]$$

$$p = -s + s^3 \quad p' = -s + s^3$$

Â» SYNC'D 27/512 , 0.05273437500

19 . Coloring, {2, 3, 6}

R: [3, 1, 2, 1, 2, 4] **B:** [5, 5, 6, 6, 3, 1]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

$$\begin{aligned} & \left['15' ('3 + \tau^2') ('-5 - \tau - 3\tau^2 + \tau^3') , -15' ('1 + \tau') ('3 + \tau^2') ('5 - 2\tau + \tau^2') , \right. \\ & -15' ('5 - \tau + 3\tau^2 + \tau^3') ('3 + \tau^2') , 15' ('-1 + \tau') ('5 - \tau + 3\tau^2 + \tau^3') ('1 + \tau') , 15' \\ & \left. ('5 + 2\tau + \tau^2') ('-1 + \tau') ('3 + \tau^2') , 30' ('-1 + \tau') ('5 - \tau + 3\tau^2 + \tau^3') \right]' \end{aligned}$$

For τ=1/2, [-637, -663, -559, -129, -325, -172] . FixedPtCheck, [637, 663, 559, 129, 325, 172]

$$\det(A + \tau \Delta) = 1' ('-1 + \tau')^2 ('1 + \tau')^3$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	4 vs 4

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0] , [1/4, 0, 0, 0, 3/4, 0] , [0, 1/4, 0, 0, 0, 3/4] , [1/4, 0, 0, 0, 0, 3/4] , [0, 1/4, 3/4, 0, 0, \\ & 0] , [3/4, 0, 0, 1/4, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, \\ & 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ = \end{aligned}$$

$$\begin{aligned} & \$ [[-9943/25733, 917158/385995, 246824/128665, 3407744/1157985, 2135168/1157985, \\ & -9991168/1157985] , [12047/25733, 300988/385995, 101784/128665, 1524704/1157985, \\ & -529792/1157985, -3278848/1157985] , [-4699/25733, -61382/385995, 89504/128665, -600256/1157985, \\ & -401152/1157985, 668672/1157985] , [29393/25733, -3031202/385995, -933256/128665, \\ & -10489216/1157985, -2079232/1157985, 28815872/1157985] , [20841/128665, -9278/385995, \\ & -323672/385995, -1417312/1157985, -318208/1157985, 524800/231597] , [-20949/128665, \\ & -205628/385995, -28704/128665, 872288/1157985, -289408/1157985, 111616/231597]] \$ x \$ [[5/2, \\ & 3/2, 3, 1/2, 9/2, 3] , [11/4, 15/8, 4, 3/4, 3, 21/8] , [21/8, 7/4, 47/16, 21/32, 111/32, 57/16] , [419/128, \\ & 205/128, 417/128, 57/64, 105/32, 345/128] , [677/256, 837/512, 1679/512, 345/512, 117/32, 1593/512] , \\ & [5961/2048, 3551/2048, 3485/1024, 1593/2048, 6573/2048, 759/256]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 0 . order: 3

$$[y_2, y_3, y_4, y_1, 0, 0]$$

$$R = \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0]] \$ = \$ [[0, -8/315, 52/315, -23/315], [0, 52/315, -23/315, -8/315], [0, -23/315, -8/315, 52/315], [0, 52/315, -23/315, -8/315], [0, -23/315, -8/315, 52/315], [1/2, -23/315, -8/315, -211/630]] \$ \times \$ [[4, 6, 3, 2, 0, 0], [8, 3, 4, 0, 0, 0], [3, 4, 8, 0, 0, 0], [4, 8, 3, 0, 0, 0]] \$$$

Omega Rank for B : cycles: {{1, 3, 5, 6}}, net cycles: 1 . order: 4

$$[y_1, 0, y_2, 0, y_3, y_4]$$

$$B = \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[4/15, -2/15, -2/15, 1/15], [4/15, -2/15, -2/15, 1/15], [-2/15, 1/15, 4/15, -2/15], [-2/15, 1/15, 4/15, -2/15], [1/15, 4/15, -2/15, -2/15], [-2/15, -2/15, 1/15, 4/15]] \$ \times \$ [[2, 0, 3, 0, 6, 4], [4, 0, 6, 0, 2, 3], [3, 0, 2, 0, 4, 6], [6, 0, 4, 0, 3, 2]] \$$$

Â» SYNC'D 135/512 , 0.2636718750

20 . Coloring, {2, 4, 5}

R: [3, 1, 6, 6, 3, 1] **B:** [5, 5, 2, 1, 2, 4]

' See graph

' ' See pair graph

Ω for A+τΔ :

$$['-15' (' 1 + \tau ')'' (' 3 + \tau ^ 2 ')', 15' (' - 1 + \tau ')'' (' 3 + \tau ^ 2 ')', -15' (' 1 + \tau ')'' (' 3 + \tau ^ 2 ')', 15' (' - 1 + \tau ')'' (' 1 + \tau ')'^ 2 , 15' (' - 1 + \tau ')'' (' 3 + \tau ^ 2 ')', -30' (' 1 + \tau ')'^ 2]'$$

For τ=1/2, [-39, -13, -39, -9, -13, -36] . FixedPtCheck, [39, 13, 39, 9, 13, 36]

$$\det(A + \tau \Delta) = 1' (' - 1 + \tau ')'^ 3 ' (' 1 + \tau ')'^ 2$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	3 vs 3	4 vs 4

bi =

$\$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 3/4, 0, 0, 0, 1/4], [3/4, 0, 0, 0, 0, 1/4], [0, 3/4, 1/4, 0, 0, 0], [1/4, 0, 0, 3/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$

$\$ [[-8541/55955, -26006/55955, 281296/167865, 212032/503595, 556288/503595, -1267712/503595], [-6291/55955, 11992/167865, 7272/55955, -317408/503595, 1028608/503595, -722432/503595], [-2151/55955, -24356/55955, -20568/55955, 2576032/503595, 1038208/503595, -3156992/503595], [-21841/55955, 42014/55955, 1552/55955, 1789792/503595, 629248/503595, -2580992/503595], [5669/55955, 4984/55955, -210752/503595, -314528/503595, -1331072/503595, 598016/167865], [55869/55955, 41064/55955, -259064/167865, -4129088/503595, -2252672/503595, 6320128/503595]] \$ \times \$ [[2, 9/2, 3/2, 3/2, 9/2, 1], [5/2, 9/2, 13/8, 3/4, 39/8, 3/4], [15/8, 39/8, 59/32, 9/16, 21/4, 19/32], [229/128, 681/128, 57/32, 57/128, 81/16, 77/128], [929/512, 657/128, 877/512, 231/512, 1365/256, 285/512], [1803/1024, 10821/2048, 3659/2048, 855/2048, 10671/2048, 277/512]] \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 6}}, net cycles: 1 . order: 3

$$[y_1, 0, y_2, 0, 0, y_3]$$

$R = \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[16/45, 1/45, -14/45], [1/45, -14/45, 16/45], [-14/45, 16/45, 1/45], [-14/45, 16/45, 1/45], [16/45, 1/45, -14/45], [1/45, -14/45, 16/45]] \$ \times \$ [[5, 0, 6, 0, 0, 4], [4, 0, 5, 0, 0, 6], [6, 0, 4, 0, 0, 5]] \$$

Omega Rank for B : cycles: {{2, 5}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, y_4, 0]$$

$B = \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0]] \$ = \$ [[0, 0, 8/15, -7/15], [0, 0, 8/15, -7/15], [0, 0, -7/15, 8/15], [0, 1/2, -7/15, 1/30], [0, 0, -7/15, 8/15], [1/2, -1/4, 1/30, -13/60]] \$ \times \$ [[1, 6, 0, 2, 6, 0], [2, 6, 0, 0, 7, 0], [0, 7, 0, 0, 8, 0], [0, 8, 0, 0, 7, 0]] \$$

Â» SYNC'D 7/128 , 0.05468750000

21 . Coloring, {2, 4, 6}

R: [3, 1, 6, 6, 2, 4] **B:** [5, 5, 2, 1, 3, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A+\tau\Delta$:

$$\begin{bmatrix} 15(\tau^3 + \tau) \tau^2 (\tau - 1 + \tau) (\tau^5 - 2\tau + \tau^2) (\tau + 1) \tau, & -15(\tau^3 + \tau) \tau^2 (\tau - 1 + \tau) (\tau^5 + 2\tau + \tau^2) \tau, & 15(\tau^5 - \tau + 3\tau^2 + \tau^3) \tau^2 (\tau + 1) \tau (\tau - 1 + \tau), & -15(\tau^5 - \tau + 3\tau^2 + \tau^3) \tau^2 (\tau + 1) \tau (\tau - 1 + \tau) \tau^2, & 15(\tau^3 + \tau) \tau^2 (\tau - 5 + \tau^2) \tau^2 (\tau - 1 + \tau) \tau^2, & -30(\tau^5 - \tau + 3\tau^2 + \tau^3) \tau^2 (\tau + 1) \tau \tau^2 \end{bmatrix}$$

For $\tau=1/2$, [-357, -175, -301, -387, -133, -516] . FixedPtCheck, [357, 175, 301, 387, 133, 516]

$$\det(A + \tau \Delta) = 1 (\tau - 1 + \tau) \tau^2 (\tau + 1) \tau^3$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	5 vs 5	4 vs 4

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 3/4, 0, 0, 0, 1/4], [3/4, 0, 0, 0, 0, 1/4], [0, 1/4, 3/4, 0, 0, 0], \\ & [3/4, 0, 0, 1/4, 0, 0]] \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], \\ & [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[-179/275, 136/55, 568/165, 2752/825, -128/275, -6656/825], [39/275, -406/825, -4712/825, -192/275, -2048/825, 512/55], \\ & [-1/25, 8/25, 16/25, 64/75, -128/75, 0], [873/275, -1948/275, -9456/275, -160/11, -2432/165, 55808/825], \\ & [89/825, -674/825, 224/275, -96/55, 128/55, -512/825], [-117/275, 362/275, 5064/275, 256/55, 1792/165, -28672/825]] \times \$ [[3, 3, 3, 1/2, 9/2, 1], [15/8, 27/8, 33/8, 1/4, 9/2, 7/8], \\ & [27/16, 135/32, 123/32, 7/32, 63/16, 35/32], [261/128, 495/128, 27/8, 35/128, 567/128, 65/64], [495/256, 1863/512, 981/256, 65/256, 567/128, 467/512], \\ & [1827/1024, 4077/1024, 3897/1024, 467/2048, 8559/2048, 523/512]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 6}}, net cycles: 0 . order: 4

$$[y_1, y_2, y_3, y_4, 0, y_5]$$

$$\begin{aligned} R = & \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0]] \\ & \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \\ & \$ = \$ [[0, 0, 1/3, -7/15, 1/5], [0, 1/3, -1/3, 1/5, -2/15], [0, 0, 0, 8/15, -7/15], [0, 0, 0, 8/15, -7/15], \\ & [1/3, -1/3, 0, -2/15, 1/5], [0, 0, 0, -7/15, 8/15]] \times \$ [[3, 3, 3, 2, 0, 4], [3, 0, 3, 4, 0, 5], [0, 0, 3, 5, 0, 7], \\ & [0, 0, 0, 7, 0, 8], [0, 0, 0, 8, 0, 7]] \$ \end{aligned}$$

Omega Rank for B : cycles: {{2, 3, 5}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, 0, y_4, 0]$$

$$\begin{aligned} B = & \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0]] \\ & \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0]] \\ & \$ = \$ [[0, 2/15, -1/5, 2/15], [0, 2/15, -1/5, 2/15], [0, -1/5, 2/15, 2/15], [1/3, -1/5, 2/15, -1/5], [0, 2/15, 2/15, -1/5], \\ & [1/3, -1/5, 2/15, -1/5]] \times \$ [[3, 3, 3, 0, 6, 0], [0, 3, 6, 0, 6, 0], [0, 6, 6, 0, 3, 0], [0, 6, 3, 0, 6, 0]] \$ \end{aligned}$$

Â» SYNC'D 77/512 , 0.1503906250

22 . Coloring, {2, 5, 6}

R: [3, 1, 6, 1, 3, 4] **B:** [5, 5, 2, 6, 2, 1]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

' ['15' ('3 + τ² ')'' ('1 + τ')' , -15' ('3 + τ² ')'' (' - 1 + τ')' , 15' ('3 + τ² ')'' ('1 + τ')' ,
15' ('1 + τ')' ' 3 , -15' ('3 + τ² ')'' (' - 1 + τ')' , 30' ('1 + τ')' ' 2 ']'

For τ=1/2, [39, 13, 39, 27, 13, 36] . FixedPtCheck, [39, 13, 39, 27, 13, 36]

det(A + τ Δ) = 1' (' - 1 + τ ')' ' 2 ' ('1 + τ')' ' 3

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	4 vs 4

bi =

\$ [[0, 0, 1/4, 0, 3/4, 0] , [1/4, 0, 0, 0, 3/4, 0] , [0, 3/4, 0, 0, 0, 1/4] , [1/4, 0, 0, 0, 0, 3/4] , [0, 3/4, 1/4, 0, 0, 0] , [3/4, 0, 0, 1/4, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ =

\$ [[6504/2185, 5251/2185, -69436/6555, -200848/6555, -832/6555, 236288/6555] , [-2588/6555, -13561/19665, 2748/2185, 29872/6555, 11456/2185, -194816/19665] , [2974/2185, 1783/6555, -25996/6555, -59728/6555, -20032/6555, 95488/6555] , [-20856/2185, -22869/2185, 261164/6555, 656432/6555, 145088/6555, -931072/6555] , [-5743/6555, -4036/19665, 6608/2185, 45472/6555, -24832/6555, -98816/19665] , [1469/2185, 16798/6555, -29536/6555, -50368/6555, -55552/6555, 114688/6555]] \$ x \$ [[5/2, 9/2, 3/2, 1/2, 9/2, 3/2] , [19/8, 9/2, 7/4, 3/8, 21/4, 3/4] , [57/32, 21/4, 61/32, 3/16, 165/32, 23/32] , [243/128, 339/64, 111/64, 23/128, 675/128, 79/128] , [469/256, 2691/512, 459/256, 79/512, 2763/512, 291/512] , [3643/2048, 11043/2048, 3701/2048, 291/2048, 10887/2048, 1155/2048]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 3, 4, 6}}, net cycles: 1 . order: 4

[y₁, 0, y₄, y₂, 0, y₃]

R = \$ [[0, 0, 1, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1]] \$ = \$ [[98/255, -52/255, 38/255, -67/255] , [-52/255, 38/255, -67/255, 98/255] , [-67/255, 98/255, -52/255, 38/255] , [-52/255, 38/255, -67/255, 98/255] , [98/255, -52/255, 38/255, -67/255] ,

$$[38/255, -67/255, 98/255, -52/255]] \$ x \$ [[4, 0, 6, 2, 0, 3], [2, 0, 4, 3, 0, 6], [3, 0, 2, 6, 0, 4], [6, 0, 3, 4, 0, 2]] \$$$

Omega Rank for B : cycles: {{2, 5}}, net cycles: 0 . order: 4

$$[y_4, y_3, 0, 0, y_2, y_1]$$

$$B = \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 0, -7/15, 8/15], [0, 0, -7/15, 8/15], [0, 0, 8/15, -7/15], [1, -2, -22/15, 38/15], [0, 0, 8/15, -7/15], [0, 1, 8/15, -22/15]] \$ x \$ [[2, 6, 0, 0, 6, 1], [1, 6, 0, 0, 8, 0], [0, 8, 0, 0, 7, 0], [0, 7, 0, 0, 8, 0]] \$$$

Â» SYNC'D 37/512 , 0.07226562500

23 . Coloring, {3, 4, 5}

R: [3, 5, 2, 6, 3, 1] **B:** [5, 1, 6, 1, 2, 4]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

$$[[-15' (' - 1 + \tau ')'' (' 5 + 2\tau + \tau ^ 2 ')'' (' 3 + \tau ^ 2 ')' , -15' (' 3 + \tau ^ 2 ')'' (' - 5 - \tau - 3\tau ^ 2 + \tau ^ 3 ')' , 15' (' 3 + \tau ^ 2 ')'' (' 1 + \tau ')'' (' 5 - 2\tau + \tau ^ 2 ')' , 15' (' - 1 + \tau ')'' ^ 2 ' (' 1 + \tau ')'' (' 5 - 2\tau + \tau ^ 2 ')' , 15' (' 5 - \tau + 3\tau ^ 2 + \tau ^ 3 ')'' (' 3 + \tau ^ 2 ')' , -30' (' - 1 + \tau ')'' (' 1 + \tau ')'' (' 5 - 2\tau + \tau ^ 2 ')'']'$$

For τ=1/2, [325, 637, 663, 51, 559, 204] . FixedPtCheck, [325, 637, 663, 51, 559, 204]

$$\det(A + \tau \Delta) = 1' (' - 1 + \tau ')' ^ 3 ' (' 1 + \tau ')' ^ 2$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	5 vs 5	5 vs 5

bi =

$$\$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 1/4, 0, 0, 0, 3/4], [3/4, 0, 0, 0, 0, 1/4], [0, 3/4, 1/4, 0, 0, 0], [1/4, 0, 0, 3/4, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$$\$ [[32321/885695, -2049784/7971255, 534728/7971255, 411968/4782753, -55533824/23913765, 58740736/23913765], [-112369/885695, -1804654/7971255, -4294432/7971255, -10154080/4782753, 25707136/23913765, 47988736/23913765], [623801/885695, -2818264/7971255, 2289488/7971255, -309856/4782753, -10158464/23913765, -1954304/23913765], [27691/885695, 1432526/7971255, 8782088/7971255, -6280960/4782753, 36670336/23913765, -35062784/23913765], [-284419/885695,$$

791876/7971255, 175088/7971255, 8852672/4782753, 35502976/23913765, -73393664/23913765],
 [40001/885695, 8104976/7971255, -2448352/7971255, 4939424/4782753, -11611904/23913765,
 -29540864/23913765]] \$ x \$ [[7/2, 3, 3/2, 3/2, 3, 5/2], [4, 21/8, 13/8, 15/8, 27/8, 3/2], [15/4, 47/16,
 59/32, 9/8, 117/32, 27/16], [111/32, 205/64, 237/128, 81/64, 227/64, 213/128], [1929/512, 1599/512,
 449/256, 639/512, 871/256, 873/512], [7587/2048, 1531/512, 3671/2048, 2619/2048, 3693/1024,
 3333/2048]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 5}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, 0, y_4, y_5]$$

R = \$ [[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0,
 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0,
 0, 0, 1]] \$ = \$ [[0, 0, -14/45, 16/45, 1/45], [0, 0, 16/45, 1/45, -14/45], [0, 0, 1/45, -14/45, 16/45], [1, -2,
 1/45, -59/45, 106/45], [0, 0, -14/45, 16/45, 1/45], [0, 1, 16/45, 1/45, -59/45]] \$ x \$ [[2, 3, 6, 0, 3, 1], [1,
 6, 5, 0, 3, 0], [0, 5, 4, 0, 6, 0], [0, 4, 6, 0, 5, 0], [0, 6, 5, 0, 4, 0]] \$

Omega Rank for B : cycles: {{1, 2, 5}}, net cycles: 0 . order: 3

$$[y_4, y_5, 0, y_3, y_1, y_2]$$

B = \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0,
 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0,
 0, 0, 1]] \$ = \$ [[0, 0, 1/45, 16/45, -14/45], [0, 0, 16/45, -14/45, 1/45], [1/3, -2/9, 1/45, 1/45, -4/45], [0, 0,
 16/45, -14/45, 1/45], [0, 0, -14/45, 1/45, 16/45], [0, 1/3, -14/45, 1/45, 1/45]] \$ x \$ [[4, 3, 0, 2, 3, 3], [5,
 3, 0, 3, 4, 0], [6, 4, 0, 0, 5, 0], [4, 5, 0, 0, 6, 0], [5, 6, 0, 0, 4, 0]] \$

Â» SYNC'D 907/8192, 0.1107177734

24 . Coloring, {3, 4, 6}

R: [3, 5, 2, 6, 2, 4] **B:** [5, 1, 6, 1, 3, 1]

' See graph

' ' See pair graph

,

Ω for A+τΔ :

['15' (' - 1 + τ ') ' (' 3 + τ ') ' (' - 5 + τ ² ') ' , 15' (' 3 + τ ') ' (' 5 - 2τ + τ ² ') ' (' 1 + τ ') ' ,
 -15' (' - 1 + τ ') ' (' 3 + τ ') ' (' 5 + 2τ + τ ² ') ' , -15' (' - 1 + τ ') ' (' 5 + 2τ + τ ² ') ' (' 1 + τ ') ' , 15'
 (' 5 - τ + 3τ ² + τ ³ ') ' (' 3 + τ ') ' , -30' (' - 1 + τ ') ' (' 5 + 2τ + τ ² ') ']'

For $\tau=1/2$, [133, 357, 175, 75, 301, 100] . FixedPtCheck, [133, 357, 175, 75, 301, 100]

$$\det(A + \tau \Delta) = 1^4 (-1 + \tau)^3 (-1 + \tau)^2$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	3 vs 5	4 vs 4

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 1/4, 0, 0, 0, 3/4], [3/4, 0, 0, 0, 0, 1/4], [0, 1/4, 3/4, 0, 0, 0], \\ & [3/4, 0, 0, 1/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[31/14625, 26864/43875, -61952/131625, -20896/131625, -54784/131625, 65536/131625], \\ & [8611/14625, 33884/43875, 45688/131625, -2176/131625, 20096/131625, -233984/131625], \\ & [-2719/14625, -10886/43875, -63952/131625, 167104/131625, -166784/131625, 129536/131625], \\ & [157/4875, -42592/14625, -101144/43875, -25312/43875, 4352/43875, 251392/43875], [821/14625, \\ & 5374/43875, 168368/131625, -105536/131625, 38656/131625, -116224/131625], [-1013/4875, \\ & -6322/14625, 6496/43875, -6592/43875, 79232/43875, -48128/43875]] \$ \times \$ [[9/2, 3/2, 3, 1/2, 3, 5/2], \\ & [27/8, 3/2, 27/8, 5/8, 15/4, 19/8], [27/8, 57/32, 117/32, 19/32, 93/32, 43/16], [243/64, 105/64, 387/128, \\ & 43/64, 381/128, 185/64], [999/256, 3/2, 1629/512, 185/256, 417/128, 1247/512], [7155/2048, 3297/2048, \\ & 3501/1024, 1247/2048, 3381/1024, 5257/2048]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 5}, {4, 6}}, net cycles: 1 . order: 2

$$\begin{aligned} & \$ [[0, 6, 3, 2, 3, 1], [0, 6, 0, 1, 6, 2], [0, 6, 0, 2, 6, 1], [0, 6, 0, 1, 6, 2], [0, 6, 0, 2, 6, 1]] \$ \\ & [0, 2y_1 + 2y_3, 2y_1 - y_2 + 2y_3, y_1, y_2, y_3] \end{aligned}$$

$$p' = -s^2 + s^4 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{1, 3, 5, 6}}, net cycles: 1 . order: 4

$$[y_1, 0, y_2, 0, y_4, y_3]$$

$$\begin{aligned} B = & \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], \\ & [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ \\ & \times \$ [[-1/15, 4/15, -1/15, -1/15], [4/15, -1/15, -1/15, -1/15], [-1/15, -1/15, -1/15, 4/15], [4/15, -1/15, -1/15, -1/15], \\ & [-1/15, -1/15, 4/15, -1/15], [4/15, -1/15, -1/15, -1/15]] \$ \times \$ [[6, 0, 3, 0, 3, 3], [3, 0, 3, 0, 6, 3], [3, 0, 6, 0, 3, 3], [3, 0, 3, 0, 3, 6]] \$ \end{aligned}$$

Â» SYNC'D 51/2048 , 0.02490234375

25 . Coloring, {3, 5, 6}

R: [3, 5, 2, 1, 3, 4] **B:** [5, 1, 6, 6, 2, 1]

' See graph

' ' See pair graph

Ω for $A+\tau\Delta$:

$$\begin{aligned} & ['15' (' - 1 + \tau ') ' (' 3 + \tau ^ 2 ') ' (' 5 + 2\tau + \tau ^ 2 ') ' , 15' (' 3 + \tau ^ 2 ') ' (' - 5 - \tau - 3\tau ^ 2 + \tau ^ 3 ') ' , \\ & -15' (' 1 + \tau ') ' (' 3 + \tau ^ 2 ') ' (' 5 - 2\tau + \tau ^ 2 ') ' , 15' (' 1 + \tau ') ' ^ 2 (' - 1 + \tau ') ' (' 5 - 2\tau + \tau ^ 2 ') ' , \\ & -15' (' 5 - \tau + 3\tau ^ 2 + \tau ^ 3 ') ' (' 3 + \tau ^ 2 ') ' , 30' (' 1 + \tau ') ' (' - 1 + \tau ') ' (' 5 - 2\tau + \tau ^ 2 ') '] ' \end{aligned}$$

For $\tau=1/2$, [-325, -637, -663, -153, -559, -204] . FixedPtCheck, [325, 637, 663, 153, 559, 204]

$$\det(A + \tau \Delta) = 1' (' 1 + \tau ') ' ^ 3 (' - 1 + \tau ') ' ^ 2$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	5 vs 5	4 vs 4

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0] , [3/4, 0, 0, 0, 1/4, 0] , [0, 1/4, 0, 0, 0, 3/4] , [1/4, 0, 0, 0, 0, 3/4] , [0, 3/4, 1/4, 0, 0, \\ & 0] , [3/4, 0, 0, 1/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, \\ & 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ = \end{aligned}$$

$$\begin{aligned} & \$ [[542698/4080055, -306499/2448033, -83188/116573, -8959472/7344099, 43328/36720495, \\ & 73119488/36720495] , [-1189652/4080055, -204193/2448033, 71548/116573, 4526512/7344099, \\ & 42611648/36720495, -71564032/36720495] , [3760018/4080055, -1210075/2448033, -262100/116573, \\ & -11783120/7344099, -18672832/36720495, 146908928/36720495] , [-1624022/4080055, \\ & 1442261/2448033, 564308/116573, 19349584/7344099, 17498048/36720495, -296572672/36720495] , \\ & [-1348547/4080055, 132980/2448033, 104176/116573, 9126496/7344099, -16945792/36720495, \\ & -48911872/36720495] , [205263/4080055, 1660550/2448033, -27808/116573, 959584/7344099, \\ & -19303552/36720495, -1042432/36720495]] \$ \times \$ [[4, 3, 3/2, 1/2, 3, 3] , [37/8, 21/8, 7/4, 3/4, 15/4, 3/2] \\ & , [105/32, 13/4, 67/32, 3/8, 33/8, 15/8] , [63/16, 463/128, 237/128, 15/32, 419/128, 237/128] , [135/32, \\ & 747/256, 923/512, 237/512, 1975/512, 891/512] , [231/64, 107/32, 4135/2048, 891/2048, 3987/1024, \\ & 435/256]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{2, 3, 5}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, y_4, y_5, 0]$$

$$\begin{aligned} & R = \$ [[0, 0, 1, 0, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, \\ & 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, \\ & 0, 0, 0]] \$ = \$ [[0, 0, 1/45, 16/45, -14/45] , [0, 0, 16/45, -14/45, 1/45] , [0, 0, -14/45, 1/45, 16/45] , [0, 1/2, \\ & 16/45, -14/45, -43/90] , [0, 0, 1/45, 16/45, -14/45] , [1/2, -1/4, -14/45, -43/90, 109/180]] \$ \times \$ [[1, 3, 6, 2, \end{aligned}$$

3, 0] , [2, 6, 4, 0, 3, 0] , [0, 4, 5, 0, 6, 0] , [0, 5, 6, 0, 4, 0] , [0, 6, 4, 0, 5, 0]] \$

Omega Rank for B : cycles: {{1, 2, 5}}, net cycles: 0 . order: 3

[y₁, y₂, 0, 0, y₃, y₄]

B = \$ [[0, 0, 0, 0, 1, 0] , [1, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 0, 1] , [0, 1, 0, 0, 0, 0] , [1, 0, 0, 0, 0, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 1/45, 17/90, -13/90] , [0, 17/90, -13/90, 1/45] , [1/4, -13/90, 1/45, -11/180] , [1/4, -13/90, 1/45, -11/180] , [0, -13/90, 1/45, 17/90] , [0, 17/90, -13/90, 1/45]] \$ x \$ [[5, 3, 0, 0, 3, 4] , [7, 3, 0, 0, 5, 0] , [3, 5, 0, 0, 7, 0] , [5, 7, 0, 0, 3, 0]] \$

Â» SYNC'D 397/1024 , 0.3876953125

26 . Coloring, {4, 5, 6}

R: [3, 5, 6, 6, 3, 4] **B:** [5, 1, 2, 1, 2, 1]

' See graph

' ' See pair graph

Ω for A+τΔ :

' ['15' ('3 + τ')'' ('-1 + τ')'' ('5 - τ + 3τ² + τ³')' , 15' ('3 + τ')'' ('-5 + τ²')'' ('-1 + τ')''² , 15' ('1 + τ')'' ('3 + τ')'' ('-1 + τ')'' ('5 - 2τ + τ²')' , -15' ('1 + τ')''³ ('5 - 2τ + τ²')' , -15' ('3 + τ')'' ('-1 + τ')''² ('5 + 2τ + τ²')' , -30' ('1 + τ')''² ('5 - 2τ + τ²')'']'

For τ=1/2, [-301, -133, -357, -459, -175, -612] . FixedPtCheck, [301, 133, 357, 459, 175, 612]

det(A + τ Δ) = 1' ('1 + τ')''³ ('-1 + τ')''²

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	3 vs 3

bi =

\$ [[0, 0, 1/4, 0, 3/4, 0] , [3/4, 0, 0, 0, 1/4, 0] , [0, 3/4, 0, 0, 0, 1/4] , [3/4, 0, 0, 0, 0, 1/4] , [0, 3/4, 1/4, 0, 0, 0] , [3/4, 0, 0, 1/4, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ = \$ [[38/975, -449/975, 1228/975, -1808/2925, 1472/975, -4864/2925] , [-32/195, 349/975, 12/65, 1712/2925, 64/975, -2816/2925] , [4/13, -717/325, 1148/195, 464/975, 1728/325, -9472/975] , [-314/325, 337/325, 5316/325, 5744/975, 4672/975, -26368/975] , [-27/325, 34/39, -2816/975, 64/975, -1024/325, 1024/195] , [271/325, 532/325, -4824/325, -3616/975, -7808/975, 23552/975]] \$ x \$ [[9/2, 9/2, 3/2, 1/2, 3, 1] , [9/2, 27/8, 15/8, 1/4, 9/2, 1/2] , [99/32, 153/32, 9/4, 1/8, 135/32, 17/32] , [261/64, 621/128, 117/64, 17/128, 225/64, 19/32] , [1071/256, 513/128, 243/128, 19/128, 2187/512, 251/512] , [7137/2048,

9477/2048, 4329/2048, 251/2048, 4239/1024, 131/256] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 6}}, net cycles: 0 . order: 4

$$[0, 0, y_3, y_4, y_1, y_2]$$

$$R = \$ [[0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0]] \$ \times \$ [[0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 1/3, 8/15, -4/5], [1/3, -2/3, -4/5, 6/5], [0, 0, -7/15, 8/15], [0, 0, -7/15, 8/15], [0, 1/3, 8/15, -4/5], [0, 0, 8/15, -7/15]] \$ \times \$ [[0, 0, 6, 2, 3, 4], [0, 0, 3, 4, 0, 8], [0, 0, 0, 8, 0, 7], [0, 0, 0, 7, 0, 8]] \$$$

Omega Rank for B : cycles: {{1, 2, 5}}, net cycles: 1 . order: 3

$$[y_1, y_2, 0, 0, y_3, 0]$$

$$B = \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0]] \$ = \$ [[-1/5, 2/15, 2/15], [2/15, 2/15, -1/5], [2/15, -1/5, 2/15], [2/15, 2/15, -1/5], [2/15, -1/5, 2/15], [2/15, 2/15, -1/5]] \$ \times \$ [[6, 6, 0, 0, 3, 0], [6, 3, 0, 0, 6, 0], [3, 6, 0, 0, 6, 0]] \$$$

Â» SYNC'D 23/64 , 0.3593750000

27 . Coloring, {2, 3, 4, 5}

R: [3, 1, 2, 6, 3, 1] **B:** [5, 5, 6, 1, 2, 4]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

$$['-15' (' 1 + \tau ')'' (' 3 + \tau ^ 2 ')'' (' 5 - 2\tau + \tau ^ 2 ')', 15' (' 3 + \tau ^ 2 ')'' (' - 5 - \tau - 3\tau ^ 2 + \tau ^ 3 ')', 15' (' 1 + \tau ')'' (' - 5 + \tau ^ 2 ')'' (' 3 + \tau ^ 2 ')', 15' (' 1 + \tau ')'' (' - 5 + \tau ^ 2 ')'' (' - 1 + \tau ')'^ 2 , 15' (' 5 + 2\tau + \tau ^ 2 ')'' (' 3 + \tau ^ 2 ')'' (' - 1 + \tau ')', -30' (' 1 + \tau ')'' (' - 5 + \tau ^ 2 ')'' (' - 1 + \tau ')'']'$$

For τ=1/2, [-663, -637, -741, -57, -325, -228] . FixedPtCheck, [663, 637, 741, 57, 325, 228]

$$\det(A + \tau \Delta) = 1' (' 1 + \tau ')'' (' - 1 + \tau ')'^ 2 ' (' 1 + 3\tau ^ 2 ')'$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	5 vs 5

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 1/4, 0, 0, 0, 3/4], [3/4, 0, 0, 0, 0, 1/4], [0, 3/4, 1/4, 0, 0, 0], \\ & [1/4, 0, 0, 3/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \\ & \$ [[292462/2646245, -3623549/7938735, 214812/529249, 15574544/23816205, \\ & -4203712/3402315, 14011136/23816205], [-118478/2646245, -2362169/7938735, 133580/529249, \\ & -22646416/23816205, -2310592/3402315, 42550016/23816205], [1072822/2646245, 750331/7938735, \\ & -240468/529249, 1657424/23816205, 5405888/3402315, -38996224/23816205], [-428088/2646245, \\ & 3023701/7938735, 407644/529249, 4979504/23816205, 2796608/3402315, -46530304/23816205], \\ & [-485313/2646245, 1010956/7938735, -667432/1587747, 17006624/23816205, 3166208/3402315, \\ & -26235904/23816205], [394927/2646245, 4824796/7938735, -31992/529249, -19878016/23816205, \\ & -4484992/3402315, 36271616/23816205]] \$ \times \$ [[2, 3, 3/2, 3/2, 9/2, 5/2], [5/2, 15/4, 13/8, 15/8, 15/4, \\ & 3/2], [87/32, 103/32, 25/16, 9/8, 75/16, 27/16], [265/128, 125/32, 237/128, 81/64, 285/64, 93/64], \\ & [293/128, 1947/512, 835/512, 279/256, 2295/512, 873/512], [2247/1024, 965/256, 3467/2048, 2619/2048, \\ & 9357/2048, 3063/2048]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{1, 2, 3}}, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, 0, 0, y_4]$$

$$\begin{aligned} R = & \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], \\ & 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, \\ & 0, 0, 1]] \$ = \$ [[0, 1/45, -14/45, 16/45], [0, -14/45, 16/45, 1/45], [0, 16/45, 1/45, -14/45], [1, 16/45, \\ & 1/45, -59/45], [0, 1/45, -14/45, 16/45], [0, -14/45, 16/45, 1/45]] \$ \times \$ [[5, 3, 6, 0, 0, 1], [4, 6, 5, 0, 0, 0], \\ & [6, 5, 4, 0, 0, 0], [5, 4, 6, 0, 0, 0]] \$ \end{aligned}$$

Omega Rank for B : cycles: {{2, 5}}, net cycles: 0 . order: 4

$$[y_1, y_2, 0, y_3, y_4, y_5]$$

$$\begin{aligned} B = & \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], \\ & 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, \\ & 0, 0, 1]] \$ = \$ [[0, 0, 0, -7/15, 8/15], [0, 0, 0, -7/15, 8/15], [1/3, -2/9, 1/27, 34/45, -113/135], [0, 0, 1/3, \\ & 8/15, -4/5], [0, 0, 0, 8/15, -7/15], [0, 1/3, -2/9, -4/5, 34/45]] \$ \times \$ [[1, 3, 0, 2, 6, 3], [2, 6, 0, 3, 4, 0], [3, \\ & 4, 0, 0, 8, 0], [0, 8, 0, 0, 7, 0], [0, 7, 0, 0, 8, 0]] \$ \end{aligned}$$

Â» SYNC'D 109/1024 , 0.1064453125

28 . Coloring, {2, 3, 4, 6}

R: [3, 1, 2, 6, 2, 4] **B:** [5, 5, 6, 1, 3, 1]

‘ See graph

‘ ‘ See pair graph

‘

Ω for $A+\tau\Delta$:

$$\begin{aligned} & [-15' (' 3 + \tau ')'' (' - 5 - \tau - 3\tau^2 + \tau^3 ')', 15' (' 1 + \tau ')'' (' 3 + \tau ')'' (' 5 - 2\tau + \tau^2 ')', 15' (' 5 - \tau + 3\tau^2 + \tau^3 ')'' (' 3 + \tau ')', 15' (' 5 - \tau + 3\tau^2 + \tau^3 ')'' (' 1 + \tau ')', -15' (' 5 + 2\tau + \tau^2 ')'' (' 3 + \tau ')'' (' - 1 + \tau ')', 30' (' 5 - \tau + 3\tau^2 + \tau^3 ')'']' \end{aligned}$$

For $\tau=1/2$, [343, 357, 301, 129, 175, 172] . FixedPtCheck, [343, 357, 301, 129, 175, 172]

$$\det(A + \tau \Delta) = 1' (' 1 + \tau ')' ^3 (' - 1 + \tau ')' ^2$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 5	4 vs 4

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 1/4, 0, 0, 0, 3/4], [3/4, 0, 0, 0, 0, 1/4], [0, 1/4, 3/4, 0, 0, 0], \\ & [3/4, 0, 0, 1/4, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \end{aligned}$$

$$\begin{aligned} & \$ [[-2952/4925, 246359/44325, 241292/44325, 41872/14775, 273088/44325, -856832/44325], \\ & [1808/4925, 117239/44325, 110132/44325, 24112/14775, 67648/44325, -380672/44325], [-1092/4925, \\ & -1261/44325, 43532/44325, -496/4925, 10048/44325, -35072/44325], [8308/4925, -736661/44325, \\ & -729668/44325, -45296/4925, -607552/44325, 2409728/44325], [4439/14775, -80746/44325, \\ & -114448/44325, -23168/14775, -121472/44325, 375808/44325], [-557/4925, -54056/44325, \\ & -55928/44325, 1984/4925, -40192/44325, 140288/44325]] \$ x \$ [[3, 3/2, 3, 1/2, 9/2, 5/2], [21/8, 15/8, \\ & 33/8, 5/8, 27/8, 19/8], [87/32, 15/8, 51/16, 19/32, 27/8, 13/4], [429/128, 105/64, 411/128, 13/16, 441/128, \\ & 325/128], [1497/512, 213/128, 219/64, 325/512, 1917/512, 1337/512], [2919/1024, 3669/2048, 453/128, \\ & 1337/2048, 7047/2048, 5581/2048]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 6}, {1, 2, 3}}, net cycles: 2 . order: 6

$$\begin{aligned} & \$ [[3, 6, 3, 2, 0, 1], [6, 3, 3, 1, 0, 2], [3, 3, 6, 2, 0, 1], [3, 6, 3, 1, 0, 2], [6, 3, 3, 2, 0, 1]] \$ \end{aligned}$$

$$[-y_1 - y_2 + 4 y_3 + 4 y_4, y_1, y_2, y_3, 0, y_4]$$

$$p = -s - s^2 + s^4 + s^5$$

Omega Rank for B : cycles: {{1, 3, 5, 6}}, net cycles: 1 . order: 4

$$[y_4, 0, y_3, 0, y_1, y_2]$$

$$\begin{aligned} & B = \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0, \\ & 0]] \$ x \$ [[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, \end{aligned}$$

0, 0, 1]] \$ = \$ [[4/15, -1/15, -1/15, -1/15] , [4/15, -1/15, -1/15, -1/15] , [-1/15, -1/15, 4/15, -1/15] , [-1/15, -1/15, -1/15, 4/15] , [-1/15, 4/15, -1/15, -1/15] , [-1/15, -1/15, -1/15, 4/15]] \$ x \$ [[3, 0, 3, 0, 6, 3] , [3, 0, 6, 0, 3, 3] , [3, 0, 3, 0, 3, 6] , [6, 0, 3, 0, 3, 3]] \$

Â» SYNC'D 315/4096 , 0.07690429688

29 . Coloring, {2, 3, 5, 6}

R: [3, 1, 2, 1, 3, 4] **B:** [5, 5, 6, 6, 2, 1]

' See graph

' ' See pair graph

Ω for A+τΔ :

' [-15' (' 1 + τ ')'' (' 3 + τ ² ')'' (' 5 - 2τ + τ ² ')' , 15' (' - 5 - τ - 3τ ² + τ ³ ')'' (' 3 + τ ² ')' , 15' (' 1 + τ ')'' (' - 5 + τ ² ')'' (' 3 + τ ² ')' , -15' (' - 1 + τ ')'' (' 1 + τ ')' ² (' - 5 + τ ² ')' , 15' (' - 1 + τ ')'' (' 3 + τ ² ')'' (' 5 + 2τ + τ ² ')' , -30' (' - 1 + τ ')'' (' 1 + τ ')'' (' - 5 + τ ² ')'']'

For τ=1/2, [-663, -637, -741, -171, -325, -228] . FixedPtCheck, [663, 637, 741, 171, 325, 228]

$$\det(A + \tau \Delta) = 1' (' 1 + \tau ')' ³ (' - 1 + \tau ')' ²$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	4 vs 4

bi =

\$ [[0, 0, 1/4, 0, 3/4, 0] , [1/4, 0, 0, 0, 3/4, 0] , [0, 1/4, 0, 0, 0, 3/4] , [1/4, 0, 0, 0, 0, 3/4] , [0, 3/4, 1/4, 0, 0, 0] , [3/4, 0, 0, 1/4, 0, 0]] \$ x \$ [[1, 0, 0, 0, 0, 0] , [0, 1, 0, 0, 0, 0] , [0, 0, 1, 0, 0, 0] , [0, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 1, 0] , [0, 0, 0, 0, 0, 1]] \$ =

\$ [[135913/51165, 12022/153495, -193016/30699, -1847840/92097, -267008/92097, 12240896/460485] , [-23147/51165, 5962/153495, 20272/30699, 160768/92097, 114688/92097, -1460224/460485] , [82393/51165, -59498/153495, -102536/30699, -639488/92097, -80768/92097, 4606976/460485] , [-407537/51165, -7988/153495, 663232/30699, 5372992/92097, 604288/92097, -36112384/460485] , [-43403/51165, -26246/153495, 314432/153495, 2815904/460485, 56576/460485, -3315712/460485] , [1717/51165, 105634/153495, -65128/153495, -207136/460485, 152576/460485, -51712/460485]] \$ x \$ [[5/2, 3, 3/2, 1/2, 9/2, 3] , [25/8, 15/4, 7/4, 3/4, 33/8, 3/2] , [9/4, 113/32, 29/16, 3/8, 165/32, 15/8] , [305/128, 553/128, 237/128, 15/32, 555/128, 105/64] , [1243/512, 951/256, 215/128, 105/256, 1287/256, 891/512] , [4785/2048, 4291/1024, 3817/2048, 891/2048, 9435/2048, 1605/1024]] \$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: $\{\{1, 2, 3\}\}$, net cycles: 0 . order: 3

$$[y_1, y_2, y_3, y_4, 0, 0]$$

$$R = \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0]] \$ = \$ [[0, -14/45, 1/45, 16/45], [0, 1/45, 16/45, -14/45], [0, 16/45, -14/45, 1/45], [0, 1/45, 16/45, -14/45], [0, -14/45, 1/45, 16/45], [1/2, 16/45, -14/45, -43/90]] \$ \times \$ [[4, 3, 6, 2, 0, 0], [5, 6, 4, 0, 0, 0], [6, 4, 5, 0, 0, 0], [4, 5, 6, 0, 0, 0]] \$$$

Omega Rank for B : cycles: $\{\{2, 5\}\}$, net cycles: 0 . order: 4

$$[y_1, y_2, 0, 0, y_3, y_4]$$

$$B = \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 0, 2/15, -1/15], [0, 0, 2/15, -1/15], [1/4, -1/8, -7/60, 7/120], [1/4, -1/8, -7/60, 7/120], [0, 0, -1/15, 2/15], [0, 1/4, -1/15, -7/60]] \$ \times \$ [[2, 3, 0, 0, 6, 4], [4, 6, 0, 0, 5, 0], [0, 5, 0, 0, 10, 0], [0, 10, 0, 0, 5, 0]] \$$$

Â» SYNC'D 19/64 , 0.2968750000

30 . Coloring, $\{2, 4, 5, 6\}$

R: [3, 1, 6, 6, 3, 4] **B:** [5, 5, 2, 1, 2, 1]

' See graph

' ' See pair graph

'

Ω for $A+\tau\Delta$:

$$[-15^{(\cdot-1+\tau^{\cdot})} (\cdot 1+\tau^{\cdot})^{(\cdot 3+\tau^{\cdot})}, 15^{(\cdot-1+\tau^{\cdot})^2} (\cdot 3+\tau^{\cdot})^{\cdot}, -15^{(\cdot-1+\tau^{\cdot})} (\cdot 1+\tau^{\cdot})^{(\cdot 3+\tau^{\cdot})}, 15^{(\cdot 1+\tau^{\cdot})^3}, 15^{(\cdot-1+\tau^{\cdot})^2} (\cdot 3+\tau^{\cdot})^{\cdot}, 30^{(\cdot 1+\tau^{\cdot})^2}]^{\cdot}$$

For $\tau=1/2$, [21, 7, 21, 27, 7, 36] . FixedPtCheck, [21, 7, 21, 27, 7, 36]

$$\det(A + \tau \Delta) = 1^{(\cdot-1+\tau^{\cdot})^2} (\cdot 1+\tau^{\cdot})^{\cdot 3}$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 4	3 vs 3

bi =

$\$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 3/4, 0, 0, 0, 1/4], [3/4, 0, 0, 0, 0, 1/4], [0, 3/4, 1/4, 0, 0, 0], [3/4, 0, 0, 1/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$

$\$ [[-501/475, 1294/1425, 16/3, 15488/1425, 256/475, -23552/1425], [29/475, 464/1425, -8/5, -8096/4275, -1792/1425, 18944/4275], [-151/475, -172/475, 56/15, 10208/1425, -384/475, -13312/1425], [1779/475, 178/475, -112/5, -24224/475, -23552/1425, 122368/1425], [427/1425, -212/475, -16/15, -15136/4275, 1408/1425, 16384/4275], [69/475, -392/475, 8/5, 3136/475, 12928/1425, -23552/1425]] \$ \times \$ [[3, 9/2, 3/2, 1/2, 9/2, 1], [9/4, 9/2, 15/8, 1/4, 45/8, 1/2], [27/16, 45/8, 63/32, 1/8, 81/16, 17/32], [243/128, 675/128, 27/16, 17/128, 351/64, 67/128], [927/512, 1377/256, 945/512, 67/512, 1377/256, 233/512], [1827/1024, 11097/2048, 3681/2048, 233/2048, 11043/2048, 253/512]] \$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 6}}, net cycles: 0 . order: 4

$$[y_1, 0, y_2, y_3, 0, y_4]$$

$R = \$ [[0, 0, 1, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 1], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 1/3, 8/15, -4/5], [1/3, -2/3, -4/5, 6/5], [0, 0, -7/15, 8/15], [0, 0, -7/15, 8/15], [0, 1/3, 8/15, -4/5], [0, 0, 8/15, -7/15]] \$ \times \$ [[3, 0, 6, 2, 0, 4], [0, 0, 3, 4, 0, 8], [0, 0, 0, 8, 0, 7], [0, 0, 0, 7, 0, 8]] \$$

Omega Rank for B : cycles: {{2, 5}}, net cycles: 0 . order: 2

$$[y_1, y_2, 0, 0, y_3, 0]$$

$B = \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0]] \$ = \$ [[0, 1/5, -2/15], [0, 1/5, -2/15], [0, -2/15, 1/5], [1/3, -2/15, -2/15], [0, -2/15, 1/5], [1/3, -2/15, -2/15]] \$ \times \$ [[3, 6, 0, 0, 6, 0], [0, 6, 0, 0, 9, 0], [0, 9, 0, 0, 6, 0]] \$$

Â» SYNC'D 3/32 , 0.09375000000

31 . Coloring, {3, 4, 5, 6}

R: [3, 5, 2, 6, 3, 4] **B:** [5, 1, 6, 1, 2, 1]

' See graph

' ' See pair graph

'

Ω for A+τΔ :

$['-15' (' 5 + 2\tau + \tau^2 ') ' (' 3 + \tau ') ' (' - 1 + \tau ') ' , -15' (' 3 + \tau ') ' (' - 5 - \tau - 3\tau^2 + \tau^3 ') ' , 15' (' 3 + \tau ') ' (' 1 + \tau ') ' (' 5 - 2\tau + \tau^2 ') ' , 15' (' 1 + \tau ') ' ^2 (' 5 - 2\tau + \tau^2 ') ' , 15' (' 5 - \tau + 3\tau^2 + \tau^3 ') ' (' 3 + \tau ') ' , 30' (' 1 + \tau ') ' (' 5 - 2\tau + \tau^2 ') '] '$

For $\tau=1/2$, [175, 343, 357, 153, 301, 204] . FixedPtCheck, [175, 343, 357, 153, 301, 204]

$$\det(A + \tau \Delta) = 1^4 (-1 + \tau)^2 (-1 + \tau)^3$$

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 5	4 vs 4

bi =

$$\$ [[0, 0, 1/4, 0, 3/4, 0], [3/4, 0, 0, 0, 1/4, 0], [0, 1/4, 0, 0, 0, 3/4], [3/4, 0, 0, 0, 0, 1/4], [0, 3/4, 1/4, 0, 0, 0], [3/4, 0, 0, 1/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ =$$

$$\$ [[15299/90825, -23036/54495, -365224/272475, -150848/90825, 68608/272475, 836608/272475], [-9337/30275, -386/54495, 419936/272475, 83872/90825, 469888/272475, -1037312/272475], [30473/30275, -64556/54495, -1133344/272475, -233888/90825, -263552/272475, 2165248/272475], [-15277/30275, 71354/54495, 2207656/272475, 106304/30275, 626048/272475, -3991552/272475], [-11247/30275, 27004/54495, 407216/272475, 49344/30275, -236672/272475, -630272/272475], [293/30275, 55784/54495, -96704/272475, 23264/30275, -370432/272475, -5632/272475]] \$ \times \$ [[9/2, 3, 3/2, 1/2, 3, 5/2], [9/2, 21/8, 15/8, 5/8, 33/8, 5/4], [27/8, 57/16, 69/32, 5/16, 129/32, 25/16], [261/64, 57/16, 237/128, 25/64, 219/64, 217/128], [2169/512, 1551/512, 15/8, 217/512, 1011/256, 761/512], [7587/2048, 3513/1024, 4191/2048, 761/2048, 4029/1024, 3097/2048]] \$$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 6}, {2, 3, 5}}, net cycles: 2 . order: 6

$$\$ [[0, 3, 6, 2, 3, 1], [0, 6, 3, 1, 3, 2], [0, 3, 3, 2, 6, 1], [0, 3, 6, 1, 3, 2], [0, 6, 3, 2, 3, 1]] \$$$

$$[0, -y_1 + 4y_4 - y_2 + 4y_3, y_1, y_4, y_2, y_3]$$

$$p = s + s^2 - s^4 - s^5$$

Omega Rank for B : cycles: {{1, 2, 5}}, net cycles: 0 . order: 3

$$[y_4, y_1, 0, 0, y_2, y_3]$$

$$B = \$ [[0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0], [0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 2/15, 2/15, -1/5], [0, 2/15, -1/5, 2/15], [1/3, -1/5, 2/15, -1/5], [0, 2/15, -1/5, 2/15], [0, -1/5, 2/15, 2/15], [0, 2/15, -1/5, 2/15]] \$ \times \$ [[6, 3, 0, 0, 3, 3], [6, 3, 0, 0, 6, 0], [3, 6, 0, 0, 6, 0], [6, 6, 0, 0, 3, 0]] \$$$

Â» SYNC'D 711/4096 , 0.1735839844

32 . Coloring, {2, 3, 4, 5, 6}

R: [3, 1, 2, 6, 3, 4] **B:** [5, 5, 6, 1, 2, 1]

‘ See graph

‘ ‘ See pair graph

Ω for A+τΔ :

$$\begin{aligned} & ['-15' (' 3 + \tau ')'' (' 5 - 2\tau + \tau^2 ')'' (' 1 + \tau ')', 15' (' 3 + \tau ')'' (' -5 - \tau - 3\tau^2 + \tau^3 ')', 15' (' 3 + \tau ')'' (' -5 + \tau^2 ')'' (' 1 + \tau ')', 15' (' -5 + \tau^2 ')'' (' 1 + \tau ')'^2, 15' (' 5 + 2\tau + \tau^2 ')'' (' 3 + \tau ')'' (' -1 + \tau ')', 30' (' -5 + \tau^2 ')'' (' 1 + \tau ')']' \end{aligned}$$

For τ=1/2, [-357, -343, -399, -171, -175, -228] . FixedPtCheck, [357, 343, 399, 171, 175, 228]

$$\det(A + \tau \Delta) = 1' (' 1 + 3\tau^2 ')'' (' -1 + \tau ')'' (' 1 + \tau ')'^2$$

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
5 vs 5	6 vs 6	6 vs 6	4 vs 5	4 vs 4

bi =

$$\begin{aligned} & \$ [[0, 0, 1/4, 0, 3/4, 0], [1/4, 0, 0, 0, 3/4, 0], [0, 1/4, 0, 0, 0, 3/4], [3/4, 0, 0, 0, 0, 1/4], [0, 3/4, 1/4, 0, 0, 0], \\ & [3/4, 0, 0, 1/4, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0], \\ & [0, 0, 0, 0, 0, 1]] \$ = \end{aligned}$$

$$\begin{aligned} & \$ [[-14834/9525, 33499/28575, 142204/28575, 87056/9525, 110528/28575, -500992/28575], \\ & [-1514/9525, 979/28575, -116/28575, -6224/9525, 49088/28575, -24832/28575], [-1814/9525, \\ & -1721/28575, 49084/28575, 58576/9525, 39488/28575, -255232/28575], [42836/9525, -82471/28575, \\ & -339316/28575, -289424/9525, -360512/28575, 1523968/28575], [5171/9525, -15056/28575, \\ & -49976/28575, -32864/9525, -82432/28575, 232448/28575], [2831/9525, 14684/28575, -42136/28575, \\ & -15104/9525, 5248/28575, 60928/28575]] \$ \times \$ [[3, 3, 3/2, 1/2, 9/2, 5/2], [3, 15/4, 15/8, 5/8, 9/2, 5/4], \\ & [75/32, 123/32, 15/8, 5/16, 81/16, 25/16], [303/128, 273/64, 237/128, 25/64, 297/64, 95/64], [633/256, \\ & 2019/512, 897/512, 95/256, 2547/512, 761/512], [609/256, 4269/1024, 3813/2048, 761/2048, 9855/2048, \\ & 2881/2048]] \$ \end{aligned}$$

Check x AllOnes: [1, 1, 1, 1, 1, 1]

Omega Rank for R : cycles: {{4, 6}, {1, 2, 3}}, net cycles: 2 . order: 6

$$\begin{aligned} & \$ [[3, 3, 6, 2, 0, 1], [3, 6, 3, 1, 0, 2], [6, 3, 3, 2, 0, 1], [3, 3, 6, 1, 0, 2], [3, 6, 3, 2, 0, 1]] \$ \end{aligned}$$

$$[-y_1 - y_2 + 4y_3 + 4y_4, y_1, y_2, y_3, 0, y_4]$$

$$p = s + s^2 - s^4 - s^5$$

Omega Rank for B : cycles: {{2, 5}}, net cycles: 0 . order: 4

$[y_1, y_2, 0, 0, y_3, y_4]$

$B = \$ [[0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 0]] \$ \times \$ [[1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 1]] \$ = \$ [[0, 0, 1/5, -2/15], [0, 0, 1/5, -2/15], [1/3, -1/3, -2/15, 1/5], [0, 1/3, -2/15, -2/15], [0, 0, -2/15, 1/5], [0, 1/3, -2/15, -2/15]] \$ \times \$ [[3, 3, 0, 0, 6, 3], [3, 6, 0, 0, 6, 0], [0, 6, 0, 0, 9, 0], [0, 9, 0, 0, 6, 0]] \$$

Â» SYNC'D 45/512 , 0.08789062500

SUMMARY	
Graph Type	NOT CC
$v(A)$	0
$v(\Delta)$	1
π	[3, 3, 3, 1, 3, 2]
Dbly Stoch	false

SANDWICH		Total 0
No .	Coloring	Rank

RT GROUPS		Total 1	
No .	Coloring	Rank	Solv
1	{}	5	Not Solvable

Δ -RANK'D	SC'D !RK'D	τ -RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
31	0	31 , 30	24 , 26	1	32	32
