

New Graph

[5, 5, 1, 5, 2], [3, 4, 5, 2, 1]

$$\pi = [2, 2, 1, 1, 3]$$

POSSIBLE RANKS

1 x 9
3 x 3

BASE DETERMINANT 351/4096, .8569335938e-1

NullSpace of Δ

{1, 2, 3, 4, 5}

Nullspace of A

$$\det(A) = -1/16$$

1 . Coloring, {}

R: [5, 5, 1, 5, 2]

B: [3, 4, 5, 2, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 5

Omega Rank for R : cycles: {{2, 5}} order: 2

See Matrix

\$ [[1, 3, 0, 0, 5] , [0, 5, 0, 0, 4] , [0, 4, 0, 0, 5]] \$

$$[y_1, y_2, 0, 0, y_3]$$

Omega Rank for B : cycles: $\{\{1, 3, 5\}, \{2, 4\}\}$

See Matrix

$$\$ [[3, 1, 2, 2, 1], [1, 2, 3, 1, 2], [2, 1, 1, 2, 3], [3, 2, 2, 1, 1], [1, 1, 3, 2, 2]] \$$$

$$[y_4, y_3, y_2, y_1, -y_4 + 2y_3 - y_2 + 2y_1]$$

$$p' = -1 - s + s^3 + s^4$$

2 . Coloring, $\{2\}$

$$\Omega p(\Delta)=0: \quad p = s + 4s^3 \quad p' = s + 4s^3$$

R: [5, 4, 1, 5, 2]

B: [3, 5, 5, 2, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 4	3 vs 5	3 vs 5	2 vs 4	2 vs 4

Omega Rank for R : cycles: $\{\{2, 4, 5\}\}$ order: 3

See Matrix

$$\$ [[1, 3, 0, 2, 3], [0, 3, 0, 3, 3], [0, 3, 0, 3, 3], [0, 3, 0, 3, 3]] \$$$

$$[y_2 - y_1, y_2, 0, y_1, y_2]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: $\{\{1, 3, 5\}\}$ order: 3

See Matrix

$$\$ [[3, 1, 2, 0, 3], [3, 0, 3, 0, 3], [3, 0, 3, 0, 3], [3, 0, 3, 0, 3]] \$$$

$$[y_2, y_1, y_2 - y_1, 0, y_2]$$

$$p' = s^2 - s^3 \quad p = s^2 - s^4$$

‘ See 3-level graph

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M N

$$\$ [[0, 1, 1, 0, 2], [1, 0, 0, 1, 2], [1, 0, 0, 0, 1], [0, 1, 0, 0, 1], [2, 2, 1, 1, 0]] \$ \quad \$ [[0, 1, 1, 0, 1], [1, 0, 0, 1, 1], [1, 0, 0, 1, 1], [0, 1, 1, 0, 1], [1, 1, 1, 1, 0]] \$$$

$$\tau = 9, r' = 2/3$$

$$\mathbf{R}: [5, 4, 1, 5, 2]$$

$$\mathbf{B}: [3, 5, 5, 2, 1]$$

Ranges

Action of R on ranges, [[3], [1], [3]]

Action of B on ranges, [[2], [2], [1]]

Cycles: R, {{2, 4, 5}}, B, {{1, 3, 5}}

$$\beta(\{1, 2, 5\}) = 1/3$$

$$\beta(\{1, 3, 5\}) = 1/3$$

$$\beta(\{2, 4, 5\}) = 1/3$$

Partitions

$$\alpha(\{5\}, \{1, 4\}, \{2, 3\}) = 1/1$$

$$b_1 = \{5\}, \quad b_2 = \{1, 4\}, \quad b_3 = \{2, 3\}$$

Action of R and B on the blocks of the partitions: = [2, 3, 1] [3, 1, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Right Group	
Coloring	{2}
Rank	3
R,B	[5, 4, 1, 5, 2], [3, 5, 5, 2, 1]
π_2	[1, 1, 0, 2, 0, 1, 2, 0, 1, 1]
u_2	[1, 1, 0, 1, 0, 1, 1, 1, 1, 1] (dim 1)
wpp	[2, 2, 2, 2, 1]
π_3	[0, 0, 1, 0, 1, 0, 0, 0, 1, 0]
u_3	[0, 0, 1, 0, 1, 0, 0, 0, 1, 1]

3 . Coloring, {3}

$$\Omega p(\Delta)=0: \quad p = s + 2s^2 - 4s^3 - 8s^4$$

R: [5, 5, 5, 5, 2]

B: [3, 4, 1, 2, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	4 vs 5	4 vs 5	2 vs 2	2 vs 4

Omega Rank for R : cycles: {{2, 5}} order: 2

See Matrix

$$\$ [[0, 3, 0, 0, 6] , [0, 6, 0, 0, 3]] \$$$

$$[0, y_1, 0, 0, y_2]$$

Omega Rank for B : cycles: {{2, 4}, {1, 3}} order: 2

See Matrix

\$ [[4, 1, 2, 2, 0] , [2, 2, 4, 1, 0] , [4, 1, 2, 2, 0] , [2, 2, 4, 1, 0]] \$

$[2 y_1, y_2, 2 y_2, y_1, 0]$

$$p' = s - s^3 \quad p = s - s^3$$

4 . Coloring, {4}

R: [5, 5, 1, 2, 2]

B: [3, 4, 5, 5, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R : cycles: {{2, 5}} order: 2

See Matrix

\$ [[1, 4, 0, 0, 4] , [0, 4, 0, 0, 5] , [0, 5, 0, 0, 4]] \$

$[y_2, y_3, 0, 0, y_1]$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3

See Matrix

\$ [[3, 0, 2, 2, 2] , [2, 0, 3, 0, 4] , [4, 0, 2, 0, 3] , [3, 0, 4, 0, 2]] \$

$[y_4, 0, y_3, y_2, y_1]$

5 . Coloring, {5}

R: [5, 5, 1, 5, 1]

B: [3, 4, 5, 2, 2]

‘ See graph

‘ ‘ See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 2	4 vs 4

Omega Rank for R : cycles: $\{\{1, 5\}\}$ order: 2
See Matrix

$$\$ [[4, 0, 0, 0, 5], [5, 0, 0, 0, 4]] \$$$

$$[y_1, 0, 0, 0, y_2]$$

Omega Rank for B : cycles: $\{\{2, 4\}\}$ order: 4
See Matrix

$$\$ [[0, 4, 2, 2, 1], [0, 3, 0, 4, 2], [0, 6, 0, 3, 0], [0, 3, 0, 6, 0]] \$$$

$$[0, y_1, y_2, y_3, y_4]$$

6 . Coloring, $\{2, 3\}$

R: [5, 4, 5, 5, 2]

B: [3, 5, 1, 2, 1]

‘ See graph

‘ ‘ See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R : cycles: $\{\{2, 4, 5\}\}$ order: 3
See Matrix

$$\$ [[0, 3, 0, 2, 4], [0, 4, 0, 3, 2], [0, 2, 0, 4, 3]] \$$$

$$[0, y_3, 0, y_1, y_2]$$

Omega Rank for B : cycles: $\{\{1, 3\}\}$ order: 4

See Matrix

$$\$ [[4, 1, 2, 0, 2], [4, 0, 4, 0, 1], [5, 0, 4, 0, 0], [4, 0, 5, 0, 0]] \$$$

$$[y_1, y_2, y_3, 0, y_4]$$

7 . Coloring, $\{2, 4\}$

R: $[5, 4, 1, 2, 2]$

B: $[3, 5, 5, 5, 1]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	3 vs 3

Omega Rank for R : cycles: $\{\{2, 4\}\}$ order: 4

See Matrix

$$\$ [[1, 4, 0, 2, 2], [0, 4, 0, 4, 1], [0, 5, 0, 4, 0], [0, 4, 0, 5, 0]] \$$$

$$[y_1, y_2, 0, y_3, y_4]$$

Omega Rank for B : cycles: $\{\{1, 3, 5\}\}$ order: 3

See Matrix

$$\$ [[3, 0, 2, 0, 4], [4, 0, 3, 0, 2], [2, 0, 4, 0, 3]] \$$$

$$[y_1, 0, y_2, 0, y_3]$$

8 . Coloring, $\{2, 5\}$

R: $[5, 4, 1, 5, 1]$

B: $[3, 5, 5, 2, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	3 vs 3

Omega Rank for R : cycles: $\{\{1, 5\}\}$ order: 2

See Matrix

$$\$ [[4, 0, 0, 2, 3], [3, 0, 0, 0, 6], [6, 0, 0, 0, 3]] \$$$

$$[y_3, 0, 0, y_2, y_1]$$

Omega Rank for B : cycles: $\{\{2, 5\}\}$ order: 2

See Matrix

$$\$ [[0, 4, 2, 0, 3], [0, 3, 0, 0, 6], [0, 6, 0, 0, 3]] \$$$

$$[0, y_3, y_1, 0, y_2]$$

9 . Coloring, $\{3, 4\}$

R: [5, 5, 5, 2, 2]

B: [3, 4, 1, 5, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 2	4 vs 4

Omega Rank for R : cycles: $\{\{2, 5\}\}$ order: 2

See Matrix

$$\$ [[0, 4, 0, 0, 5], [0, 5, 0, 0, 4]] \$$$

$$[0, y_1, 0, 0, y_2]$$

Omega Rank for B : cycles: $\{\{1, 3\}\}$ order: 4
See Matrix

$$\$ [[4, 0, 2, 2, 1], [3, 0, 4, 0, 2], [6, 0, 3, 0, 0], [3, 0, 6, 0, 0]] \$$$

$$[y_4, 0, y_1, y_2, y_3]$$

10 . Coloring, $\{3, 5\}$

$$\Omega p(\Delta)=0: \quad p = s + 2s^2 - 4s^3 - 8s^4$$

R: $[5, 5, 5, 5, 1]$

B: $[3, 4, 1, 2, 2]$

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	4 vs 5	4 vs 5	2 vs 2	2 vs 4

Omega Rank for R : cycles: $\{\{1, 5\}\}$ order: 2
See Matrix

$$\$ [[3, 0, 0, 0, 6], [6, 0, 0, 0, 3]] \$$$

$$[y_1, 0, 0, 0, y_2]$$

Omega Rank for B : cycles: $\{\{1, 3\}, \{2, 4\}\}$ order: 2
See Matrix

$$\$ [[1, 4, 2, 2, 0], [2, 2, 1, 4, 0], [1, 4, 2, 2, 0], [2, 2, 1, 4, 0]] \$$$

$$[y_2, 2y_1, y_1, 2y_2, 0]$$

$$p = -s + s^3 \quad p' = s - s^3$$

11 . Coloring, {4, 5}

R: [5, 5, 1, 2, 1]

B: [3, 4, 5, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R : cycles: {{1, 5}} order: 2

See Matrix

$$\$ [[4, 1, 0, 0, 4] , [4, 0, 0, 0, 5] , [5, 0, 0, 0, 4]] \$$$

$$[y_2, y_3, 0, 0, y_1]$$

Omega Rank for B : cycles: {{2, 4, 5}} order: 3

See Matrix

$$\$ [[0, 3, 2, 2, 2] , [0, 2, 0, 3, 4] , [0, 4, 0, 2, 3] , [0, 3, 0, 4, 2]] \$$$

$$[0, y_3, y_4, y_2, y_1]$$

12 . Coloring, {2, 3, 4}

R: [5, 4, 5, 2, 2]

B: [3, 5, 1, 5, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	3 vs 3

Omega Rank for R : cycles: $\{\{2, 4\}\}$ order: 2
 See Matrix

$$\$ [[0, 4, 0, 2, 3], [0, 5, 0, 4, 0], [0, 4, 0, 5, 0]] \$$$

$$[0, y_3, 0, y_2, y_1]$$

Omega Rank for B : cycles: $\{\{1, 3\}\}$ order: 2
 See Matrix

$$\$ [[4, 0, 2, 0, 3], [5, 0, 4, 0, 0], [4, 0, 5, 0, 0]] \$$$

$$[y_1, 0, y_2, 0, y_3]$$

13 . Coloring, $\{2, 3, 5\}$

R: [5, 4, 5, 5, 1]

B: [3, 5, 1, 2, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	2 vs 4

Omega Rank for R : cycles: $\{\{1, 5\}\}$ order: 2
 See Matrix

$$\$ [[3, 0, 0, 2, 4], [4, 0, 0, 0, 5], [5, 0, 0, 0, 4]] \$$$

$$[y_3, 0, 0, y_1, y_2]$$

Omega Rank for B : cycles: $\{\{1, 3\}, \{2, 5\}\}$ order: 2
 See Matrix

$$\$ [[1, 4, 2, 0, 2], [2, 2, 1, 0, 4], [1, 4, 2, 0, 2], [2, 2, 1, 0, 4]] \$$$

$$[y_1, 2y_2, y_2, 0, 2y_1]$$

$$p' = -s + s^3 \quad p = -s + s^3$$

14 . Coloring, {2, 4, 5}

R: [5, 4, 1, 2, 1]

B: [3, 5, 5, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 3

Omega Rank for R : cycles: {{1, 5}, {2, 4}} order: 2

See Matrix

$$\$ [[4, 1, 0, 2, 2], [2, 2, 0, 1, 4], [4, 1, 0, 2, 2], [2, 2, 0, 1, 4]] \$$$

$$[2y_2, y_1, 0, y_2, 2y_1]$$

$$p = s - s^3 \quad p' = s - s^3$$

Omega Rank for B : cycles: {{2, 5}} order: 2

See Matrix

$$\$ [[0, 3, 2, 0, 4], [0, 4, 0, 0, 5], [0, 5, 0, 0, 4]] \$$$

$$[0, y_1, y_2, 0, y_3]$$

15 . Coloring, {3, 4, 5}

R: [5, 5, 5, 2, 1]

B: [3, 4, 1, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 5

Omega Rank for R : cycles: $\{\{1, 5\}\}$ order: 2
See Matrix

$$\$ [[3, 1, 0, 0, 5], [5, 0, 0, 0, 4], [4, 0, 0, 0, 5]] \$$$

$$[y_1, y_2, 0, 0, y_3]$$

Omega Rank for B : cycles: $\{\{1, 3\}, \{2, 4, 5\}\}$
See Matrix

$$\$ [[1, 3, 2, 2, 1], [2, 1, 1, 3, 2], [1, 2, 2, 1, 3], [2, 3, 1, 2, 1], [1, 1, 2, 3, 2]] \$$$

$$[y_1, 2y_1 + 2y_2 - y_4 - y_3, y_2, y_4, y_3]$$

$$p' = -1 - s + s^3 + s^4$$

16 . Coloring, $\{2, 3, 4, 5\}$

$$\Omega p(\Delta)=0: \quad p = s - 4s^3 \quad p' = s - 4s^3$$

R: [5, 4, 5, 2, 1]

B: [3, 5, 1, 5, 2]

‘ See graph

‘ ‘ See pair graph

‘

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
2 vs 4	3 vs 5	3 vs 5	2 vs 4	2 vs 4

Omega Rank for R : cycles: $\{\{1, 5\}, \{2, 4\}\}$ order: 2
See Matrix

$$\$ [[3, 1, 0, 2, 3], [3, 2, 0, 1, 3], [3, 1, 0, 2, 3], [3, 2, 0, 1, 3]] \$$$

$$[y_2, y_1, 0, y_2 - y_1, y_2]$$

$$p = -s + s^3 \quad p' = -s + s^3$$

Omega Rank for B : cycles: $\{\{1, 3\}, \{2, 5\}\}$ order: 2

See Matrix

$$\$ [[1, 3, 2, 0, 3], [2, 3, 1, 0, 3], [1, 3, 2, 0, 3], [2, 3, 1, 0, 3]] \$$$

$$[y_2 - y_1, y_2, y_1, 0, y_2]$$

$$p = s - s^3 \quad p' = -s + s^3$$

‘ See 3-level graph

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M N

$$\$ [[0, 1, 0, 1, 2], [1, 0, 1, 0, 2], [0, 1, 0, 0, 1], [1, 0, 0, 0, 1], [2, 2, 1, 1, 0]] \$ \quad \$ [[0, 1, 0, 1, 1], [1, 0, 1, 0, 1], [0, 1, 0, 1, 1], [1, 0, 1, 0, 1], [1, 1, 1, 1, 0]] \$$$

$$\tau = 9, r' = 2/3$$

$$\mathbf{R}: [5, 4, 5, 2, 1]$$

$$\mathbf{B}: [3, 5, 1, 5, 2]$$

Ranges

Action of R on ranges, $[[2], [1], [2]]$

Action of B on ranges, $[[3], [3], [1]]$

Cycles: R, $\{\{1, 5\}, \{2, 4\}\}$, B, $\{\{1, 3\}, \{2, 5\}\}$

$$\beta(\{1, 2, 5\}) = 1/3$$

$$\beta(\{1, 4, 5\}) = 1/3$$

$$\beta(\{2, 3, 5\}) = 1/3$$

Partitions

$$\alpha(\{\{1, 3\}, \{5\}, \{2, 4\}\}) = 1/1$$

$$b1 = \{1, 3\} \text{ ‘ , ‘ } b2 = \{5\} \text{ ‘ , ‘ } b3 = \{2, 4\}$$

Action of R and B on the blocks of the partitions: = [2, 1, 3] [1, 3, 2]
 with invariant measure [1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Right Group	
Coloring	{2, 3, 4, 5}
Rank	3
R,B	[5, 4, 5, 2, 1], [3, 5, 1, 5, 2]
π_2	[1, 0, 1, 2, 1, 0, 2, 0, 1, 1]
u_2	[1, 0, 1, 1, 1, 0, 1, 1, 1, 1] (dim 1)
wpp	[2, 2, 2, 2, 1]
π_3	[0, 0, 1, 0, 0, 1, 0, 1, 0, 0]
u_3	[0, 0, 1, 0, 0, 1, 0, 1, 0, 1]

SUMMARY	
Graph Type	NOT CC
$v(A)$	0
$v(\Delta)$	1
π	[2, 2, 1, 1, 3]
Dbly Stoch	false

SANDWICH		Total 0
No .	Coloring	Rank

RT GROUPS		Total 2	
No .	Coloring	Rank	Solv
1	{2}	3	Not Solvable
2	{2, 3, 4, 5}	3	Not Solvable

Δ-RANK'D	SC'D !RK'D	τ-RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
12	0	12 , 12	13 , 9	2	16	16
