

# New Graph

[3, 3, 5, 5, 1, 1], [2, 4, 6, 6, 4, 2]

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$$\pi = [1, 1, 1, 1, 1, 1]$$

POSSIBLE RANKS

1 x 6  
2 x 3

BASE DETERMINANT 91/512, .1777343750

*NullSpace* of  $\Delta$

{1, 2, 3, 4}, {5, 6}

Nullspace of A

[[6],[5]] ‘ ‘ [[2, 4],[1, 3]]

1 . Coloring, { }

$$\Omega p(\Delta)=0: \quad p' = s^3 \quad p = s^2 \quad p' = s^2$$

**R:** [3, 3, 5, 5, 1, 1]

**B:** [2, 4, 6, 6, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	A+(1/2) $\Delta$	A-(1/2) $\Delta$	<b>R</b>	<b>B</b>
1 vs 4	1 vs 4	1 vs 4	1 vs 3	1 vs 3

Omega Rank for R : cycles: {{1, 3, 5}} order: 3

See Matrix

\$ [ [2, 0, 2, 0, 2, 0] , [2, 0, 2, 0, 2, 0] , [2, 0, 2, 0, 2, 0] ] \$

$$[y_1, 0, y_1, 0, y_1, 0]$$

$$p = -s + s^3 \quad p' = -s + s^2$$

Omega Rank for B : cycles:  $\{\{2, 4, 6\}\}$  order: 3

See Matrix

$$\$ [ [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2] ] \$$$

$$[0, y_1, 0, y_1, 0, y_1]$$

$$p = s - s^3 \quad p' = s - s^2$$

‘ See 3-level graph

‘

M            N

$$\begin{aligned} & \$ [ [0, 0, 1, 0, 1, 0], [0, 0, 0, 1, 0, 1], [1, 0, 0, 0, 1, 0], [0, 1, 0, 0, 0, 1], [1, 0, 1, 0, 0, 0], [0, 1, 0, 1, 0, 0] ] \\ & \$ \quad \$ [ [0, 1, 2, 2, 2, 1], [1, 0, 2, 2, 1, 2], [2, 2, 0, 0, 2, 2], [2, 2, 0, 0, 2, 2], [2, 1, 2, 2, 0, 1], [1, 2, 2, 2, 1, \\ & \quad \quad \quad 0] ] \$ \end{aligned}$$

$$\tau = 12, r' = 2/3$$

$$\mathbf{R}: [3, 3, 5, 5, 1, 1]$$

$$\mathbf{B}: [2, 4, 6, 6, 4, 2]$$

Ranges

Action of R on ranges,  $[[1], [1]]$

Action of B on ranges,  $[[2], [2]]$

Cycles: R ,  $\{\{1, 3, 5\}\}$ , B ,  $\{\{2, 4, 6\}\}$

$$\beta(\{1, 3, 5\}) = 1/2$$

$$\beta(\{2, 4, 6\}) = 1/2$$

Partitions

Action of R on partitions,  $[[1], [1]]$

Action of B on partitions,  $[[2], [2]]$

$$\alpha(\{\{1, 2\}, \{5, 6\}, \{3, 4\}\}) = 1/2$$

$$\alpha(\{\{3, 4\}, \{2, 5\}, \{1, 6\}\}) = 1/2$$

$b_1 = \{1, 2\}$  , ,  $b_2 = \{5, 6\}$  , ,  $b_3 = \{3, 4\}$  , ,  $b_4 = \{2, 5\}$  , ,  $b_5 = \{1, 6\}$

Action of R and B on the blocks of the partitions: = [2, 3, 1, 3, 2] [5, 3, 4, 5, 3]  
with invariant measure [1, 1, 2, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Sandwich	
<b>Coloring</b>	{ }
<b>Rank</b>	3
<b>R,B</b>	[3, 3, 5, 5, 1, 1], [2, 4, 6, 6, 4, 2]
$\pi_2$	[0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0]
$u_2$	[1, 2, 2, 2, 1, 2, 2, 1, 2, 0, 2, 2, 2, 2, 1] (dim 1)
<b>wpp</b>	[2, 2, 2, 2, 2, 2]
$\pi_3$	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]
$u_3$	[1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 0, 0, 0, 1, 1]

2 . Coloring, {2}

$$\Omega p(\Delta)=0: \quad p = s^3 \quad p' = s^3$$

**R:** [3, 4, 5, 5, 1, 1]

**B:** [2, 3, 6, 6, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
2 vs 4	2 vs 5	2 vs 5	2 vs 4	2 vs 4

Omega Rank for R : cycles:  $\{\{1, 3, 5\}\}$  order: 3  
 See Matrix

$$\$ [ [2, 0, 1, 1, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0] ] \$$$

$$[y_1, 0, y_1 - y_2, y_2, y_1, 0]$$

$$p = -s^2 + s^4 \quad p = -s^2 + s^3$$

Omega Rank for B : cycles:  $\{\{2, 3, 6\}\}$  order: 3  
 See Matrix

$$\$ [ [0, 2, 1, 1, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2] ] \$$$

$$[0, y_2, -y_1 + y_2, y_1, 0, y_2]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4$$

‘ See 3-level graph

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M N

$$\$ [ [0, 0, 1, 1, 2, 0], [0, 0, 1, 1, 0, 2], [1, 1, 0, 0, 1, 1], [1, 1, 0, 0, 1, 1], [2, 0, 1, 1, 0, 0], [0, 2, 1, 1, 0, 0] ]$$

$$\$ \quad \$ [ [0, 1, 2, 2, 2, 1], [1, 0, 2, 2, 1, 2], [2, 2, 0, 0, 2, 2], [2, 2, 0, 0, 2, 2], [2, 1, 2, 2, 0, 1], [1, 2, 2, 2, 1, 0] ] \$$$

$$\tau = 12, r' = 2/3$$

$$\mathbf{R}: [3, 4, 5, 5, 1, 1]$$

$$\mathbf{B}: [2, 3, 6, 6, 4, 2]$$

Ranges

Action of R on ranges,  $[[1], [1], [2], [2]]$

Action of B on ranges,  $[[4], [4], [3], [3]]$

Cycles: R ,  $\{\{1, 3, 5\}\}$ , B ,  $\{\{2, 3, 6\}\}$

$$\beta(\{1, 3, 5\}) = 1/4$$

$$\beta(\{1, 4, 5\}) = 1/4$$

$$\beta(\{2, 3, 6\}) = 1/4$$

$$\beta(\{2, 4, 6\}) = 1/4$$

Partitions

Action of R on partitions, [[2], [2]]

Action of B on partitions, [[1], [1]]

$$\alpha(\{\{3, 4\}, \{2, 5\}, \{1, 6\}\}) = 1/2$$

$$\alpha(\{\{1, 2\}, \{5, 6\}, \{3, 4\}\}) = 1/2$$

$$b_1 = \{1, 2\} \text{ , , } b_2 = \{5, 6\} \text{ , , } b_3 = \{3, 4\} \text{ , , } b_4 = \{2, 5\} \text{ , , } b_5 = \{1, 6\}$$

Action of R and B on the blocks of the partitions: = [2, 3, 1, 3, 2] [5, 3, 4, 5, 3]

with invariant measure [1, 1, 2, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Sandwich	
<b>Coloring</b>	{2}
<b>Rank</b>	3
<b>R,B</b>	[3, 4, 5, 5, 1, 1], [2, 3, 6, 6, 4, 2]
$\pi_2$	[0, 1, 1, 2, 0, 1, 1, 0, 2, 0, 1, 1, 1, 1, 0]
$u_2$	[1, 2, 2, 2, 1, 2, 2, 1, 2, 0, 2, 2, 2, 2, 1] (dim 1)
<b>wpp</b>	[2, 2, 2, 2, 2, 2]
$\pi_3$	[0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0]
$u_3$	[1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 0, 0, 0, 1, 1]

3 . Coloring, {3}

**R:** [3, 3, 6, 5, 1, 1]

**B:** [2, 4, 5, 6, 4, 2]

‘ See graph

‘ ‘ See pair graph

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	4 vs 5	5 vs 5	2 vs 4	4 vs 4

Omega Rank for R : cycles:  $\{\{1, 3, 6\}\}$  order: 3  
See Matrix

$$\$ [ [2, 0, 2, 0, 1, 1], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2] ] \$$$

$$[y_1, 0, y_1, 0, y_1 - y_2, y_2]$$

$$p = -s^2 + s^4 \quad p = -s^2 + s^3$$

Omega Rank for B : cycles:  $\{\{2, 4, 6\}\}$  order: 3  
See Matrix

$$\$ [ [0, 2, 0, 2, 1, 1], [0, 1, 0, 3, 0, 2], [0, 2, 0, 1, 0, 3], [0, 3, 0, 2, 0, 1] ] \$$$

$$[0, y_4, 0, y_3, y_2, y_1]$$

4 . Coloring,  $\{4\}$

**R**: [3, 3, 5, 6, 1, 1]

**B**: [2, 4, 6, 5, 4, 2]

‘ See graph

‘ ‘ See pair graph

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 4

Omega Rank for R : cycles:  $\{\{1, 3, 5\}\}$  order: 3  
See Matrix

$$\$ [ [2, 0, 2, 0, 1, 1], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0] ] \$$$

$$[y_2, 0, y_2, 0, y_2 - y_1, y_1]$$

$$p = -s^2 + s^3 \quad p = -s^2 + s^4$$

Omega Rank for B : cycles: {{4, 5}} order: 4

See Matrix

$$\$ [ [0, 2, 0, 2, 1, 1], [0, 1, 0, 3, 2, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0] ] \$$$

$$[0, y_1 - y_2 + y_3, 0, y_1, y_2, y_3]$$

$$p = -s^3 + s^4$$

5 . Coloring, {5}

**R**: [3, 3, 5, 5, 4, 1]

**B**: [2, 4, 6, 6, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	4 vs 5	5 vs 5	3 vs 4	4 vs 4

Omega Rank for R : cycles: {{4, 5}} order: 4

See Matrix

$$\$ [ [1, 0, 2, 1, 2, 0], [0, 0, 1, 2, 3, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0] ] \$$$

$$[y_1, 0, y_1 - y_3 + y_2, y_3, y_2, 0]$$

$$p = -s^3 + s^4$$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3

See Matrix

$$\$ [ [1, 2, 0, 1, 0, 2], [0, 3, 0, 2, 0, 1], [0, 1, 0, 3, 0, 2], [0, 2, 0, 1, 0, 3] ] \$$$

$$[y_4, y_3, 0, y_1, 0, y_2]$$

6 . Coloring, {6}

**R:** [3, 3, 5, 5, 1, 2]

**B:** [2, 4, 6, 6, 4, 1]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 4

Omega Rank for R : cycles: {{1, 3, 5}} order: 3

See Matrix

$$\$ [ [1, 1, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0] ] \$$$

$$[-y_1 + y_2, y_1, y_2, 0, y_2, 0]$$

$$p = -s^2 + s^4 \quad p = -s^2 + s^3$$

Omega Rank for B : cycles: {{1, 2, 4, 6}} order: 4

See Matrix

$$\$ [ [1, 1, 0, 2, 0, 2], [2, 1, 0, 1, 0, 2], [2, 2, 0, 1, 0, 1], [1, 2, 0, 2, 0, 1] ] \$$$

$$[y_2, y_3, 0, -y_2 + y_3 + y_1, 0, y_1]$$

$$p = -s + s^2 - s^3 + s^4$$

7 . Coloring, {2, 3}

**R:** [3, 4, 6, 5, 1, 1]

**B:** [2, 3, 5, 6, 4, 2]

‘ See graph



‘ ‘ See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	6 vs 6	6 vs 6	3 vs 5	5 vs 5

Omega Rank for R : cycles:  $\{\{1, 3, 6\}\}$  order: 3  
See Matrix

$$\$ [ [2, 0, 1, 1, 1, 1], [2, 0, 2, 0, 1, 1], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2] ] \$$$

$$[y_2 + y_3, 0, y_2 + y_3 - y_1, y_1, y_2, y_3]$$

$$p = s^3 - s^4 \quad p' = -s^3 + s^4$$

Omega Rank for B : cycles:  $\{\{2, 3, 4, 5, 6\}\}$  order: 5  
See Matrix

$$\$ [ [0, 2, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1], [0, 1, 1, 1, 2, 1], [0, 1, 1, 2, 1, 1], [0, 1, 1, 1, 1, 2] ] \$$$

$$[0, y_5, y_4, y_2, y_3, y_1]$$

8. Coloring,  $\{2, 4\}$

**R**: [3, 4, 5, 6, 1, 1]

**B**: [2, 3, 6, 5, 4, 2]

‘ See graph

‘ ‘ See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	6 vs 6	6 vs 6	3 vs 5	3 vs 5

Omega Rank for R : cycles:  $\{\{1, 3, 5\}\}$  order: 3  
See Matrix

$$\$ [ [2, 0, 1, 1, 1, 1], [2, 0, 2, 0, 1, 1], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0] ] \$$$

$$[y_3 + y_2, 0, y_3 + y_2 - y_1, y_1, y_3, y_2]$$

$$p = -s^3 + s^5 \quad p = -s^3 + s^4$$

Omega Rank for B : cycles: {{4, 5}, {2, 3, 6}} order: 6

See Matrix

$$\$ [ [0, 2, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1], [0, 1, 1, 1, 1, 2], [0, 2, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1] ] \$$$

$$[0, -y_2 + 4y_1 - y_3, y_2, y_1, y_1, y_3]$$

$$p' = -s + s^4 \quad p = s - s^4$$

9 . Coloring, {2, 5}

**R**: [3, 4, 5, 5, 4, 1]

**B**: [2, 3, 6, 6, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	3 vs 4	4 vs 4	3 vs 4	4 vs 4

Omega Rank for R : cycles: {{4, 5}} order: 4

See Matrix

$$\$ [ [1, 0, 1, 2, 2, 0], [0, 0, 1, 2, 3, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0] ] \$$$

$$[y_3, 0, y_3 - y_1 + y_2, y_1, y_2, 0]$$

$$p = -s^3 + s^4$$

Omega Rank for B : cycles: {{2, 3, 6}} order: 3

See Matrix

$$\$ [ [1, 2, 1, 0, 0, 2], [0, 3, 2, 0, 0, 1], [0, 1, 3, 0, 0, 2], [0, 2, 1, 0, 0, 3] ] \$$$

$$[y_2, y_1, y_3, 0, 0, y_4]$$

10 . Coloring, {2, 6}

**R:** [3, 4, 5, 5, 1, 2]

**B:** [2, 3, 6, 6, 4, 1]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	3 vs 5	4 vs 5

Omega Rank for R : cycles: {{1, 3, 5}} order: 3

See Matrix

$$\$ [ [1, 1, 1, 1, 2, 0], [2, 0, 1, 1, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0] ] \$$$

$$[-y_1 + y_3, y_1, -y_2 + y_3, y_2, y_3, 0]$$

$$p = -s^3 + s^5 \quad p = -s^3 + s^4$$

Omega Rank for B : cycles: {{1, 2, 3, 6}} order: 4

See Matrix

$$\$ [ [1, 1, 1, 1, 0, 2], [2, 1, 1, 0, 0, 2], [2, 2, 1, 0, 0, 1], [1, 2, 2, 0, 0, 1], [1, 1, 2, 0, 0, 2] ] \$$$

$$[y_1, y_1 + y_3 + y_2 - y_4, y_3, y_2, 0, y_4]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

11 . Coloring, {3, 4}

$$\Omega p(\Delta)=0: \quad p = s^2 - 4s^4$$

**R:** [3, 3, 6, 6, 1, 1]

**B:** [2, 4, 5, 5, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 4	4 vs 4	4 vs 4	1 vs 3	3 vs 3

Omega Rank for R : cycles:  $\{\{1, 3, 6\}\}$  order: 3

See Matrix

$$\$ [ [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2] ] \$$$

$$[y_1, 0, y_1, 0, 0, y_1]$$

$$p = -s + s^2 \quad p = -s + s^3$$

Omega Rank for B : cycles:  $\{\{4, 5\}\}$  order: 2

See Matrix

$$\$ [ [0, 2, 0, 2, 2, 0], [0, 0, 0, 4, 2, 0], [0, 0, 0, 2, 4, 0] ] \$$$

$$[0, y_1, 0, y_3, y_2, 0]$$

12 . Coloring,  $\{3, 5\}$

**R**: [3, 3, 6, 5, 4, 1]

**B**: [2, 4, 5, 6, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	6 vs 6	6 vs 6	3 vs 5	3 vs 5

Omega Rank for R : cycles:  $\{\{4, 5\}, \{1, 3, 6\}\}$  order: 6

See Matrix

$$\$ [ [1, 0, 2, 1, 1, 1], [1, 0, 1, 1, 1, 2], [2, 0, 1, 1, 1, 1], [1, 0, 2, 1, 1, 1], [1, 0, 1, 1, 1, 2] ] \$$$

$$[-y_1 + 4y_2 - y_3, 0, y_1, y_2, y_2, y_3]$$

$$p' = s - s^4 \quad p = s - s^4$$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3

See Matrix

$$\$ [ [1, 2, 0, 1, 1, 1], [1, 2, 0, 2, 0, 1], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2] ] \$$$

$$[y_1, y_1 + y_3, 0, y_1 + y_3 - y_2, y_2, y_3]$$

$$p' = s^3 - s^4 \quad p = s^3 - s^5$$

13 . Coloring, {3, 6}

**R**: [3, 3, 6, 5, 1, 2]

**B**: [2, 4, 5, 6, 4, 1]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	6 vs 6	6 vs 6	3 vs 5	4 vs 5

Omega Rank for R : cycles: {{2, 3, 6}} order: 3

See Matrix

$$\$ [ [1, 1, 2, 0, 1, 1], [1, 1, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2] ] \$$$

$$[-y_1 + y_2 + y_3, y_1, y_2 + y_3, 0, y_2, y_3]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles: {{1, 2, 4, 6}} order: 4

See Matrix

$$\$ [ [1, 1, 0, 2, 1, 1], [1, 1, 0, 2, 0, 2], [2, 1, 0, 1, 0, 2], [2, 2, 0, 1, 0, 1], [1, 2, 0, 2, 0, 1] ] \$$$

$$[y_1 - y_2 + y_3 + y_4, y_1, 0, y_2, y_3, y_4]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

14 . Coloring, {4, 5}

**R:** [3, 3, 5, 6, 4, 1]

**B:** [2, 4, 6, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	6 vs 6	6 vs 6	5 vs 5	4 vs 5

Omega Rank for R : cycles: {{1, 3, 4, 5, 6}} order: 5

See Matrix

$$\$ [ [1, 0, 2, 1, 1, 1], [1, 0, 1, 1, 2, 1], [1, 0, 1, 2, 1, 1], [1, 0, 1, 1, 1, 2], [2, 0, 1, 1, 1, 1] ] \$$$

$$[y_5, 0, y_4, y_3, y_1, y_2]$$

Omega Rank for B : cycles: {{1, 2, 4, 5}} order: 4

See Matrix

$$\$ [ [1, 2, 0, 1, 1, 1], [1, 2, 0, 2, 1, 0], [1, 1, 0, 2, 2, 0], [2, 1, 0, 1, 2, 0], [2, 2, 0, 1, 1, 0] ] \$$$

$$[y_1 - y_2 + y_3 - y_4, y_1, 0, y_2, y_3, y_4]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

15 . Coloring, {4, 6}

**R:** [3, 3, 5, 6, 1, 2]

**B:** [2, 4, 6, 5, 4, 1]

‘ See graph

‘ ‘ See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	6 vs 6	6 vs 6	3 vs 5	4 vs 5

Omega Rank for R : cycles:  $\{\{1, 3, 5\}\}$  order: 3  
See Matrix

$$\$ [ [1, 1, 2, 0, 1, 1], [1, 1, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0], [2, 0, 2, 0, 2, 0] ] \$$$

$$[y_1, -y_1 + y_3 + y_2, y_3 + y_2, 0, y_3, y_2]$$

$$p' = s^3 - s^4 \quad p = s^3 - s^5$$

Omega Rank for B : cycles:  $\{\{4, 5\}\}$  order: 4  
See Matrix

$$\$ [ [1, 1, 0, 2, 1, 1], [1, 1, 0, 2, 2, 0], [0, 1, 0, 3, 2, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0] ] \$$$

$$[y_1 - y_4 + y_2 + y_3, y_1, 0, y_4, y_2, y_3]$$

$$p = -s^4 + s^5$$

16 . Coloring,  $\{5, 6\}$

$$\Omega p(\Delta)=0: \quad p = s^2 - 4s^4$$

**R:** [3, 3, 5, 5, 4, 2]

**B:** [2, 4, 6, 6, 1, 1]

‘ See graph

‘ ‘ See pair graph

,

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 4	4 vs 4	4 vs 4	3 vs 4	3 vs 4

Omega Rank for R : cycles: {{4, 5}} order: 4  
See Matrix

$$\$ [ [0, 1, 2, 1, 2, 0], [0, 0, 1, 2, 3, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0] ] \$$$

$$[0, y_1, y_2, y_3, -y_1 + y_2 + y_3, 0]$$

$$p = -s^3 + s^4$$

Omega Rank for B : cycles: {{1, 2, 4, 6}} order: 4  
See Matrix

$$\$ [ [2, 1, 0, 1, 0, 2], [2, 2, 0, 1, 0, 1], [1, 2, 0, 2, 0, 1], [1, 1, 0, 2, 0, 2] ] \$$$

$$[y_3, y_2, 0, -y_3 + y_2 + y_1, 0, y_1]$$

$$p = -s + s^2 - s^3 + s^4$$

17 . Coloring, {2, 3, 4}

**R**: [3, 4, 6, 6, 1, 1]

**B**: [2, 3, 5, 5, 4, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	2 vs 4	4 vs 4

Omega Rank for R : cycles: {{1, 3, 6}} order: 3  
See Matrix

$$\$ [ [2, 0, 1, 1, 0, 2], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2], [2, 0, 2, 0, 0, 2] ] \$$$

$$[y_2, 0, -y_1 + y_2, y_1, 0, y_2]$$

$$p = s^2 - s^4 \quad p' = s^2 - s^3$$

Omega Rank for B : cycles: {{4, 5}} order: 4  
See Matrix



$$\$ [ [0, 2, 1, 1, 2, 0], [0, 0, 2, 2, 2, 0], [0, 0, 0, 2, 4, 0], [0, 0, 0, 4, 2, 0] ] \$$$

$$[0, y_4, y_3, y_2, y_1, 0]$$

18 . Coloring, {2, 3, 5}

$$\Omega p(\Delta)=0: \quad p = s + 3s^2 + 4s^3 + 4s^4$$

**R:** [3, 4, 6, 5, 4, 1]

**B:** [2, 3, 5, 6, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 4	4 vs 5	4 vs 5	2 vs 5	4 vs 5

Omega Rank for R : cycles: {{4, 5}, {1, 3, 6}} order: 6

See Matrix

$$\$ [ [1, 0, 1, 2, 1, 1], [1, 0, 1, 1, 2, 1], [1, 0, 1, 2, 1, 1], [1, 0, 1, 1, 2, 1], [1, 0, 1, 2, 1, 1] ] \$$$

$$[y_2, 0, y_2, 3y_2 - y_1, y_1, y_2]$$

$$p = s - s^5 \quad p' = s^2 - s^4 \quad p'' = -s + s^3$$

Omega Rank for B : cycles: {{1, 2, 3, 5}} order: 4

See Matrix

$$\$ [ [1, 2, 1, 0, 1, 1], [1, 2, 2, 0, 1, 0], [1, 1, 2, 0, 2, 0], [2, 1, 1, 0, 2, 0], [2, 2, 1, 0, 1, 0] ] \$$$

$$[y_4 - y_3 + y_2 - y_1, y_4, y_3, 0, y_2, y_1]$$

$$p = s^2 - s^3 + s^4 - s^5$$

M N

$$\$ [ [0, 1, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1], [1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0] ] \$$$

$$\$ [ [0, 1, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0], [1, 0, 1, 0, 0, 1], [1, 0, 1, 0, 0, 1], [0, 1, 0, 1, 1, 0] ] \$$$

$$\tau = 18, r' = 1/2$$

**R:** [3, 4, 6, 5, 4, 1]

**B:** [2, 3, 5, 6, 1, 2]

Ranges

Action of R on ranges, [[6], [7], [6], [8], [2], [9], [8], [3], [2]]

Action of B on ranges, [[4], [5], [1], [7], [4], [9], [3], [5], [1]]

Cycles: R, {{4, 5}, {1, 3, 6}}, B, {{1, 2, 3, 5}}

$$\beta(\{1, 2\}) = 1/9$$

$$\beta(\{1, 4\}) = 1/9$$

$$\beta(\{1, 5\}) = 1/9$$

$$\beta(\{2, 3\}) = 1/9$$

$$\beta(\{2, 6\}) = 1/9$$

$$\beta(\{3, 4\}) = 1/9$$

$$\beta(\{3, 5\}) = 1/9$$

$$\beta(\{4, 6\}) = 1/9$$

$$\beta(\{5, 6\}) = 1/9$$

Partitions

$$\alpha(\{\{1, 3, 6\}, \{2, 4, 5\}\}) = 1/1$$

$$b_1 = \{1, 3, 6\}, b_2 = \{2, 4, 5\}$$

Action of R and B on the blocks of the partitions: = [1, 2] [2, 1]

with invariant measure [1, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-2 partition graph.

'

<b>Right Group</b>	
<b>Coloring</b>	{2, 3, 5}
<b>Rank</b>	2
<b>R,B</b>	[3, 4, 6, 5, 4, 1], [2, 3, 5, 6, 1, 2]
$\pi_2$	[1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1]
$u_2$	[1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1] (dim 1)
<b>wpp</b>	[3, 3, 3, 3, 3, 3]

19 . Coloring, {2, 3, 6}

$$\Omega p(\Delta)=0: \quad p = s \quad p' = s \quad p' = s^2 \quad p' = s^3$$

**R:** [3, 4, 6, 5, 1, 2]

**B:** [2, 3, 5, 6, 4, 1]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
0 vs 4	1 vs 6	1 vs 6	1 vs 6	1 vs 6

Omega Rank for R : cycles: {{1, 2, 3, 4, 5, 6}} order: 6

See Matrix

$$\$ [ [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1] ]$$

\$

$$[y_1, y_1, y_1, y_1, y_1, y_1]$$

$$p' = -1 + s^4 \quad p' = -1 + s \quad p' = -1 + s^3 \quad p' = -1 + s^5 \quad p' = -1 + s^2$$

Omega Rank for B : cycles: {{1, 2, 3, 4, 5, 6}} order: 6

See Matrix

$$\$ [ [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1] ]$$

\$

$$[y_1, y_1, y_1, y_1, y_1, y_1]$$

$$p' = 1 - s \quad p' = -s + s^5 \quad p' = -s + s^4 \quad p' = -s + s^3 \quad p' = -s + s^2$$

‘ See 6-level graph

‘

M

N

$\$ [ [0, 1, 1, 1, 1, 1], [1, 0, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1], [1, 1, 1, 0, 1, 1], [1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 0] ]$   
 $\$ \quad \$ [ [0, 1, 1, 1, 1, 1], [1, 0, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1], [1, 1, 1, 0, 1, 1], [1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 0] ] \$$

$\tau = 6, r' = 5/6$

**R:** [3, 4, 6, 5, 1, 2]

**B:** [2, 3, 5, 6, 4, 1]

Ranges

Action of R on ranges, [[1]]

Action of B on ranges, [[1]]

Cycles: R, {{1, 2, 3, 4, 5, 6}}, B, {{1, 2, 3, 4, 5, 6}}

$\beta(\{1, 2, 3, 4, 5, 6\}) = 1/1$

Partitions

$\alpha(\{\{2\}, \{1\}, \{5\}, \{6\}, \{3\}, \{4\}\}) = 1/1$

$b_1 = \{2\}, b_2 = \{1\}, b_3 = \{5\}, b_4 = \{6\}, b_5 = \{3\}, b_6 = \{4\}$

Action of R and B on the blocks of the partitions: = [4, 3, 6, 5, 2, 1] [2, 4, 5, 6, 1, 3]  
with invariant measure [1, 1, 1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-6 partition graph.

Right Group	
<b>Coloring</b>	{2, 3, 6}
<b>Rank</b>	6
<b>R,B</b>	[3, 4, 6, 5, 1, 2], [2, 3, 5, 6, 4, 1]
$\pi_2$	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
$u_2$	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] (dim 1)
<b>wpp</b>	[1, 1, 1, 1, 1, 1]
$\pi_6$	[1]
$u_6$	[1]

20 . Coloring, {2, 4, 5}

**R:** [3, 4, 5, 6, 4, 1]

**B:** [2, 3, 6, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	5 vs 5	3 vs 5

Omega Rank for R : cycles: {{1, 3, 4, 5, 6}} order: 5

See Matrix

\$ [ [1, 0, 1, 2, 1, 1] , [1, 0, 1, 1, 1, 2] , [2, 0, 1, 1, 1, 1] , [1, 0, 2, 1, 1, 1] , [1, 0, 1, 1, 2, 1] ] \$

$[y_3, 0, y_1, y_2, y_4, y_5]$

Omega Rank for B : cycles: {{2, 3, 6}} order: 3

See Matrix

\$ [ [1, 2, 1, 0, 1, 1] , [1, 2, 2, 0, 0, 1] , [0, 2, 2, 0, 0, 2] , [0, 2, 2, 0, 0, 2] , [0, 2, 2, 0, 0, 2] ] \$

$[y_2 + y_3 - y_1, y_2 + y_3, y_2, 0, y_3, y_1]$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

21 . Coloring, {2, 4, 6}

$\Omega p(\Delta)=0: \quad p' = s^2 \quad p' = s^3 \quad p' = s \quad p = s$

**R:** [3, 4, 5, 6, 1, 2]

**B:** [2, 3, 6, 5, 4, 1]

‘ See graph

‘ ‘ See pair graph

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
0 vs 4	1 vs 6	1 vs 6	1 vs 6	1 vs 6

Omega Rank for R : cycles:  $\{\{1, 3, 5\}, \{2, 4, 6\}\}$  order: 3  
See Matrix

$\$ [ [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1] ]$   
\$

$[y_1, y_1, y_1, y_1, y_1, y_1]$

$$p' = -s^4 + s^5 \quad p' = 1 - s^4 \quad p' = s - s^4 \quad p' = s^2 - s^4 \quad p' = s^3 - s^4$$

Omega Rank for B : cycles:  $\{\{1, 2, 3, 6\}, \{4, 5\}\}$  order: 4  
See Matrix

$\$ [ [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1] ]$   
\$

$[y_1, y_1, y_1, y_1, y_1, y_1]$

$$p' = -1 + s \quad p' = -1 + s^2 \quad p' = -1 + s^3 \quad p' = -1 + s^4 \quad p' = -1 + s^5$$

‘ See 6-level graph

M N

$\$ [ [0, 1, 1, 1, 1, 1], [1, 0, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1], [1, 1, 1, 0, 1, 1], [1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 0] ]$   
\$ \$ [ [0, 1, 1, 1, 1, 1], [1, 0, 1, 1, 1, 1], [1, 1, 0, 1, 1, 1], [1, 1, 1, 0, 1, 1], [1, 1, 1, 1, 0, 1], [1, 1, 1, 1, 1, 0] ] \$

$$\tau = 6, r' = 5/6$$

**R**: [3, 4, 5, 6, 1, 2]

**B**: [2, 3, 6, 5, 4, 1]

Ranges

Action of R on ranges, [[1]]

Action of B on ranges, [[1]]

Cycles:  $R, \{\{1, 3, 5\}, \{2, 4, 6\}\}, B, \{\{1, 2, 3, 6\}, \{4, 5\}\}$

$$\beta(\{1, 2, 3, 4, 5, 6\}) = 1/1$$

Partitions

$$\alpha(\{\{2\}, \{1\}, \{5\}, \{6\}, \{3\}, \{4\}\}) = 1/1$$

$$b_1 = \{2\} \text{ ' , ' } b_2 = \{1\} \text{ ' , ' } b_3 = \{5\} \text{ ' , ' } b_4 = \{6\} \text{ ' , ' } b_5 = \{3\} \text{ ' , ' } b_6 = \{4\}$$

Action of R and B on the blocks of the partitions: = [4, 3, 5, 6, 2, 1] [2, 4, 6, 5, 1, 3]  
with invariant measure [1, 1, 1, 1, 1, 1]

N by blocks, check: true . ' See partition graph.

' ' See level-6 partition graph.

'

<b>Right Group</b>	
<b>Coloring</b>	{2, 4, 6}
<b>Rank</b>	6
<b>R,B</b>	[3, 4, 5, 6, 1, 2], [2, 3, 6, 5, 4, 1]
$\pi_2$	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
$u_2$	[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] (dim 2)
<b>wpp</b>	[1, 1, 1, 1, 1, 1]
$\pi_6$	[1]
$u_6$	[1]

22 . Coloring, {2, 5, 6}

**R:** [3, 4, 5, 5, 4, 2]

**B:** [2, 3, 6, 6, 1, 1]

' See graph

' ' See pair graph

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 4

Omega Rank for R : cycles: {{4, 5}} order: 2  
See Matrix

$$\$ [ [0, 1, 1, 2, 2, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0] ] \$$$

$$[0, y_1, y_1, y_2, y_2, 0]$$

$$p' = s^2 - s^3 \quad p = s^2 - s^4$$

Omega Rank for B : cycles: {{1, 2, 3, 6}} order: 4  
See Matrix

$$\$ [ [2, 1, 1, 0, 0, 2], [2, 2, 1, 0, 0, 1], [1, 2, 2, 0, 0, 1], [1, 1, 2, 0, 0, 2] ] \$$$

$$[y_1 - y_3 + y_2, y_1, y_3, 0, 0, y_2]$$

$$p = -s + s^2 - s^3 + s^4$$

23 . Coloring, {3, 4, 5}

**R**: [3, 3, 6, 6, 4, 1]

**B**: [2, 4, 5, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	4 vs 4	2 vs 4

Omega Rank for R : cycles: {{1, 3, 6}} order: 3  
See Matrix

$$\$ [ [1, 0, 2, 1, 0, 2], [2, 0, 1, 0, 0, 3], [3, 0, 2, 0, 0, 1], [1, 0, 3, 0, 0, 2] ] \$$$



$$[y_4, 0, y_3, y_2, 0, y_1]$$

Omega Rank for B : cycles: {{1, 2, 4, 5}} order: 4

See Matrix

$$\$ [ [1, 2, 0, 1, 2, 0], [2, 1, 0, 2, 1, 0], [1, 2, 0, 1, 2, 0], [2, 1, 0, 2, 1, 0] ] \$$$

$$[y_1, y_2, 0, y_1, y_2, 0]$$

$$p' = s - s^3 \quad p = s - s^3$$

24 . Coloring, {3, 4, 6}

**R:** [3, 3, 6, 6, 1, 2]

**B:** [2, 4, 5, 5, 4, 1]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 4

Omega Rank for R : cycles: {{2, 3, 6}} order: 3

See Matrix

$$\$ [ [1, 1, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2], [0, 2, 2, 0, 0, 2] ] \$$$

$$[-y_1 + y_2, y_1, y_2, 0, 0, y_2]$$

$$p' = s^2 - s^3 \quad p = s^2 - s^4$$

Omega Rank for B : cycles: {{4, 5}} order: 4

See Matrix

$$\$ [ [1, 1, 0, 2, 2, 0], [0, 1, 0, 3, 2, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0] ] \$$$

$$[y_1 - y_2 + y_3, y_1, 0, y_2, y_3, 0]$$

$$p = -s^3 + s^4$$

25 . Coloring, {3, 5, 6}

**R:** [3, 3, 6, 5, 4, 2]

**B:** [2, 4, 5, 6, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	3 vs 5	4 vs 5

Omega Rank for R : cycles: {{2, 3, 6}, {4, 5}} order: 6

See Matrix

$\$ [ [0, 1, 2, 1, 1, 1], [0, 1, 1, 1, 1, 2], [0, 2, 1, 1, 1, 1], [0, 1, 2, 1, 1, 1], [0, 1, 1, 1, 1, 2] ] \$$

$[0, -y_3 + 4y_2 - y_1, y_3, y_2, y_2, y_1]$

$$p = -s + s^4 \quad p' = -s + s^4$$

Omega Rank for B : cycles: {{1, 2, 4, 6}} order: 4

See Matrix

$\$ [ [2, 1, 0, 1, 1, 1], [2, 2, 0, 1, 0, 1], [1, 2, 0, 2, 0, 1], [1, 1, 0, 2, 0, 2], [2, 1, 0, 1, 0, 2] ] \$$

$[y_1 - y_2 + y_3 + y_4, y_1, 0, y_2, y_3, y_4]$

$$p = -s^2 + s^3 - s^4 + s^5$$

26 . Coloring, {4, 5, 6}

$$\Omega p(\Delta)=0: \quad p' = s^2 + 2s^3 \quad p = s^2 - 4s^4$$

**R:** [3, 3, 5, 6, 4, 2]

**B:** [2, 4, 6, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
2 vs 4	5 vs 5	5 vs 5	5 vs 5	4 vs 5

Omega Rank for R : cycles:  $\{\{2, 3, 4, 5, 6\}\}$  order: 5  
See Matrix

$$\$ [ [0, 1, 2, 1, 1, 1], [0, 1, 1, 1, 2, 1], [0, 1, 1, 2, 1, 1], [0, 1, 1, 1, 1, 2], [0, 2, 1, 1, 1, 1] ] \$$$

$$[0, y_5, y_4, y_1, y_2, y_3]$$

Omega Rank for B : cycles:  $\{\{1, 2, 4, 5\}\}$  order: 4  
See Matrix

$$\$ [ [2, 1, 0, 1, 1, 1], [2, 2, 0, 1, 1, 0], [1, 2, 0, 2, 1, 0], [1, 1, 0, 2, 2, 0], [2, 1, 0, 1, 2, 0] ] \$$$

$$[y_1, y_1 + y_4 - y_3 - y_2, 0, y_4, y_3, y_2]$$

$$p = s^2 - s^3 + s^4 - s^5$$

27 . Coloring,  $\{2, 3, 4, 5\}$

**R**: [3, 4, 6, 6, 4, 1]

**B**: [2, 3, 5, 5, 1, 2]

‘ See graph

‘ ‘ See pair graph

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	4 vs 4	4 vs 4	4 vs 4	2 vs 4

Omega Rank for R : cycles:  $\{\{1, 3, 6\}\}$  order: 3  
See Matrix

$$\$ [ [1, 0, 1, 2, 0, 2], [2, 0, 1, 0, 0, 3], [3, 0, 2, 0, 0, 1], [1, 0, 3, 0, 0, 2] ] \$$$

$$[y_1, 0, y_2, y_4, 0, y_3]$$

Omega Rank for B : cycles:  $\{\{1, 2, 3, 5\}\}$  order: 4

See Matrix

$$\$ [ [1, 2, 1, 0, 2, 0], [2, 1, 2, 0, 1, 0], [1, 2, 1, 0, 2, 0], [2, 1, 2, 0, 1, 0] ] \$$$

$$[y_2, y_1, y_2, 0, y_1, 0]$$

$$p = s - s^3 \quad p' = s - s^3$$

28 . Coloring,  $\{2, 3, 4, 6\}$

**R:**  $[3, 4, 6, 6, 1, 2]$

**B:**  $[2, 3, 5, 5, 4, 1]$

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	3 vs 5	4 vs 5

Omega Rank for R : cycles:  $\{\{2, 4, 6\}\}$  order: 3

See Matrix

$$\$ [ [1, 1, 1, 1, 0, 2], [0, 2, 1, 1, 0, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2] ] \$$$

$$[-y_1 + y_3, y_1, -y_2 + y_3, y_2, 0, y_3]$$

$$p = -s^3 + s^4 \quad p = -s^3 + s^5$$

Omega Rank for B : cycles:  $\{\{4, 5\}\}$  order: 4

See Matrix

$$\$ [ [1, 1, 1, 1, 2, 0], [0, 1, 1, 2, 2, 0], [0, 0, 1, 2, 3, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0] ] \$$$

$$[y_1 - y_2 - y_3 + y_4, y_1, y_2, y_3, y_4, 0]$$

$$p = s^4 - s^5$$

29 . Coloring, {2, 3, 5, 6}

**R:** [3, 4, 6, 5, 4, 2]

**B:** [2, 3, 5, 6, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	6 vs 6	6 vs 6	4 vs 5	4 vs 5

Omega Rank for R : cycles: {{4, 5}} order: 4

See Matrix

$\$ [ [0, 1, 1, 2, 1, 1], [0, 1, 0, 2, 2, 1], [0, 1, 0, 3, 2, 0], [0, 0, 0, 3, 3, 0], [0, 0, 0, 3, 3, 0] ] \$$

$[0, y_4, -y_4 + y_1 - y_2 + y_3, y_1, y_2, y_3]$

$$p = -s^4 + s^5$$

Omega Rank for B : cycles: {{1, 2, 3, 5}} order: 4

See Matrix

$\$ [ [2, 1, 1, 0, 1, 1], [2, 2, 1, 0, 1, 0], [1, 2, 2, 0, 1, 0], [1, 1, 2, 0, 2, 0], [2, 1, 1, 0, 2, 0] ] \$$

$[y_1 - y_3 + y_2 + y_4, y_1, y_3, 0, y_2, y_4]$

$$p = -s^2 + s^3 - s^4 + s^5$$

30 . Coloring, {2, 4, 5, 6}

$$\Omega p(\Delta)=0: \quad p = s^2 - 4s^4 \quad p' = s^2 - 2s^3$$

**R:** [3, 4, 5, 6, 4, 2]

**B:** [2, 3, 6, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
2 vs 4	6 vs 6	6 vs 6	3 vs 5	4 vs 5

Omega Rank for R : cycles:  $\{\{2, 4, 6\}\}$  order: 3  
See Matrix

$$\$ [ [0, 1, 1, 2, 1, 1], [0, 1, 0, 2, 1, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2], [0, 2, 0, 2, 0, 2] ] \$$$

$$[0, y_1 - y_2, y_1 - y_3, y_1, y_2, y_3]$$

$$p' = -s^3 + s^4 \quad p = s^3 - s^4$$

Omega Rank for B : cycles:  $\{\{1, 2, 3, 6\}\}$  order: 4  
See Matrix

$$\$ [ [2, 1, 1, 0, 1, 1], [2, 2, 1, 0, 0, 1], [1, 2, 2, 0, 0, 1], [1, 1, 2, 0, 0, 2], [2, 1, 1, 0, 0, 2] ] \$$$

$$[y_2, y_3, y_1, 0, y_2 - y_3 + y_1 - y_4, y_4]$$

$$p = -s^2 + s^3 - s^4 + s^5$$

31 . Coloring,  $\{3, 4, 5, 6\}$

$$\Omega p(\Delta)=0: \quad p = s^2 + 4s^4$$

**R:** [3, 3, 6, 6, 4, 2]

**B:** [2, 4, 5, 5, 1, 1]

‘ See graph

‘ ‘ See pair graph

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
3 vs 4	4 vs 4	4 vs 4	4 vs 4	3 vs 4

Omega Rank for R : cycles:  $\{\{2, 3, 6\}\}$  order: 3  
See Matrix

$$\$ [ [0, 1, 2, 1, 0, 2], [0, 2, 1, 0, 0, 3], [0, 3, 2, 0, 0, 1], [0, 1, 3, 0, 0, 2] ] \$$$

$$[0, y_1, y_2, y_3, 0, y_4]$$

Omega Rank for B : cycles:  $\{\{1, 2, 4, 5\}\}$  order: 4  
See Matrix

$$\$ [ [2, 1, 0, 1, 2, 0], [2, 2, 0, 1, 1, 0], [1, 2, 0, 2, 1, 0], [1, 1, 0, 2, 2, 0] ] \$$$

$$[y_2, y_2 + y_1 - y_3, 0, y_1, y_3, 0]$$

$$p = -s + s^2 - s^3 + s^4$$

32 . Coloring,  $\{2, 3, 4, 5, 6\}$

**R**:  $[3, 4, 6, 6, 4, 2]$

**B**:  $[2, 3, 5, 5, 1, 1]$

‘ See graph

‘ ‘ See pair graph

‘

$\Delta$ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	<b>R</b>	<b>B</b>
4 vs 4	5 vs 5	5 vs 5	4 vs 4	3 vs 4

Omega Rank for R : cycles:  $\{\{2, 4, 6\}\}$  order: 3  
See Matrix

$$\$ [ [0, 1, 1, 2, 0, 2], [0, 2, 0, 1, 0, 3], [0, 3, 0, 2, 0, 1], [0, 1, 0, 3, 0, 2] ] \$$$

$$[0, y_4, y_3, y_2, 0, y_1]$$

Omega Rank for B : cycles:  $\{\{1, 2, 3, 5\}\}$  order: 4  
See Matrix

$$\$ [ [2, 1, 1, 0, 2, 0], [2, 2, 1, 0, 1, 0], [1, 2, 2, 0, 1, 0], [1, 1, 2, 0, 2, 0] ] \$$$

$$[y_3, y_1, y_2, 0, y_3 - y_1 + y_2, 0]$$

$$p = -s + s^2 - s^3 + s^4$$


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SUMMARY	
<b>Graph Type</b>	CC
$v(A)$	2
$v(\Delta)$	2
$\pi$	[1, 1, 1, 1, 1, 1]
<b>Dbly Stoch</b>	true

SANDWICH		Total 2
No .	Coloring	Rank
1	{}	3
2	{2}	3

RT GROUPS		Total 3	
No .	Coloring	Rank	Solv
1	{2, 3, 5}	2	Not Solvable
2	{2, 4, 6}	6	["group", Not Solvable]
3	{2, 3, 6}	6	["group", Not Solvable]

CC Colorings		Total 1
No .	Coloring	Sandwich,Rank
1	{}	true, 3



$\Delta$ -RANK'D	SC'D !RK'D	$\tau$ -RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	$2^{n-1}$
22	0	24 , 27	7 , 6	5	32	32

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