New Graph

[2, 4, 4, 2, 6, 5], [3, 6, 5, 3, 1, 4]

π = [1, 2, 2, 3, 2, 2]

POSSIBLE RANKS

1 x 12
2 x 6
3 x 4

BASE DETERMINANT 231/2048, .1127929688

NullSpace of Δ

{2, 3}, {1, 4, 5, 6}

Nullspace of A

[{3},{2}] ‘,‘ [{5, 6},{1, 4}]

1 . Coloring, {}

$\Omega p(\Delta)=0$: $p = s^3 + 2s^4$

R: [2, 4, 4, 2, 6, 5]
B: [3, 6, 5, 3, 1, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 4	3 *vs* 5	3 *vs* 5	1 *vs* 4	3 *vs* 5

Omega Rank for R : cycles: {{5, 6}, {2, 4}} order: 2
See Matrix

$ [[0, 4, 0, 4, 2, 2] , [0, 4, 0, 4, 2, 2] , [0, 4, 0, 4, 2, 2] , [0, 4, 0, 4, 2, 2]] $

$[0, 2y_1, 0, 2y_1, y_1, y_1]$

$p = s - s^4$ $p' = s - s^3$ $p' = s^2 - s^3$

Omega Rank for B : cycles: {{1, 3, 5}}   order: 3
See Matrix

$ [[2, 0, 4, 2, 2, 2] , [2, 0, 4, 2, 4, 0] , [4, 0, 4, 0, 4, 0] , [4, 0, 4, 0, 4, 0] , [4, 0, 4, 0, 4, 0]] $

$[y_3 - y_2, 0, y_3, y_2, y_1, y_3 - y_1]$

$p = -s^3 + s^5$ $p = -s^3 + s^4$

‘ See 3-level graph

‘

M N

$ [[0, 0, 2, 0, 1, 1] , [0, 0, 0, 4, 2, 2] , [2, 0, 0, 2, 2, 2] , [0, 4, 2, 0, 3, 3] , [1, 2, 2, 3, 0, 0] , [1, 2, 2, 3, 0, 0]] $ $ [[0, 1, 1, 0, 1, 1] , [1, 0, 0, 1, 1, 1] , [1, 0, 0, 1, 1, 1] , [0, 1, 1, 0, 1, 1] , [1, 1, 1, 1, 0, 0] , [1, 1, 1, 1, 0, 0]] $

τ= 12 , r’= 2/3

R: [2, 4, 4, 2, 6, 5]
B: [3, 6, 5, 3, 1, 4]

Ranges

Action of R on ranges, [[4], [3], [4], [3], [4], [3]]
Action of B on ranges, [[1], [5], [2], [6], [1], [5]]

Cycles: R , {{5, 6}, {2, 4}},   B , {{1, 3, 5}}

β({1, 3, 5}) = 1/8
β({1, 3, 6}) = 1/8
β({2, 4, 5}) = 1/4
β({2, 4, 6}) = 1/4
β({3, 4, 5}) = 1/8
β({3, 4, 6}) = 1/8

Partitions
α([{5, 6}, {1, 4}, {2, 3}]) = 1/1

b1 = {5, 6} ‘ , ‘ b2 = {1, 4} ‘ , ‘ b3 = {2, 3}

Action of R and B on the blocks of the partitions: = [1, 3, 2] [3, 1, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Right Group	
Coloring	{}
Rank	3
R,B	[2, 4, 4, 2, 6, 5], [3, 6, 5, 3, 1, 4]
π_2	[0, 2, 0, 1, 1, 0, 4, 2, 2, 2, 2, 2, 3, 3, 0]
u_2	[1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 2, 2, 0, 1, 1, 0, 0]
u_3	[0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]

2 . Coloring, {2}

R: [2, 6, 4, 2, 6, 5]
B: [3, 4, 5, 3, 1, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	4 *vs* 5	5 *vs* 5	3 *vs* 4	4 *vs* 4

Omega Rank for R : cycles: {{5, 6}} order: 4
See Matrix

$ [[0, 4, 0, 2, 2, 4] , [0, 2, 0, 0, 4, 6] , [0, 0, 0, 0, 6, 6] , [0, 0, 0, 0, 6, 6]] $

$$[0, y_2, 0, y_2 + y_1 - y_3, y_1, y_3]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3
See Matrix

$ [[2, 0, 4, 4, 2, 0] , [2, 0, 6, 0, 4, 0] , [4, 0, 2, 0, 6, 0] , [6, 0, 4, 0, 2, 0]] $

$$[y_1, 0, y_4, y_2, y_3, 0]$$

3 . Coloring, {3}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$ $p' = s^2 + 2s^3$

R: [2, 4, 5, 2, 6, 5]
B: [3, 6, 4, 3, 1, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{5, 6}, {2, 4}} order: 2
See Matrix

$ [[0, 4, 0, 2, 4, 2] , [0, 2, 0, 4, 2, 4] , [0, 4, 0, 2, 4, 2] , [0, 2, 0, 4, 2, 4]] $

$$[0, y_1, 0, y_2, y_1, y_2]$$

$p = s - s^3$ $p' = s - s^3$

Omega Rank for B : cycles: {{3, 4}} order: 2
See Matrix

$ [[2, 0, 4, 4, 0, 2] , [0, 0, 6, 6, 0, 0] , [0, 0, 6, 6, 0, 0] , [0, 0, 6, 6, 0, 0]] $

$[y_1, 0, y_2, y_2, 0, y_1]$

$p' = -s^2 + s^3$ $p = s^2 - s^3$

M N

$ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 2, 0] , [0, 0, 0, 2, 0, 0] , [0, 0, 2, 0, 0, 1] , [0, 2, 0, 0, 0, 0] , [1, 0, 0, 1, 0, 0]]
$ $ [[0, 0, 1, 0, 1, 1] , [0, 0, 1, 0, 1, 1] , [1, 1, 0, 1, 0, 0] , [0, 0, 1, 0, 1, 1] , [1, 1, 0, 1, 0, 0] , [1, 1, 0, 1, 0, 0]] $

τ= 18 , r’= 1/2

R: [2, 4, 5, 2, 6, 5]
B: [3, 6, 4, 3, 1, 4]

Ranges

Action of R on ranges, [[2], [4], [2], [2]]
Action of B on ranges, [[3], [1], [3], [3]]

Cycles: R , {{5, 6}, {2, 4}}, B , {{3, 4}}

β({1, 6}) = 1/6
β({2, 5}) = 1/3
β({3, 4}) = 1/3
β({4, 6}) = 1/6

Partitions
α([{3, 5, 6}, {1, 2, 4}]) = 1/1

b1 = {3, 5, 6} ‘ , ‘ b2 = {1, 2, 4}

Action of R and B on the blocks of the partitions: = [1, 2] [2, 1]
with invariant measure [1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Right Group	
Coloring	{3}
Rank	2
R,B	[2, 4, 5, 2, 6, 5], [3, 6, 4, 3, 1, 4]
π_2	[0, 0, 0, 0, 1, 0, 0, 2, 0, 2, 0, 0, 0, 1, 0]
u_2	[0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

4 . Coloring, {4}

$\Omega p(\Delta)=0$: $p = s + 2s^3 + 4s^4$

R: [2, 4, 4, 3, 6, 5]
B: [3, 6, 5, 2, 1, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	4 *vs* 6	4 *vs* 6	2 *vs* 5	3 *vs* 6

Omega Rank for R : cycles: {{5, 6}, {3, 4}} order: 2
See Matrix

$$ [[0, 1, 3, 4, 2, 2] , [0, 0, 4, 4, 2, 2] , [0, 0, 4, 4, 2, 2] , [0, 0, 4, 4, 2, 2] , [0, 0, 4, 4, 2, 2]] $$

$$[0, -y_1 + 2 y_2, y_1, 2 y_2, y_2, y_2]$$

$p = s^2 - s^5$ $p' = s^3 - s^4$ $p' = s^2 - s^4$

Omega Rank for B : cycles: {{1, 3, 5}, {2, 4, 6}} order: 3
See Matrix

$$ [[2, 3, 1, 2, 2, 2] , [2, 2, 2, 2, 1, 3] , [1, 2, 2, 3, 2, 2] , [2, 3, 1, 2, 2, 2] , [2, 2, 2, 2, 1, 3] , [1, 2, 2, 3, 2, 2]] $$

$[4 y_3 - 5 y_1 + 4 y_2, 3 y_3 - 4 y_1 + 4 y_2, y_3, y_1, y_2, 4 y_3 - 4 y_1 + 3 y_2]$

p’ = $s^2 - s^5$ p’ = $s - s^4$ p’ = $1 - s^3$

‘ See 3-level graph

‘

M N

$ [[0, 4, 1, 0, 2, 3] , [4, 0, 0, 6, 6, 4] , [1, 0, 0, 9, 4, 6] , [0, 6, 9, 0, 8, 7] , [2, 6, 4, 8, 0, 0] , [3, 4, 6, 7, 0, 0]]
$ $ [[0, 1, 1, 0, 1, 1] , [1, 0, 0, 1, 1, 1] , [1, 0, 0, 1, 1, 1] , [0, 1, 1, 0, 1, 1] , [1, 1, 1, 1, 0, 0] , [1, 1, 1, 1, 0, 0]] $

τ= 12 , r’= 2/3

R: [2, 4, 4, 3, 6, 5]
B: [3, 6, 5, 2, 1, 4]

Ranges

Action of R on ranges, [[5], [4], [4], [7], [6], [7], [6]]
Action of B on ranges, [[3], [7], [6], [2], [5], [1], [4]]

Cycles: R , {{5, 6}, {3, 4}}, B , {{1, 3, 5}, {2, 4, 6}}

β({1, 2, 5}) = 1/10
β({1, 2, 6}) = 1/10
β({1, 3, 6}) = 1/20
β({2, 4, 5}) = 1/5
β({2, 4, 6}) = 1/10
β({3, 4, 5}) = 1/5
β({3, 4, 6}) = 1/4

Partitions
α([{5, 6}, {1, 4}, {2, 3}]) = 1/1

b1 = {5, 6} ‘ , ‘ b2 = {1, 4} ‘ , ‘ b3 = {2, 3}

Action of R and B on the blocks of the partitions: = [1, 3, 2] [3, 1, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Right Group	
Coloring	{4}
Rank	3
R,B	[2, 4, 4, 3, 6, 5], [3, 6, 5, 2, 1, 4]
π_2	[4, 1, 0, 2, 3, 0, 6, 6, 4, 9, 4, 6, 8, 7, 0]
$\mathbf{u}_2$	[1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 2, 2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 4, 2, 0, 4, 5, 0, 0]
$\mathbf{u}_3$	[0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]

5 . Coloring, {5}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$ $p' = s^2 - 2s^3$

R: [2, 4, 4, 2, 1, 5]
B: [3, 6, 5, 3, 6, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
2 *vs* 4	3 *vs* 5	3 *vs* 5	3 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{2, 4}} order: 4
See Matrix

$ [[2, 4, 0, 4, 2, 0] , [2, 6, 0, 4, 0, 0] , [0, 6, 0, 6, 0, 0] , [0, 6, 0, 6, 0, 0]] $

$$[y_1, y_1 + y_2 - y_3, 0, y_2, y_3, 0]$$

$p = s^3 - s^4$

Omega Rank for B : cycles: {{3, 4, 5, 6}} order: 4
See Matrix

$ [[0, 0, 4, 2, 2, 4] , [0, 0, 2, 4, 4, 2] , [0, 0, 4, 2, 2, 4] , [0, 0, 2, 4, 4, 2]] $

$[0, 0, y_2, y_1, y_1, y_2]$

$p = s - s^3$ $p' = s - s^3$

M N

$ [[0, 1, 0, 0, 0, 0] , [1, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 2] , [0, 1, 0, 0, 2, 0] , [0, 0, 0, 2, 0, 0] , [0, 0, 2, 0, 0, 0]]
$ $ [[0, 3, 2, 0, 3, 1] , [3, 0, 1, 3, 0, 2] , [2, 1, 0, 2, 1, 3] , [0, 3, 2, 0, 3, 1] , [3, 0, 1, 3, 0, 2] , [1, 2, 3, 1, 2, 0]] $

τ= 18 , r'= 1/2

R: [2, 4, 4, 2, 1, 5]
B: [3, 6, 5, 3, 6, 4]

Ranges

Action of R on ranges, [[2], [2], [4], [1]]
Action of B on ranges, [[3], [3], [4], [3]]

Cycles: R , {{2, 4}}, B , {{3, 4, 5, 6}}

β({1, 2}) = 1/6
β({2, 4}) = 1/6
β({3, 6}) = 1/3
β({4, 5}) = 1/3

Partitions

Action of R on partitions, [[2], [2]]
Action of B on partitions, [[2], [1]]

α([{2, 5, 6}, {1, 3, 4}]) = 1/3
α([{2, 3, 5}, {1, 4, 6}]) = 2/3

b1 = {2, 5, 6} ‘ , ‘ b2 = {1, 3, 4} ‘ , ‘ b3 = {2, 3, 5} ‘ , ‘ b4 = {1, 4, 6}

Action of R and B on the blocks of the partitions: = [4, 3, 4, 3] [3, 4, 2, 1]
with invariant measure [1, 1, 2, 2]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{5}
Rank	2
R,B	[2, 4, 4, 2, 1, 5], [3, 6, 5, 3, 6, 4]
π_2	[1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 2, 0, 0]
$\mathbf{u}_2$	[3, 2, 0, 3, 1, 1, 3, 0, 2, 2, 1, 3, 3, 1, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

6 . Coloring, {6}

R: [2, 4, 4, 2, 6, 4]
B: [3, 6, 5, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	3 *vs* 4	4 *vs* 4	2 *vs* 3	4 *vs* 4

Omega Rank for R : cycles: {{2, 4}} order: 2
See Matrix

$ [[0, 4, 0, 6, 0, 2] , [0, 6, 0, 6, 0, 0] , [0, 6, 0, 6, 0, 0]] $

$$[0, y_1 - y_2, 0, y_1, 0, y_2]$$

$p = - s^2 + s^3$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3
See Matrix

$ [[2, 0, 4, 0, 4, 2] , [4, 0, 2, 0, 6, 0] , [6, 0, 4, 0, 2, 0] , [2, 0, 6, 0, 4, 0]] $

$[y_1, 0, y_4, 0, y_3, y_2]$

7 . Coloring, {2, 3}

R: [2, 6, 5, 2, 6, 5]
B: [3, 4, 4, 3, 1, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 4	4 *vs* 4	3 *vs* 3	3 *vs* 3

Omega Rank for R : cycles: {{5, 6}} order: 2
See Matrix

$ [[0, 4, 0, 0, 4, 4] , [0, 0, 0, 0, 4, 8] , [0, 0, 0, 0, 8, 4]] $

$[0, y_2, 0, 0, y_1, y_3]$

Omega Rank for B : cycles: {{3, 4}} order: 2
See Matrix

$ [[2, 0, 4, 6, 0, 0] , [0, 0, 8, 4, 0, 0] , [0, 0, 4, 8, 0, 0]] $

$[y_3, 0, y_1, y_2, 0, 0]$

8 . Coloring, {2, 4}

R: [2, 6, 4, 3, 6, 5]
B: [3, 4, 5, 2, 1, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	3 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{3, 4}, {5, 6}} order: 2
See Matrix

$ [[0, 1, 3, 2, 2, 4] , [0, 0, 2, 3, 4, 3] , [0, 0, 3, 2, 3, 4] , [0, 0, 2, 3, 4, 3] , [0, 0, 3, 2, 3, 4]] $

$$[0, -7 y_3 - y_2 + 6 y_1, y_3, -6 y_3 + 5 y_1, y_2, y_1]$$

$p = - s^2 + s^4$ $p' = s^2 - s^4$

Omega Rank for B : cycles: {{2, 4}, {1, 3, 5}} order: 6
See Matrix

$ [[2, 3, 1, 4, 2, 0] , [2, 4, 2, 3, 1, 0] , [1, 3, 2, 4, 2, 0] , [2, 4, 1, 3, 2, 0] , [2, 3, 2, 4, 1, 0]] $

$$[5 y_2, 7 y_2 + 7 y_1 - 5 y_3 + 7 y_4, 5 y_1, 5 y_3, 5 y_4, 0]$$

$p = s + s^2 - s^4 - s^5$

9 . Coloring, {2, 5}

R: [2, 6, 4, 2, 1, 5]
B: [3, 4, 5, 3, 6, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 5	3 *vs* 4

Omega Rank for R : cycles: {{1, 2, 5, 6}} order: 4
See Matrix

$ [[2, 4, 0, 2, 2, 2] , [2, 4, 0, 0, 2, 4] , [2, 2, 0, 0, 4, 4] , [4, 2, 0, 0, 4, 2] , [4, 4, 0, 0, 2, 2]] $

$$[y_1, y_1 + y_2 - y_3 + y_4, 0, y_2, y_3, y_4]$$

$p = - s^2 + s^3 - s^4 + s^5$

Omega Rank for B : cycles: {{3, 4, 5, 6}} order: 4
See Matrix

$$[[0, 0, 4, 4, 2, 2] , [0, 0, 4, 2, 4, 2] , [0, 0, 2, 2, 4, 4] , [0, 0, 2, 4, 2, 4]]$$

$$[0, 0, y_1 + y_2 - y_3, y_1, y_2, y_3]$$

$p = s - s^2 + s^3 - s^4$

10 . Coloring, {2, 6}

Ωp(Δ)=0: $p = s^3$ $p' = s^3$

R: [2, 6, 4, 2, 6, 4]
B: [3, 4, 5, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 4	2 *vs* 4	2 *vs* 4	1 *vs* 3	2 *vs* 4

Omega Rank for R : cycles: {{2, 4, 6}} order: 3
See Matrix

$$[[0, 4, 0, 4, 0, 4] , [0, 4, 0, 4, 0, 4] , [0, 4, 0, 4, 0, 4]]$$

$$[0, y_1, 0, y_1, 0, y_1]$$

$p = - s + s^2$ $p = - s + s^3$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3
See Matrix

$$[[2, 0, 4, 2, 4, 0] , [4, 0, 4, 0, 4, 0] , [4, 0, 4, 0, 4, 0] , [4, 0, 4, 0, 4, 0]]$$

$[y_2 - y_1, 0, y_2, y_1, y_2, 0]$

$p = -s^2 + s^4$ $p = -s^2 + s^3$

‘ See 3-level graph

‘

M N

$ [[0, 0, 1, 0, 1, 0] , [0, 0, 0, 2, 0, 2] , [1, 0, 0, 1, 2, 0] , [0, 2, 1, 0, 1, 2] , [1, 0, 2, 1, 0, 0] , [0, 2, 0, 2, 0, 0]]
$ $ [[0, 1, 1, 0, 1, 1] , [1, 0, 1, 1, 0, 1] , [1, 1, 0, 1, 1, 0] , [0, 1, 1, 0, 1, 1] , [1, 0, 1, 1, 0, 1] , [1, 1, 0, 1, 1, 0]] $

τ= 12 , r’= 2/3

R: [2, 6, 4, 2, 6, 4]
B: [3, 4, 5, 3, 1, 5]

Ranges

Action of R on ranges, [[2], [2], [2]]
Action of B on ranges, [[1], [3], [1]]

Cycles: R , {{2, 4, 6}}, B , {{1, 3, 5}}

β({1, 3, 5}) = 1/4
β({2, 4, 6}) = 1/2
β({3, 4, 5}) = 1/4

Partitions
α([{2, 5}, {3, 6}, {1, 4}]) = 1/1

b1 = {2, 5} ‘ , ‘ b2 = {3, 6} ‘ , ‘ b3 = {1, 4}

Action of R and B on the blocks of the partitions: = [3, 1, 2] [2, 3, 1]
with invariant measure [1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Right Group	
Coloring	{2, 6}
Rank	3
R,B	[2, 6, 4, 2, 6, 4], [3, 4, 5, 3, 1, 5]
π_2	[0, 1, 0, 1, 0, 0, 2, 0, 2, 1, 2, 0, 1, 2, 0]
$\mathbf{u}_2$	[1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0]
$\mathbf{u}_3$	[1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1]

11 . Coloring, {3, 4}

R: [2, 4, 5, 3, 6, 5]
B: [3, 6, 4, 2, 1, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{5, 6}} order: 4
See Matrix

$ [[0, 1, 3, 2, 4, 2] , [0, 0, 2, 1, 5, 4] , [0, 0, 1, 0, 6, 5] , [0, 0, 0, 0, 6, 6] , [0, 0, 0, 0, 6, 6]] $

$$[0, y_4, y_3, y_1, y_2, -y_4 - y_3 + y_1 + y_2]$$

$p = - s^4 + s^5$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3
See Matrix

$ [[2, 3, 1, 4, 0, 2] , [0, 4, 2, 3, 0, 3] , [0, 3, 0, 5, 0, 4] , [0, 5, 0, 4, 0, 3] , [0, 4, 0, 3, 0, 5]] $

$[y_1, y_3, y_4, y_5, 0, y_2]$

12 . Coloring, {3, 5}

R: [2, 4, 5, 2, 1, 5]
B: [3, 6, 4, 3, 6, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	4 *vs* 4	4 *vs* 4	4 *vs* 4	3 *vs* 3

Omega Rank for R : cycles: {{2, 4}}   order: 4
See Matrix

$ [[2, 4, 0, 2, 4, 0] , [4, 4, 0, 4, 0, 0] , [0, 8, 0, 4, 0, 0] , [0, 4, 0, 8, 0, 0]] $

$[y_1, y_2, 0, y_3, y_4, 0]$

Omega Rank for B : cycles: {{3, 4}}   order: 2
See Matrix

$ [[0, 0, 4, 4, 0, 4] , [0, 0, 4, 8, 0, 0] , [0, 0, 8, 4, 0, 0]] $

$[0, 0, y_3, y_2, 0, y_1]$

13 . Coloring, {3, 6}

R: [2, 4, 5, 2, 6, 4]
B: [3, 6, 4, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 4	4 *vs* 5

Omega Rank for R : cycles: {{2, 4}} order: 4
See Matrix

$ [[0, 4, 0, 4, 2, 2] , [0, 4, 0, 6, 0, 2] , [0, 6, 0, 6, 0, 0] , [0, 6, 0, 6, 0, 0]] $

$$[0, y_1 + y_2 - y_3, 0, y_1, y_2, y_3]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{3, 4}} order: 4
See Matrix

$ [[2, 0, 4, 2, 2, 2] , [2, 0, 4, 4, 2, 0] , [2, 0, 6, 4, 0, 0] , [0, 0, 6, 6, 0, 0] , [0, 0, 6, 6, 0, 0]] $

$$[y_1 - y_4 + y_3 - y_2, 0, y_1, y_4, y_3, y_2]$$

$p = s^4 - s^5$

14 . Coloring, {4, 5}

R: [2, 4, 4, 3, 1, 5]
B: [3, 6, 5, 2, 6, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 5	5 *vs* 5

Omega Rank for R : cycles: {{3, 4}} order: 4
See Matrix

$ [[2, 1, 3, 4, 2, 0] , [2, 2, 4, 4, 0, 0] , [0, 2, 4, 6, 0, 0] , [0, 0, 6, 6, 0, 0] , [0, 0, 6, 6, 0, 0]] $

$$[y_3, y_3 - y_1 + y_2 - y_4, y_1, y_2, y_4, 0]$$

$p = - s^4 + s^5$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3
See Matrix

$ [[0, 3, 1, 2, 2, 4] , [0, 2, 0, 4, 1, 5] , [0, 4, 0, 5, 0, 3] , [0, 5, 0, 3, 0, 4] , [0, 3, 0, 4, 0, 5]] $

$$[0, y_5, y_4, y_3, y_2, y_1]$$

15 . Coloring, {4, 6}

R: [2, 4, 4, 3, 6, 4]
B: [3, 6, 5, 2, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	2 *vs* 4	5 *vs* 5

Omega Rank for R : cycles: {{3, 4}} order: 2
See Matrix

$ [[0, 1, 3, 6, 0, 2] , [0, 0, 6, 6, 0, 0] , [0, 0, 6, 6, 0, 0] , [0, 0, 6, 6, 0, 0]] $

$$[0, y_2, y_1, 3 y_2 + y_1, 0, 2 y_2]$$

$p = - s^2 + s^3$ $p = - s^2 + s^4$

Omega Rank for B : cycles: {{1, 3, 5}} order: 3
See Matrix

$ [[2, 3, 1, 0, 4, 2] , [4, 0, 2, 0, 3, 3] , [3, 0, 4, 0, 5, 0] , [5, 0, 3, 0, 4, 0] , [4, 0, 5, 0, 3, 0]] $

$$[y_2, y_1, y_5, 0, y_3, y_4]$$

16 . Coloring, {5, 6}

R: [2, 4, 4, 2, 1, 4]
B: [3, 6, 5, 3, 6, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 4	4 *vs* 4	3 *vs* 3	3 *vs* 3

Omega Rank for R : cycles: {{2, 4}} order: 2
See Matrix

$ [[2, 4, 0, 6, 0, 0] , [0, 8, 0, 4, 0, 0] , [0, 4, 0, 8, 0, 0]] $

$[y_1, y_3, 0, y_2, 0, 0]$

Omega Rank for B : cycles: {{5, 6}} order: 2
See Matrix

$ [[0, 0, 4, 0, 4, 4] , [0, 0, 0, 0, 8, 4] , [0, 0, 0, 0, 4, 8]] $

$[0, 0, y_1, 0, y_3, y_2]$

17 . Coloring, {2, 3, 4}

Ωp(Δ)=0: $p = s - 2s^3 - 4s^4$

R: [2, 6, 5, 3, 6, 5]
B: [3, 4, 4, 2, 1, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	5 *vs* 5	5 *vs* 5	3 *vs* 4	4 *vs* 4

Omega Rank for R : cycles: {{5, 6}}   order: 2
See Matrix

$ [[0, 1, 3, 0, 4, 4] , [0, 0, 0, 0, 7, 5] , [0, 0, 0, 0, 5, 7] , [0, 0, 0, 0, 7, 5]] $

$$[0, y_3, 3 y_3, 0, y_1, y_2]$$

$p = - s^2 + s^4$

Omega Rank for B : cycles: {{2, 4}}   order: 4
See Matrix

$ [[2, 3, 1, 6, 0, 0] , [0, 6, 2, 4, 0, 0] , [0, 4, 0, 8, 0, 0] , [0, 8, 0, 4, 0, 0]] $

$$[y_2, y_1, y_3, y_4, 0, 0]$$

18 . Coloring, {2, 3, 5}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$ $p' = s^2 + 2s^3$

R: [2, 6, 5, 2, 1, 5]
B: [3, 4, 4, 3, 6, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 4	2 *vs* 3

Omega Rank for R : cycles: {{1, 2, 5, 6}}   order: 4
See Matrix

$ [[2, 4, 0, 0, 4, 2] , [4, 2, 0, 0, 2, 4] , [2, 4, 0, 0, 4, 2] , [4, 2, 0, 0, 2, 4]] $

$$[y_2, y_1, 0, 0, y_1, y_2]$$

$p = - s + s^3$ $p' = - s + s^3$

Omega Rank for B : cycles: {{3, 4}}   order: 2
See Matrix

$ [[0, 0, 4, 6, 0, 2] , [0, 0, 6, 6, 0, 0] , [0, 0, 6, 6, 0, 0]] $

$[0, 0, y_2, y_1, 0, -y_2 + y_1]$

$p = - s^2 + s^3$

M N

$ [[0, 0, 0, 0, 0, 1] , [0, 0, 0, 0, 2, 0] , [0, 0, 0, 2, 0, 0] , [0, 0, 2, 0, 0, 1] , [0, 2, 0, 0, 0, 0] , [1, 0, 0, 1, 0, 0]]
$ $ [[0, 2, 3, 0, 1, 3] , [2, 0, 1, 2, 3, 1] , [3, 1, 0, 3, 2, 0] , [0, 2, 3, 0, 1, 3] , [1, 3, 2, 1, 0, 2] , [3, 1, 0, 3, 2, 0]] $

τ= 18 , r'= 1/2

R: [2, 6, 5, 2, 1, 5]
B: [3, 4, 4, 3, 6, 4]

Ranges

Action of R on ranges, [[2], [1], [2], [2]]
Action of B on ranges, [[3], [4], [3], [3]]

Cycles: R , {{1, 2, 5, 6}}, B , {{3, 4}}

β({1, 6}) = 1/6
β({2, 5}) = 1/3
β({3, 4}) = 1/3
β({4, 6}) = 1/6

Partitions

Action of R on partitions, [[2], [1]]
Action of B on partitions, [[2], [2]]

α([{1, 2, 4}, {3, 5, 6}]) = 1/3
α([{2, 3, 6}, {1, 4, 5}]) = 2/3

b1 = {1, 2, 4} ' , ' b2 = {3, 5, 6} ' , ' b3 = {2, 3, 6} ' , ' b4 = {1, 4, 5}

Action of R and B on the blocks of the partitions: = [4, 3, 1, 2] [3, 4, 4, 3]
with invariant measure [1, 1, 2, 2]

N by blocks, check: true . ' See partition graph.

' ' See level-2 partition graph.

‘

Sandwich	
Coloring	{2, 3, 5}
Rank	2
R,B	[2, 6, 5, 2, 1, 5], [3, 4, 4, 3, 6, 4]
π_2	[0, 0, 0, 0, 1, 0, 0, 2, 0, 2, 0, 0, 0, 1, 0]
$\mathbf{u}_2$	[2, 3, 0, 1, 3, 1, 2, 3, 1, 3, 2, 0, 1, 3, 2] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

19 . Coloring, {2, 3, 6}

R: [2, 6, 5, 2, 6, 4]
B: [3, 4, 4, 3, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	4 *vs* 5	4 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{2, 4, 6}} order: 3
See Matrix

$ [[0, 4, 0, 2, 2, 4] , [0, 2, 0, 4, 0, 6] , [0, 4, 0, 6, 0, 2] , [0, 6, 0, 2, 0, 4]] $

$$[0, y_4, 0, y_3, y_1, y_2]$$

Omega Rank for B : cycles: {{3, 4}} order: 4
See Matrix

$ [[2, 0, 4, 4, 2, 0] , [2, 0, 6, 4, 0, 0] , [0, 0, 6, 6, 0, 0] , [0, 0, 6, 6, 0, 0]] $

$$[y_1 - y_2 + y_3, 0, y_1, y_2, y_3, 0]$$

$p = - s^3 + s^4$

20 . Coloring, {2, 4, 5}

R: [2, 6, 4, 3, 1, 5]
B: [3, 4, 5, 2, 6, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 6	4 *vs* 5

Omega Rank for R : cycles: {{1, 2, 5, 6}, {3, 4}} order: 4
See Matrix

$$[[2, 1, 3, 2, 2, 2] , [2, 2, 2, 3, 2, 1] , [2, 2, 3, 2, 1, 2] , [1, 2, 2, 3, 2, 2] , [2, 1, 3, 2, 2, 2] , [2, 2, 2, 3, 2, 1]]$$

$$[y_2, 6 y_2 - 7 y_1 - y_3 + 6 y_4, y_1, 5 y_2 - 6 y_1 + 5 y_4, y_3, y_4]$$

$p' = - 1 + s^4$ $p' = - s + s^5$

Omega Rank for B : cycles: {{2, 4}} order: 4
See Matrix

$$[[0, 3, 1, 4, 2, 2] , [0, 4, 0, 5, 1, 2] , [0, 5, 0, 6, 0, 1] , [0, 6, 0, 6, 0, 0] , [0, 6, 0, 6, 0, 0]]$$

$$[0, y_4, y_3, y_2, y_1, -y_4 - y_3 + y_2 + y_1]$$

$p = s^4 - s^5$

21 . Coloring, {2, 4, 6}

R: [2, 6, 4, 3, 6, 4]
B: [3, 4, 5, 2, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	4 *vs* 5

Omega Rank for R : cycles: {{3, 4}} order: 4
See Matrix

$ [[0, 1, 3, 4, 0, 4] , [0, 0, 4, 7, 0, 1] , [0, 0, 7, 5, 0, 0] , [0, 0, 5, 7, 0, 0]] $

$$[0, y_1, y_4, y_3, 0, y_2]$$

Omega Rank for B : cycles: {{1, 3, 5}, {2, 4}} order: 6
See Matrix

$ [[2, 3, 1, 2, 4, 0] , [4, 2, 2, 3, 1, 0] , [1, 3, 4, 2, 2, 0] , [2, 2, 1, 3, 4, 0] , [4, 3, 2, 2, 1, 0]] $

$$[7 y_1 - 5 y_4 + 7 y_3 - 5 y_2, 5 y_1, 5 y_4, 5 y_3, 5 y_2, 0]$$

$p = - s - s^2 + s^4 + s^5$

22 . Coloring, {2, 5, 6}

R: [2, 6, 4, 2, 1, 4]
B: [3, 4, 5, 3, 6, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	4 *vs* 4	3 *vs* 4	4 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{2, 4, 6}} order: 3
See Matrix

$ [[2, 4, 0, 4, 0, 2] , [0, 6, 0, 2, 0, 4] , [0, 2, 0, 4, 0, 6] , [0, 4, 0, 6, 0, 2]] $

$$[y_1, y_2, 0, y_3, 0, y_4]$$

Omega Rank for B : cycles: {{5, 6}} order: 4
See Matrix

$ [[0, 0, 4, 2, 4, 2] , [0, 0, 2, 0, 6, 4] , [0, 0, 0, 0, 6, 6] , [0, 0, 0, 0, 6, 6]] $

$$[0, 0, y_1 + y_2 - y_3, y_1, y_2, y_3]$$

$p = - s^3 + s^4$

23 . Coloring, {3, 4, 5}

R: [2, 4, 5, 3, 1, 5]
B: [3, 6, 4, 2, 6, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	5 *vs* 5	4 *vs* 4

Omega Rank for R : cycles: {{1, 2, 3, 4, 5}} order: 5
See Matrix

$ [[2, 1, 3, 2, 4, 0] , [4, 2, 2, 1, 3, 0] , [3, 4, 1, 2, 2, 0] , [2, 3, 2, 4, 1, 0] , [1, 2, 4, 3, 2, 0]] $

$$[y_3, y_4, y_5, y_2, y_1, 0]$$

Omega Rank for B : cycles: {{2, 4, 6}} order: 3
See Matrix

$ [[0, 3, 1, 4, 0, 4] , [0, 4, 0, 5, 0, 3] , [0, 5, 0, 3, 0, 4] , [0, 3, 0, 4, 0, 5]] $

$$[0, y_1, y_2, y_3, 0, y_4]$$

24 . Coloring, {3, 4, 6}

R: [2, 4, 5, 3, 6, 4]
B: [3, 6, 4, 2, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	4 *vs* 5	5 *vs* 6

Omega Rank for R : cycles: {{3, 4, 5, 6}} order: 4
See Matrix

$ [[0, 1, 3, 4, 2, 2] , [0, 0, 4, 3, 3, 2] , [0, 0, 3, 2, 4, 3] , [0, 0, 2, 3, 3, 4] , [0, 0, 3, 4, 2, 3]] $

$$[0, y_2, y_3, y_2 + y_3 - y_1 + y_4, y_1, y_4]$$

$p = - s^2 + s^3 - s^4 + s^5$

Omega Rank for B : cycles: {{1, 2, 3, 4, 5, 6}} order: 6
See Matrix

$ [[2, 3, 1, 2, 2, 2] , [2, 2, 2, 1, 2, 3] , [2, 1, 2, 2, 3, 2] , [3, 2, 2, 2, 2, 1] , [2, 2, 3, 2, 1, 2] , [1, 2, 2, 3, 2, 2]] $

$$[y_2, y_1, y_2 - y_1 + y_3 - y_4 + y_5, y_3, y_4, y_5]$$

$p' = 1 - s + s^2 - s^3 + s^4 - s^5$

25 . Coloring, {3, 5, 6}

$\Omega p(\Delta)=0$: $p = s^2 - 4s^4$ $p' = s^2 - 2s^3$

R: [2, 4, 5, 2, 1, 4]
B: [3, 6, 4, 3, 6, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
2 *vs* 4	3 *vs* 5	3 *vs* 5	3 *vs* 4	2 *vs* 4

Omega Rank for R : cycles: {{2, 4}} order: 4
See Matrix

$ [[2, 4, 0, 4, 2, 0] , [2, 6, 0, 4, 0, 0] , [0, 6, 0, 6, 0, 0] , [0, 6, 0, 6, 0, 0]] $

$$[y_2, y_1 + y_2 - y_3, 0, y_1, y_3, 0]$$

$p = - s^3 + s^4$

Omega Rank for B : cycles: {{5, 6}, {3, 4}} order: 2
See Matrix

$ [[0, 0, 4, 2, 2, 4] , [0, 0, 2, 4, 4, 2] , [0, 0, 4, 2, 2, 4] , [0, 0, 2, 4, 4, 2]] $

$$[0, 0, y_2, y_1, y_1, y_2]$$

$p = s - s^3$ $p' = s - s^3$

M N

$ [[0, 1, 0, 0, 0, 0] , [1, 0, 0, 1, 0, 0] , [0, 0, 0, 0, 0, 2] , [0, 1, 0, 0, 2, 0] , [0, 0, 0, 2, 0, 0] , [0, 0, 2, 0, 0, 0]] $ $ [[0, 1, 0, 0, 1, 1] , [1, 0, 1, 1, 0, 0] , [0, 1, 0, 0, 1, 1] , [0, 1, 0, 0, 1, 1] , [1, 0, 1, 1, 0, 0] , [1, 0, 1, 1, 0, 0]] $

τ= 18 , r’= 1/2

R: [2, 4, 5, 2, 1, 4]
B: [3, 6, 4, 3, 6, 5]

Ranges

Action of R on ranges, [[2], [2], [4], [1]]
Action of B on ranges, [[3], [3], [4], [3]]

Cycles: R , {{2, 4}}, B , {{5, 6}, {3, 4}}

β({1, 2}) = 1/6
β({2, 4}) = 1/6
β({3, 6}) = 1/3

β({4, 5}) = 1/3

Partitions
α([{2, 5, 6}, {1, 3, 4}]) = 1/1

b1 = {2, 5, 6} ‘ , ‘ b2 = {1, 3, 4}

Action of R and B on the blocks of the partitions: = [2, 1] [1, 2]
with invariant measure [1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Right Group	
Coloring	{3, 5, 6}
Rank	2
R,B	[2, 4, 5, 2, 1, 4], [3, 6, 4, 3, 6, 5]
π_2	[1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 2, 0, 0]
$\mathbf{u}_2$	[1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0] (dim 1)
wpp	[3, 3, 3, 3, 3, 3]

26 . Coloring, {4, 5, 6}

R: [2, 4, 4, 3, 1, 4]
B: [3, 6, 5, 2, 6, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	5 *vs* 5	5 *vs* 5	4 *vs* 4	3 *vs* 4

Omega Rank for R : cycles: {{3, 4}}   order: 4
See Matrix

$ [[2, 1, 3, 6, 0, 0] , [0, 2, 6, 4, 0, 0] , [0, 0, 4, 8, 0, 0] , [0, 0, 8, 4, 0, 0]] $

$[y_1, y_2, y_3, y_4, 0, 0]$

Omega Rank for B : cycles: {{5, 6}}   order: 2
See Matrix

$ [[0, 3, 1, 0, 4, 4] , [0, 0, 0, 0, 5, 7] , [0, 0, 0, 0, 7, 5] , [0, 0, 0, 0, 5, 7]] $

$[0, 3 y_1, y_1, 0, y_2, y_3]$

$p = s^2 - s^4$

27 . Coloring, {2, 3, 4, 5}

R: [2, 6, 5, 3, 1, 5]
B: [3, 4, 4, 2, 6, 4]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	5 *vs* 5	2 *vs* 4

Omega Rank for R : cycles: {{1, 2, 5, 6}}   order: 4
See Matrix

$ [[2, 1, 3, 0, 4, 2] , [4, 2, 0, 0, 5, 1] , [5, 4, 0, 0, 1, 2] , [1, 5, 0, 0, 2, 4] , [2, 1, 0, 0, 4, 5]] $

$[y_3, y_4, y_5, 0, y_1, y_2]$

Omega Rank for B : cycles: {{2, 4}}   order: 2
See Matrix

$ [[0, 3, 1, 6, 0, 2] , [0, 6, 0, 6, 0, 0] , [0, 6, 0, 6, 0, 0] , [0, 6, 0, 6, 0, 0]] $

$$[0, -3 y_2 + y_1, y_2, y_1, 0, 2 y_2]$$

$p = - s^2 + s^3$ $p = - s^2 + s^4$

28 . Coloring, {2, 3, 4, 6}

R: [2, 6, 5, 3, 6, 4]
B: [3, 4, 4, 2, 1, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{3, 4, 5, 6}}   order: 4
See Matrix

$$[[0, 1, 3, 2, 2, 4] , [0, 0, 2, 4, 3, 3] , [0, 0, 4, 3, 2, 3] , [0, 0, 3, 3, 4, 2] , [0, 0, 3, 2, 3, 4]]$$

$$[0, y_1, y_2, y_3, y_4, y_5]$$

Omega Rank for B : cycles: {{2, 4}}   order: 4
See Matrix

$$[[2, 3, 1, 4, 2, 0] , [2, 4, 2, 4, 0, 0] , [0, 4, 2, 6, 0, 0] , [0, 6, 0, 6, 0, 0] , [0, 6, 0, 6, 0, 0]]$$

$$[y_1 + y_2 - y_3 + y_4, y_1, y_2, y_3, y_4, 0]$$

$p = - s^4 + s^5$

29 . Coloring, {2, 3, 5, 6}

$\Omega p(\Delta)=0$: $p = s^3 - 2s^4$

R: [2, 6, 5, 2, 1, 4]
B: [3, 4, 4, 3, 6, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 4	3 *vs* 5	3 *vs* 5	3 *vs* 5	1 *vs* 4

Omega Rank for R : cycles: {{2, 4, 6}} order: 3
See Matrix

$ [[2, 4, 0, 2, 2, 2] , [2, 4, 0, 2, 0, 4] , [0, 4, 0, 4, 0, 4] , [0, 4, 0, 4, 0, 4] , [0, 4, 0, 4, 0, 4]] $

$$[y_2 + y_3 - y_1, y_2 + y_3, 0, y_1, y_2, y_3]$$

$p = - s^3 + s^5$ $p = - s^3 + s^4$

Omega Rank for B : cycles: {{5, 6}, {3, 4}} order: 2
See Matrix

$ [[0, 0, 4, 4, 2, 2] , [0, 0, 4, 4, 2, 2] , [0, 0, 4, 4, 2, 2] , [0, 0, 4, 4, 2, 2]] $

$$[0, 0, 2 y_1, 2 y_1, y_1, y_1]$$

$p' = - s + s^2$ $p = s - s^2$ $p' = - s + s^3$

‘ See 3-level graph

‘

M N

$ [[0, 2, 0, 0, 1, 1] , [2, 0, 0, 2, 2, 2] , [0, 0, 0, 4, 2, 2] , [0, 2, 4, 0, 3, 3] , [1, 2, 2, 3, 0, 0] , [1, 2, 2, 3, 0, 0]] $ $ [[0, 1, 1, 0, 1, 1] , [1, 0, 0, 1, 1, 1] , [1, 0, 0, 1, 1, 1] , [0, 1, 1, 0, 1, 1] , [1, 1, 1, 1, 0, 0] , [1, 1, 1, 1, 0, 0]] $

τ= 12 , r’= 2/3

R: [2, 6, 5, 2, 1, 4]
B: [3, 4, 4, 3, 6, 5]

Ranges

Action of R on ranges, [[2], [4], [2], [4], [1], [3]]
Action of B on ranges, [[6], [5], [6], [5], [6], [5]]

Cycles: R , {{2, 4, 6}}, B , {{5, 6}, {3, 4}}

β({1, 2, 5}) = 1/8
β({1, 2, 6}) = 1/8
β({2, 4, 5}) = 1/8
β({2, 4, 6}) = 1/8
β({3, 4, 5}) = 1/4
β({3, 4, 6}) = 1/4

Partitions
α([{5, 6}, {1, 4}, {2, 3}]) = 1/1

b1 = {5, 6} ‘ , ‘ b2 = {1, 4} ‘ , ‘ b3 = {2, 3}

Action of R and B on the blocks of the partitions: = [3, 1, 2] [1, 3, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Right Group	
Coloring	{2, 3, 5, 6}
Rank	3
R,B	[2, 6, 5, 2, 1, 4], [3, 4, 4, 3, 6, 5]
π_2	[2, 0, 0, 1, 1, 0, 2, 2, 2, 4, 2, 2, 3, 3, 0]
$\mathbf{u}_2$	[1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 2, 2, 0, 0]
$\mathbf{u}_3$	[0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]

30 . Coloring, {2, 4, 5, 6}

R: [2, 6, 4, 3, 1, 4]
B: [3, 4, 5, 2, 6, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	5 *vs* 5	5 *vs* 5	5 *vs* 5	3 *vs* 5

Omega Rank for R : cycles: {{3, 4}} order: 4
See Matrix

$ [[2, 1, 3, 4, 0, 2] , [0, 2, 4, 5, 0, 1] , [0, 0, 5, 5, 0, 2] , [0, 0, 5, 7, 0, 0] , [0, 0, 7, 5, 0, 0]] $

$$[y_4, y_1, y_2, y_3, 0, y_5]$$

Omega Rank for B : cycles: {{5, 6}, {2, 4}} order: 2
See Matrix

$ [[0, 3, 1, 2, 4, 2] , [0, 2, 0, 3, 3, 4] , [0, 3, 0, 2, 4, 3] , [0, 2, 0, 3, 3, 4] , [0, 3, 0, 2, 4, 3]] $

$$[0, 5 y_2 - 6 y_1 + 5 y_3, y_2, y_1, 6 y_2 - 7 y_1 + 6 y_3, y_3]$$

$p = - s^2 + s^4$ $p' = - s^2 + s^4$

31 . Coloring, {3, 4, 5, 6}

R: [2, 4, 5, 3, 1, 4]
B: [3, 6, 4, 2, 6, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
4 *vs* 4	6 *vs* 6	6 *vs* 6	5 *vs* 5	4 *vs* 5

Omega Rank for R : cycles: {{1, 2, 3, 4, 5}} order: 5
See Matrix

$ [[2, 1, 3, 4, 2, 0] , [2, 2, 4, 1, 3, 0] , [3, 2, 1, 2, 4, 0] , [4, 3, 2, 2, 1, 0] , [1, 4, 2, 3, 2, 0]] $

$$[y_3, y_2, y_1, y_5, y_4, 0]$$

Omega Rank for B : cycles: {{5, 6}} order: 4
See Matrix

$ [[0, 3, 1, 2, 2, 4] , [0, 2, 0, 1, 4, 5] , [0, 1, 0, 0, 5, 6] , [0, 0, 0, 0, 6, 6] , [0, 0, 0, 0, 6, 6]] $

$$[0, -y_1 + y_3 - y_4 + y_2, y_1, y_3, y_4, y_2]$$

$p = s^4 - s^5$

32 . Coloring, {2, 3, 4, 5, 6}

Ωp(Δ)=0: $p = s - 2s^3 + 4s^4$

R: [2, 6, 5, 3, 1, 4]
B: [3, 4, 4, 2, 6, 5]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 4	4 *vs* 6	4 *vs* 6	4 *vs* 6	2 *vs* 5

Omega Rank for R : cycles: {{1, 2, 3, 4, 5, 6}} order: 6
See Matrix

$ [[2, 1, 3, 2, 2, 2] , [2, 2, 2, 2, 3, 1] , [3, 2, 2, 1, 2, 2] , [2, 3, 1, 2, 2, 2] , [2, 2, 2, 2, 1, 3] , [1, 2, 2, 3, 2, 2]] $

$$[y_2 + y_3 - y_4, y_1, -y_1 + y_2 + y_3, y_4, y_2, y_3]$$

$p' = s - s^2 + s^4 - s^5$ $p' = 1 - s^2 + s^3 - s^5$

Omega Rank for B : cycles: {{5, 6}, {2, 4}} order: 2
See Matrix

$ [[0, 3, 1, 4, 2, 2] , [0, 4, 0, 4, 2, 2] , [0, 4, 0, 4, 2, 2] , [0, 4, 0, 4, 2, 2] , [0, 4, 0, 4, 2, 2]] $

$[0, y_1, -y_1 + 2 y_2, 2 y_2, y_2, y_2]$

$p = s^2 - s^4$ $p' = s^2 - s^3$ $p' = - s^3 + s^4$

‘ See 3-level graph

‘

M N

$ [[0, 2, 7, 0, 4, 5] , [2, 0, 0, 16, 10, 8] , [7, 0, 0, 11, 8, 10] , [0, 16, 11, 0, 14, 13] , [4, 10, 8, 14, 0, 0] , [5, 8, 10, 13, 0, 0]] $ $ [[0, 1, 1, 0, 1, 1] , [1, 0, 0, 1, 1, 1] , [1, 0, 0, 1, 1, 1] , [0, 1, 1, 0, 1, 1] , [1, 1, 1, 1, 0, 0] , [1, 1, 1, 1, 0, 0]] $

τ= 12 , r’= 2/3

R: [2, 6, 5, 3, 1, 4]
B: [3, 4, 4, 2, 6, 5]

Ranges

Action of R on ranges, [[2], [6], [1], [5], [4], [8], [3], [7]]
Action of B on ranges, [[8], [7], [8], [7], [6], [5], [6], [5]]

Cycles: R , {{1, 2, 3, 4, 5, 6}}, B , {{5, 6}, {2, 4}}

β({1, 2, 5}) = 1/27
β({1, 2, 6}) = 1/54
β({1, 3, 5}) = 2/27
β({1, 3, 6}) = 13/108
β({2, 4, 5}) = 13/54
β({2, 4, 6}) = 11/54
β({3, 4, 5}) = 4/27
β({3, 4, 6}) = 17/108

Partitions
α([{5, 6}, {1, 4}, {2, 3}]) = 1/1

b1 = {5, 6} ‘ , ‘ b2 = {1, 4} ‘ , ‘ b3 = {2, 3}

Action of R and B on the blocks of the partitions: = [3, 1, 2] [1, 3, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-3 partition graph.

‘

Right Group	
Coloring	{2, 3, 4, 5, 6}
Rank	3
R,B	[2, 6, 5, 3, 1, 4], [3, 4, 4, 2, 6, 5]
π_2	[2, 7, 0, 4, 5, 0, 16, 10, 8, 11, 8, 10, 14, 13, 0]
$\mathbf{u}_2$	[1, 1, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0] (dim 1)
wpp	[2, 2, 2, 2, 2, 2]
π_3	[0, 0, 4, 2, 0, 8, 13, 0, 0, 0, 0, 0, 0, 26, 22, 0, 16, 17, 0, 0]
$\mathbf{u}_3$	[0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0]

SUMMARY		
Graph Type		CC
ν**(A)**		2
ν**(Δ)**		2
π		[1, 2, 2, 3, 2, 2]
Dbly Stoch		false

SANDWICH		Total 2
No .	**Coloring**	**Rank**
1	{5}	2
2	{2, 3, 5}	2

RT GROUPS		Total 7	
No .	**Coloring**	**Rank**	**Solv**
1	{3}	2	Solvable
2	{}	3	Not Solvable
3	{2, 3, 4, 5, 6}	3	Not Solvable
4	{2, 6}	3	Solvable
5	{2, 3, 5, 6}	3	Not Solvable
6	{3, 5, 6}	2	Solvable
7	{4}	3	Not Solvable

Δ-RANK'D	**SC'D !RK'D**	**τ-RANK'D**	**R/B RANK'D**	**NOT SYNC'D**	**Total Runs**	**2^{n-1}**
22	0	21 , 21	12 , 10	9	32	32