New Graph

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[2, 3, 2, 3], [4, 4, 1, 1]
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π = [1, 1, 1, 1]

POSSIBLE RANKS

1 x 4
2 x 2

BASE DETERMINANT 117/512, .2285156250

NullSpace of Δ

{1, 2, 3, 4}

Nullspace of A

[{1, 4},{2, 3}]

1 . Coloring, {}

$\Omega p(\Delta)=0$: p' = s^2 p = s^2

R: [2, 3, 2, 3]
B: [4, 4, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
1 *vs* 3	1 *vs* 3	1 *vs* 3	1 *vs* 2	1 *vs* 2

Omega Rank for R : cycles: {{2, 3}} order: 2
See Matrix

$ [[0, 2, 2, 0] , [0, 2, 2, 0]] $

$$[0, y_1, y_1, 0]$$

$p = - s + s^2$

Omega Rank for B : cycles: {{1, 4}} order: 2
See Matrix

$$\$ [[2, 0, 0, 2] , [2, 0, 0, 2]] \$$$

$$[y_1, 0, 0, y_1]$$

$p = - s + s^2$

M N

$ [[0, 0, 0, 1] , [0, 0, 1, 0] , [0, 1, 0, 0] , [1, 0, 0, 0]] $ $ [[0, 1, 1, 2] , [1, 0, 2, 1] , [1, 2, 0, 1] , [2, 1, 1, 0]] $

$\tau = 8$, r'= 1/2

R: [2, 3, 2, 3]
B: [4, 4, 1, 1]

Ranges

Action of R on ranges, [[2], [2]]
Action of B on ranges, [[1], [1]]

Cycles: R , {{2, 3}}, B , {{1, 4}}

$\beta(\{1, 4\}) = 1/2$
$\beta(\{2, 3\}) = 1/2$

Partitions

Action of R on partitions, [[1], [1]]
Action of B on partitions, [[2], [2]]

$\alpha([\{1, 3\}, \{2, 4\}]) = 1/2$
$\alpha([\{1, 2\}, \{3, 4\}]) = 1/2$

b1 = {1, 2} ' , ' b2 = {1, 3} ' , ' b3 = {3, 4} ' , ' b4 = {2, 4}

Action of R and B on the blocks of the partitions: = [2, 4, 4, 2] [3, 3, 1, 1]
with invariant measure [1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-2 partition graph.

‘

Sandwich	
Coloring	{}
Rank	2
R,B	[2, 3, 2, 3], [4, 4, 1, 1]
π_2	[0, 0, 1, 1, 0, 0]
$\mathbf{u}_2$	[1, 1, 2, 2, 1, 1] (dim 1)
wpp	[2, 2, 2, 2]

2 . Coloring, {2}

R: [2, 4, 2, 3]
B: [4, 3, 1, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 3	4 *vs* 4	4 *vs* 4	3 *vs* 3	2 *vs* 3

Omega Rank for R : cycles: {{2, 3, 4}} order: 3
See Matrix

$$ [[0, 2, 1, 1] , [0, 1, 1, 2] , [0, 1, 2, 1]] $$

$$[0, y_2, y_3, y_1]$$

Omega Rank for B : cycles: {{1, 4}} order: 2
See Matrix

$ [[2, 0, 1, 1] , [2, 0, 0, 2] , [2, 0, 0, 2]] $

$$[y_2, 0, y_2 - y_1, y_1]$$

$$p = - s^2 + s^3$$

3 . Coloring, {3}

R: [2, 3, 1, 3]
B: [4, 4, 2, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 3	4 *vs* 4	4 *vs* 4	3 *vs* 3	2 *vs* 3

Omega Rank for R : cycles: {{1, 2, 3}} order: 3
See Matrix

$ [[1, 1, 2, 0] , [2, 1, 1, 0] , [1, 2, 1, 0]] $

$$[y_2, y_3, y_1, 0]$$

Omega Rank for B : cycles: {{1, 4}} order: 2
See Matrix

$ [[1, 1, 0, 2] , [2, 0, 0, 2] , [2, 0, 0, 2]] $

$$[-y_1 + y_2, y_1, 0, y_2]$$

$$p = s^2 - s^3$$

4 . Coloring, {4}

R: [2, 3, 2, 1]
B: [4, 4, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 3	4 *vs* 4	4 *vs* 4	2 *vs* 3	3 *vs* 3

Omega Rank for R : cycles: {{2, 3}} order: 2
See Matrix

$$ \$ [[1, 2, 1, 0] , [0, 2, 2, 0] , [0, 2, 2, 0]] \$ $$

$$ [y_1 - y_2, y_1, y_2, 0] $$

$p = s^2 - s^3$

Omega Rank for B : cycles: {{1, 3, 4}} order: 3
See Matrix

$$ \$ [[1, 0, 1, 2] , [1, 0, 2, 1] , [2, 0, 1, 1]] \$ $$

$$ [y_1, 0, y_3, y_2] $$

5 . Coloring, {2, 3}

Ωp(Δ)=0: p’ = s^2 p’ = s p = s

R: [2, 4, 1, 3]
B: [4, 3, 2, 1]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
0 *vs* 3	1 *vs* 4	1 *vs* 4	1 *vs* 4	1 *vs* 4

Omega Rank for R : cycles: {{1, 2, 3, 4}} order: 4
See Matrix

$$ [[1, 1, 1, 1] , [1, 1, 1, 1] , [1, 1, 1, 1] , [1, 1, 1, 1]] $$

$$[y_1, y_1, y_1, y_1]$$

$p' = -1 + s$ $p' = -1 + s^3$ $p' = -1 + s^2$

Omega Rank for B : cycles: {{1, 4}, {2, 3}} order: 2
See Matrix

$$ [[1, 1, 1, 1] , [1, 1, 1, 1] , [1, 1, 1, 1] , [1, 1, 1, 1]] $$

$$[y_1, y_1, y_1, y_1]$$

$p' = -1 + s$ $p' = -1 + s^2$ $p' = -1 + s^3$

‘ See 4-level graph

‘

M N

$$ [[0, 1, 1, 1] , [1, 0, 1, 1] , [1, 1, 0, 1] , [1, 1, 1, 0]] $$ $$ [[0, 1, 1, 1] , [1, 0, 1, 1] , [1, 1, 0, 1] , [1, 1, 1, 0]] $$

$\tau = 4$, $r' = 3/4$

R: [2, 4, 1, 3]
B: [4, 3, 2, 1]

Ranges

Action of R on ranges, [[1]]
Action of B on ranges, [[1]]

Cycles: R , {{1, 2, 3, 4}}, B , {{1, 4}, {2, 3}}

$\beta(\{1, 2, 3, 4\}) = 1/1$

Partitions
$\alpha([\{1\}, \{2\}, \{3\}, \{4\}]) = 1/1$

b1 = {1} ‘ , ‘ b2 = {2} ‘ , ‘ b3 = {3} ‘ , ‘ b4 = {4}

Action of R and B on the blocks of the partitions: = [3, 1, 4, 2] [4, 3, 2, 1]
with invariant measure [1, 1, 1, 1]

N by blocks, check: true . ‘ See partition graph.

‘ ‘ See level-4 partition graph.

‘

Right Group	
Coloring	{2, 3}
Rank	4
R,B	[2, 4, 1, 3], [4, 3, 2, 1]
π_2	[1, 1, 1, 1, 1, 1]
$\mathbf{u}_2$	[1, 1, 1, 1, 1, 1] (dim 2)
wpp	[1, 1, 1, 1]
π_4	[1]
$\mathbf{u}_4$	[1]

6 . Coloring, {2, 4}

R: [2, 4, 2, 1]
B: [4, 3, 1, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 3	3 *vs* 3	3 *vs* 3	3 *vs* 3	3 *vs* 3

Omega Rank for R : cycles: {{1, 2, 4}} order: 3
See Matrix

$$ [[1, 2, 0, 1] , [1, 1, 0, 2] , [2, 1, 0, 1]] $$

$$[y_2 , y_3 , 0, y_1]$$

Omega Rank for B : cycles: {{1, 3, 4}} order: 3
See Matrix

$$ [[1, 0, 2, 1] , [2, 0, 1, 1] , [1, 0, 1, 2]] $$

$$[y_1 , 0, y_2 , y_3]$$

7 . Coloring, {3, 4}

R: [2, 3, 1, 1]
B: [4, 4, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 3	3 *vs* 3	3 *vs* 3	3 *vs* 3	3 *vs* 3

Omega Rank for R : cycles: {{1, 2, 3}} order: 3
See Matrix

$$ [[2, 1, 1, 0] , [1, 2, 1, 0] , [1, 1, 2, 0]] $$

$$[y_1 , y_3 , y_2 , 0]$$

Omega Rank for B : cycles: {{2, 3, 4}} order: 3
See Matrix

$$ [[0, 1, 1, 2] , [0, 1, 2, 1] , [0, 2, 1, 1]] $$

$$[0, y_3 , y_2 , y_1]$$

8 . Coloring, {2, 3, 4}

R: [2, 4, 1, 1]
B: [4, 3, 2, 3]

‘ See graph

‘ ‘ See pair graph

‘

Δ-Rank	**A+(1/2)Δ**	**A-(1/2)Δ**	**R**	**B**
3 *vs* 3	4 *vs* 4	4 *vs* 4	3 *vs* 3	2 *vs* 3

Omega Rank for R : cycles: {{1, 2, 4}} order: 3
See Matrix

$ [[2, 1, 0, 1] , [1, 2, 0, 1] , [1, 1, 0, 2]] $

$$[y_1, y_3, 0, y_2]$$

Omega Rank for B : cycles: {{2, 3}} order: 2
See Matrix

$ [[0, 1, 2, 1] , [0, 2, 2, 0] , [0, 2, 2, 0]] $

$$[0, y_1, y_2, -y_1 + y_2]$$

$p = s^2 - s^3$

SUMMARY		
Graph Type		CC
ν(A)		1
ν(Δ)		1
π		[1, 1, 1, 1]
Dbly Stoch		true

SANDWICH		Total 1
No .	**Coloring**	**Rank**
1	{}	2

RT GROUPS		Total 1	
No .	**Coloring**	**Rank**	**Solv**
1	{2, 3}	4	["group", Not Solvable]

CC Colorings		Total 1
No .	**Coloring**	**Sandwich,Rank**
1	{}	true, 2

Δ-RANK'D	**SC'D !RK'D**	**τ-RANK'D**	**R/B RANK'D**	**NOT SYNC'D**	**Total Runs**	**2^{n-1}**
6	0	6 , 6	5 , 3	2	8	8
