

New Graph

[5, 5, 5, 5, 4], [4, 4, 1, 2, 3]

$$\pi = [3, 5, 6, 10, 12]$$

$$\delta = [1, 1, 1, 3, 4]$$

POSSIBLE RANKS

- 1 x 36
- 2 x 18
- 3 x 12
- 4 x 9
- 6 x 6

BASE DETERMINANT 39/256, .1523437500

NullSpace of Δ

{1, 2, 3, 4, 5}

Nullspace of A

[{3, 5}, {1, 2, 4}]

1 . Coloring, {}

R: [5, 5, 5, 5, 4]
 B: [4, 4, 1, 2, 3]

` See graph

` ` See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	2 vs 2	4 vs 4

Omega Rank for R :

$$-t^+ t^3$$

, cycles: {{4, 5}} order: 2

$$\begin{pmatrix} 0 & 0 & 0 & 12 & 24 \\ 0 & 0 & 0 & 24 & 12 \end{pmatrix}$$

$$[0, 0, 0, y_1, y_2]$$

Omega Rank for B :

$$-t^3 t^5$$

, cycles: {{2, 4}} order: 4

$$\begin{pmatrix} 6 & 10 & 12 & 8 & 0 \\ 12 & 8 & 0 & 16 & 0 \\ 0 & 16 & 0 & 20 & 0 \\ 0 & 20 & 0 & 16 & 0 \end{pmatrix}$$

$$[y_1, y_2, y_3, y_4, 0]$$

2. Coloring, {2}

R: [5, 4, 5, 5, 4]
 B: [4, 5, 1, 2, 3]

` See graph

` ` See pair graph

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Δ -Rank	A+(1/2) Δ	A-(1/2) Δ	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 2	5 vs 5

Ω_{\mp} Rank for R :

$$-t \quad t^3$$

,
 cycles: {{4, 5}} order: 2

$$\begin{pmatrix} 0 & 0 & 0 & 17 & 19 \\ 0 & 0 & 0 & 19 & 17 \end{pmatrix}$$

$$[0, 0, 0, y_1, y_2]$$

Ω_{\mp} Rank for B :

$$-1 \quad t^5$$

,
 cycles: {{1, 2, 3, 4, 5}} order: 5

$$\begin{pmatrix} 6 & 10 & 12 & 3 & 5 \\ 12 & 3 & 5 & 6 & 10 \\ (5 & 6 & 10 & 12 & 3) \\ 10 & 12 & 3 & 5 & 6 \\ 3 & 5 & 6 & 10 & 12 \end{pmatrix}$$

$$[y_1, y_2, y_3, y_4, y_5]$$

3. Coloring, {3}

$$\Omega p(\Delta)=0: \quad p = s^2 - 4s^3 + 4s^4$$

R: [5, 5, 1, 5, 4]
 B: [4, 4, 5, 2, 3]

` See graph

See pair graph

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
3 vs 4	3 vs 4	3 vs 4	2 vs 3	2 vs 4

Omega Rank for R :
 $-t^2 \quad t^4$

cycles: {{4, 5}} order: 2

$$\begin{pmatrix} 6 & 0 & 0 & 12 & 18 \\ 0 & 0 & 0 & 18 & 18 \\ 0 & 0 & 0 & 18 & 18 \end{pmatrix}$$

$$[y_1, 0, 0, y_2, y_1 + y_2]$$

$$p = -s^2 \quad s^3$$

Omega Rank for B :
 $-t \quad t^3$

cycles: {{3, 5}, {2, 4}} order: 2

$$\begin{pmatrix} 0 & 10 & 12 & 8 & 6 \\ 0 & 8 & 6 & 10 & 12 \\ 0 & 10 & 12 & 8 & 6 \\ 0 & 8 & 6 & 10 & 12 \end{pmatrix}$$

$$[0, y_1, 2y_1 - y_2, y_2, -y_1 + 2y_2]$$

$$p = -s \quad s^3 \quad p' = -s \quad s^3$$

$$\begin{matrix} & & & & M & & N & & & & \\ 0 & 0 & 0 & 0 & 3 & & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 4 & 0 & 1 & & 0 & 0 & 1 & 0 & 1 \\ (0 & 4 & 0 & 2 & 0) & & (1 & 1 & 0 & 1 & 0) \\ 0 & 0 & 2 & 0 & 8 & & 0 & 0 & 1 & 0 & 1 \\ 3 & 1 & 0 & 8 & 0 & & 1 & 1 & 0 & 1 & 0 \\ & & & & & & NM & & & & \\ & & & & & & 3 & 5 & 0 & 10 & 0 \\ & & & & & & 3 & 5 & 0 & 10 & 0 \\ & & & & & & (0 & 0 & 6 & 0 & 12) \\ & & & & & & 3 & 5 & 0 & 10 & 0 \\ & & & & & & 0 & 0 & 6 & 0 & 12 \end{matrix}$$

$\tau = 13, r' = 1/2$

R: [5, 5, 1, 5, 4]
 B: [4, 4, 5, 2, 3]

Ranges

Action of R on ranges, [[5], [1], [5], [1], [5]]
 Action of B on ranges, [[4], [5], [4], [3], [2]]

Cycles: R , {{4, 5}}, B , {{3, 5}, {2, 4}}

$$\beta(\{1, 5\}) = 1/6$$

$$\beta(\{2, 3\}) = 2/9$$

$$\beta(\{2, 5\}) = 1/18$$

$$\beta(\{3, 4\}) = 1/9$$

$$\beta(\{4, 5\}) = 4/9$$

Partitions

$$\alpha(\{\{1, 2, 4\}, \{3, 5\}\}) = 1/1$$

$$b_1 = \{1, 2, 4\}, b_2 = \{3, 5\}$$

Action of R and B on the blocks of the partitions: = $[2, 1]$ $[1, 2]$
with invariant measure $[1, 1]$

N by blocks, check: true . See partition graph.

See level-2 partition graph.

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Right Group	
Coloring	{3}
Rank	2
R,B	[5, 5, 1, 5, 4], [4, 4, 5, 2, 3]
π_2	[0, 0, 0, 3, 4, 0, 1, 2, 0, 8]
u_2	[0, 1, 0, 1, 1, 0, 1, 1, 0, 1] (dim 1)
wpp	[3, 3, 2, 3, 2]

4 . Coloring, {4}

$$R: [5, 5, 5, 2, 4]$$

$$B: [4, 4, 1, 5, 3]$$

See graph

See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	3 vs 3	4 vs 4

Ω_+ Rank for R :

$$-t \quad t^4$$

cycles: {{2, 4, 5}} order: 3

$$\begin{pmatrix} 0 & 10 & 0 & 12 & 14 \\ 0 & 12 & 0 & 14 & 10 \\ 0 & 14 & 0 & 10 & 12 \end{pmatrix}$$

$$[0, y_1, 0, y_2, y_3]$$

Omega Rank for B :

$$-t \quad t^5$$

, cycles: {{1, 3, 4, 5}} order: 4

$$\begin{pmatrix} 6 & 0 & 12 & 8 & 10 \\ 12 & 0 & 10 & 6 & 8 \\ 10 & 0 & 8 & 12 & 6 \\ 8 & 0 & 6 & 10 & 12 \end{pmatrix}$$

$$[y_4, 0, y_1, y_2, y_3]$$

5 . Coloring, {5}

$$R: [5, 5, 5, 5, 3]$$

$$B: [4, 4, 1, 2, 4]$$

` See graph

` ` See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	2 vs 2	3 vs 3

Omega Rank for R :

$$-t \quad t^3$$

, cycles: {{3, 5}} order: 2

$$\begin{pmatrix} 0 & 0 & 12 & 0 & 24 \\ 0 & 0 & 24 & 0 & 12 \end{pmatrix}$$

$$[0, 0, y_1, 0, y_2]$$

Omega Rank for B :

$$-t^2 \quad t^4$$

, cycles: {{2, 4}} order: 2

$$\begin{pmatrix} 6 & 10 & 0 & 20 & 0 \\ 0 & 20 & 0 & 16 & 0 \\ 0 & 16 & 0 & 20 & 0 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3, 0]$$

6 . Coloring, {2, 3}

R: [5, 4, 1, 5, 4]
 B: [4, 5, 5, 2, 3]

` See graph

` ` See pair graph

`

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R :

$$-t^2 + t^4$$

, cycles: {{4, 5}} order: 2

$$\begin{pmatrix} 6 & 0 & 0 & 17 & 13 \\ 0 & 0 & 0 & 13 & 23 \\ 0 & 0 & 0 & 23 & 13 \end{pmatrix}$$

$$[y_1, 0, 0, y_3, y_2]$$

Omega Rank for B :

$$-t^3 + t^5$$

, cycles: {{3, 5}} order: 4

$$\begin{pmatrix} 0 & 10 & 12 & 3 & 11 \\ 0 & 3 & 11 & 0 & 22 \\ 0 & 0 & 22 & 0 & 14 \\ 0 & 0 & 14 & 0 & 22 \end{pmatrix}$$

$$[0, y_4, y_3, y_2, y_1]$$

7 . Coloring, {2, 4}

R: [5, 4, 5, 2, 4]
 B: [4, 5, 1, 5, 3]

` See graph

` ` See pair graph

`

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R :

$$-t^2 + t^4$$

, cycles: {{2, 4}} order: 2

$$\begin{pmatrix} 0 & 10 & 0 & 17 & 9 \\ 0 & 17 & 0 & 19 & 0 \\ 0 & 19 & 0 & 17 & 0 \end{pmatrix}$$

$$[0, y_1, 0, y_2, y_3]$$

Omega Rank for B :

$$-t^+ t^5$$

, cycles: {{1, 3, 4, 5}} order: 4

$$\begin{pmatrix} 6 & 0 & 12 & 3 & 15 \\ 12 & 0 & 15 & 6 & 3 \\ 15 & 0 & 3 & 12 & 6 \\ 3 & 0 & 6 & 15 & 12 \end{pmatrix}$$

$$[y_4, 0, y_2, y_3, y_1]$$

8 . Coloring, {2, 5}

$$R: [5, 4, 5, 5, 3]$$

$$B: [4, 5, 1, 2, 4]$$

` See graph

` ` See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	3 vs 3	4 vs 4

Omega Rank for R :

$$-t^2+ t^4$$

, cycles: {{3, 5}} order: 2

$$\begin{pmatrix} 0 & 0 & 12 & 5 & 19 \\ 0 & 0 & 19 & 0 & 17 \\ 0 & 0 & 17 & 0 & 19 \end{pmatrix}$$

$$[0, 0, y_1, y_2, y_3]$$

Omega Rank for B :

$$-t^2+ t^5$$

, cycles: {{2, 4, 5}} order: 3

$$\begin{pmatrix} 6 & 10 & 0 & 15 & 5 \\ 0 & 15 & 0 & 11 & 10 \\ 0 & 11 & 0 & 10 & 15 \\ 0 & 10 & 0 & 15 & 11 \end{pmatrix}$$

$$[y_3, y_4, 0, y_2, y_1]$$

9 . Coloring, {3, 4}

R: [5, 5, 1, 2, 4]
 B: [4, 4, 5, 5, 3]

` See graph

` ` See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	4 vs 4	3 vs 3

Omega Rank for R :

$$-t^2 + t^5$$

,
 cycles: {{2, 4, 5}} order: 3

$$\begin{pmatrix} 6 & 10 & 0 & 12 & 8 \\ 0 & 12 & 0 & 8 & 16 \\ 0 & 8 & 0 & 16 & 12 \\ 0 & 16 & 0 & 12 & 8 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3, y_4]$$

Omega Rank for B :

$$-t^2 + t^4$$

,
 cycles: {{3, 5}} order: 2

$$\begin{pmatrix} 0 & 0 & 12 & 8 & 16 \\ 0 & 0 & 16 & 0 & 20 \\ 0 & 0 & 20 & 0 & 16 \end{pmatrix}$$

$$[0, 0, y_3, y_2, y_1]$$

10 . Coloring, {3, 5}

R: [5, 5, 1, 5, 3]
 B: [4, 4, 5, 2, 4]

` See graph

` ` See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	3 vs 4	3 vs 3	3 vs 3

Omega Rank for R :

$$-t^{\tau} t^4$$

, cycles: {{1, 3, 5}} order: 3

$$\begin{pmatrix} 6 & 0 & 12 & 0 & 18 \\ 12 & 0 & 18 & 0 & 6 \\ 18 & 0 & 6 & 0 & 12 \end{pmatrix}$$

$$[y_1, 0, y_2, 0, y_3]$$

Omega Rank for B :

$$-t^{2+} t^4$$

, cycles: {{2, 4}} order: 2

$$\begin{pmatrix} 0 & 10 & 0 & 20 & 6 \\ 0 & 20 & 0 & 16 & 0 \\ 0 & 16 & 0 & 20 & 0 \end{pmatrix}$$

$$[0, y_3, 0, y_2, y_1]$$

11 . Coloring, {4, 5}

R: [5, 5, 5, 2, 3]

B: [4, 4, 1, 5, 4]

` See graph

`` See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	3 vs 3	3 vs 3

Omega Rank for R :

$$-t^{2+} t^4$$

, cycles: {{3, 5}} order: 2

$$\begin{pmatrix} 0 & 10 & 12 & 0 & 14 \\ 0 & 0 & 14 & 0 & 22 \\ 0 & 0 & 22 & 0 & 14 \end{pmatrix}$$

$$[0, y_1, y_2, 0, y_3]$$

Omega Rank for B :

$$-t^{2+} t^4$$

, cycles: {{4, 5}} order: 2

$$\begin{pmatrix} 6 & 0 & 0 & 20 & 10 \\ 0 & 0 & 0 & 16 & 20 \\ 0 & 0 & 0 & 20 & 16 \end{pmatrix}$$

$$[y_1, 0, 0, y_2, y_3]$$

12 . Coloring, {2, 3, 4}

R: [5, 4, 1, 2, 4]
 B: [4, 5, 5, 5, 3]

` See graph

` ` See pair graph

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Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	3 vs 3

Omega Rank for R :

$$-t^3 + t^5$$

,
 cycles: {{2, 4}} order: 4

$$\begin{pmatrix} 6 & 10 & 0 & 17 & 3 \\ 0 & 17 & 0 & 13 & 6 \\ 0 & 13 & 0 & 23 & 0 \\ 0 & 23 & 0 & 13 & 0 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3, y_4]$$

Omega Rank for B :

$$-t^2 + t^4$$

,
 cycles: {{3, 5}} order: 2

$$\begin{pmatrix} 0 & 0 & 12 & 3 & 21 \\ 0 & 0 & 21 & 0 & 15 \\ 0 & 0 & 15 & 0 & 21 \end{pmatrix}$$

$$[0, 0, y_1, y_2, y_3]$$

13 . Coloring, {2, 3, 5}

R: [5, 4, 1, 5, 3]
 B: [4, 5, 5, 2, 4]

` See graph

` ` See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 4	3 vs 3

Omega Rank for R :

$$-t^{2^+} t^5$$

, cycles: {{1, 3, 5}} order: 3

$$\begin{pmatrix} 6 & 0 & 12 & 5 & 13 \\ 12 & 0 & 13 & 0 & 11 \\ 13 & 0 & 11 & 0 & 12 \\ 11 & 0 & 12 & 0 & 13 \end{pmatrix}$$

$$[y_1, 0, y_2, y_3, y_4]$$

Omega Rank for B :

$$-t^+ t^4$$

, cycles: {{2, 4, 5}} order: 3

$$\begin{pmatrix} 0 & 10 & 0 & 15 & 11 \\ 0 & 15 & 0 & 11 & 10 \\ 0 & 11 & 0 & 10 & 15 \end{pmatrix}$$

$$[0, y_1, 0, y_2, y_3]$$

14 . Coloring, {2, 4, 5}

$$R: [5, 4, 5, 2, 3]$$

$$B: [4, 5, 1, 5, 4]$$

` See graph

` ` See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	2 vs 4	3 vs 3

Omega Rank for R :

$$-t^+ t^3$$

, cycles: {{3, 5}, {2, 4}} order: 2

$$\begin{pmatrix} 0 & 10 & 12 & 5 & 9 \\ 0 & 5 & 9 & 10 & 12 \\ 0 & 10 & 12 & 5 & 9 \\ 0 & 5 & 9 & 10 & 12 \end{pmatrix}$$

$$[0, -25 y_1 + 25 y_2, -21 y_1 + 25 y_2, 10 y_1, 10 y_2]$$

$$p = -s^+ s^3 \quad p^l = -s^+ s^3$$

Omega Rank for B :

$$-t^{2^+} t^4$$

, cycles: {{4, 5}} order: 2

$$\begin{pmatrix} 6 & 0 & 0 & 15 & 15 \\ 0 & 0 & 0 & 21 & 15 \\ 0 & 0 & 0 & 15 & 21 \end{pmatrix}$$

$$[y_1, 0, 0, y_2, y_3]$$

15 . Coloring, {3, 4, 5}

$$\begin{aligned} R: & [5, 5, 1, 2, 3] \\ B: & [4, 4, 5, 5, 4] \end{aligned}$$

` See graph

`` See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	4 vs 4	4 vs 4	4 vs 4	2 vs 2

Omega Rank for R :

$$-t^2 \quad t^5$$

, cycles: {{1, 3, 5}} order: 3

$$\begin{pmatrix} 6 & 10 & 12 & 0 & 8 \\ 12 & 0 & 8 & 0 & 16 \\ 8 & 0 & 16 & 0 & 12 \\ 16 & 0 & 12 & 0 & 8 \end{pmatrix}$$

$$[y_1, y_2, y_3, 0, y_4]$$

Omega Rank for B :

$$-t \quad t^3$$

, cycles: {{4, 5}} order: 2

$$\begin{pmatrix} 0 & 0 & 0 & 20 & 16 \\ 0 & 0 & 0 & 16 & 20 \end{pmatrix}$$

$$[0, 0, 0, y_1, y_2]$$

16 . Coloring, {2, 3, 4, 5}

$$\begin{aligned} R: & [5, 4, 1, 2, 3] \\ B: & [4, 5, 5, 5, 4] \end{aligned}$$

` See graph

`` See pair graph

,

Δ -Rank	$A+(1/2)\Delta$	$A-(1/2)\Delta$	R	B
4 vs 4	5 vs 5	5 vs 5	4 vs 5	2 vs 2

Omega Rank for R :

$$-1 - t^3 + t^4$$

cycles: {{2, 4}, {1, 3, 5}}

$$\begin{pmatrix} 6 & 10 & 12 & 5 & 3 \\ 12 & 5 & 3 & 10 & 6 \\ (3 & 10 & 6 & 5 & 12) \\ 6 & 5 & 12 & 10 & 3 \\ 12 & 10 & 3 & 5 & 6 \end{pmatrix}$$

$$p' = -1 - s^3 + s^4$$

$$[7y_1 - 5y_2 + 7y_3 - 5y_4, 5y_1, 5y_2, 5y_3, 5y_4]$$

Omega Rank for B :

$$-t^3$$

cycles: {{4, 5}} order: 2

$$\begin{pmatrix} 0 & 0 & 0 & 15 & 21 \\ 0 & 0 & 0 & 21 & 15 \end{pmatrix}$$

$$[0, 0, 0, y_1, y_2]$$

SUMMARY	
Graph Type	CC
$v(A)$	1
$v(\Delta)$	1
π	[3, 5, 6, 10, 12]
Dbly Stoch	false

SANDWICH		Total 0
No .	Coloring	Rank

RT GROUPS		Total 1	
No .	Coloring	Rank	Solv
1	{3}	2	Solvable

Δ -RANK'D	SC'D !RK'D	τ -RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
15	0	15 , 14	13 , 15	1	16	16
