New Graph

[4, 4, 4, 3], [3, 3, 2, 1]

π = [1, 1, 2, 2]
δ= [1, 1, 3, 3]

POSSIBLE RANKS

1 x 6
2 x 3

BASE DETERMINANT 3/16, .1875000000

NullSpace of Δ

{1, 2, 3, 4}

Nullspace of A

[{1, 4},{2, 3}]

1 . Coloring, {}

R: [4, 4, 4, 3]
B: [3, 3, 2, 1]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 3	3 *vs* 3	2 *vs* 2	3 *vs* 3

Omega Rank for R :
$-t + t^3$

,
cycles: {{3, 4}}   order: 2

$$\begin{pmatrix} 0 & 0 & 2 & 4 \\ 0 & 0 & 4 & 2 \end{pmatrix}$$

$$[0, 0, y_1, y_2]$$

Omega Rank for B :
$-t^2 + t^4$

,
cycles: {{2, 3}}   order: 2

$$\begin{pmatrix} 2 & 2 & 2 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 4 & 2 & 0 \end{pmatrix}$$

$$[y_3, y_1, y_2, 0]$$

2 . Coloring, {2}

$\Omega p(\Delta)=0$: $p = s - 4s^3$ $p' = s - 2s^2$

R: [4, 3, 4, 3]
B: [3, 4, 2, 1]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 *vs* 3	2 *vs* 4	2 *vs* 4	1 *vs* 2	2 *vs* 4

Omega Rank for R :
$-t + t^3$
,
cycles: {{3, 4}} order: 2

$$\begin{pmatrix} 0 & 0 & 3 & 3 \\ 0 & 0 & 3 & 3 \end{pmatrix}$$

$$[0, 0, y_1, y_1]$$

$p = -s + s^2$

Omega Rank for B :
$-1 + t^4$
,
cycles: {{1, 2, 3, 4}} order: 4

$$\begin{pmatrix} 2 & 2 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 2 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{pmatrix}$$

$$[y_1, y_1, y_2, y_2]$$

$p' = -s + s^3$ $p' = -1 + s^2$

M N

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 3 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 1 & 2 & 0 & 3 \\ 2 & 1 & 3 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 3 & 0 & 4 & 2 \\ 0 & 3 & 2 & 4 \\ 2 & 1 & 6 & 0 \\ 1 & 2 & 0 & 6 \end{pmatrix}$$

τ= 8 , r'= 1/2

R: [4, 3, 4, 3]
B: [3, 4, 2, 1]

Ranges

Action of R on ranges, [[2], [2]]
Action of B on ranges, [[2], [1]]

Cycles: R , {{3, 4}}, B , {{1, 2, 3, 4}}

β({1, 2}) = 1/3
β({3, 4}) = 2/3

Partitions

Action of R on partitions, [[1], [1]]
Action of B on partitions, [[2], [1]]

α([{1, 3}, {2, 4}]) = 2/3
α([{2, 3}, {1, 4}]) = 1/3

b1 = {1, 3} ` , ` b2 = {2, 4} ` , ` b3 = {2, 3} ` , ` b4 = {1, 4}

Action of R and B on the blocks of the partitions: = [2, 1, 2, 1] [4, 3, 1, 2]
with invariant measure [2, 2, 1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-2 partition graph.

`

Sandwich	
Coloring	{2}
Rank	2
R,B	[4, 3, 4, 3], [3, 4, 2, 1]
π_2	[1, 0, 0, 0, 0, 2]
u_2	[3, 1, 2, 2, 1, 3] (dim 1)
wpp	[2, 2, 2, 2]

3 . Coloring, {3}

Ωp(Δ)=0: $p = s^2$ $p' = s^2$

R: [4, 4, 2, 3]

B: [3, 3, 4, 1]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 *vs* 3	1 *vs* 3	1 *vs* 3	1 *vs* 3	1 *vs* 3

Omega Rank for R :
$-t + t^4$
,
cycles: {{2, 3, 4}}   order: 3

$$\begin{pmatrix} 0 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \end{pmatrix}$$

$[0, y_1, y_1, y_1]$

$p = -s + s^3 \quad p = -s + s^2$

Omega Rank for B :
$-t + t^4$
,
cycles: {{1, 3, 4}}   order: 3

$$\begin{pmatrix} 2 & 0 & 2 & 2 \\ 2 & 0 & 2 & 2 \\ 2 & 0 & 2 & 2 \end{pmatrix}$$

$[y_1, 0, y_1, y_1]$

$p = -s + s^2 \quad p = -s + s^3$

` See 3-level graph

`

M N

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

NM

$$\begin{pmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 1 & 1 & 4 & 2 \\ 1 & 1 & 2 & 4 \end{pmatrix}$$

τ= 6 , r'= 2/3

R: [4, 4, 2, 3]
B: [3, 3, 4, 1]

Ranges

Action of R on ranges, [[2], [2]]
Action of B on ranges, [[1], [1]]

Cycles: R , {{2, 3, 4}}, B , {{1, 3, 4}}

β({1, 3, 4}) = 1/2
β({2, 3, 4}) = 1/2

Partitions
α([{1, 2}, {3}, {4}]) = 1/1

b1 = {1, 2} ` , ` b2 = {3} ` , ` b3 = {4}

Action of R and B on the blocks of the partitions: = [2, 3, 1] [3, 1, 2]
with invariant measure [1, 1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-3 partition graph.

`

Right Group	
Coloring	{3}
Rank	3
R,B	[4, 4, 2, 3], [3, 3, 4, 1]
π_2	[0, 1, 1, 1, 1, 2]
u_2	[0, 1, 1, 1, 1, 1] (dim 1)
wpp	[2, 2, 1, 1]
π_3	[0, 0, 1, 1]
u_3	[0, 0, 1, 1]

4 . Coloring, {4}

R: [4, 4, 4, 1]
B: [3, 3, 2, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 3	3 *vs* 3	2 *vs* 2	2 *vs* 2

Omega Rank for R :
$-t + t^3$
,
cycles: {{1, 4}} order: 2

$$\begin{pmatrix} 2 & 0 & 0 & 4 \\ 4 & 0 & 0 & 2 \end{pmatrix}$$

$[y_2, 0, 0, y_1]$

Omega Rank for B :
$-t + t^3$
,
cycles: {{2, 3}} order: 2

$$\begin{pmatrix} 0 & 2 & 4 & 0 \\ 0 & 4 & 2 & 0 \end{pmatrix}$$

$[0, y_1, y_2, 0]$

5 . Coloring, {2, 3}

R: [4, 3, 2, 3]
B: [3, 4, 4, 1]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 4	4 *vs* 4	2 *vs* 3	3 *vs* 3

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{2, 3}} order: 2

$$\begin{pmatrix} 0 & 2 & 3 & 1 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \end{pmatrix}$$

$[0, y_1 - y_2, y_1, y_2]$

p = $-s^2 + s^3$

Omega Rank for B :
$-t + t^4$
,
cycles: {{1, 3, 4}} order: 3

$$\begin{pmatrix} 2 & 0 & 1 & 3 \\ 3 & 0 & 2 & 1 \\ 1 & 0 & 3 & 2 \end{pmatrix}$$

$[y_1, 0, y_3, y_2]$

6 . Coloring, {2, 4}

R: [4, 3, 4, 1]
B: [3, 4, 2, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 4	4 *vs* 4	2 *vs* 3	3 *vs* 3

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{1, 4}} order: 2

$$\begin{matrix} 2 & 0 & 1 & 3 \\ (3 & 0 & 0 & 3) \\ 3 & 0 & 0 & 3 \end{matrix}$$

$$[-y_1 + y_2, 0, y_1, y_2]$$

$p = -s^2 + s^3$

Omega Rank for B :
$-t + t^4$
,
cycles: {{2, 3, 4}} order: 3

$$\begin{matrix} 0 & 2 & 3 & 1 \\ (0 & 3 & 1 & 2) \\ 0 & 1 & 2 & 3 \end{matrix}$$

$$[0, y_1, y_2, y_3]$$

7 . Coloring, {3, 4}

R: [4, 4, 2, 1]
B: [3, 3, 4, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
3 *vs* 3	3 *vs* 3	3 *vs* 3	3 *vs* 3	2 *vs* 2

Omega Rank for R :
$-t^2 + t^4$
,
cycles: {{1, 4}} order: 2

$$\begin{pmatrix} 2 & 2 & 0 & 2 \\ 2 & 0 & 0 & 4 \\ 4 & 0 & 0 & 2 \end{pmatrix}$$

$$[y_1, y_2, 0, y_3]$$

Omega Rank for B :
$-t + t^3$
,
cycles: {{3, 4}} order: 2

$$\begin{pmatrix} 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 4 \end{pmatrix}$$

$$[0, 0, y_1, y_2]$$

8 . Coloring, {2, 3, 4}

Ωp(Δ)=0: $p' = s + 2s^2$ $p = s - 4s^3$

R: [4, 3, 2, 1]
B: [3, 4, 4, 3]

` See graph

` ` See pair graph

`

Δ-Rank	A+(1/2)Δ	A-(1/2)Δ	R	B
1 *vs* 3	2 *vs* 4	2 *vs* 4	2 *vs* 4	1 *vs* 2

Omega Rank for R :
$-1 + t^2$
,
cycles: {{2, 3}, {1, 4}} order: 2

$$\begin{pmatrix} 2 & 2 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 2 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{pmatrix}$$

$$[y_1, y_1, y_2, y_2]$$

$p' = -s + s^3$ $p' = -1 + s^2$

Omega Rank for B :
$-t + t^3$
,
cycles: {{3, 4}} order: 2

$$\begin{pmatrix} 0 & 0 & 3 & 3 \\ 0 & 0 & 3 & 3 \end{pmatrix}$$

$$[0, 0, y_1, y_1]$$

p = $-s' \; s^2$

$$M \quad N$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

$$NM$$

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{pmatrix}$$

τ= 8 , r'= 1/2

R: [4, 3, 2, 1]
B: [3, 4, 4, 3]

Ranges

Action of R on ranges, [[2], [1]]
Action of B on ranges, [[2], [2]]

Cycles: R , {{2, 3}, {1, 4}}, B , {{3, 4}}

β({1, 2}) = 1/3
β({3, 4}) = 2/3

Partitions
α([{2, 3}, {1, 4}]) = 1/1

b1 = {2, 3} ` , ` b2 = {1, 4}

Action of R and B on the blocks of the partitions: = [1, 2] [2, 1]
with invariant measure [1, 1]

N by blocks, check: true . ` See partition graph.

` ` See level-2 partition graph.

`

Right Group	
Coloring	{2, 3, 4}
Rank	2
R,B	[4, 3, 2, 1], [3, 4, 4, 3]
π_2	[1, 0, 0, 0, 0, 2]
u_2	[1, 1, 0, 0, 1, 1] (dim 1)
wpp	[2, 2, 2, 2]

SUMMARY	
Graph Type	CC
ν(A)	1
ν(Δ)	1
π	[1, 1, 2, 2]
Dbly Stoch	false

SANDWICH		Total 1
No .	Coloring	Rank
1	{2}	2

RT GROUPS		Total 2	
No .	Coloring	Rank	Solv
1	{2, 3, 4}	2	Solvable
2	{3}	3	Solvable

Δ-RANK'D	SC'D !RK'D	τ-RANK'D	R/B RANK'D	NOT SYNC'D	Total Runs	2^{n-1}
5	0	3 , 5	3 , 5	3	8	8
