```
> #with(linalg):
> #read `/home/ph/maple/jac.txt`:
> #read `/home/ph/maple/lie.txt`:
> assume(sigma>0):
> ##########################################################
> ######### PROCEDURE STARTS HERE ############################
> ##########################################################
> #unassign('A','AA','C','B','alpha','XX','GR','R','YY','V','x','delta'):
> #dJ:=doJacobians();canvars(dJ[3]);
> ##########################################################################
  ##################################################
```

Lie Algebras start here

```
> ######################
  ##########################################################################
```

Here is HEISENBERG

```
> X:=matrix(3,3,[0,a[3],a[2],0,0,a[1],0,0,0]):
> d:=3;n:=3;lie(X,d,n,0):dJ:=doJacobians();coordmap();canvars(dJ[3]);
  #kirlv();duals();
```

$$d := 3$$

$$n := 3$$

the group element is computed

solving equations for pi-dagger

solving equations for pi-star

piadj available

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & A_1 \\ 0 & 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} 1 & 0 & 0 \\ A_3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_1(t) = \alpha_1\, t + x_1,\ A_2(t) = \frac{1}{2}\,\alpha_1\,\alpha_3\, t^2 + \alpha_2\, t + t\,\alpha_3\, x_1 + x_2,\ A_3(t) = \alpha_3\, t + x_3$$

$$\text{"DalphaDA"}, \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2}\alpha_3 & 1 & -\frac{1}{2}\alpha_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{"Jacobian of the group law="}, \begin{bmatrix} 1 & 0 & 0 \\ A_3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{"DADalpha"}, \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2}A_3 & 1 & \frac{1}{2}A_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$dJ := DalphaDA, DDADalpha, AA$$

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & A_1 \\ 0 & 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} 1 & 0 & 0 \\ A_3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"for zero initial conditions we get the coordinate map"

$$A_1 = \alpha_1, A_2 = \frac{1}{2}\alpha_1\,\alpha_3 + \alpha_2, A_3 = \alpha_3$$

"Raising operators"

$$Y_1 = x_1 + \frac{1}{2}x_2\,\delta_3$$

$$Y_2 = x_2$$

$$Y_3 = \frac{1}{2}x_2\,\delta_1 + x_3$$

"Recursion operators"

$$x_1 = R_1 - \frac{1}{2} R_2 V_3$$

$$x_2 = R_2$$

$$x_3 = -\frac{1}{2} R_2 V_1 + R_3$$

"Representation with abelian coherent state"

$$xi\text{-}hat_1 = x_1 - \frac{1}{2} x_2 \delta_3$$

$$xi\text{-}hat_2 = x_2$$

$$xi\text{-}hat_3 = \frac{1}{2} x_2 \delta_1 + x_3$$

```
> #### OSCILLATOR
```

```
> X:=matrix(3,3,[0,a[3],a[2],0,a[4],a[1],0,0,0]);d:=4;n:=3;lie(X
  n,0):dJ:=doJacobians();coordmap();canvars(dJ[3]);#
  kirlv();adjrep();#duals();
```

$$X := \begin{bmatrix} 0 & a_3 & a_2 \\ 0 & a_4 & a_1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$d := 4$$

$$n := 3$$

the group element is computed

solving equations for pi-dagger

solving equations for pi-star

piadj available

```
> X:=matrix(2,2,[[a[2],a[1]],[0,0]]);
```

$$X := \begin{bmatrix} a_2 & a_1 \\ 0 & 0 \end{bmatrix}$$

```
> d:=2;n:=2;lie(X,d,n,0);dJ:=doJacobians();coordmap();canvars(dJ[3]);#
  kirlv();#duals();
```

$$d := 2$$

$$n := 2$$

the group element is computed

solving equations for pi-dagger

solving equations for pi-star

piadj available

gg, pidgr, pistar

$$\text{"pihat"} = \begin{bmatrix} 1 & A_1 \\ 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} \mathbf{e}^{A_2} & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_1(t) = \frac{-\alpha_1 + \mathbf{e}^{(\alpha_2 t)} \alpha_1 + \mathbf{e}^{(\alpha_2 t)} \alpha_2 x_1}{\alpha_2}, A_2(t) = \alpha_2 t + x_2$$

$$\text{"DalphaDA"}, \begin{bmatrix} \frac{\alpha_2}{-1 + \mathbf{e}^{\alpha_2}} & -\frac{\alpha_1 \left(\mathbf{e}^{\alpha_2} \alpha_2 + 1 - \mathbf{e}^{\alpha_2} \right)}{\alpha_2 \left(-1 + \mathbf{e}^{\alpha_2} \right)} \\ 0 & 1 \end{bmatrix}$$

$$\text{"Jacobian of the group law="}, \begin{bmatrix} \mathbf{e}^{A_2} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{"DADalpha"}, \begin{bmatrix} \frac{-1 + \mathbf{e}^{A_2}}{A_2} & \frac{A_1 \left(1 - \mathbf{e}^{A_2} + A_2 \mathbf{e}^{A_2} \right)}{A_2 \left(-1 + \mathbf{e}^{A_2} \right)} \\ 0 & 1 \end{bmatrix}$$

$$dJ := DalphaDA, DDADalpha, AA$$

$$\text{"pihat"} = \begin{bmatrix} 1 & A_1 \\ 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} \mathbf{e}^{A_2} & 0 \\ 0 & 1 \end{bmatrix}$$

"for zero initial conditions we get the coordinate map"

$$A_1 = \frac{\alpha_1 \left(-1 + \mathbf{e}^{\alpha_2} \right)}{\alpha_2}, A_2 = \alpha_2$$

"Raising operators"

$$Y_1 = \frac{x_1 \left(-1 + \mathbf{e}^{\delta_2} \right)}{\delta_2}$$

$$Y_2 = \frac{x_1 \, \delta_1 \left(1 - \mathbf{e}^{\delta_2} + \delta_2 \, \mathbf{e}^{\delta_2} \right)}{\delta_2 \left(-1 + \mathbf{e}^{\delta_2} \right)} + x_2$$

"Recursion operators"

$$x_1 = \frac{R_1 \, V_2}{-1 + \mathbf{e}^{V_2}}$$

$$x_2 = -\frac{R_1 \, V_1 \left(\mathbf{e}^{V_2} \, V_2 + 1 - \mathbf{e}^{V_2} \right)}{V_2 \left(-1 + \mathbf{e}^{V_2} \right)} + R_2$$

"Representation with abelian coherent state"

$$xi\text{-}hat_1 = \frac{x_1\,\delta_2}{-1 + \mathbf{e}^{\delta_2}}$$

$$xi\text{-}hat_2 = x_1\,\delta_1 - \frac{x_1\,\delta_1\left(1 - \mathbf{e}^{\delta_2} + \delta_2\,\mathbf{e}^{\delta_2}\right)}{\delta_2\left(-1 + \mathbf{e}^{\delta_2}\right)} + x_2$$

Here is N5

```
> #X:=matrix(5,5,[[0,a[1],a[2],a[3],a[4]],[0,0,a[5],a[6],a[7]],[0
  ,0,a[8],a[9]],[0,0,0,0,a[10]],[0,0,0,0,0]]);
```

```
> #d:=10;n:=5;lie(X,d,n,0);
```

```
> ########  E2
```

```
> X:=matrix(3,3,[[0,a[3],a[1]],[-a[3],0,(a[2])],[0,0,0]]);
```

$$X := \begin{bmatrix} 0 & a_3 & a_1 \\ -a_3 & 0 & a_2 \\ 0 & 0 & 0 \end{bmatrix}$$

```
> d:=3:n:=3:lie(X,d,n,1);dJ:=doJacobians();coordmap();canvars(dJ[3]);#
  kirlv();duals();
```

the group element is computed

solving equations for pi-dagger

solving equations for pi-star

piadj available

hk, pidgr, pistar

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & A_2 \\ 0 & 1 & -A_1 \\ 0 & 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} \cos(A_3) & \sin(A_3) & 0 \\ -\sin(A_3) & \cos(A_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_1(t) = \frac{-\cos(\alpha_3\,t)\,\alpha_2 + \cos(\alpha_3\,t)\,\alpha_3\,x_1 + \sin(\alpha_3\,t)\,\alpha_1 + \sin(\alpha_3\,t)\,\alpha_3\,x_2 + \alpha_2}{\alpha_3},$$