```
> #with(linalg):
> #read `/home/ph/maple/jac.txt`:
> #read `/home/ph/maple/lie.txt`:
> assume(sigma>0):
> ##########################################################
> ######### PROCEDURE STARTS HERE ############################
> ##########################################################
> #unassign('A','AA','C','B','alpha','XX','GR','R','YY','V','x','delta'):
> #dJ:=doJacobians();canvars(dJ[3]);
> ##########################################################################
  ##################################################
```

Lie Algebras start here

```
> ######################
  ##########################################################################
```

Here is HEISENBERG

```
> X:=matrix(3,3,[0,a[3],a[2],0,0,a[1],0,0,0]):
> d:=3;n:=3;lie(X,d,n,0):dJ:=doJacobians();coordmap();canvars(dJ[3]);
  #kirlv();duals();
```

$$d := 3$$

$$n := 3$$

the group element is computed

solving equations for pi-dagger

solving equations for pi-star

piadj available

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & A_1 \\ 0 & 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} 1 & 0 & 0 \\ A_3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_1(t) = \alpha_1\, t + x_1,\ A_2(t) = \frac{1}{2}\,\alpha_1\,\alpha_3\, t^2 + \alpha_2\, t + t\,\alpha_3\, x_1 + x_2,\ A_3(t) = \alpha_3\, t + x_3$$

$$\text{"DalphaDA"}, \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2}\alpha_3 & 1 & -\frac{1}{2}\alpha_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{"Jacobian of the group law="}, \begin{bmatrix} 1 & 0 & 0 \\ A_3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{"DADalpha"}, \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2}A_3 & 1 & \frac{1}{2}A_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$dJ := DalphaDA, DDADalpha, AA$$

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & A_1 \\ 0 & 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} 1 & 0 & 0 \\ A_3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"for zero initial conditions we get the coordinate map"

$$A_1 = \alpha_1, A_2 = \frac{1}{2}\alpha_1\,\alpha_3 + \alpha_2, A_3 = \alpha_3$$

"Raising operators"

$$Y_1 = x_1 + \frac{1}{2}x_2\,\delta_3$$

$$Y_2 = x_2$$

$$Y_3 = \frac{1}{2}x_2\,\delta_1 + x_3$$

"Recursion operators"

$$x_1 = R_1 - \frac{1}{2} R_2 V_3$$

$$x_2 = R_2$$

$$x_3 = -\frac{1}{2} R_2 V_1 + R_3$$

"Representation with abelian coherent state"

$$xi\text{-}hat_1 = x_1 - \frac{1}{2} x_2 \delta_3$$

$$xi\text{-}hat_2 = x_2$$

$$xi\text{-}hat_3 = \frac{1}{2} x_2 \delta_1 + x_3$$

```
> #### OSCILLATOR
```

```
> X:=matrix(3,3,[0,a[3],a[2],0,a[4],a[1],0,0,0]);d:=4;n:=3;lie(X
  n,0):dJ:=doJacobians();coordmap();canvars(dJ[3]);#
  kirlv();adjrep();#duals();
```

$$X := \begin{bmatrix} 0 & a_3 & a_2 \\ 0 & a_4 & a_1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$d := 4$$

$$n := 3$$

the group element is computed

solving equations for pi-dagger

solving equations for pi-star

piadj available

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & 0 & A_1 \\ 0 & 1 & A_1 & 0 \\ 0 & 0 & 1 & -A_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} \mathbf{e}^{A_4} & 0 & 0 & 0 \\ A_3\,\mathbf{e}^{A_4} & 1 & 0 & 0 \\ 0 & 0 & \mathbf{e}^{(-A_4)} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1(t) = \frac{\mathbf{e}^{(\alpha_4\,t)}\,\alpha_1 + \mathbf{e}^{(\alpha_4\,t)}\,\alpha_4\,x_1 - \alpha_1}{\alpha_4}, A_2(t) =$$

$$\frac{\mathbf{e}^{(\alpha_4\,t)}\,\alpha_3\,\alpha_1 + \mathbf{e}^{(\alpha_4\,t)}\,\alpha_3\,\alpha_4\,x_1 - \alpha_1\,\alpha_3\,t\,\alpha_4 + t\,{\alpha_4}^2\,\alpha_2 + x_2\,{\alpha_4}^2 - \alpha_1\,\alpha_3 - x_1\,\alpha_3\,\alpha_4}{{\alpha_4}^2},$$

$$A_3(t) = \frac{\alpha_3 - \mathbf{e}^{(-\alpha_4\,t)}\,\alpha_3 + \mathbf{e}^{(-\alpha_4\,t)}\,\alpha_4\,x_3}{\alpha_4}, A_4(t) = \alpha_4\,t + x_4$$

$$\text{"DalphaDA"}, \left[\left[\frac{\alpha_4}{\mathbf{e}^{\alpha_4} - 1}, 0, 0, -\frac{\alpha_1\left(\alpha_4\,\mathbf{e}^{\alpha_4} - \mathbf{e}^{\alpha_4} + 1\right)}{\alpha_4\left(\mathbf{e}^{\alpha_4} - 1\right)} \right], \left[-\frac{\alpha_3\left(\mathbf{e}^{\alpha_4} - \alpha_4 - 1\right)}{\alpha_4\left(\mathbf{e}^{\alpha_4} - 1\right)}, 1, \right.\right.$$

$$\left.\left. \frac{\alpha_1\left(\mathbf{e}^{\alpha_4} - \alpha_4 - 1\right)}{\alpha_4\left(-1 + \mathbf{e}^{(-\alpha_4)}\right)}, \frac{\alpha_1\,\alpha_3\left(-2\,\mathbf{e}^{\alpha_4} + 4 - 2\,\mathbf{e}^{(-\alpha_4)} + \alpha_4\,\mathbf{e}^{\alpha_4} - \mathbf{e}^{(-\alpha_4)}\,\alpha_4\right)}{{\alpha_4}^2\left(\mathbf{e}^{\alpha_4} - 1\right)\left(-1 + \mathbf{e}^{(-\alpha_4)}\right)} \right], \right.$$

$$\left. \left[0, 0, -\frac{\alpha_4}{-1 + \mathbf{e}^{(-\alpha_4)}}, \frac{\alpha_3\left(\mathbf{e}^{(-\alpha_4)}\,\alpha_4 - 1 + \mathbf{e}^{(-\alpha_4)}\right)}{\alpha_4\left(-1 + \mathbf{e}^{(-\alpha_4)}\right)} \right], [0, 0, 0, 1] \right]$$

"Jacobian of the group law=" ,
$$\begin{bmatrix} e^{A_4} & 0 & 0 & 0 \\ A_3\, e^{A_4} & 1 & 0 & 0 \\ 0 & 0 & e^{(-A_4)} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"DADalpha" ,
$$\left[\left[\frac{e^{A_4} - 1}{A_4},\ 0,\ 0,\ \frac{A_1 \left(-e^{A_4} + 1 + A_4\, e^{A_4} \right)}{A_4 \left(e^{A_4} - 1 \right)} \right], \left[\frac{A_3\, e^{A_4} \left(e^{A_4} - A_4 - 1 \right)}{A_4 \left(e^{A_4} - 1 \right)},\ 1,\ \frac{A_1 \left(e^{A_4} - A_4 - 1 \right)}{A_4 \left(e^{A_4} - 1 \right)},\ \frac{e^{A_4} A_1 A_3 \left(A_4\, e^{A_4} - 2\, e^{A_4} + A_4 + 2 \right)}{A_4 \left(e^{A_4} - 1 \right)^2} \right],$$
$$\left[0,\ 0,\ \frac{\left(e^{A_4} - 1 \right) e^{(-A_4)}}{A_4},\ -\frac{A_3 \left(e^{A_4} - A_4 - 1 \right)}{A_4 \left(e^{A_4} - 1 \right)} \right], [0, 0, 0, 1] \Bigg]$$

$$dJ := DalphaDA,\ DDADalpha,\ AA$$

"pihat" $= \begin{bmatrix} 1 & 0 & 0 & A_1 \\ 0 & 1 & A_1 & 0 \\ 0 & 0 & 1 & -A_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, "pihatstar" $= \begin{bmatrix} e^{A_4} & 0 & 0 & 0 \\ A_3\, e^{A_4} & 1 & 0 & 0 \\ 0 & 0 & e^{(-A_4)} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

"for zero initial conditions we get the coordinate map"

$$A_1 = \frac{\alpha_1 \left(e^{\alpha_4} - 1 \right)}{\alpha_4},\ A_2 = \frac{e^{\alpha_4} \alpha_3\, \alpha_1 - \alpha_1\, \alpha_3\, \alpha_4 + \alpha_4^{\,2}\, \alpha_2 - \alpha_1\, \alpha_3}{\alpha_4^{\,2}},$$

$$A_3 = -\frac{\alpha_3\left(-1 + \mathbf{e}^{(-\alpha_4)}\right)}{\alpha_4},\ A_4 = \alpha_4$$

"Raising operators"

$$Y_1 = \frac{x_1\left(\mathbf{e}^{\delta_4} - 1\right)}{\delta_4} + \frac{x_2\,\delta_3\,\mathbf{e}^{\delta_4}\left(\mathbf{e}^{\delta_4} - \delta_4 - 1\right)}{\delta_4\left(\mathbf{e}^{\delta_4} - 1\right)}$$

$$Y_2 = x_2$$

$$Y_3 = \frac{x_2\,\delta_1\left(\mathbf{e}^{\delta_4} - \delta_4 - 1\right)}{\delta_4\left(\mathbf{e}^{\delta_4} - 1\right)} + \frac{x_3\left(\mathbf{e}^{\delta_4} - 1\right)\mathbf{e}^{(-\delta_4)}}{\delta_4}$$

$$Y_4 = \frac{x_1\,\delta_1\left(-\mathbf{e}^{\delta_4} + 1 + \delta_4\,\mathbf{e}^{\delta_4}\right)}{\delta_4\left(\mathbf{e}^{\delta_4} - 1\right)} + \frac{x_2\,\mathbf{e}^{\delta_4}\,\delta_1\,\delta_3\left(\delta_4\,\mathbf{e}^{\delta_4} - 2\,\mathbf{e}^{\delta_4} + \delta_4 + 2\right)}{\delta_4\left(\mathbf{e}^{\delta_4} - 1\right)^2}$$
$$- \frac{x_3\,\delta_3\left(\mathbf{e}^{\delta_4} - \delta_4 - 1\right)}{\delta_4\left(\mathbf{e}^{\delta_4} - 1\right)} + x_4$$

"Recursion operators"

$$x_1 = \frac{R_1\,V_4}{\mathbf{e}^{V_4} - 1} - \frac{R_2\,V_3\left(\mathbf{e}^{V_4} - V_4 - 1\right)}{V_4\left(\mathbf{e}^{V_4} - 1\right)}$$

$$x_2 = R_2$$

$$x_3 = \frac{R_2\,V_1\left(\mathbf{e}^{V_4} - V_4 - 1\right)}{V_4\left(-1 + \mathbf{e}^{(-V_4)}\right)} - \frac{R_3\,V_4}{-1 + \mathbf{e}^{(-V_4)}}$$

$$x_4 = -\frac{R_1\, V_1 \left(V_4\, \mathbf{e}^{V_4} - \mathbf{e}^{V_4} + 1 \right)}{V_4 \left(\mathbf{e}^{V_4} - 1 \right)} + \frac{R_2\, V_1\, V_3 \left(-2\, \mathbf{e}^{V_4} + 4 - 2\, \mathbf{e}^{(-V_4)} + V_4\, \mathbf{e}^{V_4} - \mathbf{e}^{(-V_4)}\, V_4 \right)}{V_4^{\,2} \left(\mathbf{e}^{V_4} - 1 \right) \left(-1 + \mathbf{e}^{(-V_4)} \right)} + \frac{R_3\, V_3 \left(\mathbf{e}^{(-V_4)}\, V_4 - 1 + \mathbf{e}^{(-V_4)} \right)}{V_4 \left(-1 + \mathbf{e}^{(-V_4)} \right)} + R_4$$

"Representation with abelian coherent state"

$$xi\text{-}hat_1 = \frac{x_1\, \delta_4}{\mathbf{e}^{\delta_4} - 1} - \frac{x_2\, \delta_3 \left(\mathbf{e}^{\delta_4} - \delta_4 - 1 \right)}{\delta_4 \left(\mathbf{e}^{\delta_4} - 1 \right)}$$

$$xi\text{-}hat_2 = x_2$$

$$xi\text{-}hat_3 = \frac{x_2\, \delta_1 \left(\mathbf{e}^{\delta_4} - 1 \right)}{\delta_4} + \frac{x_2\, \delta_1 \left(\mathbf{e}^{\delta_4} - \delta_4 - 1 \right)}{\delta_4 \left(-1 + \mathbf{e}^{(-\delta_4)} \right)} - \frac{x_3\, \delta_4}{-1 + \mathbf{e}^{(-\delta_4)}}$$

$$xi\text{-}hat_4 = \frac{\delta_1 \left(x_1\, \delta_4^{\,2} - \mathbf{e}^{\delta_4}\, x_2\, \delta_3 + x_2\, \delta_3\, \delta_4 + x_2\, \delta_3 \right)}{\delta_4^{\,2}} + \frac{\delta_3 \left(\delta_1\, \mathbf{e}^{\delta_4}\, x_2 - x_2\, \delta_1\, \delta_4 - x_2\, \delta_1 - x_3\, \delta_4^{\,2} \right)}{\delta_4^{\,2}} - \frac{x_1\, \delta_1 \left(-\mathbf{e}^{\delta_4} + 1 + \delta_4\, \mathbf{e}^{\delta_4} \right)}{\delta_4 \left(\mathbf{e}^{\delta_4} - 1 \right)} + \frac{x_2\, \delta_1\, \delta_3 \left(-2\, \mathbf{e}^{\delta_4} + 4 - 2\, \mathbf{e}^{(-\delta_4)} + \delta_4\, \mathbf{e}^{\delta_4} - \mathbf{e}^{(-\delta_4)}\, \delta_4 \right)}{\delta_4^{\,2} \left(\mathbf{e}^{\delta_4} - 1 \right) \left(-1 + \mathbf{e}^{(-\delta_4)} \right)}$$

$$+ \frac{x_3\,\delta_3\left(\mathbf{e}^{(-\delta_4)}\,\delta_4 - 1 + \mathbf{e}^{(-\delta_4)}\right)}{\delta_4\left(-1 + \mathbf{e}^{(-\delta_4)}\right)} + x_4$$

```
> ####### SU(2)
> #X:=matrix(2,2,[a[2],a[1],-a[3],-a[2]]);
> #d:=3:n:=2:lie(X,d,n,0);coordmap();#kirlv();duals();
> #for i to d do AX[i]:=simplify(subs(alpha[1]*alpha[3]-alpha
  [2]^2=sigma^2,AA[i])); print(AX[i]) od;
```

Here is ABCD

```
> X:=matrix(4,4,[[0,a[4],0,a[3]],[0,0,a[4],a[2]],[0,0,0,a[1]
  ],[0,0,0,0]]);
```

$$X := \begin{bmatrix} 0 & a_4 & 0 & a_3 \\ 0 & 0 & a_4 & a_2 \\ 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> d:=4;n:=4;lie(X,d,n,0);dJ:=doJacobians();coordmap();canvars(dJ[3]);
```

$$d := 4$$

$$n := 4$$

the group element is computed

solving equations for pi-dagger

solving equations for pi-star

piadj available

gg, pidgr, pistar

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & A_1 \\ 0 & 0 & 1 & A_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ A_4 & 1 & 0 & 0 \\ \frac{1}{2}A_4^{\,2} & A_4 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1(t) = \alpha_1\, t + x_1,\ A_2(t) = \frac{1}{2}\,\alpha_4\,\alpha_1\, t^2 + \alpha_2\, t + t\, x_1\,\alpha_4 + x_2,$$

$$A_3(t) = \frac{1}{6}\,\alpha_1\,\alpha_4^{\,2}\, t^3 + \frac{1}{2}\, t^2\,\alpha_4\,\alpha_2 + \frac{1}{2}\, t^2\, x_1\,\alpha_4^{\,2} + \alpha_3\, t + t\,\alpha_4\, x_2 + x_3,\ A_4(t) = \alpha_4\, t + x_4$$

$$\text{"DalphaDA"},\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2}\,\alpha_4 & 1 & 0 & -\frac{1}{2}\,\alpha_1 \\ \frac{1}{12}\,\alpha_4^{\,2} & -\frac{1}{2}\,\alpha_4 & 1 & -\frac{1}{12}\,\alpha_4\,\alpha_1 - \frac{1}{2}\,\alpha_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{"Jacobian of the group law="},\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ A_4 & 1 & 0 & 0 \\ \frac{1}{2}\,A_4^{\,2} & A_4 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{"DADalpha"},\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2}\,A_4 & 1 & 0 & \frac{1}{2}\,A_1 \\ \frac{1}{6}\,A_4^{\,2} & \frac{1}{2}\,A_4 & 1 & \frac{1}{12}\,A_4\,A_1 + \frac{1}{2}\,A_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$dJ := DalphaDA,\ DDADalpha,\ AA$$

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & A_1 \\ 0 & 0 & 1 & A_2 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ \text{"pihatstar"} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ A_4 & 1 & 0 & 0 \\ \frac{1}{2}\,A_4^{\,2} & A_4 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"for zero initial conditions we get the coordinate map"

$$A_1 = \alpha_1,\ A_2 = \frac{1}{2}\,\alpha_4\,\alpha_1 + \alpha_2,\ A_3 = \frac{1}{6}\,\alpha_1\,\alpha_4^{\,2} + \frac{1}{2}\,\alpha_4\,\alpha_2 + \alpha_3,\ A_4 = \alpha_4$$

"Raising operators"

$$Y_1 = x_1 + \frac{1}{2}\,x_2\,\delta_4 + \frac{1}{6}\,x_3\,\delta_4^{\,2}$$

$$Y_2 = x_2 + \frac{1}{2}\,x_3\,\delta_4$$

$$Y_3 = x_3$$

$$Y_4 = \frac{1}{2}\,x_2\,\delta_1 + \frac{1}{12}\,x_3\,(\delta_4\,\delta_1 + 6\,\delta_2) + x_4$$

"Recursion operators"

$$x_1 = R_1 - \frac{1}{2}\,R_2\,V_4 + \frac{1}{12}\,R_3\,V_4^{\,2}$$

$$x_2 = R_2 - \frac{1}{2}\,R_3\,V_4$$

$$x_3 = R_3$$

$$x_4 = -\frac{1}{2}\,R_2\,V_1 - \frac{1}{12}\,R_3\,(V_4\,V_1 + 6\,V_2) + R_4$$

"Representation with abelian coherent state"

$$xi\text{-}hat_1 = x_1 - \frac{1}{2}\,x_2\,\delta_4 + \frac{1}{12}\,x_3\,\delta_4^{\,2}$$

$$xi\text{-}hat_2 = x_2 - \frac{1}{2}\,x_3\,\delta_4$$

$$xi\text{-}hat_3 = x_3$$

$$xi\text{-}hat_4 = -\frac{1}{2}\,(-2\,x_2 + x_3\,\delta_4)\,\delta_1 + \frac{1}{2}\,x_3\,(\delta_4\,\delta_1 + 2\,\delta_2) - \frac{1}{2}\,x_2\,\delta_1 - \frac{1}{12}\,x_3\,(\delta_4\,\delta_1 + 6\,\delta_2) + x_4$$

Here is aff(2)

```
> X:=matrix(2,2,[[a[2],a[1]],[0,0]]);
```

$$X := \begin{bmatrix} a_2 & a_1 \\ 0 & 0 \end{bmatrix}$$

```
> d:=2;n:=2;lie(X,d,n,0);dJ:=doJacobians();coordmap();canvars(dJ[3]);#
  kirlv();#duals();
```

$$d := 2$$

$$n := 2$$

the group element is computed

solving equations for pi-dagger

solving equations for pi-star

piadj available

gg, pidgr, pistar

$$\text{"pihat"} = \begin{bmatrix} 1 & A_1 \\ 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} \mathbf{e}^{A_2} & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_1(t) = \frac{-\alpha_1 + \mathbf{e}^{(\alpha_2 t)} \alpha_1 + \mathbf{e}^{(\alpha_2 t)} \alpha_2 x_1}{\alpha_2}, A_2(t) = \alpha_2 t + x_2$$

$$\text{"DalphaDA"}, \begin{bmatrix} \frac{\alpha_2}{-1 + \mathbf{e}^{\alpha_2}} & -\frac{\alpha_1 \left(\mathbf{e}^{\alpha_2} \alpha_2 + 1 - \mathbf{e}^{\alpha_2} \right)}{\alpha_2 \left(-1 + \mathbf{e}^{\alpha_2} \right)} \\ 0 & 1 \end{bmatrix}$$

$$\text{"Jacobian of the group law="}, \begin{bmatrix} \mathbf{e}^{A_2} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{"DADalpha"}, \begin{bmatrix} \frac{-1 + \mathbf{e}^{A_2}}{A_2} & \frac{A_1 \left(1 - \mathbf{e}^{A_2} + A_2 \mathbf{e}^{A_2} \right)}{A_2 \left(-1 + \mathbf{e}^{A_2} \right)} \\ 0 & 1 \end{bmatrix}$$

$$dJ := DalphaDA, DDADalpha, AA$$

$$\text{"pihat"} = \begin{bmatrix} 1 & A_1 \\ 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} \mathbf{e}^{A_2} & 0 \\ 0 & 1 \end{bmatrix}$$

"for zero initial conditions we get the coordinate map"

$$A_1 = \frac{\alpha_1 \left(-1 + \mathbf{e}^{\alpha_2} \right)}{\alpha_2}, A_2 = \alpha_2$$

"Raising operators"

$$Y_1 = \frac{x_1 \left(-1 + \mathbf{e}^{\delta_2} \right)}{\delta_2}$$

$$Y_2 = \frac{x_1 \, \delta_1 \left(1 - \mathbf{e}^{\delta_2} + \delta_2 \, \mathbf{e}^{\delta_2} \right)}{\delta_2 \left(-1 + \mathbf{e}^{\delta_2} \right)} + x_2$$

"Recursion operators"

$$x_1 = \frac{R_1 \, V_2}{-1 + \mathbf{e}^{V_2}}$$

$$x_2 = -\frac{R_1 \, V_1 \left(\mathbf{e}^{V_2} \, V_2 + 1 - \mathbf{e}^{V_2} \right)}{V_2 \left(-1 + \mathbf{e}^{V_2} \right)} + R_2$$

"Representation with abelian coherent state"

$$xi\text{-}hat_1 = \frac{x_1\,\delta_2}{-1 + \mathbf{e}^{\delta_2}}$$

$$xi\text{-}hat_2 = x_1\,\delta_1 - \frac{x_1\,\delta_1\left(1 - \mathbf{e}^{\delta_2} + \delta_2\,\mathbf{e}^{\delta_2}\right)}{\delta_2\left(-1 + \mathbf{e}^{\delta_2}\right)} + x_2$$

Here is N5

```
> #X:=matrix(5,5,[[0,a[1],a[2],a[3],a[4]],[0,0,a[5],a[6],a[7]],[0
  ,0,a[8],a[9]],[0,0,0,0,a[10]],[0,0,0,0,0]]);
```

```
> #d:=10;n:=5;lie(X,d,n,0);
```

```
> ########  E2
```

```
> X:=matrix(3,3,[[0,a[3],a[1]],[-a[3],0,(a[2])],[0,0,0]]);
```

$$X := \begin{bmatrix} 0 & a_3 & a_1 \\ -a_3 & 0 & a_2 \\ 0 & 0 & 0 \end{bmatrix}$$

```
> d:=3:n:=3:lie(X,d,n,1);dJ:=doJacobians();coordmap();canvars(dJ[3]);#
  kirlv();duals();
```

the group element is computed

solving equations for pi-dagger

solving equations for pi-star

piadj available

hk, pidgr, pistar

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & A_2 \\ 0 & 1 & -A_1 \\ 0 & 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} \cos(A_3) & \sin(A_3) & 0 \\ -\sin(A_3) & \cos(A_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_1(t) = \frac{-\cos(\alpha_3\,t)\,\alpha_2 + \cos(\alpha_3\,t)\,\alpha_3\,x_1 + \sin(\alpha_3\,t)\,\alpha_1 + \sin(\alpha_3\,t)\,\alpha_3\,x_2 + \alpha_2}{\alpha_3},$$

$$A_2(t) = \frac{\sin(\alpha_3\ t)\ \alpha_2 - \sin(\alpha_3\ t)\ \alpha_3\ x_1 + \cos(\alpha_3\ t)\ \alpha_1 + \cos(\alpha_3\ t)\ \alpha_3\ x_2 - \alpha_1}{\alpha_3},$$

$$A_3(t) = \alpha_3\ t + x_3$$

$$\text{"DalphaDA"}, \left[\left[-\frac{\sin(\alpha_3)\ \alpha_3}{2\ (\cos(\alpha_3) - 1)}, -\frac{1}{2}\ \alpha_3, -\frac{\alpha_3\ \cos(\alpha_3)\ \alpha_1 - 2\ \sin(\alpha_3)\ \alpha_1 + \alpha_3\ \sin(\alpha_3)\ \alpha_2 + \alpha_1\ \alpha_3}{2\ \sin(\alpha_3)\ \alpha_3} \right], \left[\frac{1}{2}\ \alpha_3, -\frac{\sin(\alpha_3)\ \alpha_3}{2\ (\cos(\alpha_3) - 1)}, -\frac{\alpha_3\ \cos(\alpha_3)\ \alpha_2 - \alpha_3\ \sin(\alpha_3)\ \alpha_1 - 2\ \sin(\alpha_3)\ \alpha_2 + \alpha_3\ \alpha_2}{2\ \sin(\alpha_3)\ \alpha_3} \right], [0, 0, 1] \right]$$

$$\text{"Jacobian of the group law="}, \begin{bmatrix} \cos(A_3) & \sin(A_3) & 0 \\ -\sin(A_3) & \cos(A_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{"DADalpha"}, \left[\left[\frac{\sin(A_3)}{A_3}, -\frac{\cos(A_3) - 1}{A_3}, \frac{A_3\ \cos(A_3)\ A_1 - 2\ A_1\ \sin(A_3) + \sin(A_3)\ A_3\ A_2 + A_1\ A_3}{2\ A_3\ \sin(A_3)} \right], \left[\frac{\cos(A_3) - 1}{A_3}, \frac{\sin(A_3)}{A_3}, -\frac{-A_3\ A_2\ \cos(A_3) + A_1\ A_3\ \sin(A_3) + 2\ A_2\ \sin(A_3) - A_2\ A_3}{2\ \sin(A_3)\ A_3} \right], [0, 0, 1] \right]$$

$$dJ := DalphaDA, DDADalpha, AA$$

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & A_2 \\ 0 & 1 & -A_1 \\ 0 & 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} \cos(A_3) & \sin(A_3) & 0 \\ -\sin(A_3) & \cos(A_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"for zero initial conditions we get the coordinate map"

$$A_1 = \frac{-\cos(\alpha_3)\,\alpha_2 + \sin(\alpha_3)\,\alpha_1 + \alpha_2}{\alpha_3},\ A_2 = \frac{\sin(\alpha_3)\,\alpha_2 + \cos(\alpha_3)\,\alpha_1 - \alpha_1}{\alpha_3},\ A_3 = \alpha_3$$

"Raising operators"

$$Y_1 = \frac{x_1 \sin(\delta_3)}{\delta_3} + \frac{x_2\,(\cos(\delta_3) - 1)}{\delta_3}$$

$$Y_2 = -\frac{x_1\,(\cos(\delta_3) - 1)}{\delta_3} + \frac{x_2 \sin(\delta_3)}{\delta_3}$$

$$Y_3 = \frac{x_1\,(\delta_3 \cos(\delta_3)\,\delta_1 - 2\,\delta_1 \sin(\delta_3) + \sin(\delta_3)\,\delta_3\,\delta_2 + \delta_1\,\delta_3)}{2\,\delta_3 \sin(\delta_3)} + \frac{x_2\,(\delta_3\,\delta_2 \cos(\delta_3) - \delta_1\,\delta_3 \sin(\delta_3) - 2\,\delta_2 \sin(\delta_3) + \delta_2\,\delta_3)}{2\,\delta_3 \sin(\delta_3)} + x_3$$

"Recursion operators"

$$x_1 = -\frac{R_1 \sin(V_3)\,V_3}{2\,(\cos(V_3) - 1)} + \frac{1}{2} R_2\,V_3$$

$$x_2 = -\frac{1}{2} R_1\,V_3 - \frac{R_2 \sin(V_3)\,V_3}{2\,(\cos(V_3) - 1)}$$

$$x_3 = -\frac{R_1\,(V_3 \cos(V_3)\,V_1 - 2 \sin(V_3)\,V_1 + V_3 \sin(V_3)\,V_2 + V_1\,V_3)}{2 \sin(V_3)\,V_3} + \frac{R_2\,(-V_3 \cos(V_3)\,V_2 + V_3 \sin(V_3)\,V_1 + 2 \sin(V_3)\,V_2 - V_3\,V_2)}{2\,V_3 \sin(V_3)} + R_3$$

"Representation with abelian coherent state"

$$xi\text{-}hat_1 = -\frac{x_1 \sin(\delta_3)\,\delta_3}{2\,(\cos(\delta_3) - 1)} + \frac{1}{2} x_2\,\delta_3$$

$$xi\text{-}hat_2 = -\frac{1}{2}\,x_1\,\delta_3 - \frac{x_2\,\sin(\delta_3)\,\delta_3}{2\,(\cos(\delta_3)-1)}$$

$$xi\text{-}hat_3 = -x_2\,\delta_1 + x_1\,\delta_2 - \frac{x_1\,(\delta_3\,\cos(\delta_3)\,\delta_1 - 2\,\delta_1\,\sin(\delta_3) + \sin(\delta_3)\,\delta_3\,\delta_2 + \delta_1\,\delta_3)}{2\,\delta_3\,\sin(\delta_3)} + \frac{x_2\,(-\delta_3\,\delta_2\,\cos(\delta_3) + \delta_1\,\delta_3\,\sin(\delta_3) + 2\,\delta_2\,\sin(\delta_3) - \delta_2\,\delta_3)}{2\,\sin(\delta_3)\,\delta_3} + x_3$$

```
> ############## SO(3)
> #X:=matrix(3,3,[[0,-a[3],a[2]],[a[3],0,-a[1]],[-a[2],a[1],0]])
  =3:n:=3:lie(X,d,n,1);
```

here is FINITE DIFFERENCE

```
> X:=matrix(2,2,[[a[3],a[2]],[0,a[1]]]);
```

$$X := \begin{bmatrix} a_3 & a_2 \\ 0 & a_1 \end{bmatrix}$$

```
> d:=3:n:=2:lie(X,d,n,0);dJ:=doJacobians();coordmap();canvars(dJ[3]);#
  kirlv();duals();
```

the group element is computed

solving equations for pi-dagger

solving equations for pi-star

piadj available

gg, pidgr, pistar

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathbf{e}^{A_1} & A_2 \\ 0 & 0 & 1 \end{bmatrix},\ \text{"pihatstar"} = \begin{bmatrix} 1 & 0 & 0 \\ A_2 & \mathbf{e}^{A_3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_1(t) = \alpha_1\,t + x_1,\ A_2(t) = \frac{\left(\alpha_2\,\mathbf{e}^{(\alpha_1\,t + x_1 - \alpha_3\,t)} - \alpha_2\,\mathbf{e}^{x_1} + x_2\,\alpha_1 - \alpha_3\,x_2\right)\mathbf{e}^{(\alpha_3\,t)}}{\alpha_1 - \alpha_3},$$

$$A_3(t) = \alpha_3\,t + x_3$$

$$\text{"DalphaDA"},\left[\,[1,\,0,\,0],\left[-\frac{\alpha_2\left(\mathbf{e}^{(\alpha_1-\alpha_3)}\,\alpha_1 - \mathbf{e}^{(\alpha_1-\alpha_3)}\,\alpha_3 - \mathbf{e}^{(\alpha_1-\alpha_3)} + 1\right)}{(\alpha_1-\alpha_3)\left(\mathbf{e}^{(\alpha_1-\alpha_3)} - 1\right)},\right.\right.$$

$$\frac{e^{(-\alpha_3)}(\alpha_1 - \alpha_3)}{e^{(\alpha_1 - \alpha_3)} - 1}, -\frac{\alpha_2\left(-\alpha_1 + \alpha_3 + e^{(\alpha_1 - \alpha_3)} - 1\right)}{(\alpha_1 - \alpha_3)\left(e^{(\alpha_1 - \alpha_3)} - 1\right)}\Bigg], [0, 0, 1]\Bigg]$$

"Jacobian of the group law=" ,

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{e^{A_3}\left(-e^{(-2A_3 + A_1 + x_1)} + e^{(-A_3 + x_1)}\right) A_2}{e^{(A_1 - A_3)} - 1} & e^{A_3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{"DADalpha"}, \left[[1, 0, 0], \left[\frac{A_2\left(-e^{(A_1 - A_3)} + 1 + e^{(A_1 - A_3)} A_1 - e^{(A_1 - A_3)} A_3\right)}{\left(e^{(A_1 - A_3)} - 1\right)(A_1 - A_3)}, \right.\right.$$

$$\left.\left. \frac{e^{A_3}\left(e^{(A_1 - A_3)} - 1\right)}{A_1 - A_3}, -\frac{A_2\left(-e^{(A_1 - A_3)} + 1 + A_1 - A_3\right)}{\left(e^{(A_1 - A_3)} - 1\right)(A_1 - A_3)} \right], [0, 0, 1] \right]$$

$$dJ := DalphaDA, DDADalpha, AA$$

$$\text{"pihat"} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{A_1} & A_2 \\ 0 & 0 & 1 \end{bmatrix}, \text{"pihatstar"} = \begin{bmatrix} 1 & 0 & 0 \\ A_2 & e^{A_3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"for zero initial conditions we get the coordinate map"

$$A_1 = \alpha_1, A_2 = \frac{\alpha_2\left(e^{(\alpha_1 - \alpha_3)} - 1\right) e^{\alpha_3}}{\alpha_1 - \alpha_3}, A_3 = \alpha_3$$

"Raising operators"

$$Y_1 = x_1 + \frac{x_2\,\delta_2\left(-\mathbf{e}^{(\delta_1-\delta_3)} + 1 + \mathbf{e}^{(\delta_1-\delta_3)}\,\delta_1 - \mathbf{e}^{(\delta_1-\delta_3)}\,\delta_3\right)}{\left(\mathbf{e}^{(\delta_1-\delta_3)} - 1\right)(\delta_1-\delta_3)}$$

$$Y_2 = \frac{x_2\,\mathbf{e}^{\delta_3}\left(\mathbf{e}^{(\delta_1-\delta_3)} - 1\right)}{\delta_1-\delta_3}$$

$$Y_3 = \frac{x_2\,\delta_2\left(\mathbf{e}^{(\delta_1-\delta_3)} - 1 - \delta_1 + \delta_3\right)}{\left(\mathbf{e}^{(\delta_1-\delta_3)} - 1\right)(\delta_1-\delta_3)} + x_3$$

"Recursion operators"

$$x_1 = R_1 - \frac{R_2\,V_2\left(\mathbf{e}^{(V_1-V_3)}\,V_1 - \mathbf{e}^{(V_1-V_3)}\,V_3 - \mathbf{e}^{(V_1-V_3)} + 1\right)}{(V_1-V_3)\left(\mathbf{e}^{(V_1-V_3)} - 1\right)}$$

$$x_2 = \frac{R_2\,\mathbf{e}^{(-V_3)}\,(V_1-V_3)}{\mathbf{e}^{(V_1-V_3)} - 1}$$

$$x_3 = \frac{R_2\,V_2\left(V_1 - V_3 - \mathbf{e}^{(V_1-V_3)} + 1\right)}{(V_1-V_3)\left(\mathbf{e}^{(V_1-V_3)} - 1\right)} + R_3$$

"Representation with abelian coherent state"

$$xi\text{-}hat_1 = x_1 - \frac{x_2\,\delta_2\left(-\mathbf{e}^{(\delta_1-\delta_3)} + 1 + \mathbf{e}^{(\delta_1-\delta_3)}\,\delta_1 - \mathbf{e}^{(\delta_1-\delta_3)}\,\delta_3\right)}{\left(\mathbf{e}^{(\delta_1-\delta_3)} - 1\right)(\delta_1-\delta_3)}$$

$$xi\text{-}hat_2 = \frac{x_2\, \mathbf{e}^{(-\delta_3)}\, (\delta_1 - \delta_3)\, \mathbf{e}^{\delta_1}}{\mathbf{e}^{(\delta_1 - \delta_3)} - 1}$$

$$xi\text{-}hat_3 = x_2\, \delta_2 - \frac{x_2\, \delta_2 \left(\mathbf{e}^{(\delta_1 - \delta_3)} - 1 - \delta_1 + \delta_3 \right)}{\left(\mathbf{e}^{(\delta_1 - \delta_3)} - 1 \right) (\delta_1 - \delta_3)} + x_3$$

here is Schrodinger

```
> #d:=6;n:=4;X := matrix([[0, a[2], a[3], -2*a[1]], [0, a[5], a[6], a[3]], [0
  -a[4], -a[5], -a[2]], [0, 0, 0, 0]]);

> #lie(X,d,n,0);#coordmap();#kirlv();duals();

> #restart;

>
```