```
> with(linalg):
> with(Ore_algebra);
```

[*OA_Internals*, *Ore_to_DESol*, *Ore_to_RESol*, *Ore_to_diff*, *Ore_to_shift*, *annihilators*, *applyopr*, *diff_algebra*, *dual_algebra*, *dual_polynomial*, *poly_algebra*, *qshift_algebra*, *rand_skew_poly*, *reverse_algebra*, *reverse_polynomial*, *shift_algebra*, *skew_algebra*, *skew_elim*, *skew_gcdex*, *skew_pdiv*, *skew_power*, *skew_prem*, *skew_product*]

```
>
```

One Variable

```
>
> n:=10;
```

$$n := 10$$

```
> unassign('z','y','Y','x','V','W'):
> V:=z-z^2/2;
```

$$V := z - \frac{1}{2} z^2$$

```
> W:=diff(V,z)^(-1);
> W:=convert(taylor(W,z=0,n),polynom);
```

$$W := \frac{1}{1 - z}$$

$$W := 1 + z + z^2 + z^3 + z^4 + z^5 + z^6 + z^7 + z^8 + z^9$$

```
> ############ ORE ALGEBRA STARTS HERE ####################
> A:=diff_algebra([z,x]);
```

$$A := Ore_algebra$$

```
> YY:=x*W;
```

$$YY := x\,(1 + z + z^2 + z^3 + z^4 + z^5 + z^6 + z^7 + z^8 + z^9)$$

```
> y[0]:=1:
> for i from 1 to n-1 do y[i]:=simplify(applyopr(YY,y[i-1],A)) od;
```

$$y_1 := x$$

$$y_2 := x^2 + x$$

$$y_3 := x^3 + 3\,x^2 + 3\,x$$

$$y_4 := x^4 + 6\,x^3 + 15\,x^2 + 15\,x$$

$$y_5 := x^5 + 10\,x^4 + 45\,x^3 + 105\,x^2 + 105\,x$$

$$y_6 := x^6 + 15\,x^5 + 105\,x^4 + 420\,x^3 + 945\,x^2 + 945\,x$$

$$y_7 := 10395\,x + x^7 + 210\,x^5 + 4725\,x^3 + 21\,x^6 + 1260\,x^4 + 10395\,x^2$$

$$y_8 := 135135\,x + 28\,x^7 + 3150\,x^5 + x^8 + 62370\,x^3 + 378\,x^6 + 17325\,x^4 + 135135\,x^2$$

$$y_9 := 2027025\,x + 630\,x^7 + 51975\,x^5 + 36\,x^8 + x^9 + 945945\,x^3 + 6930\,x^6 + 270270\,x^4 + 2027025\,x^2$$

>

Several Variables

Example in two variables from an analytic function

>

```
> n:=4;
```

$$n := 4$$

```
> ################################################
> unassign('z','y','c','v','Y','x','zc','V','W','YY','i','j','k','l','a','b
  '):
> f:=z-(4+2*I)*z^2;zc:=z1+I*z2;f1:=subs(z=zc,diff(f,z));
> V1:=evalc(Re(expand(subs(z=zc,f))));V2:=evalc(Im(expand(subs(z=zc,f))));
```

$$f := z + (-4 - 2\,I)\,z^2$$

$$zc := z1 + I\,z2$$

$$f1 := 1 + (-8 - 4\,I)\,(z1 + I\,z2)$$

$$V1 := z1 - 4\,z1^2 + 4\,z1\,z2 + 4\,z2^2$$

$$V2 := z2 - 2\,z1^2 - 8\,z1\,z2 + 2\,z2^2$$

```
> Jac:=jacobian([V1,V2],[z1,z2]):
> W:=evalm(Jac^(-1)):adj(Jac),factor(det(Jac));
> WMat:=map(mtaylor,W,[z1=0,z2=0],n);
```

$$\begin{bmatrix} 1 - 8\,z1 + 4\,z2 & -4\,z1 - 8\,z2 \\ 4\,z1 + 8\,z2 & 1 - 8\,z1 + 4\,z2 \end{bmatrix}, 1 - 16\,z1 + 8\,z2 + 80\,z1^2 + 80\,z2^2$$

$$WMat := [[1 + 8\,z1 - 4\,z2 + 48\,z1^2 - 48\,z2^2 - 128\,z1\,z2 + 128\,z1^3 - 384\,z1\,z2^2 - 2112\,z2\,z1^2 + 704\,z2^3, -4\,z1 - 8\,z2 - 64\,z1^2 - 96\,z1\,z2 + 64\,z2^2 - 704\,z1^3 - 384\,z2\,z1^2 + 2112\,z1\,z2^2 + 128\,z2^3], [4\,z1 + 8\,z2 + 64\,z1^2 + 96\,z1\,z2 - 64\,z2^2 + 704\,z1^3 + 384\,z2\,z1^2 - 2112\,z1\,z2^2 - 128\,z2^3, 1 + 8\,z1 - 4\,z2 + 48\,z1^2 - 48\,z2^2 - 128\,z1\,z2 + 128\,z1^3 - 384\,z1\,z2^2 - 2112\,z2\,z1^2 + 704\,z2^3]]$$

```
> unassign('i','j','k'):
```

```
> ############ ORE ALGEBRA STARTS HERE ####################
> A:=diff_algebra([z1,x1],[z2,x2]);
```

$$A := Ore_algebra$$

```
> for ix to N do YY[ix]:=simplify(add(x||k*WMat[k,ix],k=1..N)) od;
```

$$YY_1 := x1 + 8\,x1\,z1 - 4\,x1\,z2 + 48\,x1\,z1^2 - 48\,x1\,z2^2 - 128\,x1\,z1\,z2 + 128\,x1\,z1^3 - 384\,x1\,z1\,z2^2 - 2112\,x1\,z2\,z1^2 + 704\,x1\,z2^3 + 4\,x2\,z1 + 8\,x2\,z2 + 64\,x2\,z1^2 + 96\,x2\,z1\,z2 - 64\,x2\,z2^2 + 704\,x2\,z1^3 + 384\,x2\,z2\,z1^2 - 2112\,x2\,z1\,z2^2 - 128\,x2\,z2^3$$

$$YY_2 := -4\,x1\,z1 - 8\,x1\,z2 - 64\,x1\,z1^2 - 96\,x1\,z1\,z2 + 64\,x1\,z2^2 - 704\,x1\,z1^3 - 384\,x1\,z2\,z1^2 + 2112\,x1\,z1\,z2^2 + 128\,x1\,z2^3 + x2 + 8\,x2\,z1 - 4\,x2\,z2 + 48\,x2\,z1^2 - 48\,x2\,z2^2 - 128\,x2\,z1\,z2 + 128\,x2\,z1^3 - 384\,x2\,z1\,z2^2 - 2112\,x2\,z2\,z1^2 + 704\,x2\,z2^3$$

```
> y[0,0]:=1:
> for i from 0 to n-1 do j:=0; if(i>0) then
  y[i,0]:=simplify(applyopr(YY[1],y[i-1,0],A)) fi; for j from 1 to n-1 do
  y[i,j]:=simplify(applyopr(YY[2],y[i,j-1],A));print("y["||i||","||j||"]=",
  [i,j]) od;  od;
```

$$j := 0$$

$$\text{"y[0,1]="},\ x2$$

$$\text{"y[0,2]="},\ x2^2 - 4\,x2 - 8\,x1$$

$$\text{"y[0,3]="},\ x2^3 - 12\,x2^2 - 24\,x2\,x1 + 192\,x1 - 144\,x2$$

$$j := 0$$

$$\text{"y[1,1]="},\ x2\,x1 - 4\,x1 + 8\,x2$$

$$\text{"y[1,2]="},\ x2^2\,x1 - 12\,x2\,x1 + 16\,x2^2 - 144\,x1 - 192\,x2 - 8\,x1^2$$

$$\text{"y[1,3]="},\ 10560\,x1 - 1920\,x2 - 24\,x2^2\,x1 + 24\,x2^3 + x2^3\,x1 - 24\,x2\,x1^2 - 720\,x2\,x1 - 672\,x2^2 + 288\,x1^2$$

$$j := 0$$

$$\text{"y[2,1]="},\ x2\,x1^2 + 24\,x2\,x1 + 4\,x2^2 - 8\,x1^2 - 192\,x1 + 144\,x2$$

$$\text{"y[2,2]="},\ -1920\,x1 - 10560\,x2 + 40\,x2^2\,x1 - 8\,x1^3 + 4\,x2^3 + x2^2\,x1^2 - 20\,x2\,x1^2 - 960\,x2\,x1 + 400\,x2^2 - 320\,x1^2$$

$$\text{"y[2,3]="},\ 497664\,x1 + 145152\,x2 + x2^3\,x1^2 - 24\,x2\,x1^3 - 2496\,x2^2\,x1 + 384\,x1^3 + 768\,x2^3 - 36\,x2^2\,x1^2 + 56\,x2^3\,x1 - 1392\,x2\,x1^2 + 4\,x2^4 - 13440\,x2\,x1 - 43200\,x2^2 + 30720\,x1^2$$

$$j := 0$$

"y[3,1]=", $x2\,x1^3 + 48\,x2\,x1^2 + 12\,x2^2\,x1 + 720\,x2\,x1 + 288\,x2^2 - 12\,x1^3 - 672\,x1^2 - 10560\,x1 + 1920\,x2$

"y[3,2]=", $145152\,x1 - 497664\,x2 - 28\,x2\,x1^3 + 1632\,x2^2\,x1 - 528\,x1^3 + 384\,x2^3 + 72\,x2^2\,x1^2 - 8\,x1^4 + 12\,x2^3\,x1 - 2496\,x2\,x1^2 + x2^2\,x1^3 - 73920\,x2\,x1 + 7680\,x2^2 - 5760\,x1^2$

"y[3,3]=", $23553024\,x1 + 21829632\,x2 + 96\,x2^3\,x1^2 - 2160\,x2\,x1^3 - 233280\,x2^2\,x1 + 62400\,x1^3 + 19200\,x2^3 - 5760\,x2^2\,x1^2 + 480\,x1^4 - 24\,x2\,x1^4 + 2880\,x2^3\,x1 - 34560\,x2\,x1^2 - 48\,x2^2\,x1^3 + x2^3\,x1^3 + 480\,x2^4 + 1435392\,x2\,x1 + 12\,x2^4\,x1 - 2575872\,x2^2 + 2345472\,x1^2$

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