

Grassmann Integral Representation for Spanning Hyperforests

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Abstract

Given a hypergraph G , we introduce a Grassmann algebra over the vertex set, and show that a class of Grassmann integrals permits an expansion in terms of spanning hyperforests. Special cases provide the generating functions for rooted and unrooted spanning (hyper)forests and spanning (hyper)trees. All these results are generalizations of Kirchhoff's matrix-tree theorem. Furthermore, we show that the class of integrals describing unrooted spanning (hyper)forests is induced by a theory with an underlying $OSP(1|2)$ supersymmetry.

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